Large-N mesons

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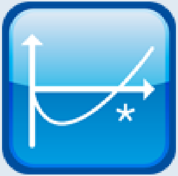
Mainz, August 2, 2013

- Large-N QCD: motivation
- Lattice simulation: details and techniques
- Results
  - Chiral logs and meson masses
  - NP renormalization, chiral condensate, decay constants
  - Spectrum
  - Continuum limit
- Conclusions



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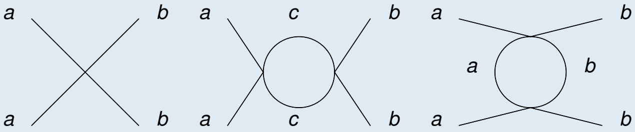
Example: scalar field theory with N-component field ϕ^a, $a = 1, \ldots, N$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu^2\phi^a\phi^a - \frac{1}{8}g^2(\phi^a\phi^a)^2 \,.$$

We define the 't Hooft coupling $\lambda = g^2 N$:



$g^2 = \frac{\lambda}{N}$ $\qquad$ $g^4 N = \frac{\lambda^2}{N}$ $\qquad$ $g^4 = \frac{\lambda^2}{N^2}$

Now we take the limit $g^2 \to 0$ and $N \to \infty$ at fixed λ ('t Hooft limit).
Obviously, this leads to simplifications!

Some properties of large-N QCD

- Sea quark effects $\propto 1/N \Rightarrow$ The $N=\infty$ limit is quenched.
- Mixing glueballs-mesons $\propto 1/\sqrt{N} \Rightarrow$ No mixing between glueballs and mesons at $N=\infty$.
- OZI rule exact at $N=\infty$.

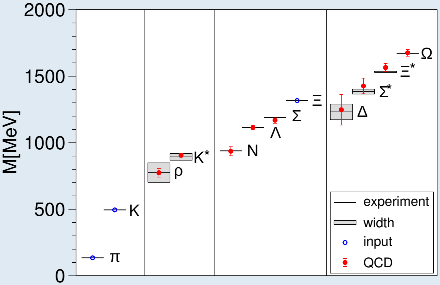
Is $N=\infty$ close to $N=3$ QCD?

AdS/QFT starts from $N=\infty$. Also many simplifications in chiral EFT!

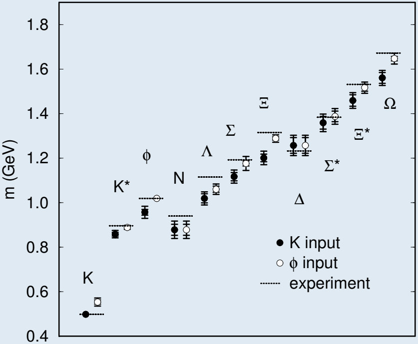
But $N=\infty$ QCD is far from being solved!

Light hadrons: $1/N^2 = 1/9 \stackrel{?}{\approx} 3/3 = N_f/N = 3/3$

If $1/9 \approx 0$ then $\exists$ evidence that $1 \approx 0$:



Full SU(3) QCD BMW-c: S Dürr et al 08



Quenched SU(3) PACS-CS: S Aoki et al 02

Obviously cannot work for flavour singlets $(f_0(500), \eta', \omega)$ but still ...

Lattice parameters

- Volumes:

N	vol
2,3	$16^3 \times 32$, $24^3 \times 48$, $32^3 \times 64$
4,5,6,7	$24^3 \times 48$
17	$12^3 \times 24$

- 200 configs for each N and volume (80 configs for $N = 17$)
- lattice spacing $a \approx 0.093\,\mathrm{fm}$
- pion mass as low as $m_\pi \approx 230\,\mathrm{MeV}$
- Wilson gluon and quark actions

GB, F Bursa, L Castagnini, S Collins, L Del Debbio, B Lucini, M Panero, JHEP 1306 071

Recently added

- 3 additional lattice spacings
- Non-perturbative renormalization

Matching the scale

- Inverse coupling $\beta = 2N/g^2 = 2N^2/\lambda$ is fixed by imposing $a\sqrt{\sigma} \approx 0.2093$ for all $\mathrm{SU}(N)$. (Lattice spacing $a \approx 0.093$ fm is kept constant in units of the string tension $\sigma \approx 1\,\mathrm{GeV/fm}$).
- Other possible choices include $aT_c =$ const, $aF/\sqrt{N} =$ const, etc.
- The κ-parameter ($2am_q = \kappa^{-1} - \kappa_c^{-1}$) is adjusted so that our set of pseudoscalar masses matches between different N (achieved by exploratory simulations).

Plan:

- Vary κ to study $m_A(m_q, N), f_A(m_q, N)$ for each meson A.
- Extrapolate to $N = \infty$ and study $1/N^2$ corrections.

Couplings used in main set of configs

N	2	3	4	5	6	7	17
β	2.4645	6.0175	11.028	17.535	25.452	34.8343	208.45
λ	3.246	2.991	2.901	2.851	2.829	2.813	2.773

$\lambda = Ng^2 = 2N^2/\beta$.

`A Hietanen et al, PLB 674(09)80`:
SU(17) at $\beta = 208.08$ (We have slightly smaller a).

Strong/weak coupling transition at $\sqrt{\sigma}a \gtrsim 1.2$.

Deconfinement transition (similar to finite-T) at $\sqrt{\sigma}N_s a \lesssim 2$.

In principle one could take $N \to \infty$ at $\lambda =$ const. (rather than keeping $a\sqrt{\sigma} =$ const.) but:

- SU(3) at $\lambda = 2.773$ ($\beta \approx 6.47$) requires $N_s \gtrsim 20$.
- SU(17) is very coarse at $\lambda = 2.991$.

Axial and vector Takahashi-Ward identity masses

Partially conserved axial current (PCAC):

$$\sum_x \partial_4 \langle 0|A_4(x,t)|\pi\rangle = 2m_{\mathrm{PCAC}} \sum_x \langle 0|j_5(x,t)|\pi\rangle \quad \text{where} \quad \begin{cases} A_\mu(x) & = & \bar{u}(x)\gamma_\mu\gamma_5 d(x) \\ j_5(x) & = & \bar{u}(x)\gamma_5 d(x) \end{cases}$$

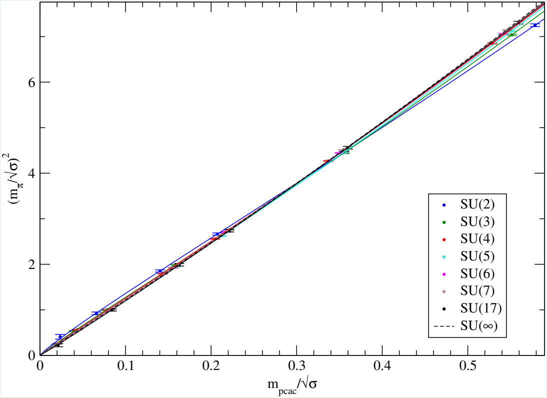
Fit:

$$am_{\mathrm{PCAC}} = \frac{Z_P}{Z_A Z_S}(1 + bam_{\mathrm{PCAC}}) \underbrace{\frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)}_{am_q}$$

Fit parameters (for each N): $Z^{-1} = Z_P/(Z_A Z_S), b, \kappa_c$.

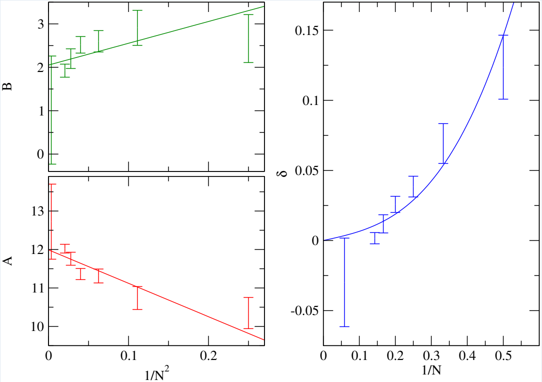
SU(3): $Z^{-1} \approx 0.75$ ($\beta = 6.0175$)
[agrees with independent determination 0.81(7) at $\beta = 6$
V Giménez et al NPB531 (98) 429]

Pion mass vs. PCAC quark mass



$$\frac{m_\pi^2}{\sigma} = A\left(\frac{m_{\mathrm{PCAC}}}{\sqrt{\sigma}}\right)^{\frac{1}{1+\delta}} + B\,\frac{m_{\mathrm{PCAC}}^2}{\sigma}$$

Pion mass: $1/N^2$ fit of the parameters



$$A = 11.99(0.10) - \frac{8.7(1.6)}{N^2}$$

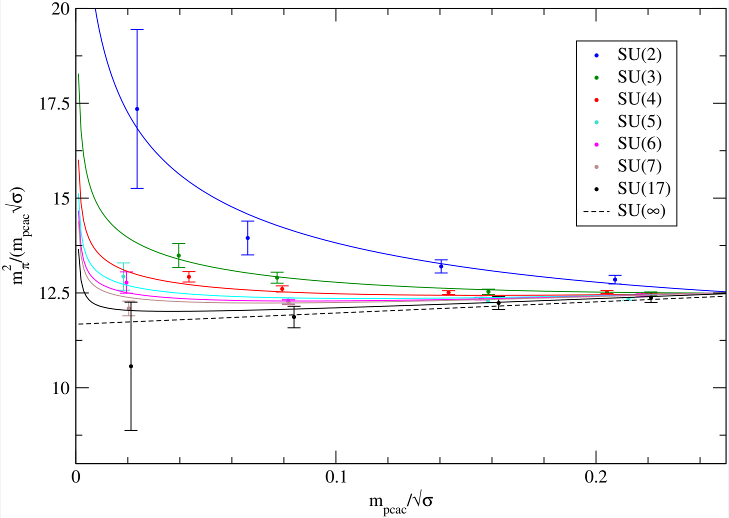
$$B = 2.05(0.13) + \frac{5.0(2.2)}{N^2}$$

$$\delta = \frac{0.056(19)}{N} + \frac{0.94(21)}{N^3}$$

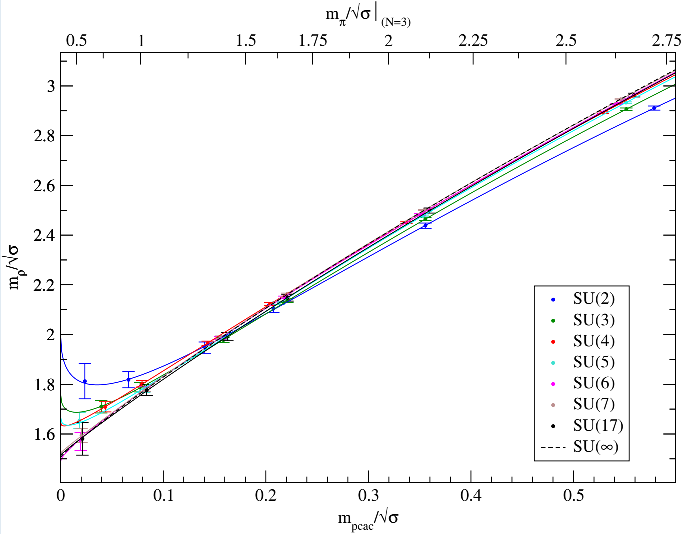
Expectation (S Sharpe PRD 46 (92) 3146):

$\delta = c_1/N + c_2/N^3 + \cdots$.

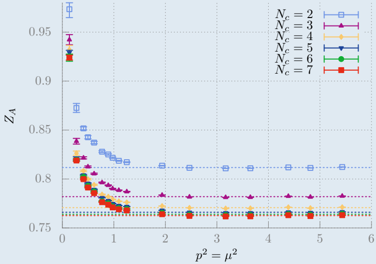
Quenched chiral logs

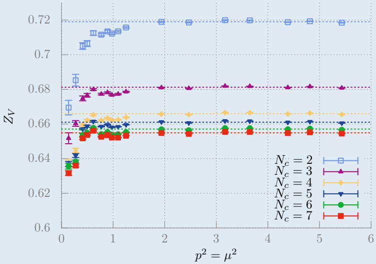


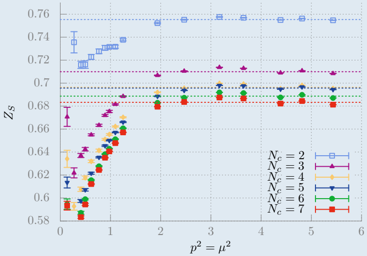
ρ vs. PCAC mass



Non-perturbative renormalization I





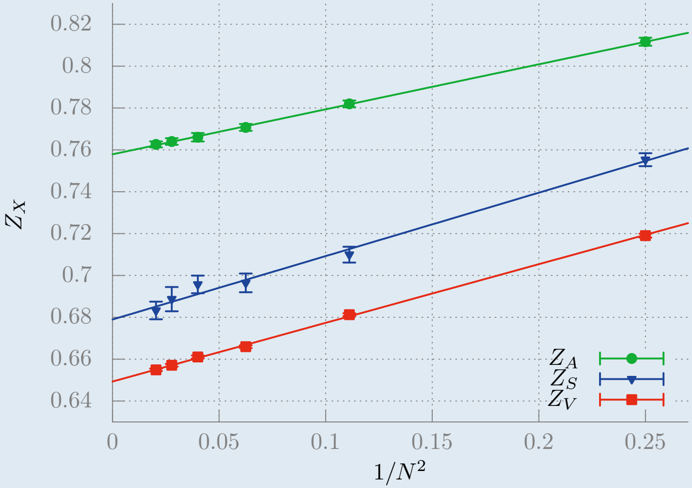


Matching to RI'MOM scheme

$$Z_S = Z_S^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$$

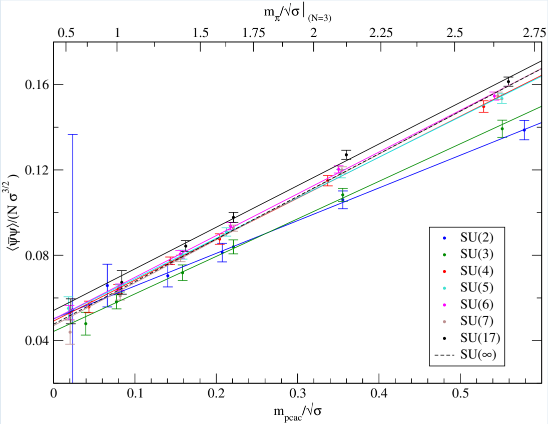
$$Z_P(\mu) = Z_A Z_S(\mu) Z$$

Non-perturbative renormalization II



Chiral condensate

$$\langle\bar\psi\psi\rangle^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = Z_S^{\overline{\mathrm{MS}}}(\mu, a) \lim_{m_q\to 0} \frac{F_\pi^2(m_q) m_\pi^2(m_q)}{2m_q} \propto N$$



Scale setting and fixing quark masses

Definitions:

$$f_X = \sqrt{2}\, F_X\,, \quad F = F_\pi(m_q = 0)\,, \quad \hat{F}_X = \sqrt{\frac{3}{N}} F_X$$

At our main lattice spacing: $\hat{F}_\infty = 0.20\sqrt{\sigma} \approx 88$ MeV.

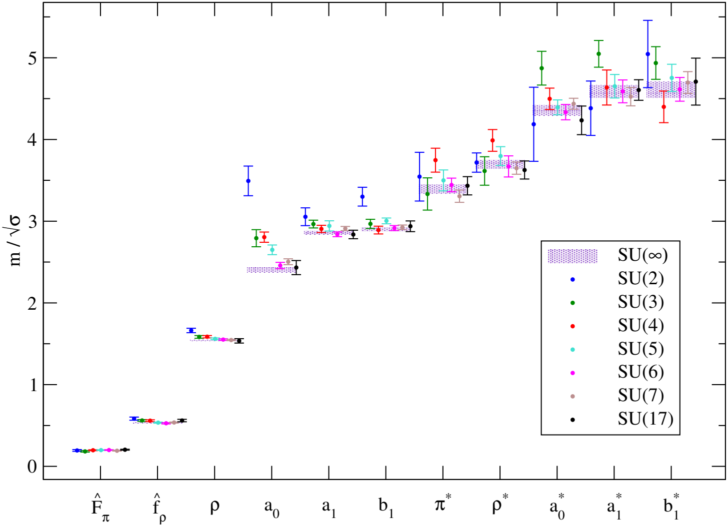
Real SU(3)-value: 85.9(1.2) MeV.

($\hat{F}/\sqrt{\sigma}$ goes down by ≈ 15 % in continuum limit.)

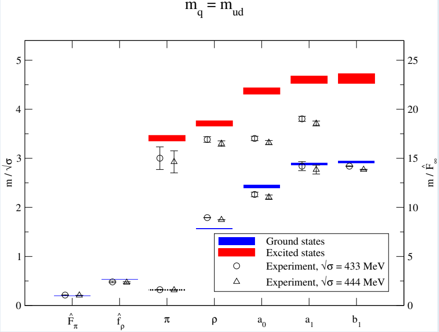
Set scale using $F = 85.9$ MeV instead of $\sqrt{\sigma} = 1$ GeV/fm ≈ 444 MeV.

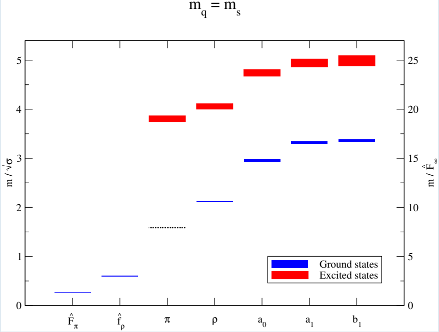
Then set m_{ud}, m_s at $N = \infty$, requiring
$m_\pi(m_{ud}) = 138$ MeV
$m_\pi(m_s) = (m_{K^\pm}^2 + m_{K^0}^2 - m_{\pi^\pm}^2)^{1/2} \approx 686.9$ MeV.

Meson spectrum and decay constants at $m_q = 0$

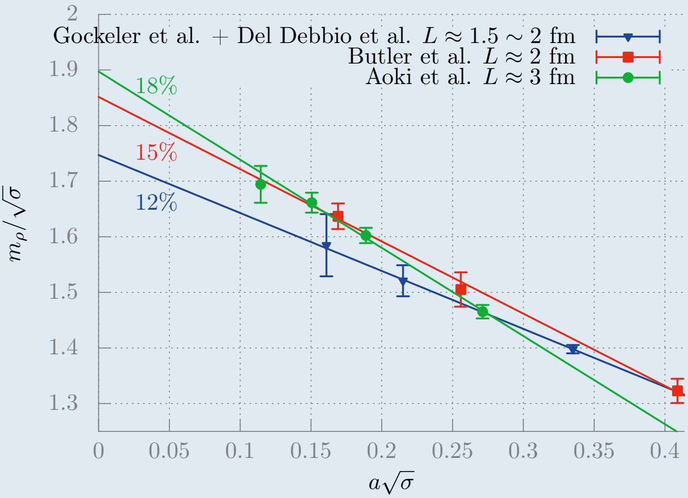


Meson spectrum and decay constants at $m_q = m_{ud}, m_s$

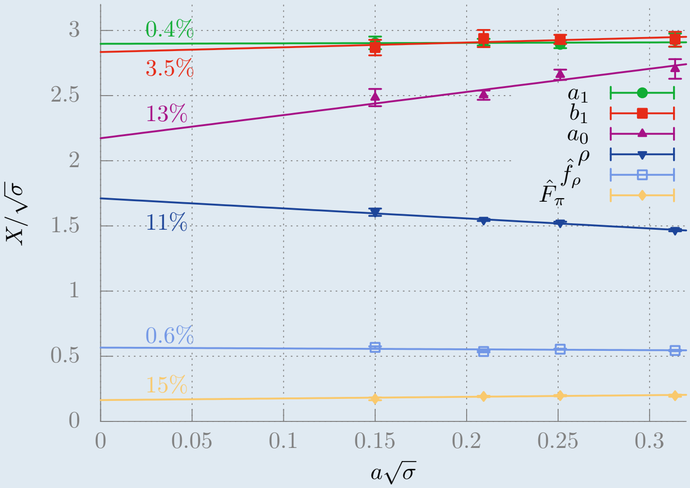




Literature: continuum limit in SU(3)



In progress: SU($7 \approx \infty$) continuum limit



Conclusions

- $N = \infty$ is a good starting point to study strong decays and mixing between different quark model sectors: glueballs, mesons, "mesons^2". Phenomenology of light scalars?
- At $N = \infty$ a connection can be made to AdS/QCD models.
- We computed the quenched meson spectrum of SU(N) for degenerate quark masses and extrapolated these to $N = \infty$.
  This limit is the same for the theory with sea quarks!
- Isovector SU(3) masses, decay constants and chiral condensate are close to the $N = \infty$ limit.
- $1/N^2$ corrections are small for $N = 3$.
- The fact that many mass ratios also differ by less than 10 % between the $N_f = 2 + 1$ theory and the quenched approximation indicates that N_f/N corrections may often be small too.
- Continuum limit in progress.