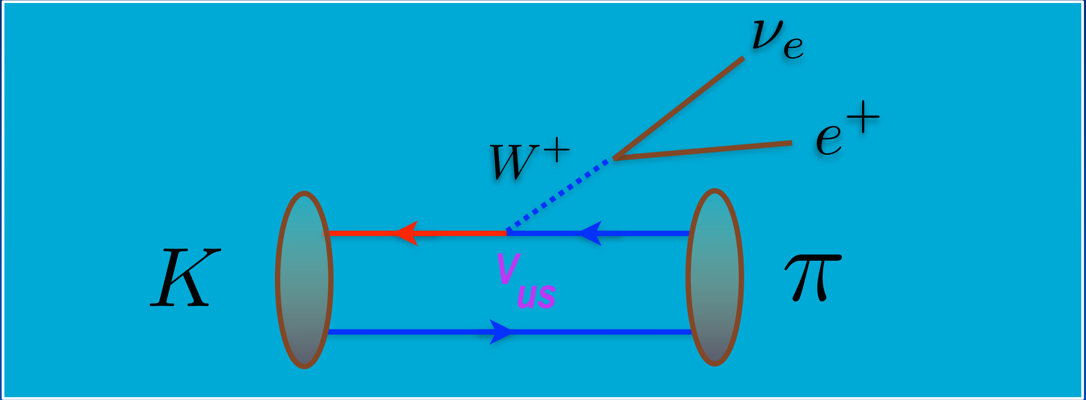
Status of Semileptonic Hyperon Decays from Lattice QCD using 2+1 flavor Domain Wall Fermions

Shoichi Sasaki (Tohoku Univ.)

V_{us} from semi-leptonic decays

Semi-leptonic Kaon decay (K_{l3} decay)



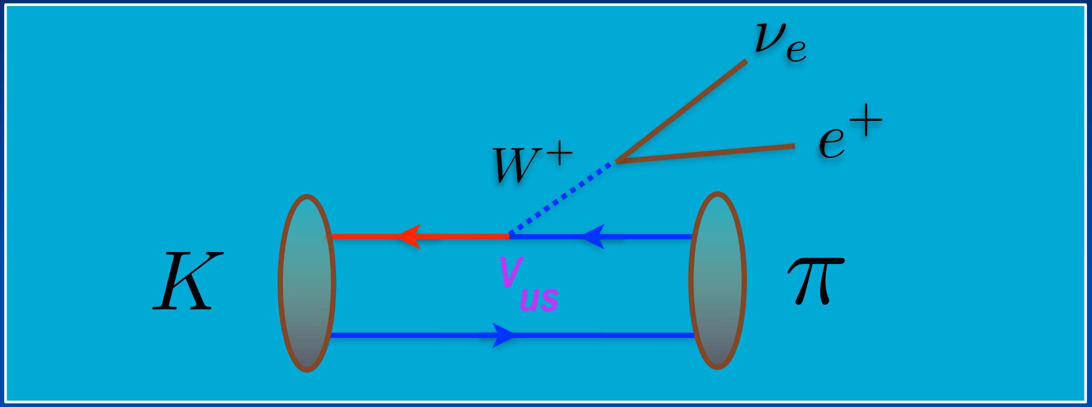
$$\text{Decay rate } \propto |V_{us}|^2 |\underline{f_+(0)}|^2$$

Quantum correction by strong interaction

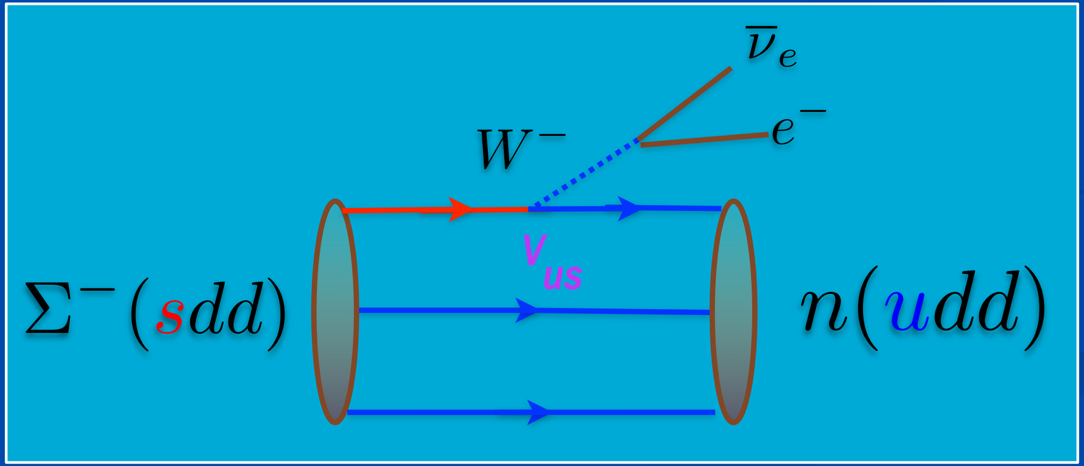
Recall:
In the exact flavor SU(3) limit, the weak vector coupling doesn't receive any quantum corrections **even inside hadrons**

V_{us} from semi-leptonic decays

Semi-leptonic Kaon decay (K_{l3} decay)



Hyperon β-decay (|Δs|=1)



Both decays can be used to determine V_{us}

Hyperon β-decay

Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \to B_2 + l + \bar{\nu}_l$$

- ✓ Simple V-A structure (weak matrix element)
- ✓ Described by six form factors

$$\langle B_2|V_\alpha - A_\alpha|B_1\rangle = \bar{u}_{B_2}(p')[\gamma_\alpha f_1(q^2) + \sigma_{\alpha\beta}q_\beta \frac{f_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha \frac{f_3(q^2)}{M_{B_1}+M_{B_2}}$$

$$+\gamma_\alpha\gamma_5 g_1(q^2) + \sigma_{\alpha\beta}q_\beta\gamma_5 \frac{g_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha\gamma_5 \frac{g_3(q^2)}{M_{B_1}+M_{B_2}}]u_{B_1}(p)$$

$$q_\alpha = (p_{B_1} - p_{B_2})_\alpha = (p_l + p_\nu)_\alpha$$

Hyperon β-decay

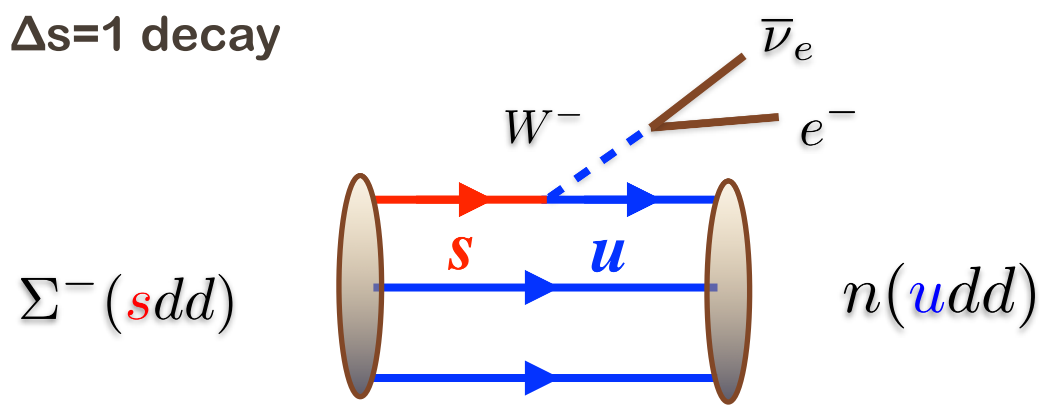
Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \to B_2 + l + \bar{\nu}_l$$

- ✓ Simple V-A structure (weak matrix element)
- ✓ Described by six form factors
- ✓ Four types of $|\Delta s|=1$ decays in the isospin limit

$$\Lambda \to p \qquad \Sigma \to n \qquad \Xi \to \Lambda \qquad \Xi \to \Sigma$$

Determination of V_{us}



Decay rate

neglecting higher orders of δ

assuming $g_2(0) = 0$

$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1} - M_{B_2})^5(1 - 3\delta)|V_{us}|^2|f_1(0)|^2\left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2 + \cdots\right]$$

① measure semi-leptonic decay width

② measure $g_1(0)/f_1(0)$

③ $f_1(0)$ from theory → $f_1(0)_{SU(3)}$ value

$$\delta = \frac{M_{B_1} - M_{B_2}}{M_{B_1} + M_{B_2}} \sim 0.1 - 0.2$$

Status of $|V_{us}|$ [PDG2012]

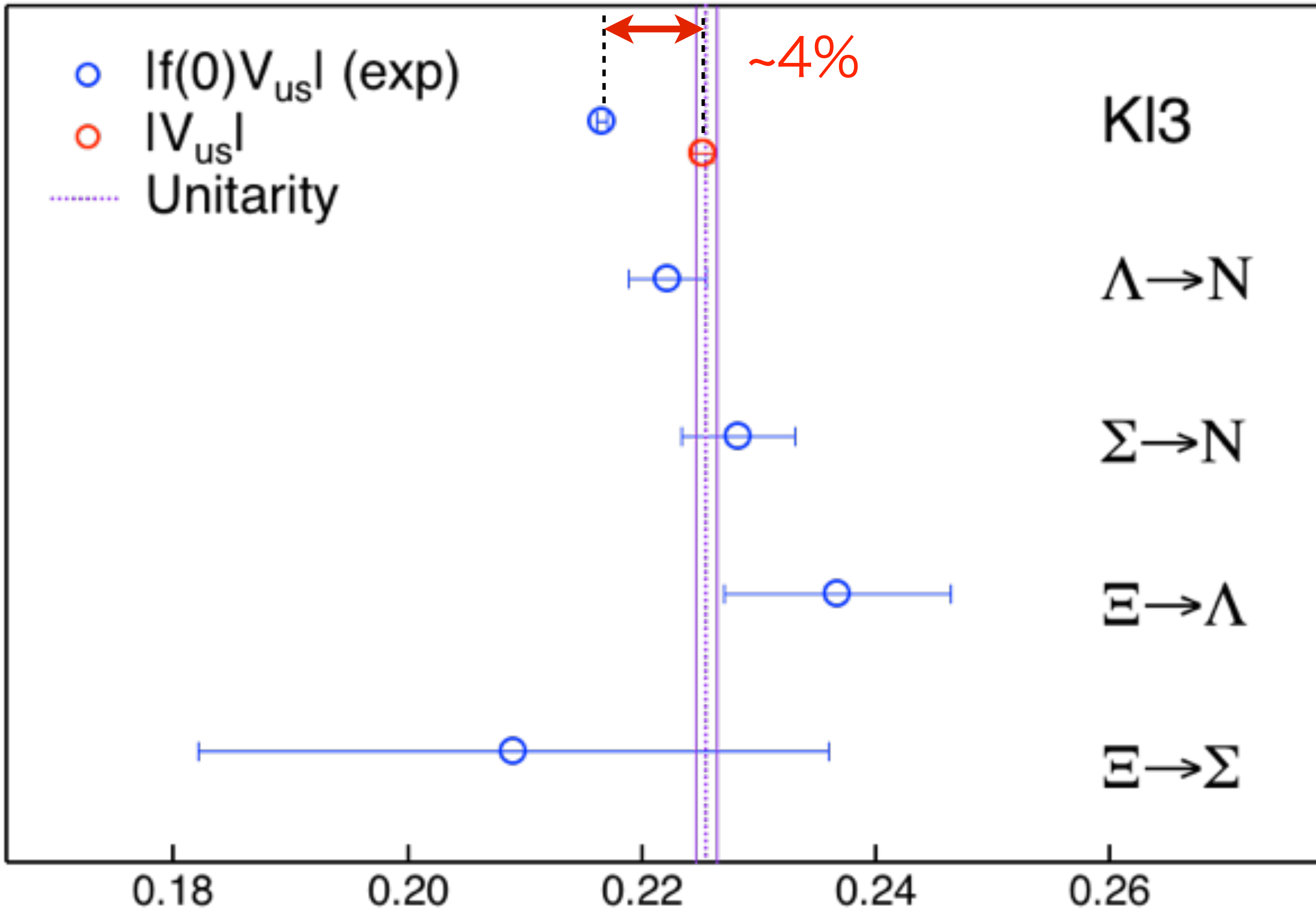
- Can be determined from the various approaches
  - **K_{l3} decays**: 0.2252(9)
  - **leptonic kaon & pion decays**: $0.2259(5)_{exp}(13)_{th}$
  - **hadronic τ-decays**: 0.2208(34)
  - **hyperon beta decays**: 0.2250(27)
  - **CKM unitarity + $|V_{ud}|$**: 0.22547(95)

the first row unitarity

$$1 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2$$

($|V_{ud}|$=0.97425(22), super-allowed nuclear β-decays, $|V_{ub}|\sim O(10^{-3})$)

SU(3) breaking corrections on V_{us}



Note that results from hyperon β decays do not include estimates of theoretical uncertainty due to second-order SU(3) breaking.

Exact SU(3) symmetry world

vector coupling

$B \to bl\nu$	$f_1^{SU(3)}$	$(g_1/f_1)^{SU(3)}$	g_1/f_1 (Exp.)
$n \to pe^-\bar{\nu}$	1	$F+D$	1.2670 ± 0.0030
$\Lambda \to pe^-\bar{\nu}$	$-\frac{\sqrt{6}}{2}$	$F+\frac{1}{3}D$	0.718 ± 0.015
$\Sigma^- \to ne^-\bar{\nu}$	-1	$F-D$	-0.340 ± 0.017
$\Xi^- \to \Lambda e^-\bar{\nu}$	$\frac{\sqrt{6}}{2}$	$F-\frac{1}{3}D$	0.25 ± 0.05
$\Xi^0 \to \Sigma^+ e^-\bar{\nu}$	1	$F+D$	1.32 ± 0.21

$$\boxed{f_1^{SU(3)}(0) = f_{\rm klm}}$$

$$g_1^{SU(3)}(0) = F f_{\rm klm} + D d_{\rm klm}$$

$$F = 0.475(4),\ \ D = 0.793(5)$$

from the conventional Cabibbo fit

Theoretical uncertainties of $f_1(0)$

SU(3) breaking starts at 2nd order (Ademollo-Gatto theorem)

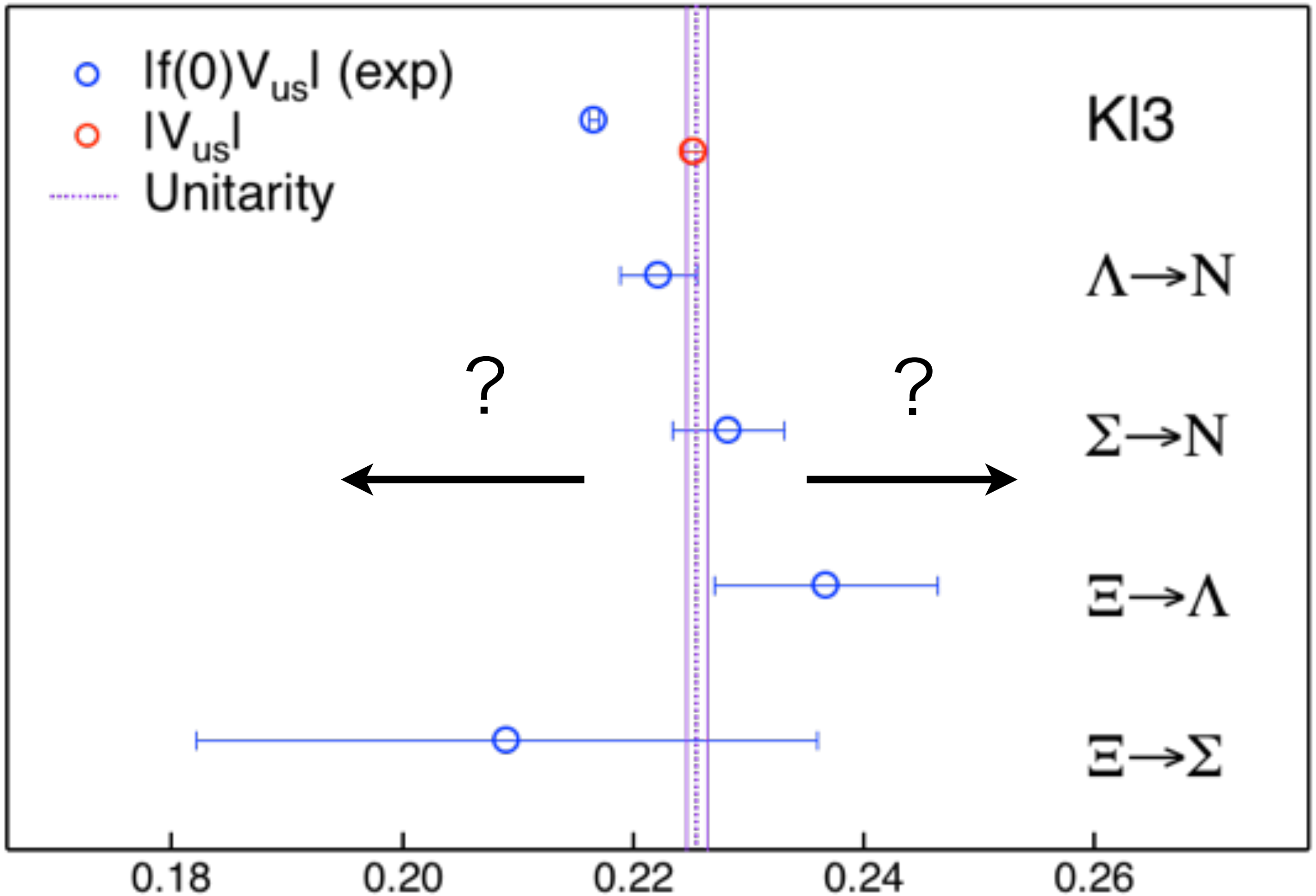
	Theoretical estimation	$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^0 \to \Sigma^+$
$\tilde{f}_1 = \lvert f_1/f_1^{\mathrm{SU(3)}} \rvert$	bag model (DH'82)	0.97	0.97	0.97	0.97
	quark model (DHK'87)	0.987	0.987	0.987	0.987
	quark model (Sch'95)	0.976	0.975	0.976	0.976
	large N_c (FMJM'98)	1.02(2)	1.04(2)	1.10(4)	1.12(5)
	full $\mathcal{O}(p^4)$ HBChPT, (V'06)	1.027	1.041	1.043	1.009
	partial $\mathcal{O}(p^5)$ HBChPT, (LKM'07)	1.066(32)	1.064(06)	1.053(22)	1.044(26)
	full $\mathcal{O}(p^4)$ CBChPT, (GCV'09)	1.001(13)	1.087(42)	1.040(28)	1.017(22)
$\lvert \tilde{f}_1 V_{us} \rvert$ (expt.)		0.2221(33)	0.2274(49)	0.2367(97)	0.209(27)

HBD (ave.): $|V_{us}| = 0.2250(50)$ with $\tilde{f}_1 = 1$

- bag model, quark model: $|f_1(0)| < |f_1(0)_{\mathrm{SU(3)}}| \Longrightarrow V_{us} \nearrow$
- large N_c, HBChPT, CBChPT: $|f_1(0)| > |f_1(0)_{\mathrm{SU(3)}}| \Longrightarrow V_{us} \searrow$

cf. CKM unitarity prediction: $|V_{us}|_{\mathrm{unitary}} = 0.22547(95)$

SU(3) breaking corrections on V_{us}



✳ Model independent evaluation of flavor SU(3)-breaking corrections is primarily required

✳ It can be achieved with high accuracy by lattice QCD

A few lattice studies

- Quench approximation
  - **$\Xi^0 \to \Sigma^+$ decay** vs. neutron β-decay
    - S. S and T. Yamazaki, Phys. Rev. D79 (09) 074508; arXiv:0811.1406
    - Domain Wall Fermion (DWF), m_π>540 MeV, m_s > 1.2 x m_s^{phys}
  - **$\Sigma^- \to$ n decay**
    - D. Guandagnoli et al., Nucl. Phys. B761 (07) 63; hep-ph/0606181
    - Clover Fermion, m_π>850 MeV, m_s > 1.3 x m_s^{phys}
- Preliminary results from 2+1 flavor dynamical simulations
  - DWF valence + MILC Staggered sea; H.W. Lin, arXiv:0812.0411
  - Clover Fermion; J. Zanotti (QCDSF/UKQCD), arXiv:1101.2806

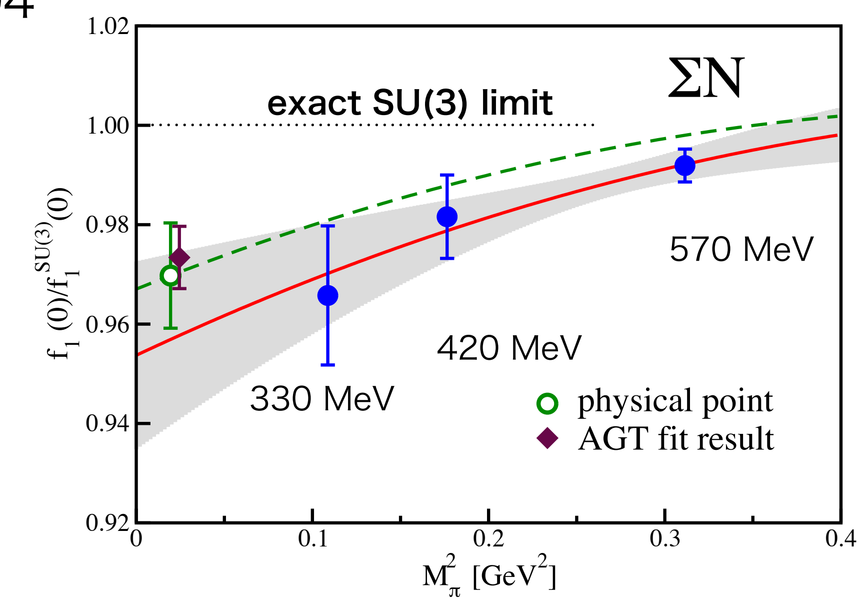
2+1 flavor DWF results

- **SS, Phys. Rev. D86, (2012) 114502**

✓ 2+1 flavor RBC+UKQCD gauge configurations

- Domain wall fermions and Iwasaki gauge action
- $L^3 \times T \times L_5$ = **$24^3 \times 64 \times 16$** (β=2.13, 1/a~1.7 GeV)
- m_{ud}=0.005, 0.01, 0.02 (3 lightest u,d quark masses)
- fixed strange quark masses at m_s=0.04
- **$\Sigma \to N$ and $\Xi \to \Sigma$ decays**

m_π [MeV]	statistics	t_{src} positions
330	240	0, 32
420	120	0, 32
570	80	0, 16, 32, 48



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$$f_1^{\Xi\to\Sigma}(0) = +0.9732(66)_{\rm stat.}(7)_{q^2}(5)_{m_q}$$

$$f_1^{\Sigma\to N}(0) = -0.9698(106)_{\rm stat.}(15)_{q^2}(36)_{m_q}$$

Simulations are performed at a single lattice spacing and in the region of m_π>330 MeV

ChPT vs. LQCD

D. Guadagonoli et al., NPB761, 63 (07)
SS, T. Yamazaki, PRD79, 074508 (09)

SU(3) corrections [%]

	$\mathcal{O}(p^4)$ HBChPT		$\mathcal{O}(p^4)$ EOMS-CBChPT		Lattice QCD	
	octet	+ decuplet	octet	+ decuplet	quench	2+1 flavor
$\Lambda \to N$	$+2.8$	$+11.2$	$-3.6^{+1.2}_{-0.9}$	$+0.1^{+1.3}_{-1.0}$	—	—
$\Sigma \to N$	$+4.1$	$+37.3$	$+3.9^{+3.8}_{-2.8}$	$+8.7^{+4.2}_{-3.1}$	-1.2 ± 2.9	-3.0 ± 1.1
$\Xi \to \Lambda$	$+4.4$	$+22.2$	$-1.2^{+2.4}_{-1.8}$	$+4.0^{+2.8}_{-2.1}$	—	—
$\Xi \to \Sigma$	$+1.0$	$+1.1$	$-1.3^{+0.3}_{-0.2}$	$+1.7^{+2.2}_{-1.6}$	-1.3 ± 1.9	-2.7 ± 0.7

G. Villadoro
PRD74, 014018 (06)

L.S. Geng et al.,
PRD79, 094022 (09)

2+1 flavor DWF calculations
SS, PRD86, 114502 (12)

- LQCD suggests small negative corrections of SU(3) breaking on hyperon vector couplings and no strong channel dependence

✓ disagrees predictions of the latest ChPT results

✓ requires further detailed study near the physical point

New results

(2+1) flavor dynamical DWF ensembles

RBC + UKQCD collaborations have been generating Domain Wall Fermion (DWF) configurations, where the chiral symmetry is well preserved. https://qcdlattices.bnl.gov/

β (1/a)	L^3 x T x L_5	La	am_s	am_{ud}	M_π (MeV)
2.13 (1.6 GeV)	16^3 x 32 x 16	1.9 fm	0.04	0.01	420
				0.02	560
				0.03	670
2.13 (1.7 GeV)	24^3 x 64 x 16	2.7 fm	0.04	0.005	330
				0.01	420
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				0.03	670
2.25 (2.3 GeV)	32^3 x 64 x 16	2.7 fm	0.03	0.004	290
				0.006	345
2.25 (2.3 GeV)				0.008	390

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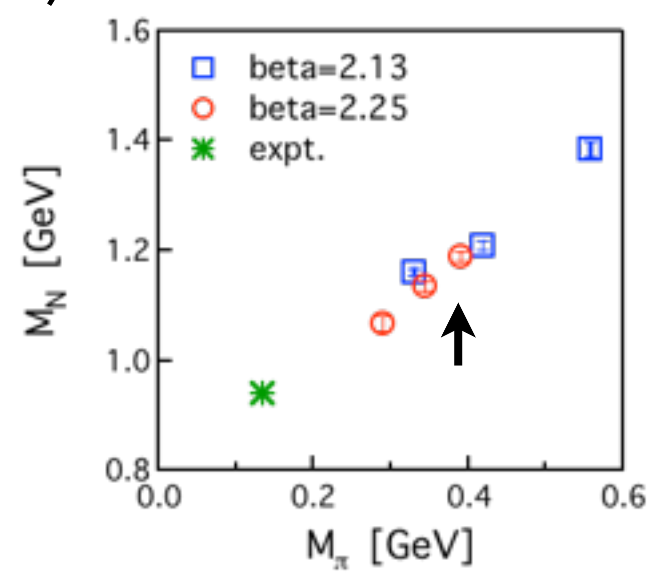
				0.006	345
2.25 (2.3 GeV)				0.008	390

➡ This work

Simulation on finer ensembles

- 2+1 flavor RBC+UKQCD gauge configurations
  - Domain wall fermions and Iwasaki gauge action
  - $L^3 \times T \times L_5 = 32^3 \times 64 \times 16$ (β=2.25, 1/a~2.3 GeV)
  - am_{ud}=0.004, 0.006, 0.008 (u,d quark masses)
  - fixed strange quark masses at am_s=0.03
  - 6 types of sequential prop (ΣN & $\Xi\Sigma$)

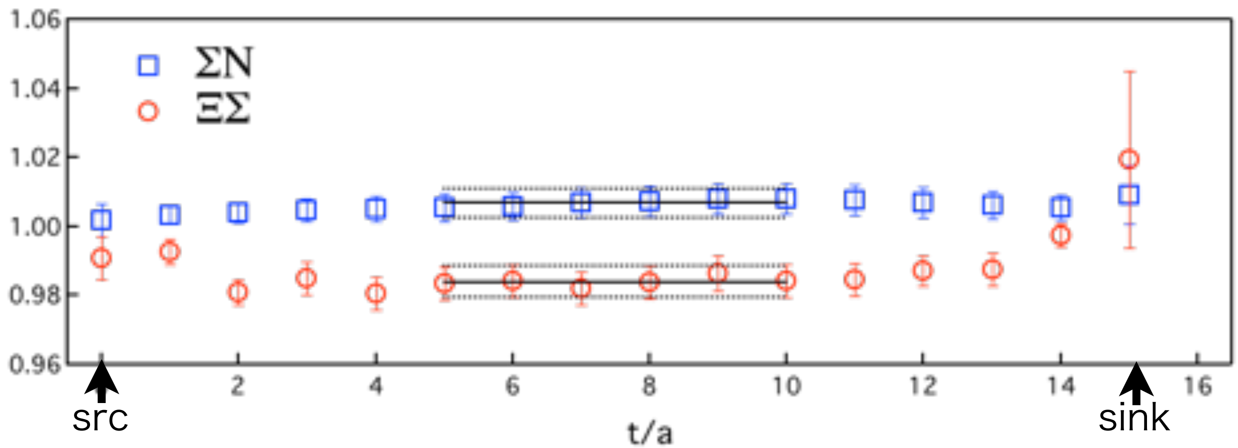
m_π [MeV]	statistics	t_{src} positions
290	in progress	0, 32
350	in progress	0, 32
390	**100**	0, 32



Precise determination of $f_S(q^2)$ at $q^2 = q^2_{max}$

- $$f_S(q^2) = f_1(q^2) + \frac{q^2}{M_B^2 - M_b^2} f_3(q^2)$$
  - At the zero three-momentum transfer (**q=0**)

$$R(t) = \frac{\langle B|\bar{s}\gamma_0 u|b\rangle\langle b|\bar{u}\gamma_0 s|B\rangle}{\langle B|\bar{s}\gamma_0 s|B\rangle\langle b|\bar{u}\gamma_0 u|b\rangle} \to |f_S(q^2_{\rm max})|^2$$



"Double ratio method"
S. Hashimoto et al.
Phys. Rev. D61 (1999) 014502

1 channel
= 4 types of sequential props

Deviation from the unity = SU(3) breaking

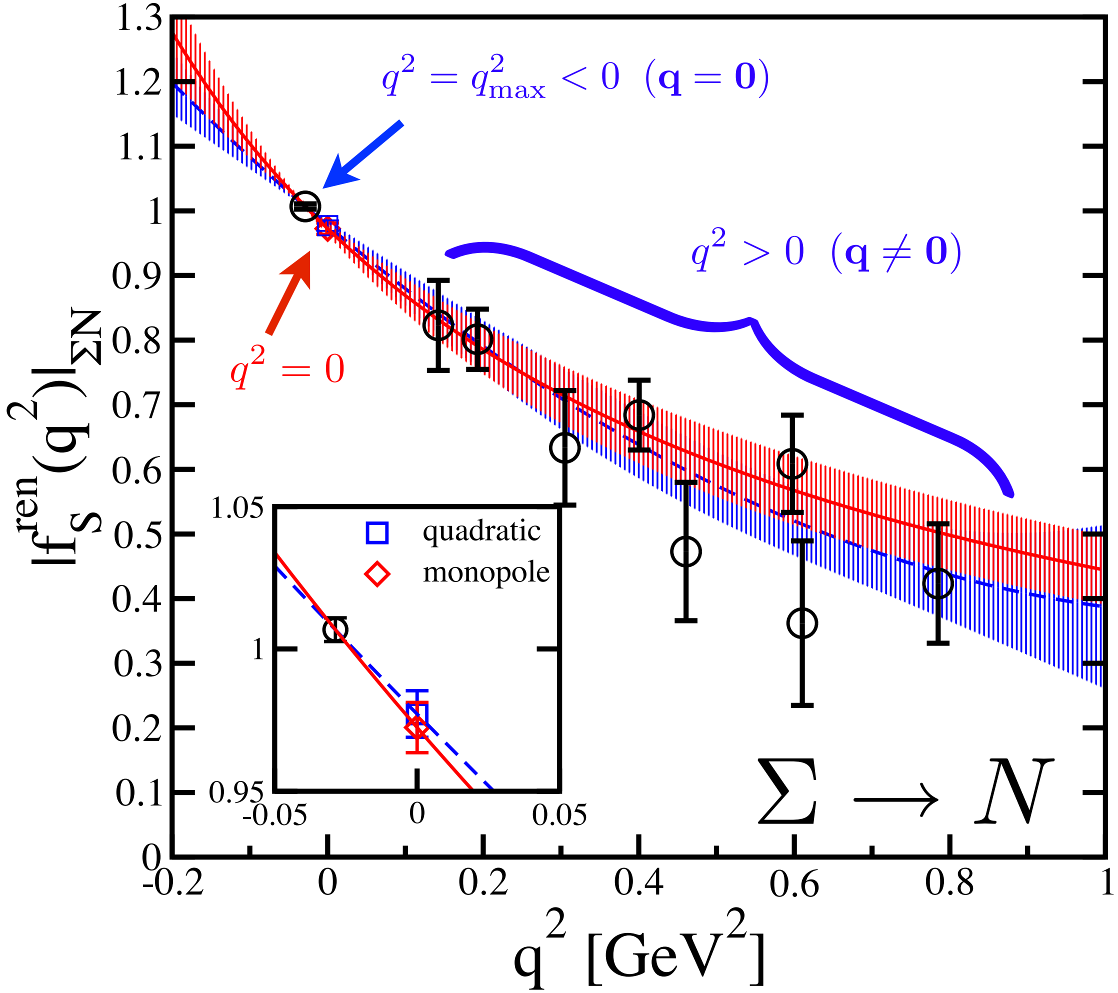
① $|q_{max}|=(M_B-M_b)\neq 0$
② $f_3(q)\neq 0$
③ Deviation from $f_1(q)_{SU(3)}$

$$[f_S(q^2_{\rm max})]_{\Sigma\to N} = 1.007(4)$$

$$[f_S(q^2_{\rm max})]_{\Xi\to\Sigma} = 0.984(7)$$

Determination of $f_S(0)=f_1(0)$

q^2 interpolation of $f_S(q^2)$ to $q^2=0$



quadratic form

$$f_S(q^2) = f_S(0)\left(1+\lambda_S^{(1)}q^2+\lambda_S^{(2)}q^4\right)$$

$f_1(0) = 0.977(5)$

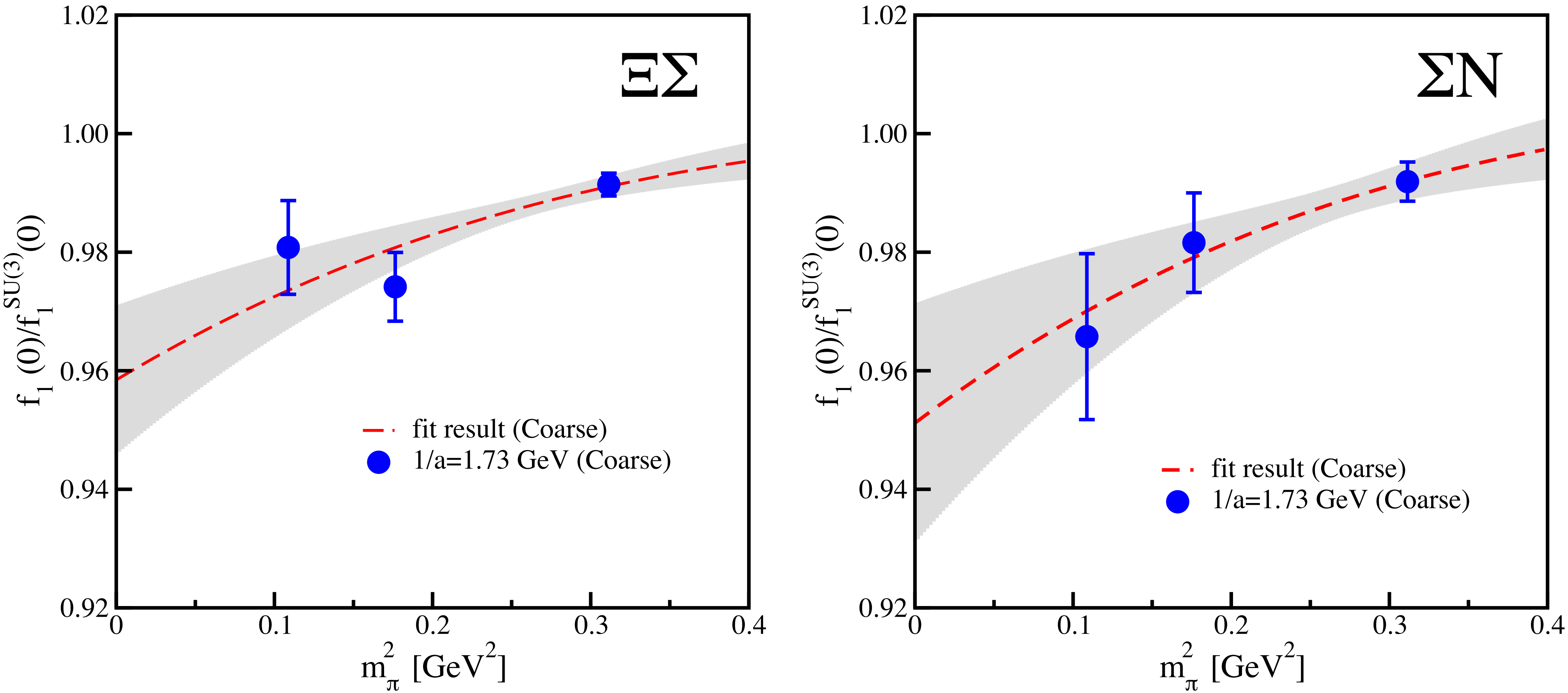
monopole form

$$f_S(q^2) = \frac{f_S(0)}{1+\lambda_S^{(1)}q^2}$$

$f_1(0) = 0.979(5)$

$$f_S(q^2) \stackrel{\mathrm{def}}{=} f_1(q^2) + \frac{q^2}{M_B^2-M_b^2} f_3(q^2)$$

Comparison of $f_1(0)$ on coarse and fine lattices



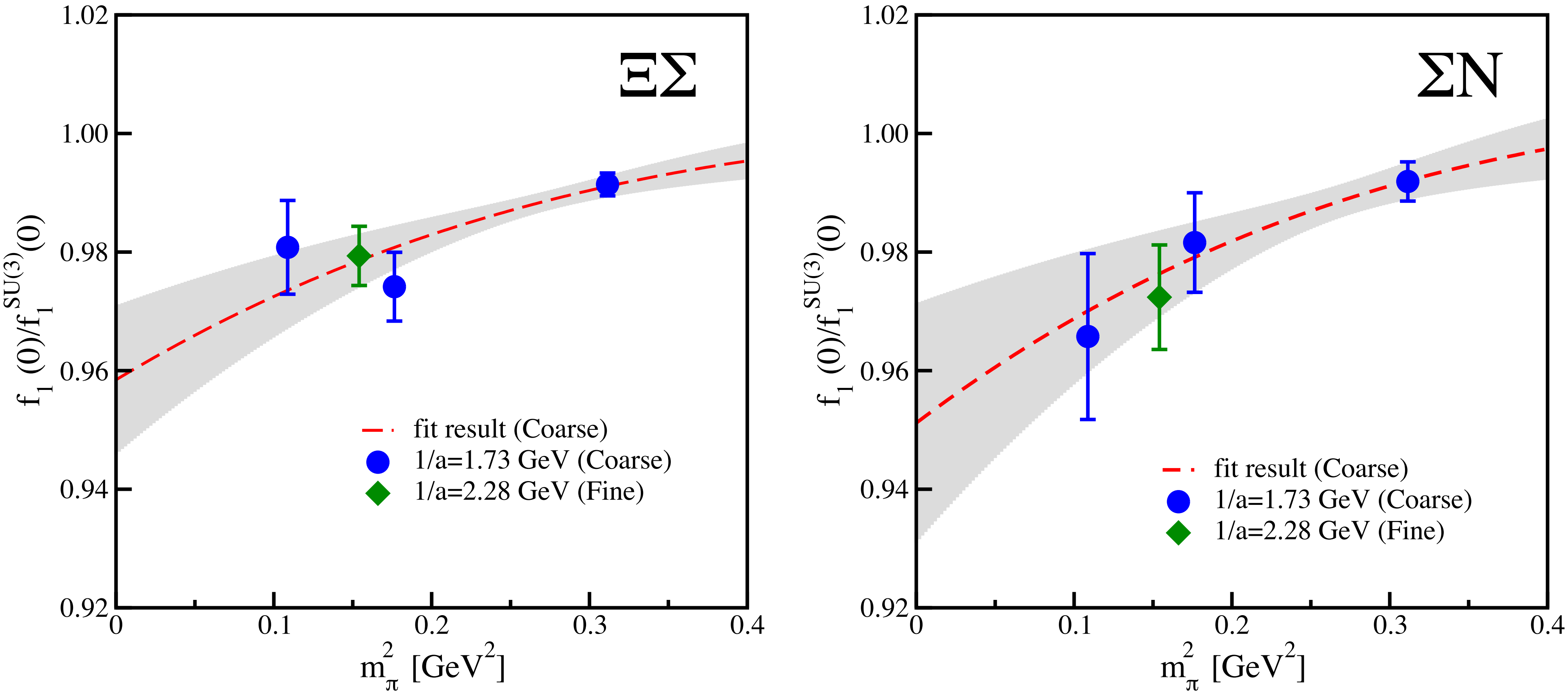
Fitting form:

$$\tilde{f}_1(0) = C_0 + (C_1 + C_2 \cdot (M_K^2 + M_\pi^2)) \cdot (M_K^2 - M_\pi^2)$$

This functional form is motivated by the Ademollo-Gatto Theorem.

Comparison of $f_1(0)$ on coarse and fine lattices

good scaling behavior (cutoff effect is small)



- needs a detailed study of **the strange quark mass dependence** since the simulated strange quark mass on both ensembles are not exactly at the physical point

Summary

We have studied the SU(3) breaking effect on hyperon vector couplings using 2+1 flavor dynamical lattice QCD.

➡ an essential theoretical input to determine **V_{us}**

$$f_1^{\Xi\to\Sigma}(0) = +0.9732(66)_{\mathrm{stat.}}(7)_{q^2}(5)_{m_q}(40)_{a^2}$$

$$f_1^{\Sigma\to N}(0) = -0.9698(106)_{\mathrm{stat.}}(15)_{q^2}(36)_{m_q}(28)_{a^2}$$

- two lattice spacing results → cutoff effect ~ 0.4 %
- statistical errors are dominant (~ 0.7-1.0 %)

Summary

We have studied the SU(3) breaking effect on hyperon vector couplings using 2+1 flavor dynamical lattice QCD.

➡ an essential theoretical input to determine **V_{us}**

✓ the tendency of the SU(3)-breaking correction disagrees with the latest **baryon ChPT** result.

SU(3) corrections [%]

	$\mathcal{O}(p^3)$ EOMS-CBChPT		$\mathcal{O}(p^4)$ EOMS-CBChPT		2+1f DWF-LQCD
	octet	+ decuplet	octet	+ decuplet	physical point
$\Lambda \to N$	-3.8	-3.1	$-3.6^{+1.2}_{-0.9}$	$+0.1^{+1.3}_{-1.0}$	—
$\Sigma \to N$	-0.8	-2.2	$+3.9^{+3.8}_{-2.8}$	$+8.7^{+4.2}_{-3.1}$	-3.0 ± 1.1
$\Xi \to \Lambda$	-2.9	-2.9	$-1.2^{+2.4}_{-1.8}$	$+4.0^{+2.8}_{-2.1}$	—
$\Xi \to \Sigma$	-3.7	-3.0	$-1.3^{+0.3}_{-0.2}$	$+1.7^{+2.2}_{-1.6}$	-2.7 ± 0.7

‣ suggests that baryon ChPT seems to have a serious convergence problem

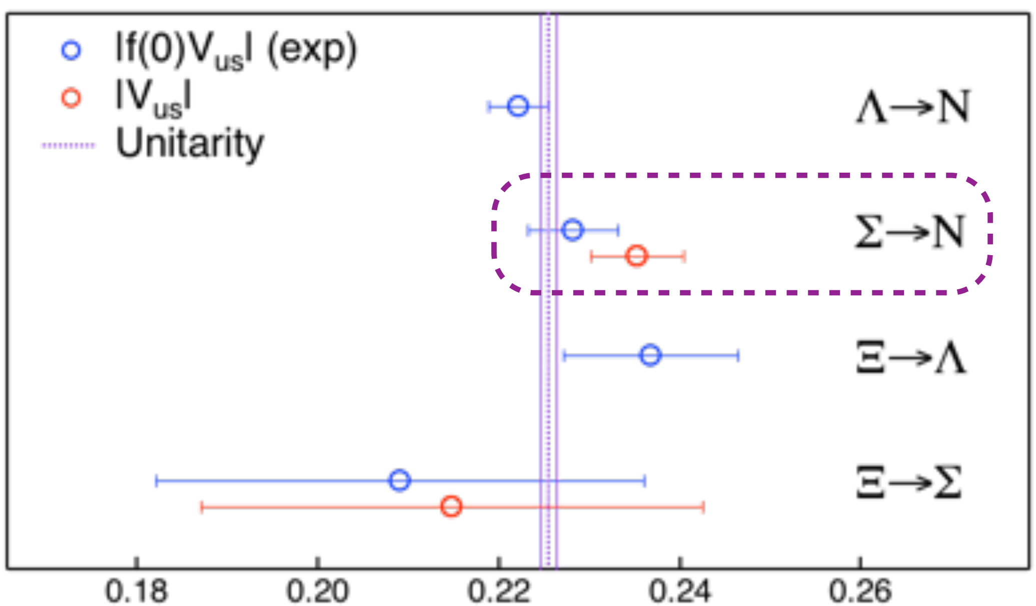
Summary

We have studied the SU(3) breaking effect on hyperon vector couplings using 2+1 flavor dynamical lattice QCD.

➡ an essential theoretical input to determine V_{us}

- ✓ the tendency of the SU(3)-breaking correction disagrees with the latest **baryon ChPT** result.

- ✓ current **Σ→N** data leaves room for the possibility of unitarity violation



➡ needs further experimental and theoretical efforts

Backup slides

SU(3) breaking correction

Ademollo-Gatto theorem:

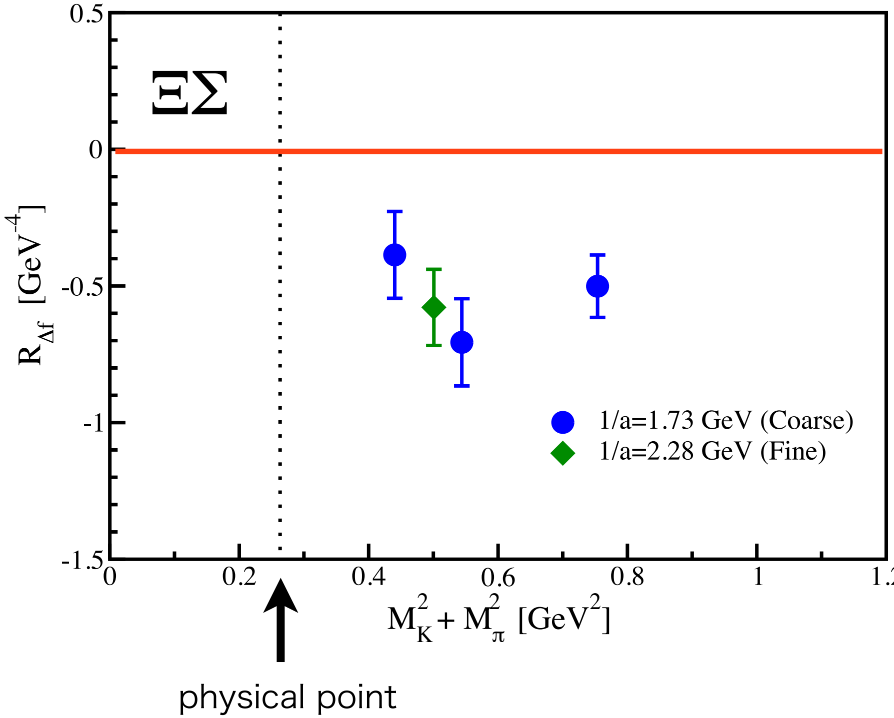
The leading symmetry breaking correction on $f_1(0)$ starts in second order

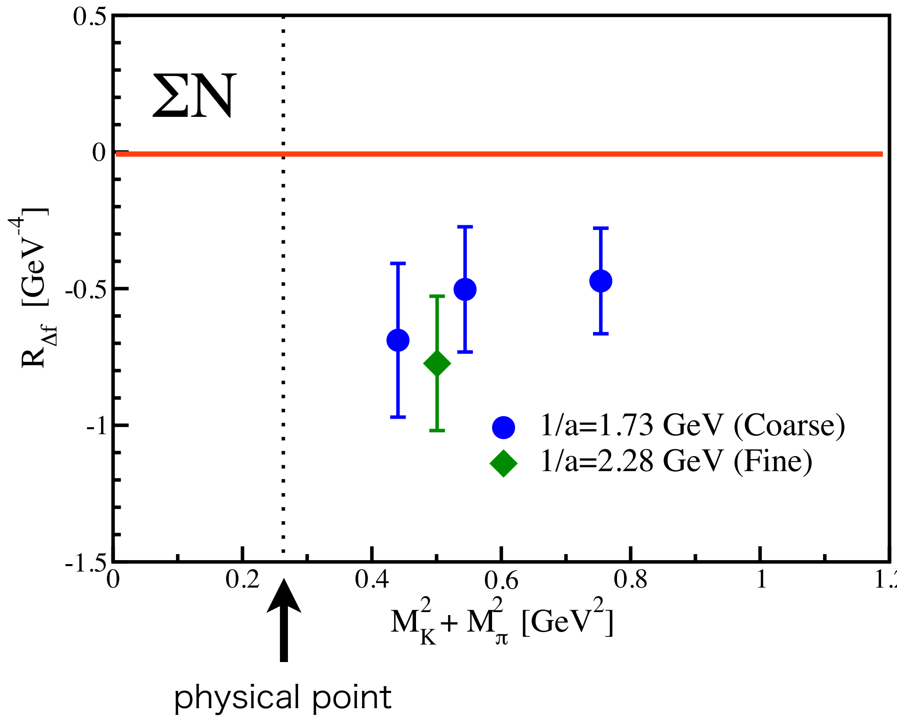
$$|f_1(0)/f_1^{\mathrm{SU}(3)}(0)| = 1 + \Delta f$$

Δf represents all SU(3) breaking corrections

Introduce $$R_{\Delta f}(M_K, M_\pi) = \frac{\Delta f}{(M_K^2 - M_\pi^2)^2}$$

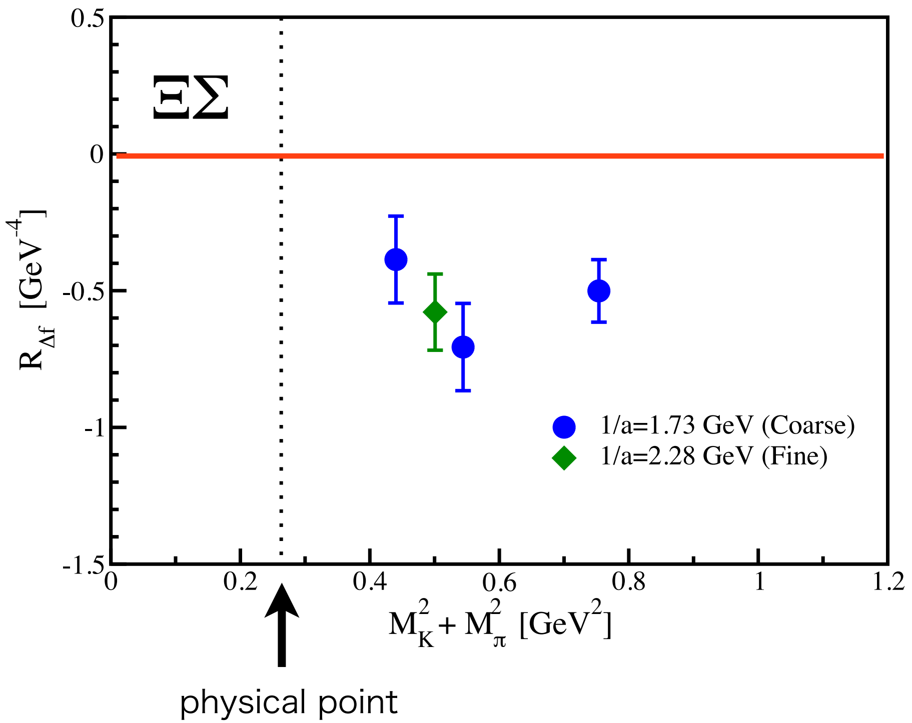
SU(3) breaking correction

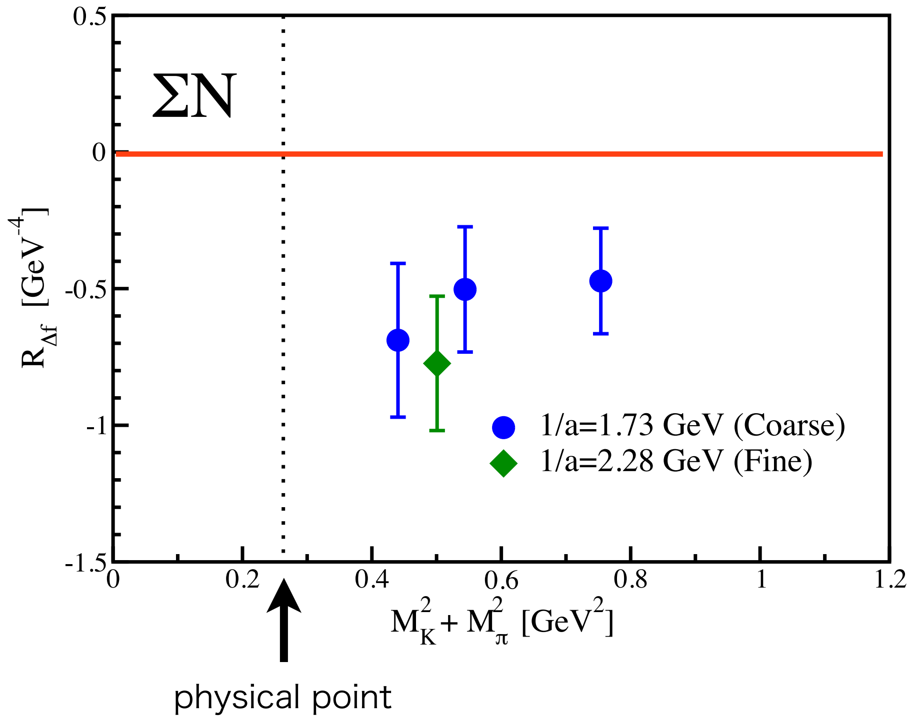




$$R_{\Delta f} < 0 \quad \leftrightarrow \quad \tilde{f}_1 = f_1(0)/f_1^{\mathrm{SU}(3)}(0) < 1$$

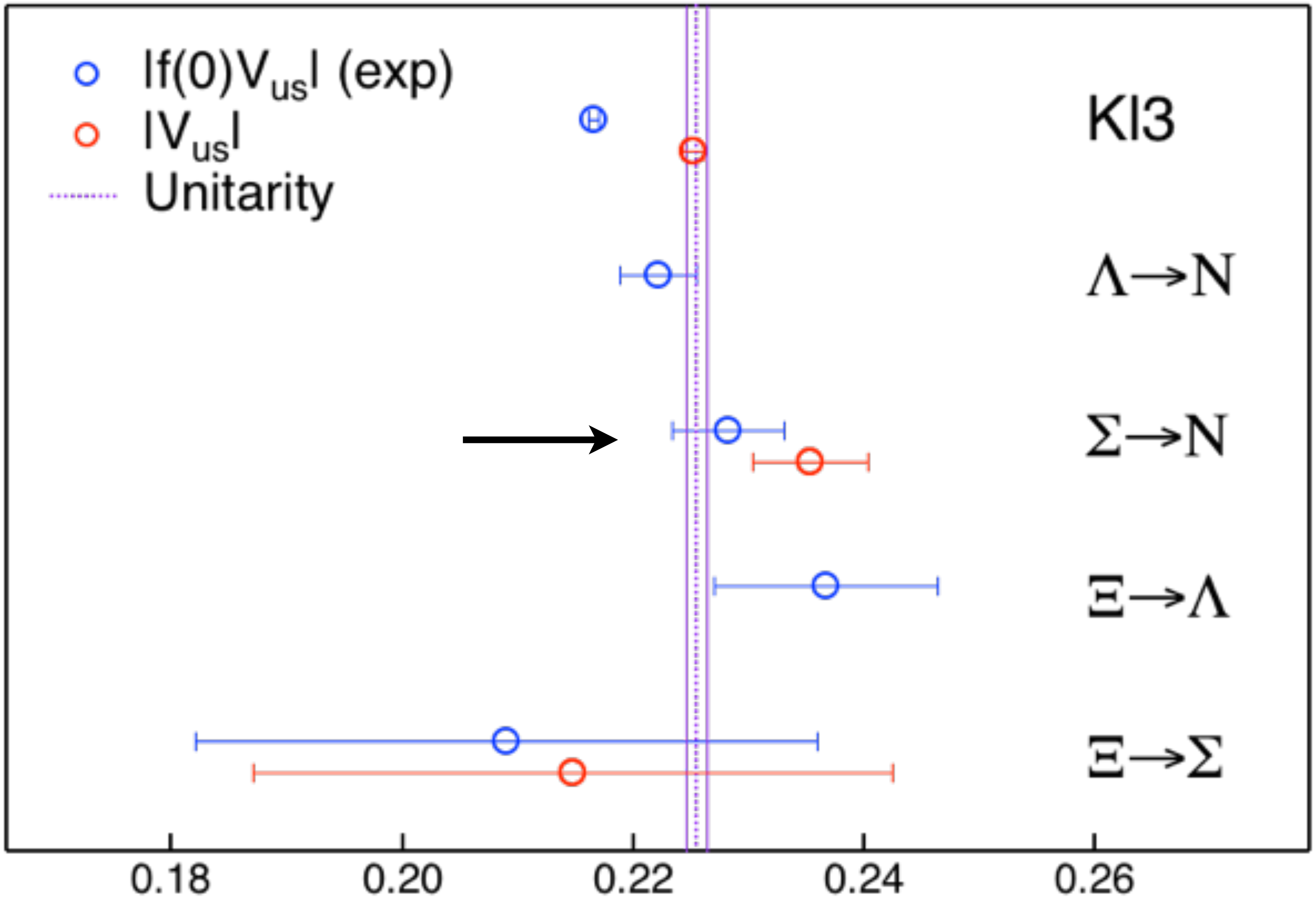
SU(3) breaking correction





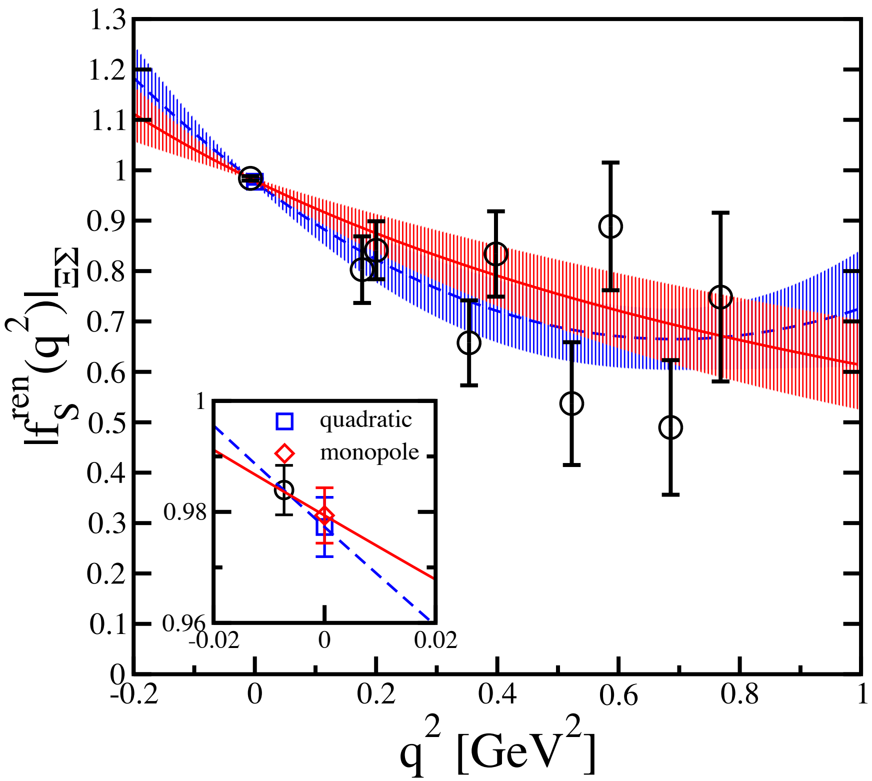
- very mild dependence on M_π and M_K
  - ✓ good AGT behavior + small higher order corrections of SU(3) breaking
- no visible dependence on lattice spacing
  - ✓ good scaling property of DWF

V_{us} is determined by combining the experimental values with **the lattice calculations of $f_1(0)$**



q^2 interpolation dependence

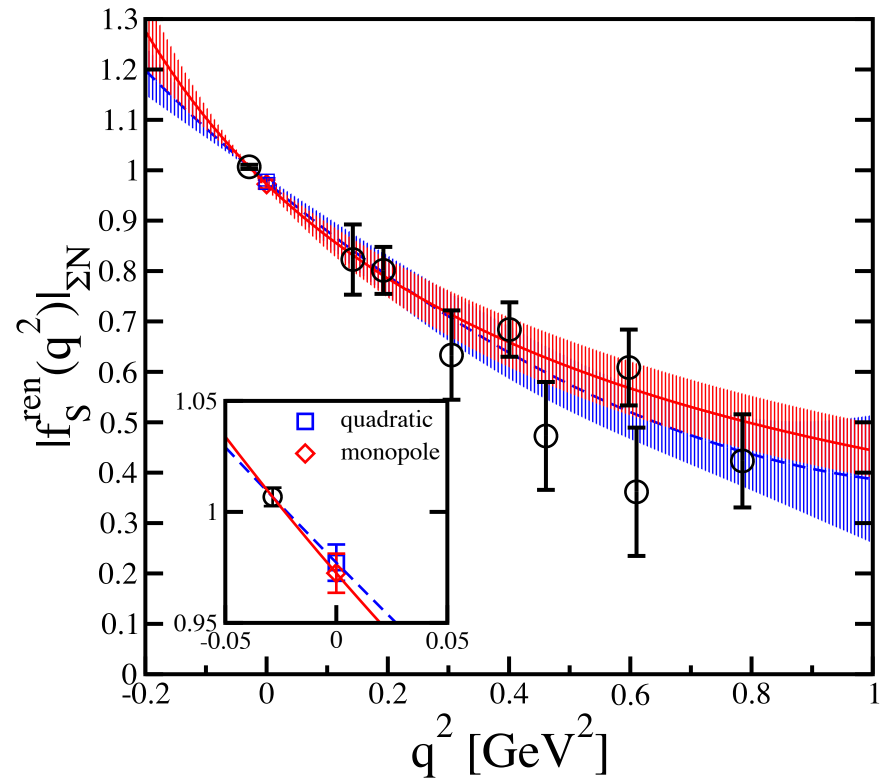
$\Xi \to \Sigma$



0.979(5) monopole

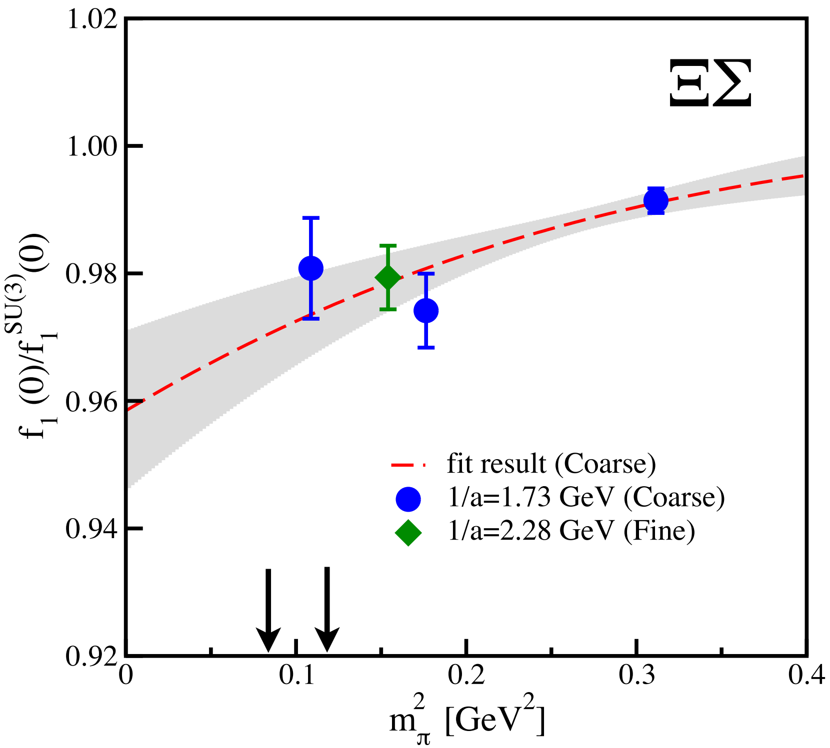
0.977(5) quadratic

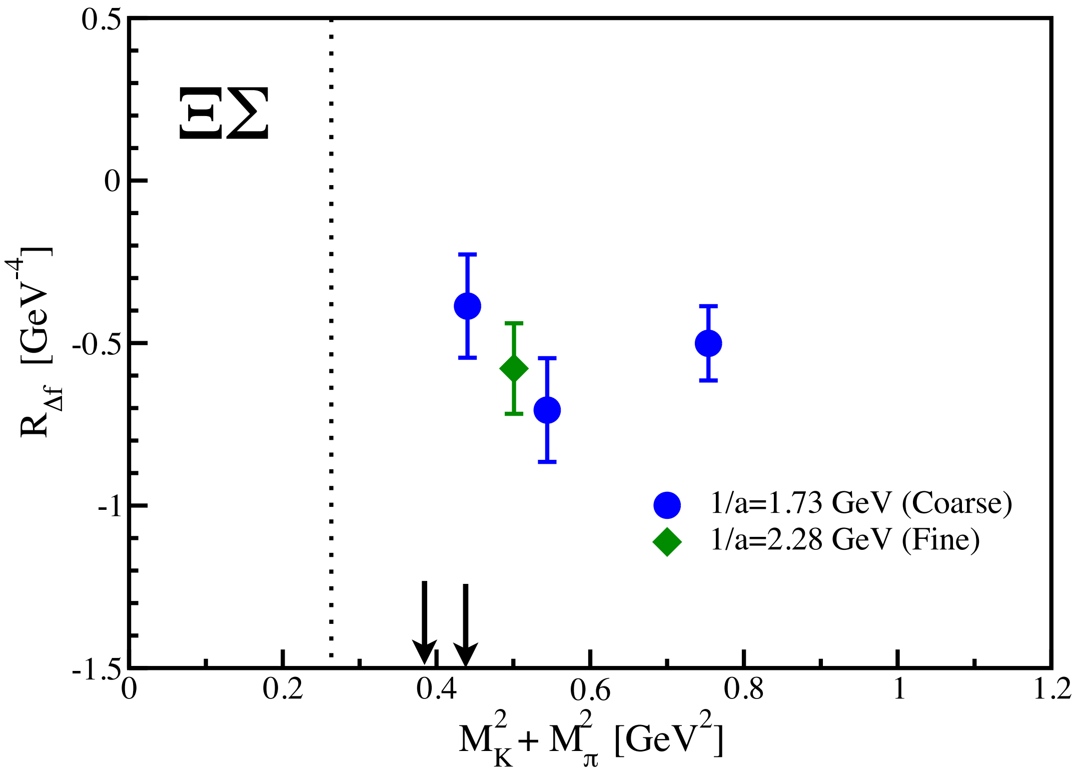
$\Sigma \to N$



0.972(9) monopole

0.977(8) quadratic





am_{ud}	m_π [MeV]	m_K [MeV]
0.004	289	554
0.006	345	570
0.008	390	688

Kl3 decay

$$f_+(0) = \overbrace{1 + \underbrace{\delta^{(2)}}_{\mathcal{O}(p^4)\ \text{ChPT}}}^{\text{no unknown LEC}} + \underbrace{\delta^{(4)}}_{\mathcal{O}(p^6)\ \text{ChPT}} + \delta^{(6)} + \cdots$$

$$\delta^{(2)} = -0.023$$

$$\delta^{(4)}_{\text{LR}} = -0.016$$ (Leutwyler-Roos) quark model

$$|\delta^{(2)}| > |\delta^{(4)}_{\text{LR}}|$$

Hyperon β-decay

odd power

$$f_1(0) = \alpha^{(1)}[\overbrace{1 + \underbrace{\alpha^{(2)}}_{\mathcal{O}(p^3)\ \text{Baryon ChPT}} + \underbrace{\alpha^{(3)}}_{\mathcal{O}(p^4)\ \text{Baryon ChPT}}}^{\text{no unknown LEC}} + \alpha^{(4)} + \cdots]$$

$$\alpha^{(1)}_{\Sigma^- n} = -1 \qquad \alpha^{(1)}_{\Lambda p} = -\sqrt{3/2}$$

$$\alpha^{(1)}_{\Xi^- \Lambda} = \sqrt{3/2} \qquad \alpha^{(1)}_{\Xi^0 \Sigma^+} = 1$$

$$|\alpha^{(2)}|_{\Sigma N} \ll |\alpha^{(3)}|_{\Sigma N}$$

Technical remark on double ratio

- $\Sigma^- \rightarrow$ n decay

$$R(t) = \frac{\langle\Sigma|\bar{s}\gamma_0 u|n\rangle\langle n|\bar{u}\gamma_0 s|\Sigma\rangle}{\langle\Sigma|\bar{s}\gamma_0 s|\Sigma\rangle\langle n|\bar{u}\gamma_0 u|n\rangle} \rightarrow 1$$

for $m_u = m_s$

- $\Xi^0 \rightarrow \Sigma^+$ decay (neutron β-decay)

$$R(t) = \frac{\langle\Xi|\bar{s}\gamma_0 u|\Sigma\rangle\langle\Sigma|\bar{u}\gamma_0 s|\Xi\rangle}{\langle\Xi|\bar{u}\gamma_0 u|\Xi\rangle\langle\Sigma|\bar{s}\gamma_0 s|\Sigma\rangle} \rightarrow 1 + \mathcal{O}(a^2, (ma)^2)$$

for $m_u = m_s$