A study of massive gauge theories on the lattice (part II)

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Lattice 2013, Mainz (Germany), August 2013

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In preparation

Let us consider a complex scalar doublet coupled to SU(2) gauge fields

$$S(\beta,\kappa,\lambda,U,\rho,\alpha) = S_{\rm gauge}(\beta,U) + \sum_x \rho(x)^2 - 3\log(\rho(x)) + \lambda(\rho(x)^2-1)^2 - \kappa \sum_{\mu>0} \rho(x)\rho(x+\hat{\mu}){\rm Tr}\left(\alpha(x+\hat{\mu})^\dagger U(x,\mu)\alpha(x)\right)$$

[Langguth, Montvay, Weisz 1985]. In the limit $\lambda \to \infty$ ρ is frozen to 1. Redefining

$$U(x,\mu) \to \alpha(x+\hat{\mu})^\dagger U(x,\mu)\alpha(x) \qquad \text{then}$$

$$S \to S_{\rm gauge}(\beta,U) - \kappa \sum_{x,\mu>0} {\rm Tr}\, U(x,\mu)$$

A massive gauge theory ($\kappa \propto m^2$) ? May it be (non-perturbatively) renormalizable ?

In perturbation theory the propagator of a massive spin 1 particle

$$\Delta_{\mu\nu} \propto \frac{1}{k^2 + m^2}\left(\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}\right)$$

does not fall off with all momentum components at large momentum. The theory is not renormalizable by power counting.

- However it is a theory made of local fields and no couplings of negative mass-dimension (operators of engineering dimension 4 at most in the action).
- The static theory is very similar in this respect. The static quark propagator doesn't fall with all momenta, still the theory is believed to have a continuum limit because dim 4 operators only appear in the static action.

Previous studies:

- Phase diagram and spectrum of SU(2)+Higgs since mid '80 (and ongoing), by now textbook studies.
- More recently: Massive gauge theories, continuum [J. Gegelia and collab. 2007 ..., R. Ferrari, 2008] and lattice [R. Ferrari and collab. 2012].

No attempt to look at scaling though.

- We start at $\beta = 2.3$, $L = 16$ and κ s.t. $m_{\rm H}/m_{\rm W} \simeq 1.4$. From [Langguth, Montvay, Weisz 1985] we know $am_{\rm W} \simeq 0.5$.
- We increase β and L, in order to keep $m_{\rm W}L > 5$ and tune κ s.t. $m_{\rm H}/m_{\rm W} \simeq 1.4$.
- We look at the scaling of $m_{\rm H}$, $m_{\rm W}$, $F_{\rm W}$ and the static potential.

Action, algorithm and observables

We adopt the SU(2) Wilson gauge action plus mass term

$$\kappa \sum_{x,\mu} \mathrm{Tr}\left[\mathcal{I} - \frac{1}{2}\left(U_\mu(x) + h.c.\right)\right] \qquad \text{site refl.pos.}$$

Heatbath and o.r. can be used by adding a term $\propto \kappa\mathcal{I}$ to the staples.

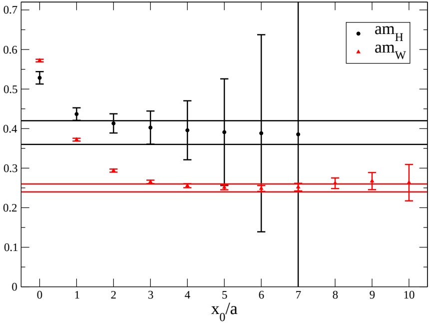
Interpolating fields. We consider connected correlators of:

$$\sum_{\vec{x}} \mathrm{Tr}\, U_\mu^{\mathrm{APE}}(x) \qquad \text{for } \mathrm{m_H}$$

$$\sum_{\vec{x}} \mathrm{Tr}(U_k^{\mathrm{APE}}(x)\tau^a) \qquad \text{for } \mathrm{m_W}, \text{ no APE for } \mathrm{F_W}$$

and in addition we considered correlators of Polyakov loops (with APE smearing) for the potential

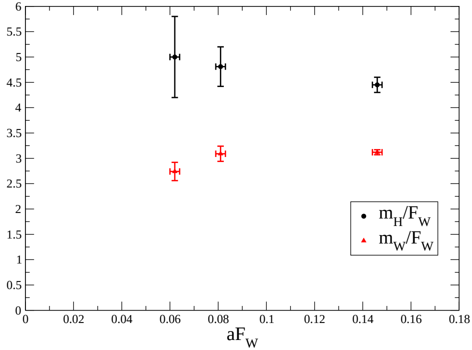
β	κ	$L^3 \times T$	am_H	am_W	aF_W	N_meas
2.3	0.405	$16^3 \times 16$	0.65(2)	0.455(5)	0.146(2)	5.4 M
2.55	0.368	$24^3 \times 24$	0.39(3)	0.25(1)	0.081(2)	1.4 M
2.75	0.356	$36^3 \times 36$	0.31(5)	0.17(1)	0.062(2)	0.7 M



4-point plateau for m_H, with $\simeq$ 10% error. Clear exponential problem, a case for the algorithm in [MDM, Giusti, 2008].

Scaling ratios

	am_{H}	am_{W}	aF_{W}
$\frac{\beta=2.3}{\beta=2.55}$	1.67(13)	1.82(7)	1.80(5)
$\frac{\beta=2.55}{\beta=2.75}$	1.26(22)	1.47(10)	1.31(5)

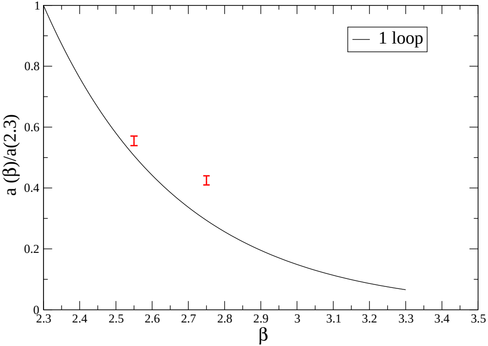


Errors on the tuning of κ still to be propagated.

$a(\beta)$

From Pilar's talk, at 1 loop

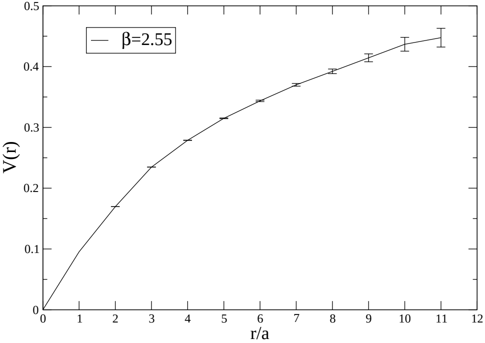
$$a\Lambda = e^{-\frac{8}{29}\pi^2\beta}$$



Static Potential

$$V(r) = -\frac{1}{T}\log C_{\mathrm{PP}}(r)_{\mathrm{connected}}$$

the mass term breaks central charge conj. $\Rightarrow \langle P\rangle \neq 0$.

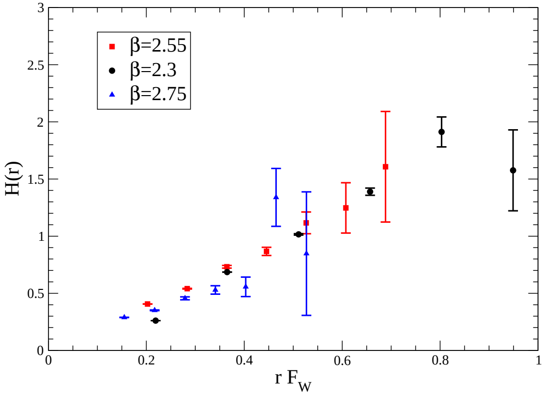


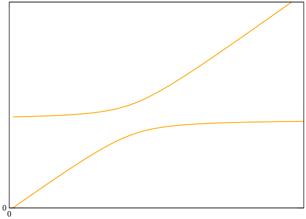
- Flattening, signaling expected string breaking due to states associated to the Ω field (in the fundamental of the gauge group).

As usual we also looked at

$$H(r) = r^2 \frac{\partial V(r)}{\partial r} \ ,$$

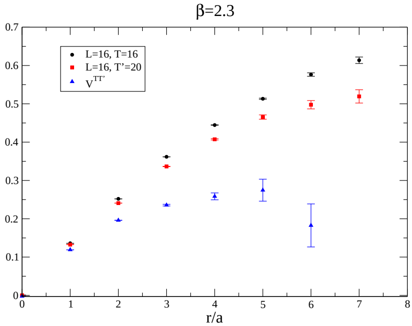
which also shows good scaling (points seem to fall on a universal curve).





Mixing problem, string and 'static-light' states. The overlap of P (stringy) on the ground state may depend on r. We also considered

$$V^{TT'}(r) = -\frac{1}{T-T'}\log\left(\frac{C_{\rm PP}(r,T)_{\rm conn}}{C_{\rm PP}(r,T')_{\rm conn}}\right)$$



T dependence of $V(r)$ is consistent with $C_{\rm PP}(r) = w_0(r)e^{-V_0(r)T}$ and $w_0(r) < 1$

Conclusions

- Massive gauge theories are theoretically interesting by their own and may offer an (Higgsless) alternative to EWSB.
- Exploratory non-perturbative study. We mostly tried to define questions (scaling region ?) and strategies (line of constant physics). A lot of room for technical improvements.
- Rich dynamics, string breaking, several interesting couplings.
- The existence of a scaling region is crucial for the model to be an alternative to the SM Higgs sector. The EFT description should be valid at least in this scaling/universality region.