Phase transitions in the three-dimensional $Z(N)$ models

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OUTLINE

1. $3d\ Z(N)$ models at zero temperature.
2. Simulation details.
3. Results.
4. Summary and perspectives

- Thorough study has been done so far only for $Z(2,3)$ and for $N \to \infty$ limit;

- For $N \geqslant 4$ only approximate locations of critical points are known

➜ *G. Bhanot, M. Creutz, Phys. Rev. D21, (1980) 2892*

- Our work for $T \neq 0$
  (was presented at a poster given by Alessandro Papa)

➜ *O. Borisenko et al., Nucl. Phys. B 870 (2013) 159*

Established facts:

- Second order phase transition (PT) in $Z(2)$

- First order PT in all standard $Z(N), N \geqslant 3$

- $U(1)$ gauge model is in confined phase for all values of β

We've performed a detailed study of $Z(N \geqslant 4)$ vector model with the goal to explore their critical properties.

- precise location of the critical points.
- critical indices and universality class.
- scaling of the critical point with N.

I. $3d$ $Z(N)$ models at zero temperature

The $3d$ $Z(N)$ gauge model on a L^3 lattice Λ is defined by following partition function

$$Z(\beta) = \sum_{\{r_l \in 0..N-1\}} \prod_{p \in \Lambda} \exp\left[\beta \cos \sum_{l \in p} \frac{2\pi r_l}{N}\right] .$$

Duality transformation turns this model into a generalized $3d$ $Z(N)$ spin model with a specific dependence of β_k on β:

$$Z(\beta) = \sum_{\{r_x \in 0..N-1\}} \prod_{(x,n) \in \Lambda^*} \exp\left[\sum_{k=1}^{N-1} \beta_k \cos \frac{2\pi k (r_x - r_{x+e_n})}{N}\right] .$$

II. Simulation details

We've simulated $3d$ $Z(N)$ models for $N = 2, 4, 5, 6, 8, 13, 20$ on the lattices up to 96^3. Since we model the dual representation, we used a cluster algorithm for updates. For most data points we used 2.5 million measurements with 10 lattice updates between measurements. For termalization 200 000 lattice updates were done.

The observables we measured include

- magnetisation: $M = \frac{1}{V} \left\langle \sum_{x \in \Lambda^*} e^{i\phi_x} \right\rangle$,
- magnetic susceptibility: $\chi_M = V \left(\left\langle |M|^2 \right\rangle - \langle |M| \rangle^2 \right)$,
- Binder cumulant U_4: $U_4 = 1 - \frac{\left\langle |M|^4 \right\rangle}{3 \left\langle |M|^2 \right\rangle^2}$,
- heat capacity: $C = \frac{d^2 \ln Z(\beta)}{d\beta^2}$.

Along with the observables we measured their derivatives (up to the third) with respect to β.

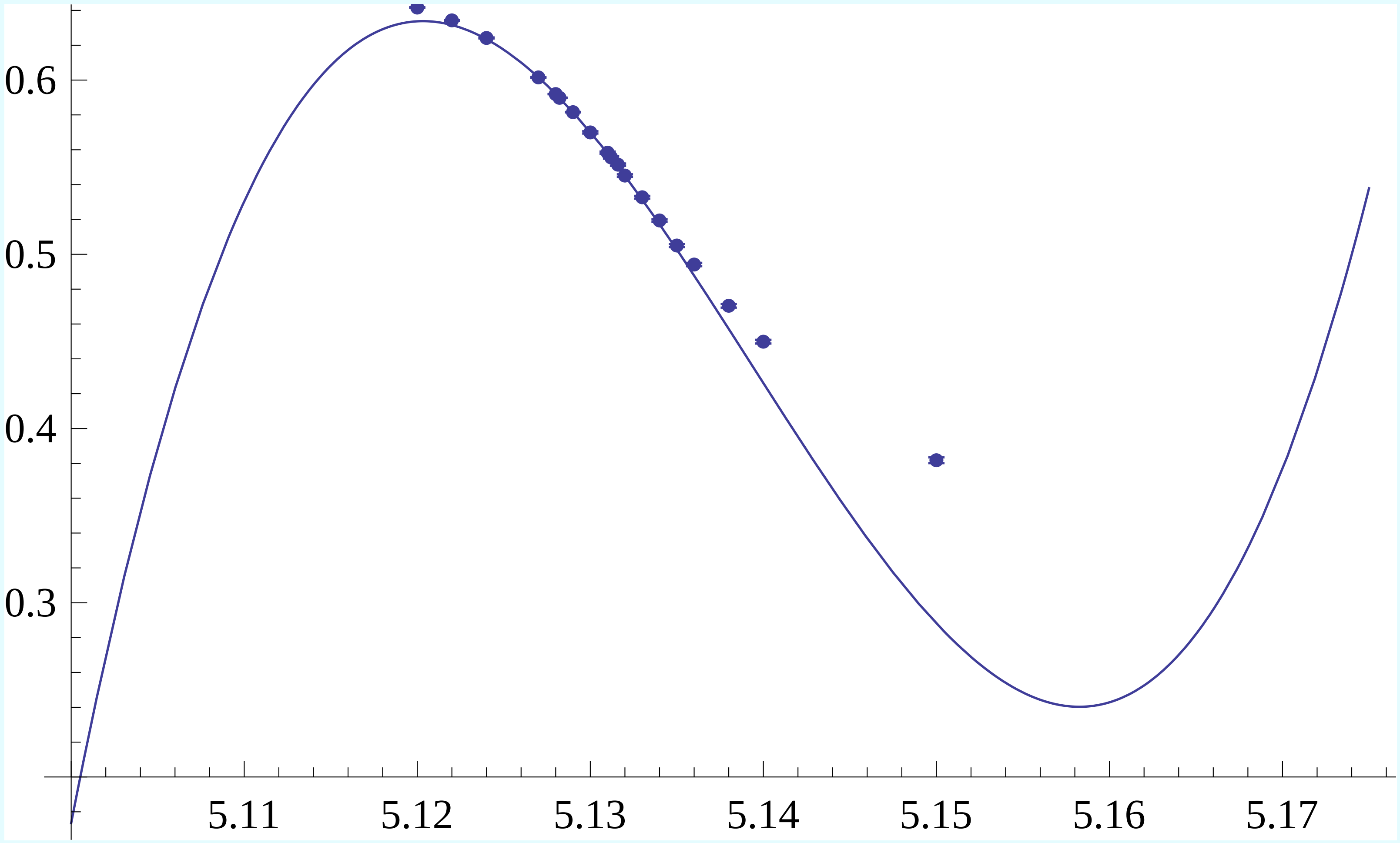
III. Results. Critical points

The position of the critical point was first localized by looking at the maximum of the magnetic susceptibility, then found more precisely by finding the intersection of the Binder cumulants on different lattice sizes, approximating them by their Taylor expansions.

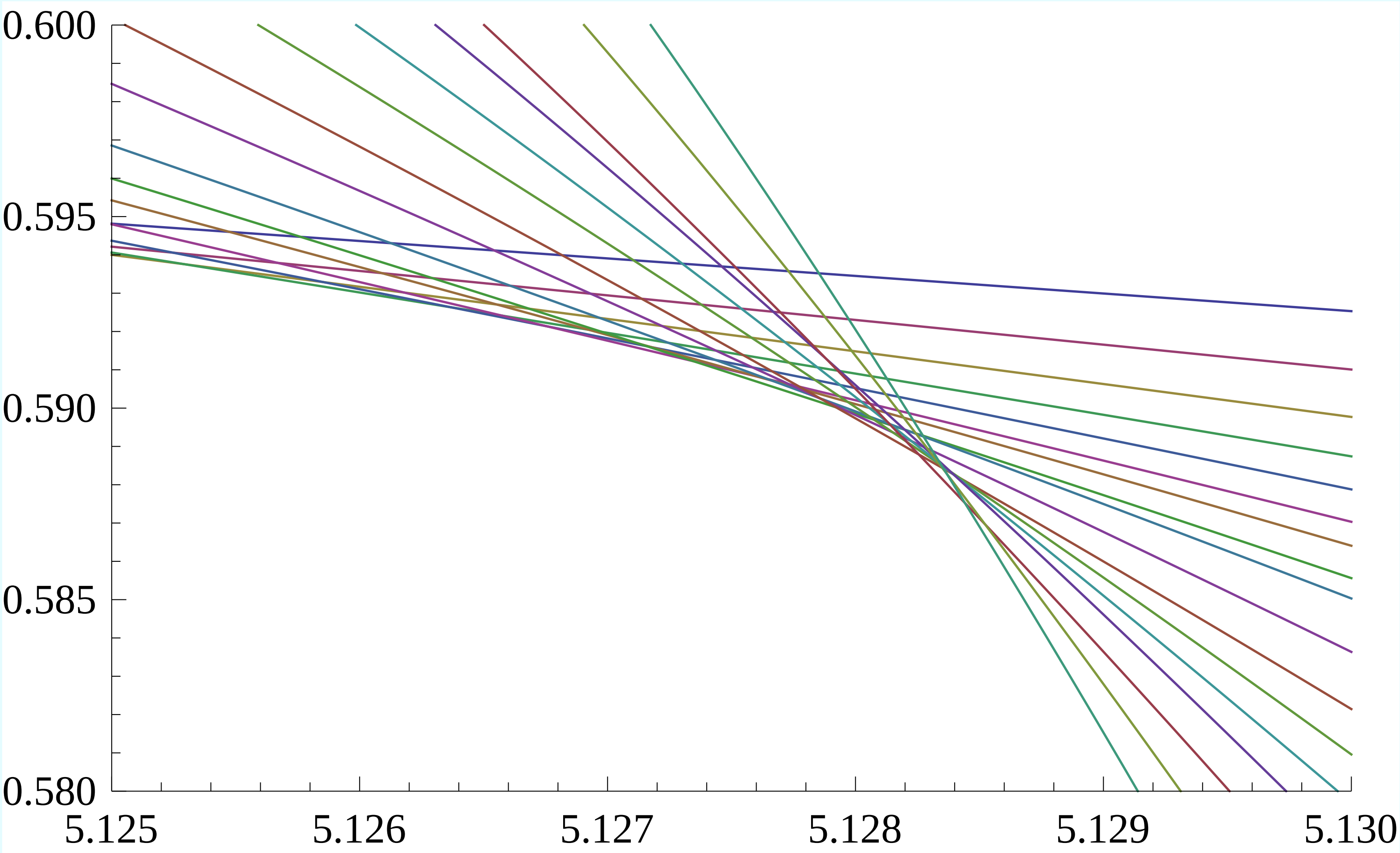
➜ *M.Campostrini et al., Phys. Rev. B63 (2001) 214503*

N	β_c
2	0.761395(4)
4	1.52276(4)
5	2.17961(10)
6	3.00683(7)
8	5.12829(13)
13	13.1077(3)
20	30.6729(5)

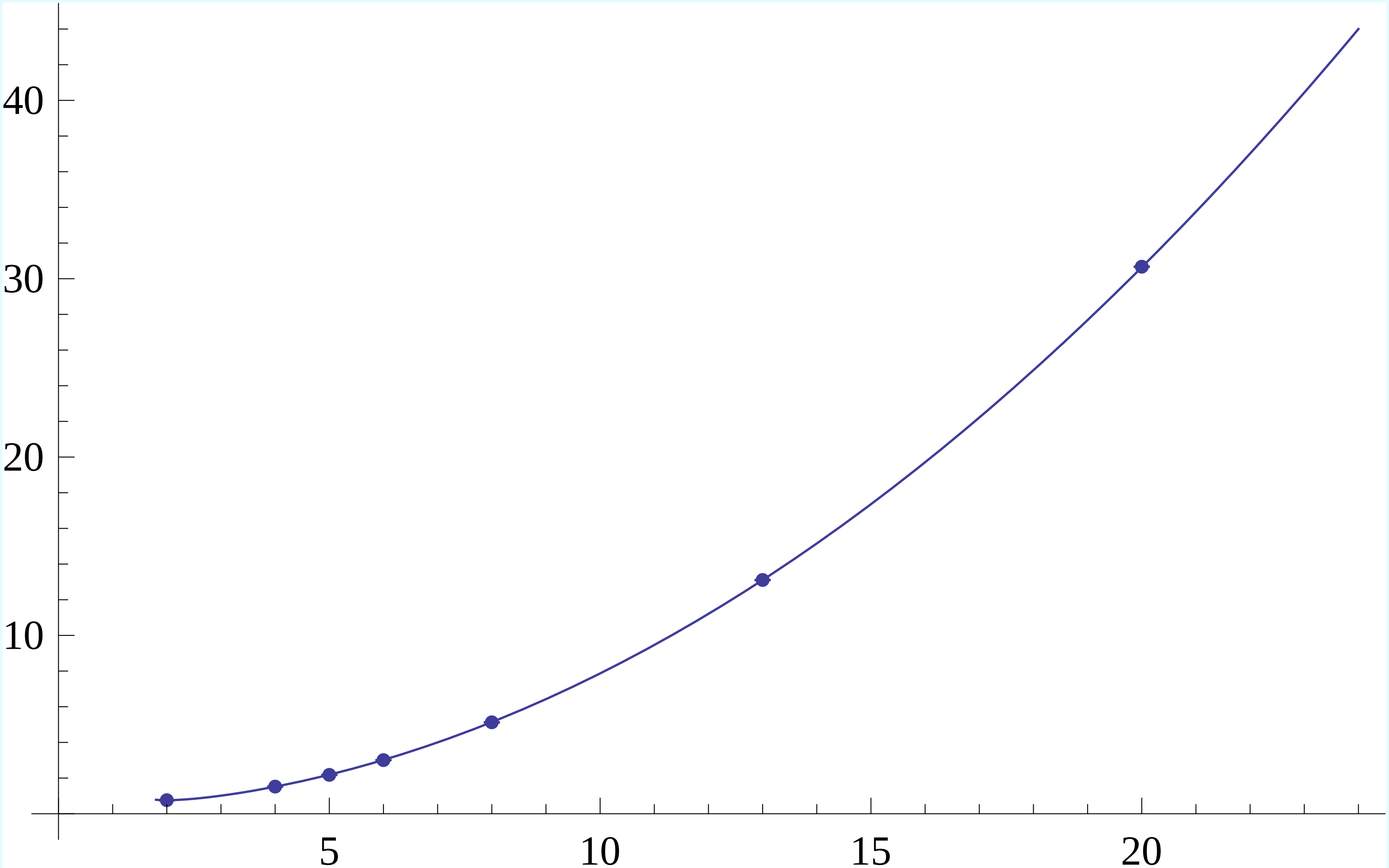
Critical β values of $3d$ $Z(N)$ models for different values of N.



Approximation of $Z(8)$ U_4 Binder cumulants on $L = 64$ around $\beta_s = 5.12822$



Taylor expansions of $Z(8)$ U_4 Binder cumulants on different lattice sizes around $\beta_s = 5.12822$



Dependence of the critical coupling β_c on N.

$$\beta_c = \frac{A}{1 - \cos\frac{2\pi}{N}} + B + C\cos\frac{2\pi}{N}$$

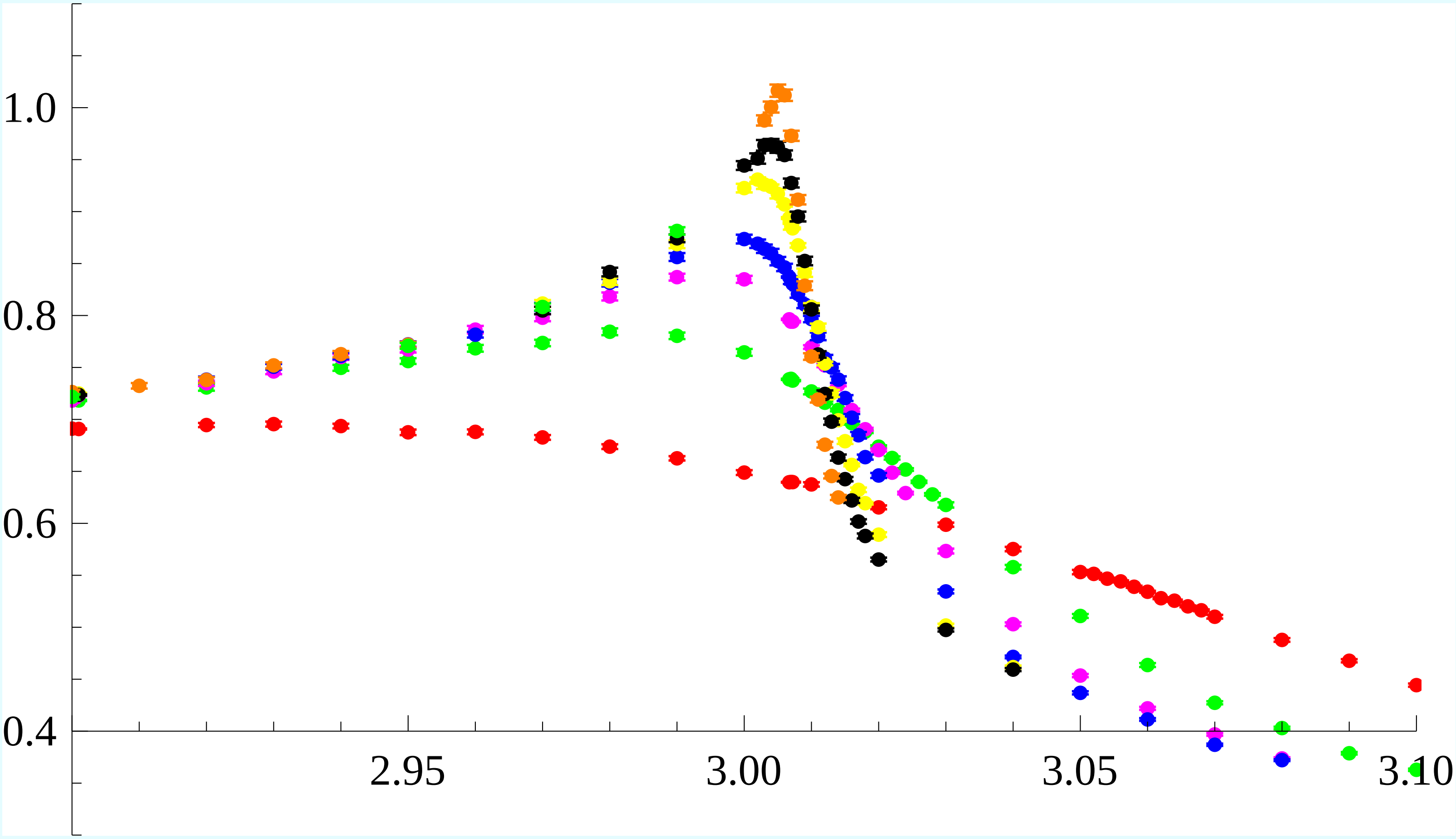
$$A = 1.499(3),\ B = 0.022(10),\ C = -0.005(4)$$

III. Results. Critical index ν.

Since the Binder cumulant U_4 around the critical point depends only on the rescaled coupling $K = (\beta - \beta_c)L^{1/\nu}$, we used this to extract the ν value: the derivative $\frac{dU_4}{dK} = \frac{dU_4}{L^{1/\nu}d\beta}$ must be the same for all L values. Corresponding fit gives us the following ν values:

N	ν
2	0.6299(10)
4	0.6312(8)
5	0.6689(14)
6	0.6739(10)
8	0.6720(4)
13	0.6723(17)
20	0.6737(7)

Note that according to these data for $N > 4$ critical index $\alpha = 2 - d\nu$ is negative, meaning we have the transition of the order higher than two.



Dependence of heat capacity in $3d$ $Z(6)$ on β

III. Results. Critical index α.

More direct extraction of the critical index α uses the behaviour of the specific heat around the critical point. We've done fitting of the maximum values of the specific heat on the lattice L^3 with following behaviour $A\,L^{-3\alpha/(2-\alpha)}$. The results are given in the following table

N	α
2	0.154(2)
4	0.150(3)
5	0.099(3)
6	0.092(2)
8	0.099(2)
13	0.098(2)
20	0.106(3)

III. Results. Critical index η.

Critical index η is obtained from the scaling of magnetisation and magnetic susceptibility

$$\begin{aligned} M &= A_M\, L^{-\beta/\nu}(1 + B_M\, L^{-\frac{\triangle}{\nu}})\,, \\ \chi_M &= A_{\chi_M}\, L^{\gamma/\nu}(1 + B_{\chi_M}\, L^{-\frac{\triangle}{\nu}})\,. \end{aligned}$$

We obtain η as $2 - \gamma/\nu$, and also check the hyperscaling relation $d = 2\beta/\nu + \gamma/\nu$

N	η	d
2	0.038(4)	2.993(11)
4	0.045(3)	2.989(8)
5	0.037(6)	2.995(9)
6	0.053(7)	2.971(9)
8	0.053(6)	2.974(9)
13	0.051(6)	2.977(10)
20	0.059(13)	2.987(18)

IV. Summary and perspectives

- Found the location of critical points in $Z(N = 4, 5, 6, 8, 13, 20)$ models.

- Obtained critical indices ν, α, η.

- The behavior of the heat capacity shows that the transition is of the second order, although data for ν index suggests that α is negative if we approach the critical point from above.

- For $N > 5$ our results suggest that the models are in $3d$ XY universality class. For $Z(5)$ model more detailed investigation is needed.

- Calculations of string tension to obtain better estimates of the ν index.