Lattice investigations of the Hosotani Mechanism

James Hetrick

University of the Pacific
Stockton, CA USA

27 Sept. 2012

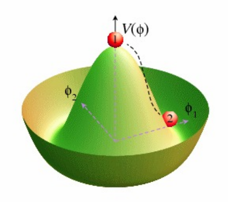
with help from G. Cossu, Y. Hosotani, E. Itou, and J.-I. Noaki

Dynamical Symmetry Breaking Mechanisms

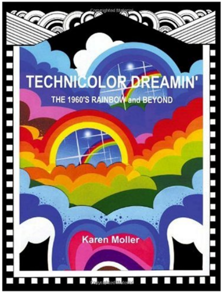
Scalar Fields

$$\mathcal{L} = \partial_\mu \varphi^\dagger \partial^\mu \varphi - V(\varphi^\dagger \varphi)$$

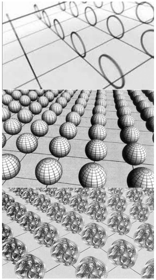
$$V(\varphi^\dagger \varphi) = \frac{1}{2}(\lambda^2 |\varphi|^4 - \mu^2 |\varphi|^2)$$



Chiral Condensates



Extra Dimensions

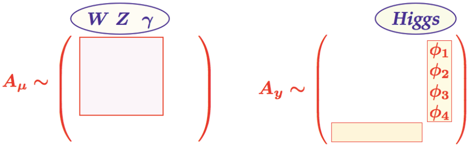


The "Hosotani Mechanism"

The basic idea

Y. Hosotani, *Phys. Lett.* **B126**, 309 (1983)

N. Manton, *Nucl. Phys.* **B158**, 141 (1979)



$A_M(x, y)$ in higher dimensions

four-dim. components A_μ $\Downarrow$ 4D gauge fields (γ, W, Z)

extra-dim. component A_y $\Downarrow$ = 4D Higgs field (H) $\Downarrow$ EW symmetry breaking

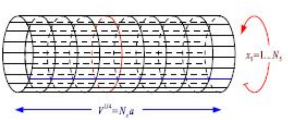
The Hosotani Mechanism v. Finite Temperature QCD?

QCD at $T \neq 0 \quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

fermions ANTI-PERIODIC



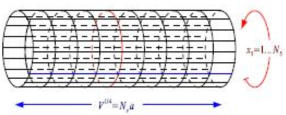
The Hosotani Mechanism v. Finite Temperature QCD?

QCD at $T \neq 0 \quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

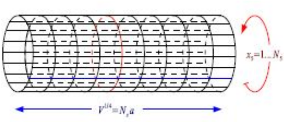
fermions ANTI-PERIODIC



Gluon Self-energy

$$\Pi_{\mu\nu}(p=0) \qquad \Rightarrow \qquad m_g^2 = \frac{1}{3} g^2 T^2 (N_c + \frac{1}{2} N_f)$$

The Hosotani Mechanism v. Finite Temperature QCD?



QCD at $T \neq 0 \quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

fermions ANTI-PERIODIC

Gluon Self-energy

$$\Pi_{\mu\nu}(p = 0) \qquad \Rightarrow \qquad m_g^2 = \tfrac{1}{3} g^2 T^2 (N_c + \tfrac{1}{2} N_f)$$

Now Consider:

QCD on $M = R^3 \times S^1$

bosons PERIODIC

fermions PERIODIC

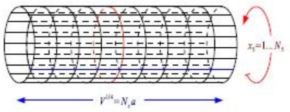
The Hosotani Mechanism v. Finite Temperature QCD?

QCD at $T \neq 0 \quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

fermions ANTI-PERIODIC



Gluon Self-energy

$$\Pi_{\mu\nu}(p=0) \quad \Rightarrow \quad m_g^2 = \frac{1}{3} g^2 T^2 (N_c + \frac{1}{2} N_f)$$

Now Consider:

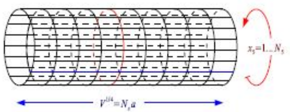
QCD on $M = R^3 \times S^1$

bosons PERIODIC

fermions PERIODIC

$$m_g^2 = \frac{1}{3} g^2 T^2 (N_c - N_f)$$

The Hosotani Mechanism v. Finite Temperature QCD?



QCD at $T \neq 0 \quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

fermions ANTI-PERIODIC

Gluon Self-energy

$$\Pi_{\mu\nu}(p=0) \qquad \Rightarrow \qquad m_g^2 = \tfrac{1}{3} g^2 T^2 (N_c + \tfrac{1}{2} N_f)$$

Now Consider:

QCD on $M = R^3 \times S^1$

bosons PERIODIC

fermions PERIODIC

$$m_g^2 = \tfrac{1}{3} g^2 T^2 (N_c - N_f)$$

What if $N_f > N_c$?

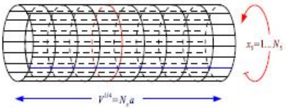
The Hosotani Mechanism v. Finite Temperature QCD?

QCD at $T \neq 0$ $\quad (M = R^3 \times S^1)$

Gross, Pisarski, Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981)

bosons PERIODIC

fermions ANTI-PERIODIC



Gluon Self-energy

$$\Pi_{\mu\nu}(p = 0) \quad \Rightarrow \quad m_g^2 = \frac{1}{3} g^2 T^2 (N_c + \frac{1}{2} N_f)$$

Now Consider:

QCD on $\quad M = R^3 \times S^1$

bosons PERIODIC

fermions PERIODIC

if $N_f > N_c$

$$m_g^2 = \frac{1}{3} g^2 T^2 (N_c - N_f)$$

$$\langle A_y \rangle \neq 0$$

Whether this occurs depends on:

- The **matter content**: number of scalar and fermion fields,
- The **boundary conditions** on the fields
- The **representation** of the fields.

★ Need matter in higher group representations to get dynamical symmetry breaking

In the compact dimension:

$$\langle A_y \rangle = \frac{1}{gL} \begin{pmatrix} \theta_1 & & \\ & \ddots & \\ & & \theta_N \end{pmatrix}, \quad \sum_{i=1}^{N} \theta_i = 0.$$

$$\langle W(x) \rangle \equiv \mathcal{P} \exp \left[ig \int_{x,y}^{x,y+L} \langle A_y \rangle dy \right] = \begin{pmatrix} \exp i\theta_1 & & \\ & \ddots & \\ & & \exp i\theta_N \end{pmatrix},$$

$F_{\mu\nu} = 0$, however: *holonomies,* θ_i*,* are physical.

$\langle \theta_i \rangle$ are determined dynamically at the quantum level, from minimization of $V_{\text{eff}}(\theta_1, \theta_2, \ldots)$.

How to calculate $V_{\text{eff}}(\theta)$?

compact dim. = S^1, $G = SU(2)$, massless matter

$$V_{\text{eff}} = \sum (\pm) \frac{i}{2} \text{tr} \ln D^M D_M, \qquad D^M D_M = \partial^\mu \partial_\mu - D_y^2, \quad (\pm) = \begin{pmatrix} \text{boson} \\ \text{fermion} \end{pmatrix}$$

In y-direction, discrete Kaluza-Klein spectrum:

$$k_n(\theta) \sim \frac{1}{R}\left(n + \frac{\theta}{2\pi}\right)$$

$$V_{\text{eff}}(\theta) = \sum (\pm) \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \sum_n \ln \left\{ p^2 + k_n^2(\theta) \right\}$$

1-loop SU(N) formulae, in $d+1$ dimensions, (massless matter):

$$V_{\rm eff} = V_{\rm eff}^{\rm gauge+ghost} + V_{\rm eff}^{\rm matter}, \qquad \theta_N = \sum_{i=1}^{N-1} -\theta_i,$$

$$V_{\rm eff}^{\rm g+gh} = -(d-2)\frac{\Gamma(d/2)}{\pi^{d/2}L^d}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\frac{\cos[n(\theta_i-\theta_j)]}{n^d},$$

$$V_{\rm eff}^{\psi,{\rm f}} = N_{\rm f}\cdot 2^{[d/2]}\frac{\Gamma(d/2)}{\pi^{d/2}L^d}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{\cos[n(\theta_i-\beta_{\rm f})]}{n^d},$$

$$V_{\rm eff}^{\psi,{\rm ad}} = N_{\rm ad}\cdot 2^{[d/2]}\frac{\Gamma(d/2)}{\pi^{d/2}L^d}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\frac{\cos[n(\theta_i-\theta_j-\beta_{\rm ad})]}{n^d}.$$

$$V_{\rm eff}^{\phi,{\rm R}} = V_{\rm eff}^{\psi,{\rm R}}(2^{[d/2]}N_{\rm R}\to -2N_{\rm R}).$$

$\beta_{\rm f}$ and $\beta_{\rm ad}$ are the (simple) B.C.'s on matter: $\psi(y+2\pi R) = e^{i\beta}\psi(y)$.

For massive fermions, use dimensional regularization

H. Hatanaka and Y. Hosotani, arXiv:1111.3756

$$
\begin{aligned}
V_{\text{eff}} &= \frac{1}{2}\int \frac{d^d p}{(2\pi)^d} \sum_n \ln(p^2 + k_n^2), \quad k_n = \sqrt{(\frac{n+x}{R})^2 + M^2}, \quad x = \frac{\theta}{2\pi} \\
&= \frac{1}{d}\frac{\Gamma(1-d/2)}{(4\pi)^{d/2}} \sum_n k_n^d
\end{aligned}
$$

$$
\begin{aligned}
\sum_n k_n^d &= -\frac{1}{2\pi i}\oint_C dz\, z^d \frac{d}{dz}\ln\rho(z), \quad \frac{\rho'(z)}{\rho(z)} = \frac{1}{z-k_n} + \ldots \\
e.g.\ \rho(z) &= 1 - \frac{\sin^2 \pi x}{\sin^2 \pi R\sqrt{z^2+M^2}}
\end{aligned}
$$

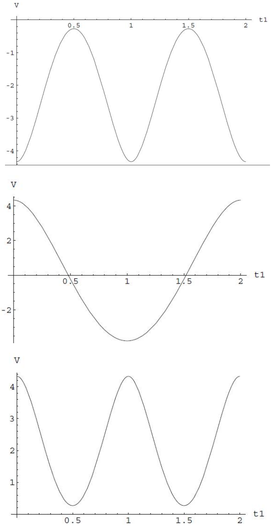
$$
\boxed{V_{\text{eff}} = \frac{1}{(4\pi)^{d/2}\Gamma(d/2)}\int_0^\infty dy y^{d-1}\ln\rho(iy)}
$$

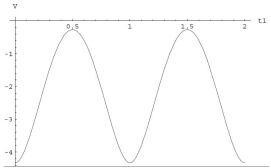
Example: $SU(2)$ with Periodic BC's

$V_{\text{eff}}^{\text{g+gh}}$

$V_{\text{eff}}^{\text{f}}$

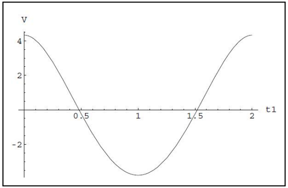
$V_{\text{eff}}^{\text{ad}}$





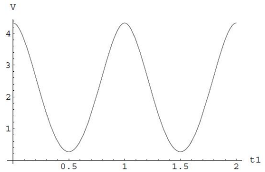
$V_{\mathrm{eff}}^{\mathrm{g+gh}}$

with Fundamental Fermions



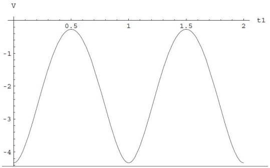
$V_{\mathrm{eff}}^{\mathrm{f}}$

If $N_f > N_c$: $\langle A_y \rangle = \pi \quad \Rightarrow W \in \mathrm{Center}_{\mathrm{SU(2)}}$

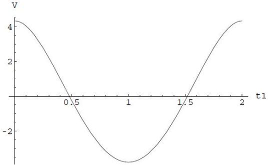


$V_{\mathrm{eff}}^{\mathrm{ad}}$

$V_{\text{eff}}^{\text{g+gh}}$

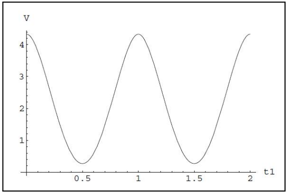


$V_{\text{eff}}^{\text{f}}$



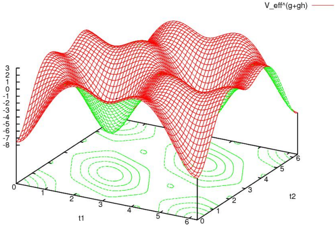
with Adjoint Fermions

$V_{\text{eff}}^{\text{ad}}$



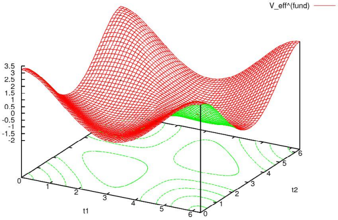
$N_f > N_c$: $\langle A_y \rangle = \pi/2 \Rightarrow W \notin \text{Center}_{SU(2)}$

Gauge Symmetry Dynamically Broken!

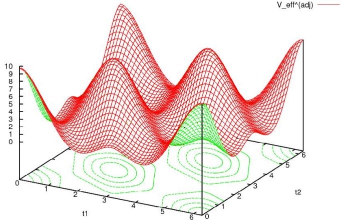


$V_{\mathrm{eff}}^{\mathrm{g+gh}}$

$SU(3)$ with Periodic BC's



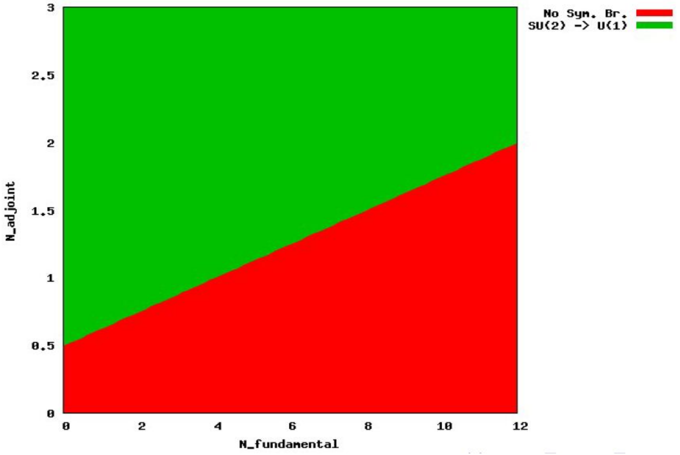
$V_{\mathrm{eff}}^{\mathrm{f}}$



$V_{\mathrm{eff}}^{\mathrm{ad}}$

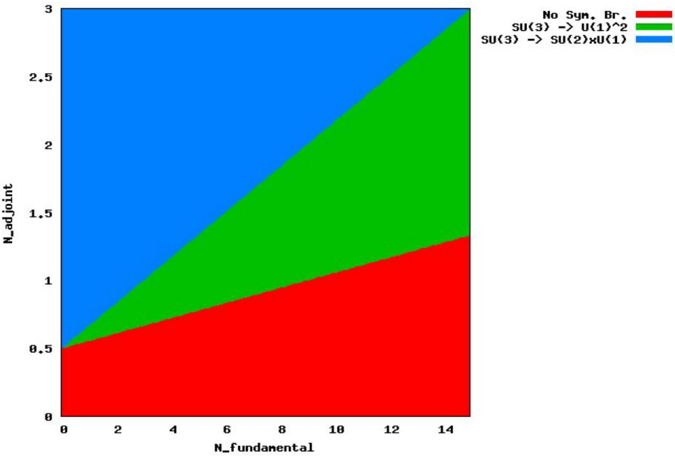
For $\underline{M^3 \times S^1}$ with Periodic BC's:

$$SU(2) \to \begin{cases} SU(2) & \text{for} \quad N_f \le 2 - 4N_{ad}, \\ U(1) & \text{for} \quad 2 - 4N_{ad} < N_f < 8N_{ad} - 4 \end{cases}$$



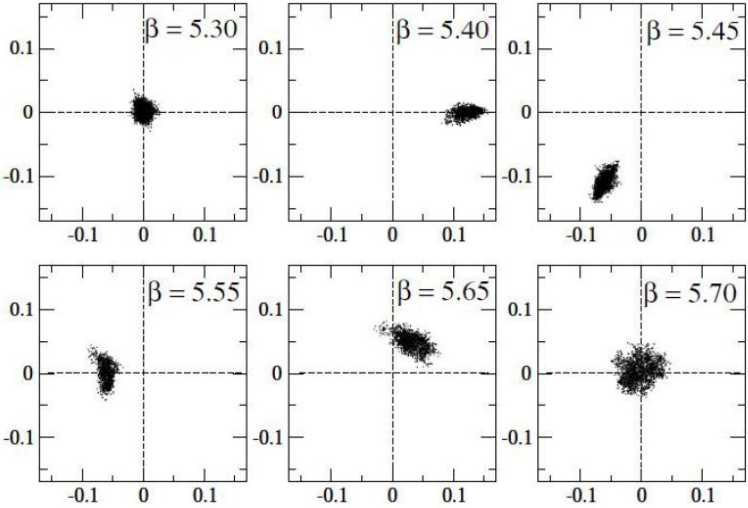
and

$$SU(3) \to \begin{cases} SU(3) & \text{for} & & & N_f & \leq & 3 - 6N_{ad}, \\ U(1)^2 & \text{for} & 3 - 6N_{ad} & < & N_f & < & 6N_{ad} - 3, \\ SU(2) \times U(1) & \text{for} & 6N_{ad} - 3 & \leq & N_f & < & 18N_{ad} - 9, \end{cases}$$



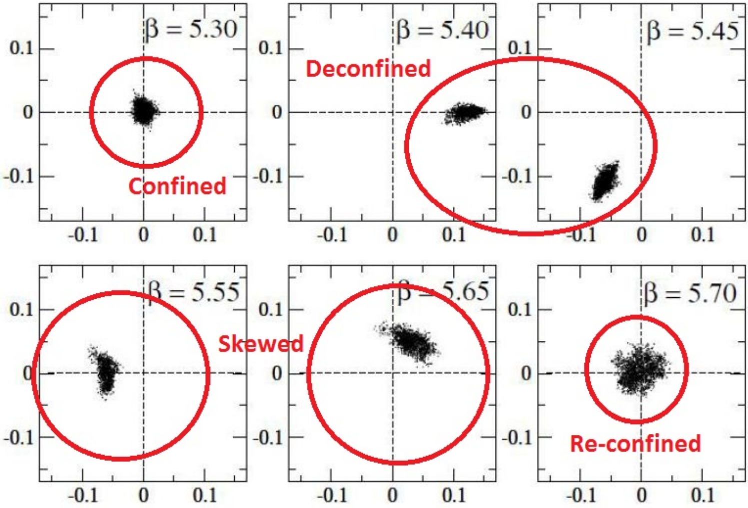
G. Cossu and M. D'Elia (**arXiv:0904.1353**)

$d = 3+1$ ($16^3 \times 4$), staggered $N_{\rm adj} = 2$, Periodic BC's



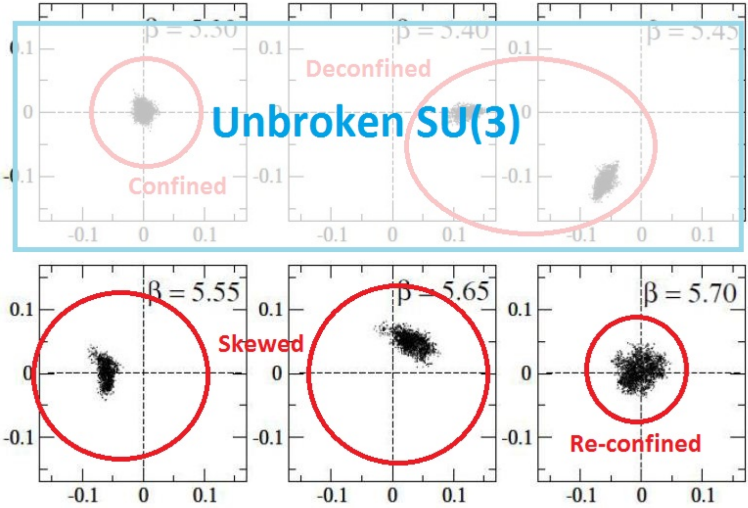
G. Cossu and M. D'Elia (**arXiv:0904.1353**)

$d = 3 + 1$ $(16^3 \times 4)$, staggered $N_{\text{adj}} = 2$, Periodic BC's



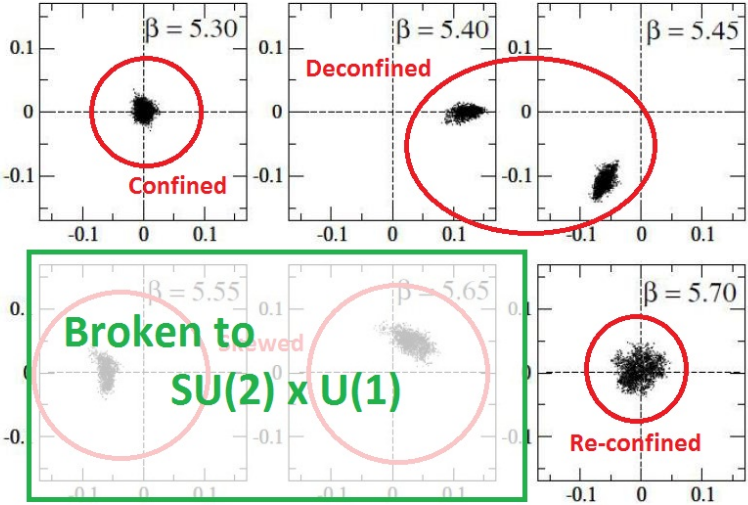
G. Cossu and M. D'Elia (**arXiv:0904.1353**)

$d = 3+1$ ($16^3 \times 4$), staggered $N_{\text{adj}} = 2$, Periodic BC's



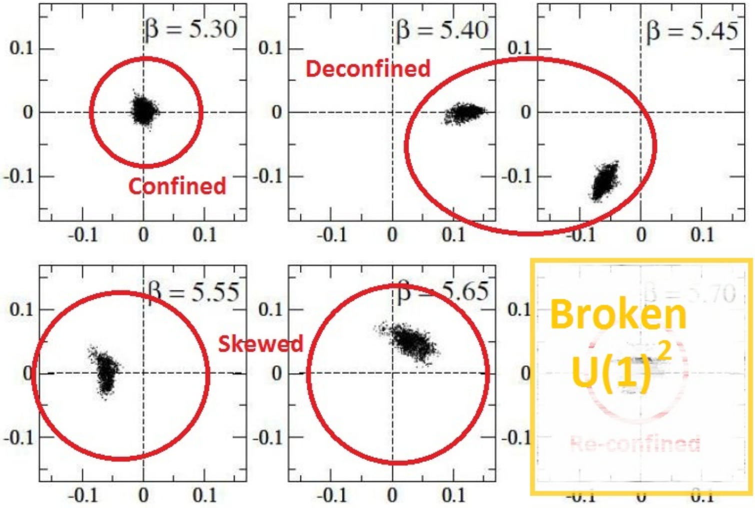
G. Cossu and M. D'Elia (**arXiv:0904.1353**)

$d = 3+1$ ($16^3 \times 4$), staggered $N_{\rm adj} = 2$, Periodic BC's



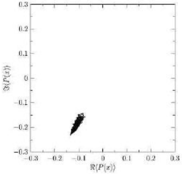
G. Cossu and M. D'Elia (**arXiv:0904.1353**)

$d = 3+1$ ($16^3 \times 4$), staggered $N_{\rm adj} = 2$, Periodic BC's

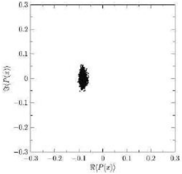


Similarly, J. Myers and M. Ogilvie (**arXiv:0707.1869v2**)

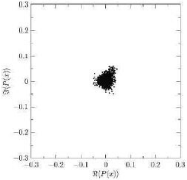
$d = 3 + 1 \ (24^3 \times 4), \ S = S_W + \sum_{\vec{x}} H_A \mathrm{Tr}_A P(\vec{x})$



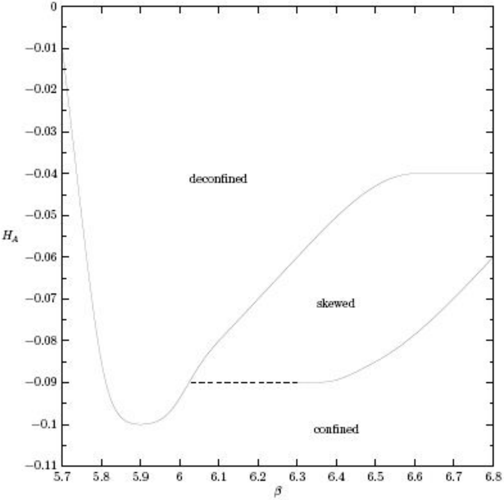
$SU(3)$ Polyakov loop histogram at $\beta = 6.5$, $H_A = -0.05$.



$SU(3)$ Polyakov loop histogram at $\beta = 6.5$, $H_A = -0.06$.

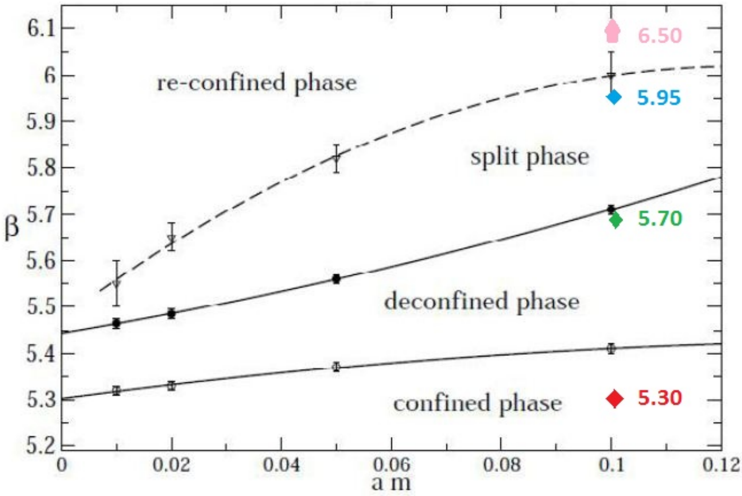


$SU(3)$ Polyakov loop histogram at $\beta = 6.5$, $H_A = -0.1$.



m-β phase diagram

from: G. Cossu and M. D'Elia (**arXiv:0904.1353**)



Lattices (♢,♢,♢,♢) marked at $am = 0.1$ provided by E. Itou.

But, *what* can we tell about **local** gauge symmetry (breaking)?

Look at correlators:

In this talk:

- Gluon Propagator

- Polyakov Loop Correlator

In future:

- Meson spectrum
- Static quark potential
- Thermodynamics

Gluon Propagator

Method:

- Lattice Landau Gauge:

  For given $U_\mu(x)$, maximize $I(U_\mu; G)$, with respect to $G(x)$

$$I(U_\mu; G) = \mathrm{Re}\, e\mathrm{Tr} \sum_x G^\dagger(x) U_\mu(x) G(x+\mu)$$

$$\frac{\delta I(U_\mu; G)}{\delta G} = 0 \quad \Rightarrow \quad \sum_\mu \left[U_\mu^G(x) - U_\mu^G(x-\mu) \right] = 0$$

- $A_\mu(x) \equiv \frac{1}{2ia} \left[U_\mu(x) - U_\mu^\dagger(x) \right]_{\mathrm{Traceless}}$

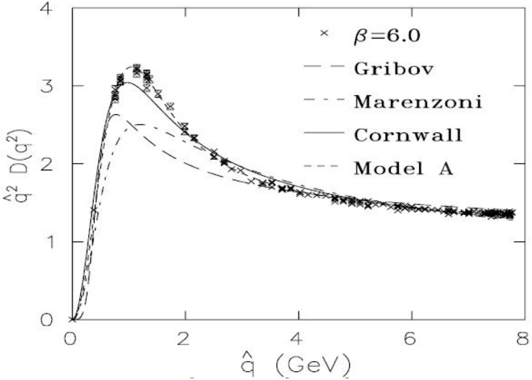
$$U_\mu(x) \sim e^{iaA_\mu(x)}$$

- Fourier transform $A_\mu(x)$
- Compute:

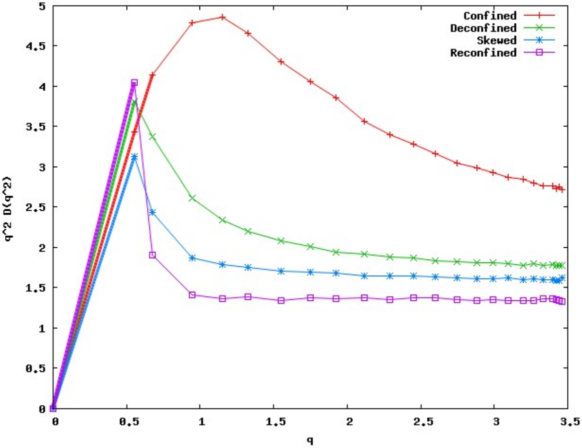
$$< \hat{A}_\mu^a(q) \hat{A}_\nu^b(q') > \equiv \mathcal{D}_{\mu\nu}^{ab}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \delta^{ab} D(q^2)$$

For comparison:
Typical gluon propagator, $q^2 D(q^2)$, in the confined phase:

C. Alexandrou, Ph. de Forcrand, and E. Follana: hep-lat/0008012

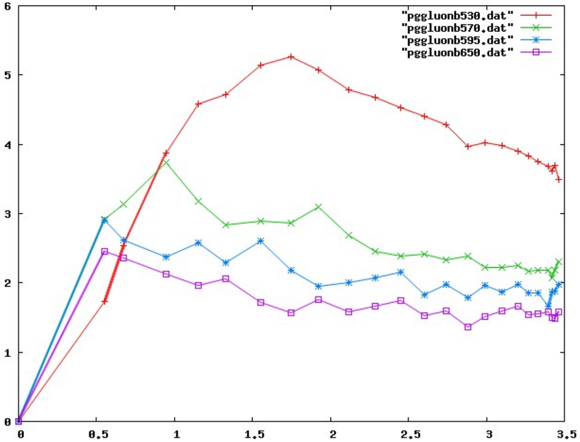


Gluon propagator, $q^2 D(q^2)$, on Adjoint fermion lattices ($V = 16^3 \times 4$)



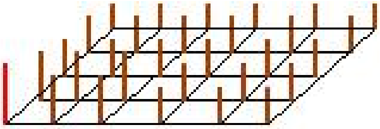
For qualitative comparison, Pure gauge correlators

$V = 16^3 \times 4$, same β values: 5.3, 5.7, 5.95, 6.5 only 100 lattices at each β



Polyakov loop correlators

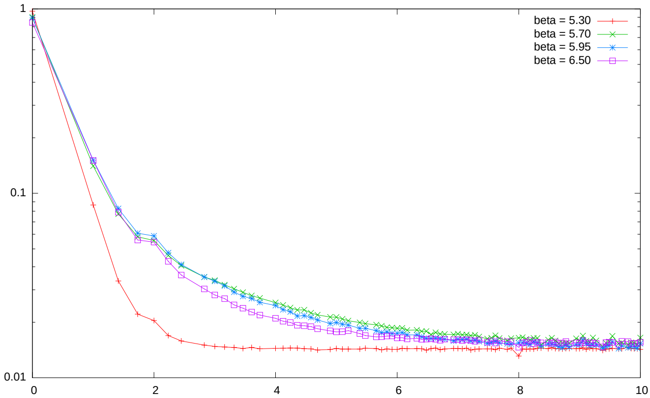
$$c(r) = \mathrm{Re} < \mathrm{Tr}\, P(x)\, \mathrm{Tr}\, P^{\dagger}(x + r) > - < \mathrm{Tr}\, P\, \mathrm{Tr}\, P^{\dagger} >$$



Polyakov loop correlators

"Simple" ($\mathrm{Tr}\, P$) version:

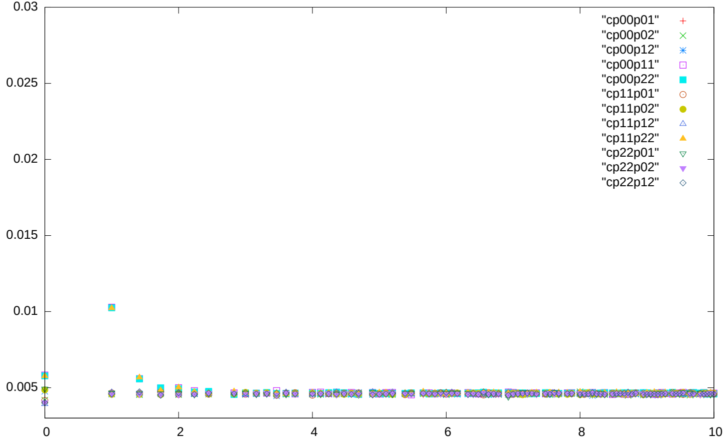
$$c(r) = < \mathrm{Tr}\, P(x)\ \mathrm{Tr}\, P^{\dagger}(x+r) > - < \mathrm{Tr}\, P\ \mathrm{Tr}\, P^{\dagger} >$$



Criticism: $\mathrm{Tr}\,P$ "*assumes*" SU(3) symmetry.

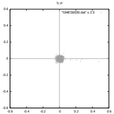
or, at least, dilutes any evidence

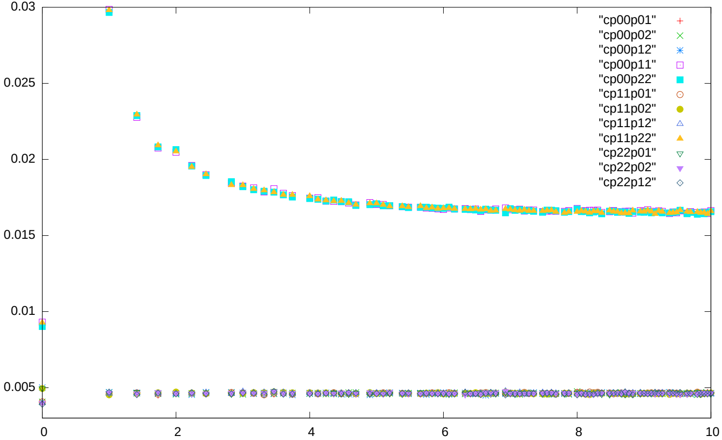
- Compute correlators of elements: $c(r) = < P_{ij}(x) P^{\dagger}_{mn}(x+r) >$



$\beta = 5.30$ *Confined Phase*

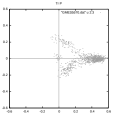
symmetry: SU(3)

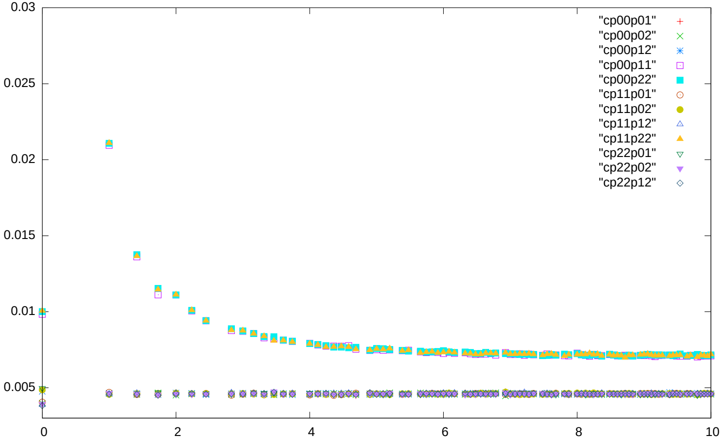




$\beta = 5.70$ *De-Confined Phase*

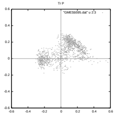
symmetry: SU(3)

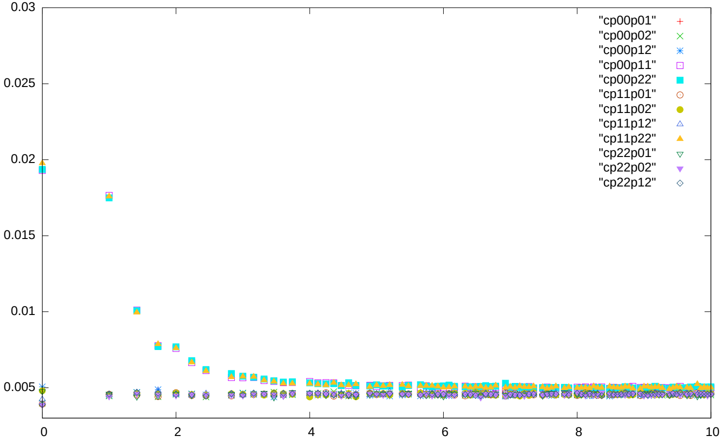




$\beta = 5.95$ *Skewed Phase*

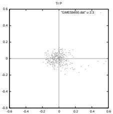
symmetry: SU(2)$\times$ U(1)





$\beta = 6.50$ *Re-Confined Phase*

symmetry: U(1)$\times$ U(1)



Conclusions:

- Starting to simulate the Hosotani Mechanism on the lattice in $(d_{\text{space}} = 3)\times S^1$
- $< P >$ loop averages agree with effective potential

  See: Guido Cossu's talk
- Gluon propagators do not show significant change from de-confined phase to Hosotani-broken symmetry phases.

  $< \hat{A}^a_\mu(q)\hat{A}^b_\nu(q') > \equiv \mathcal{D}^{ab}_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{ab} D(q^2)$

  δ^{ab} assumes SU(3) symmetry. Possibly compute: $D^{ab}(q)$?
- Same for Polyakov loop coorelator:

  including $c(r) =< P_{ij}(x)P^\dagger_{mn}(x+r) >$
- Could $d = 4 + 1$ be different?
- Need to look at Dimensionally Reduced Action for this theory.

  $S_{\text{eff}} = \frac{1}{T}\int dx^d \left\{ F^a_{ij}F^a_{ij} + \mathrm{Tr}\,[D_i, A_0]^2 + m^2\mathrm{Tr}\,A_0^2 + \lambda\mathrm{Tr}\,A_0^4 + ...\right\}$