Phase structure and Hosotani mechanism in QCD-like theory with compact dimensions

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Understanding of spontaneous gauge symmetry breaking is important key in the beyond the standard physics.

Hosotani mechanism : Y. Hosotani, Phys. Lett. B 126 (1983) 309.

When the extra dimension is **not simply connected**,
the extra-dimensional gauge field component can have the **nontrivial vacuum expectation value**.

Wilson loop in compacted direction | Eigen value

$$W = P\exp\left\{ig\int_C dy A_y\right\} \longrightarrow \mathrm{diag}[e^{2\pi i q_1}, e^{2\pi i q_2}, \cdots, e^{2\pi i q_N}]$$

$$q_i \neq q_j \quad \Rightarrow \quad m_n^2 = \frac{1}{L^2}(n + q_i - q_j)^2$$

Gauge symmetry breaking is happen

For example, $q_1 = q_2 \neq q_3$: SU(2) × U(1)

Nontrivial *q* breaks the gauge symmetry and then **Higgs can be considered as a fluctuation of A $_y$**.

$$SO(5)\times U(1) \xrightarrow{\text{Orbifold boundary condition}} SO(4)\times U(1) \xrightarrow{\text{Braine dynamics}} SU(2)\times U(1) \xrightarrow{\text{Hosotani mechanism}} U(1)$$

The gauge symmetry breaking have been investigated in the investigation of **Quantum chromodynamic**.

Adjoint fermion

M. Unsal, Phys. Rev. Lett. 100 (2008) 032005, J. C. Myers, M. C. Ogilvie, Phys. Rev. D 77 (2008) 125030.
G. Cossu, M. D'Elia, JHEP 07(2009) 048.

Adjoint fermion with periodic boundary condition leads the spontaneous gauge symmetry breaking.

Fundamental fermion

The fundamental fermion usually can not lead the gauge symmetry breaking.

But, it is possible by using **flavor twisted boundary condition**.

H. Kouno, Y. Sakai, T. Makiyama, K. Tokunaga, T. Sasaki, and M. Yahiro, J. Phys. G: Nucl. Part. Phys. **39** (2012) 085010.

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

In both case, the boundary condition of fermion plays a important role.

In this talk, we mainly discuss the gauge symmetry breaking in 4D. This system is good laboratory to understand the Hosotani mechanism.

Some our results in 5D are shown in K.K. and T. Misumi, JHEP 05 (2013) 042.

(+Chiral symmetry breaking and restoration)

Recently, effects of the boundary condition of fermion are energetically investigated in different context.

The **imaginary chemical potential** appeared in QCD is important. A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734.

Matsubara frequency with the imaginary chemical potential

Imaginary chemical potential

Fermion : $\omega_n^f = 2\pi T\,(n + 1/2) + \mu_I$

It comes from the anti-periodic boundary condition.

Imaginary chemical potential can be transformed to the boundary angle

$$\omega_n^f = 2\pi T\,(n + \phi) \longrightarrow \omega_n^{\ f} = 2\pi T\,(n + 1/2) - \pi T + 2\pi T\phi$$

Boundary angle

It can be considered as the imaginary chemical potential

Therefore, we may use knowledge obtained in investigation of QCD phase diagram to spontaneous gauge symmetry breaking phenomena.

This study is also related with the investigation of QCD structure itself.

One-loop effective potential

Perturbative one-loop potential

D. Gross, R. Pisarski, L. Yaffe, Rev. Mod. Phys 53 (1981) 43.

Here we consider 4D.
Euclidean temporal direction is treated as compact dimension.
Arbitral dimensional representation can be obtained by same way.

Gauge boson

$$\mathcal{V}_g(q) = -\frac{2}{L^4\pi^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{\cos[2n\pi q_{ij}]}{n^4}$$

Boundary angle: Fermion : 1/2; Boson : 0, π

Fundamental fermion

$$\mathcal{V}_f^{\phi}(q;N_f,m_f) = \frac{2N_f m_f^2}{\pi^2 L^2}\sum_{i=1}^{N}\sum_{n=1}^{\infty}\frac{K_2(nm_f L)}{n^2}\cos[2\pi n(q_i+\phi)]$$

Adjoint fermion

$$\mathcal{V}_a^{\phi}(q;N_a,m_a) = \frac{2N_a m_a^2}{\pi^2 L^2}\sum_{i,j=1}^{N}\sum_{n=1}^{\infty}\left(1-\frac{1}{N}\delta_{ij}\right)\frac{K_2(nm_a L)}{n^2}\cos[2\pi n(q_{ij}+\phi)]$$

Flavor number

Fermion mass

Phase : $\langle A_y\rangle = \frac{2\pi}{gL}q$

We introduce the fermion mass as a scale to control the gauge symmetry breaking.

Contour plot

K.K. and T. Misumi, JHEP 05 (2013) 042

Gauge symmetry breaking

$$q_1 + q_2 + q_3 = 0 \pmod 1$$

Gauge boson + adjoint fermion
with periodic boundary condition

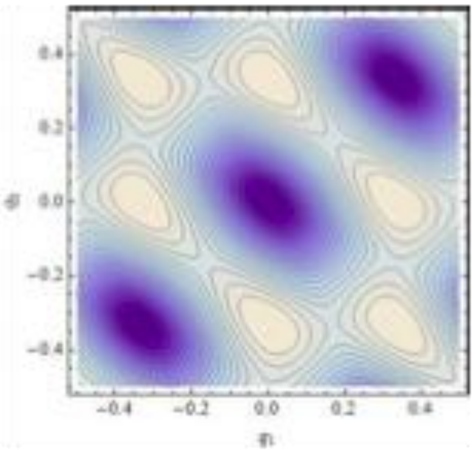
SU(3) → Deconfined phase

SU(2) × U(1) → Split phase

U(1) × U(1) → Re-confined phase

$\boxed{q_1 = q_2 = q_3}$

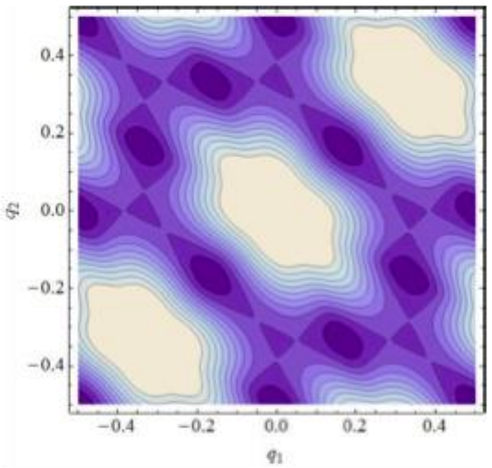
Large m



SU(3)

$\boxed{q_1 = q_2 \neq q_3}$

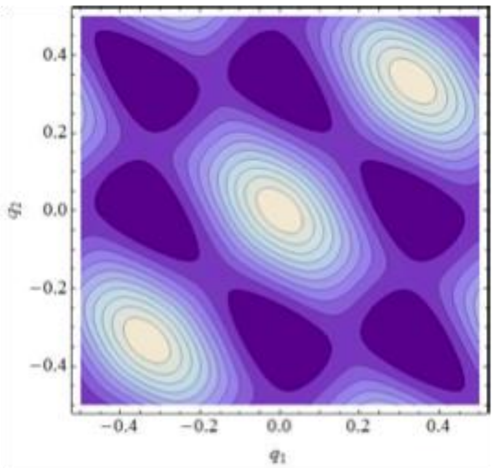
Medium m



SU(2) × U(1)

$\boxed{q_1 \neq q_2 \neq q_3}$

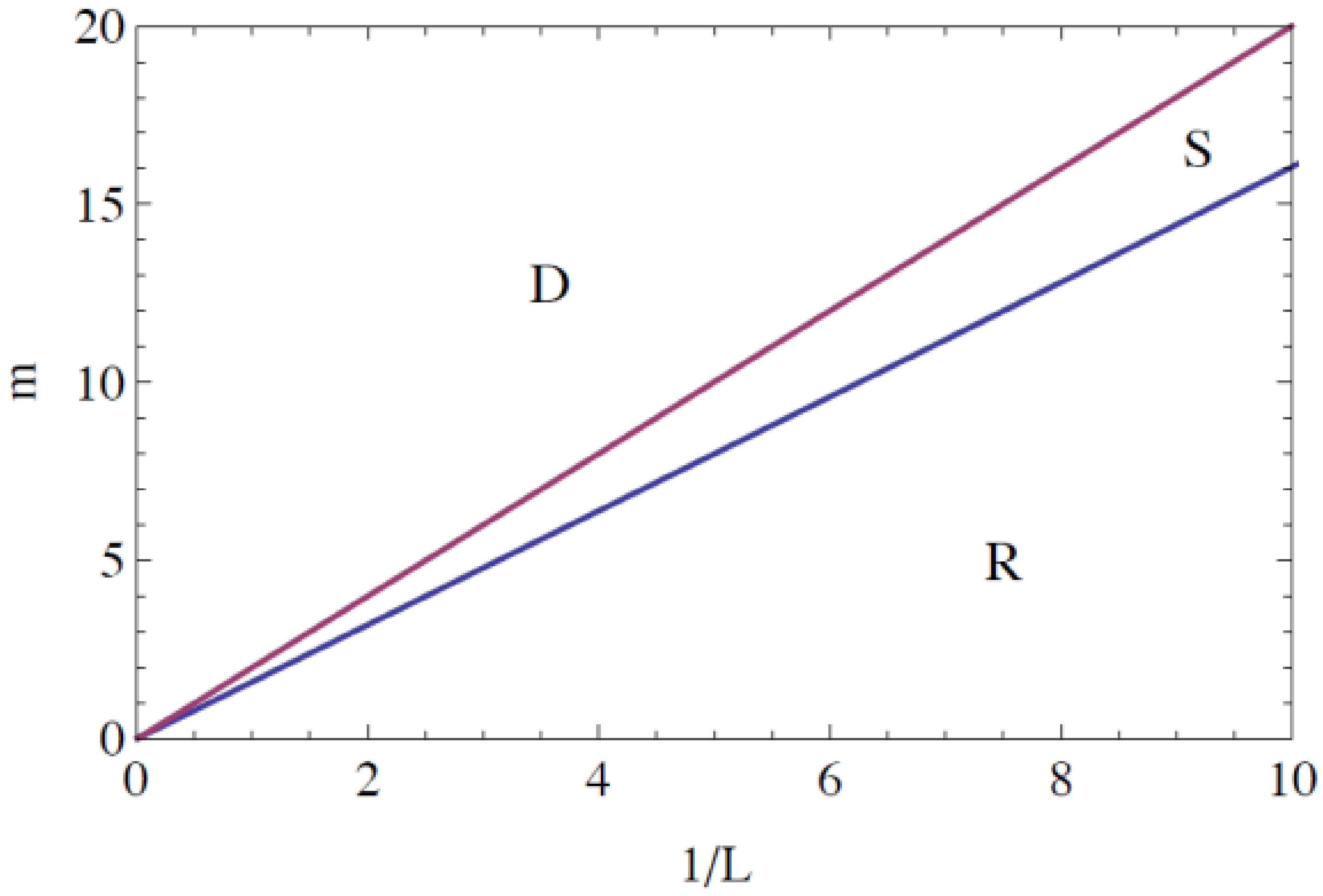
Small m



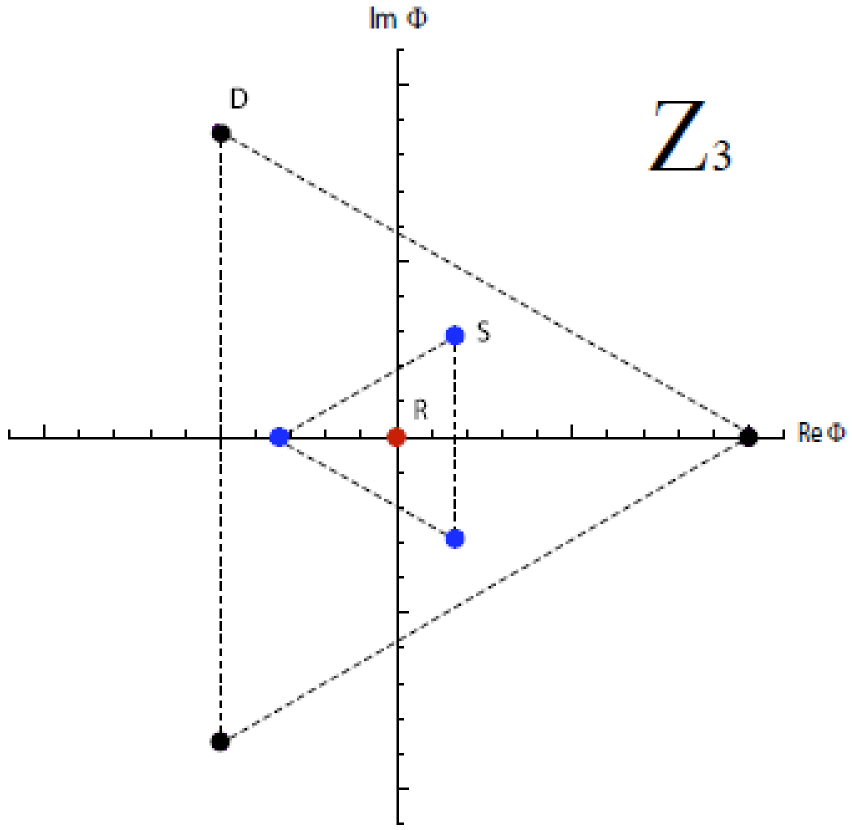
U(1) × U(1)

Phase Structure

Gauge boson + adjoint fermion
with periodic boundary condition



Distribution plot of Polyakov-loop

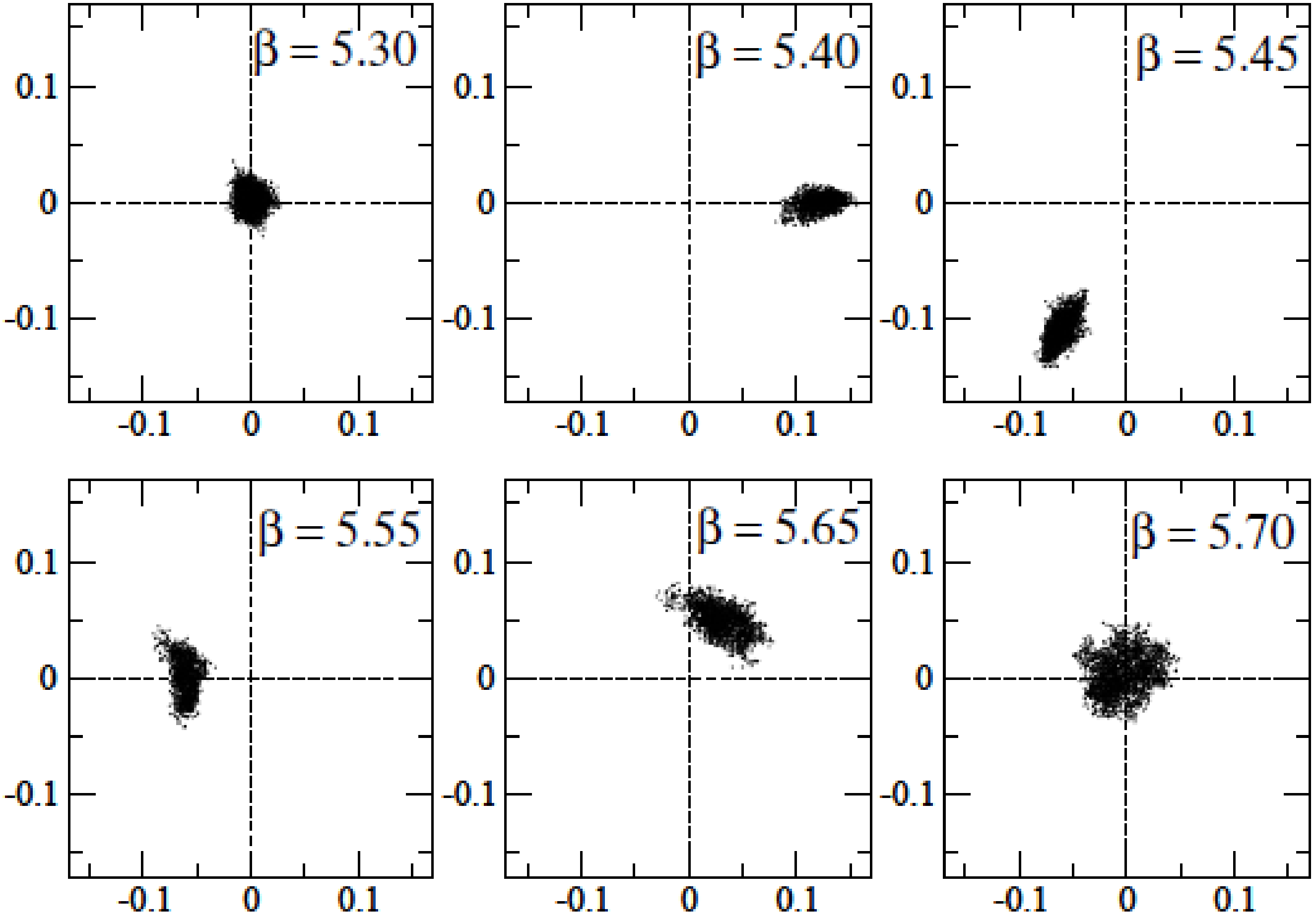


Similar phase diagram was obtained by using Adjoint fermion + gluon (model) calculation in H. Nishimura and M. Ogilvie, Phys. Rev. D 81 (2010) 014018.

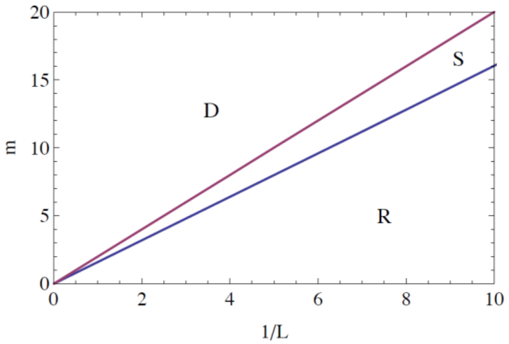
Hosotani mechanism

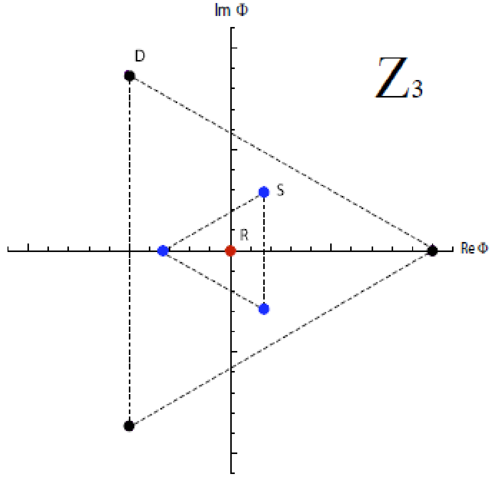
Phase Structure

Lattice data : G. Cossu, M. D'Elia, JHEP 07(2009) 048.
(4D, two flavor, three color, staggered adjoint fermion)



Phase diagram





Phase Structure

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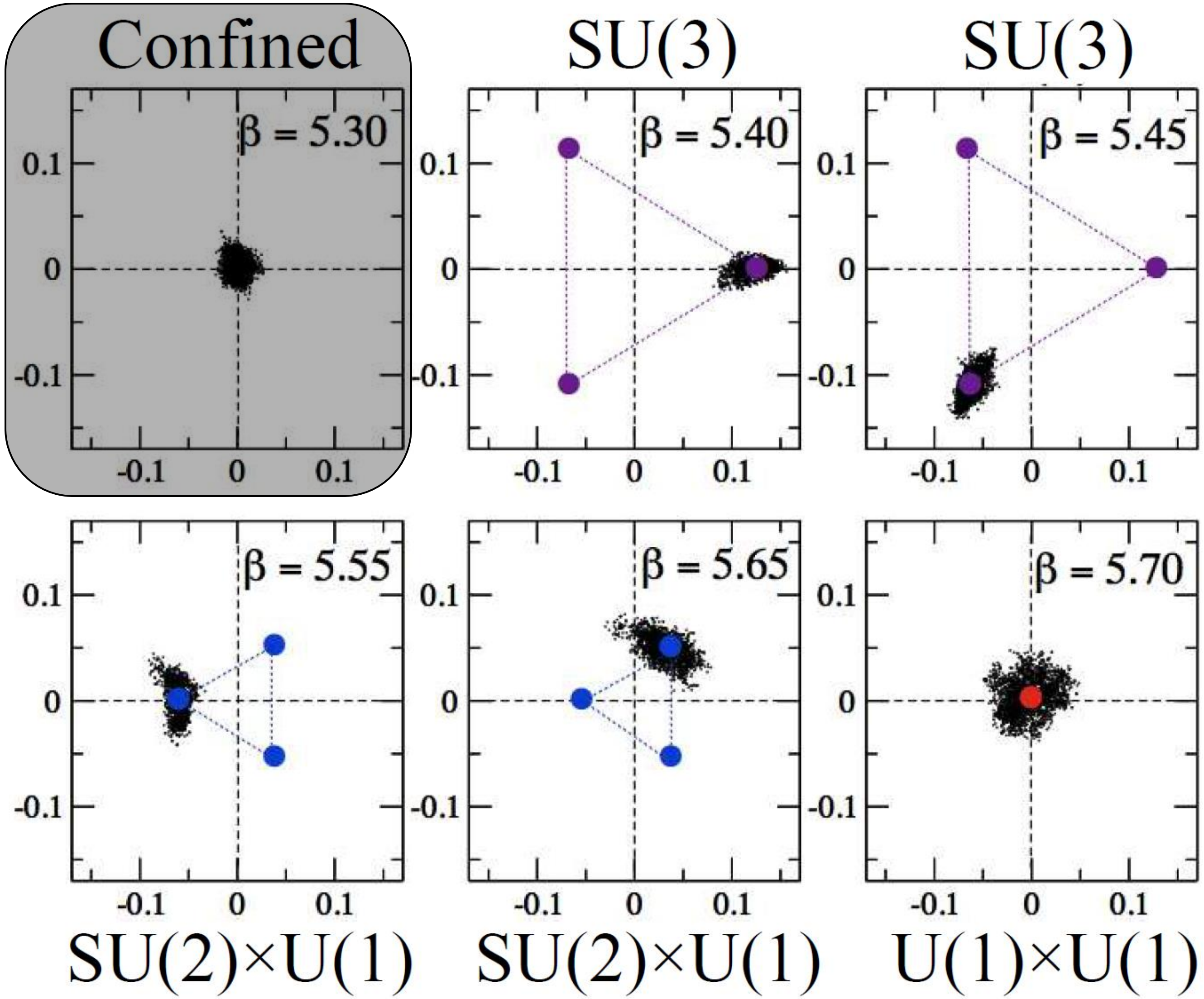
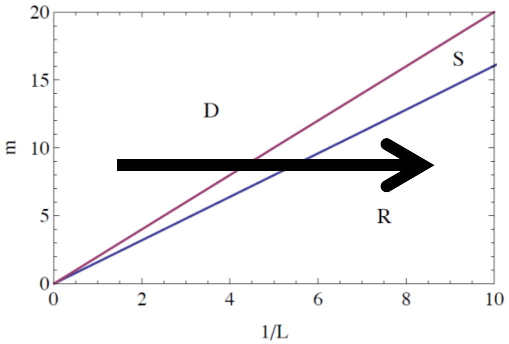
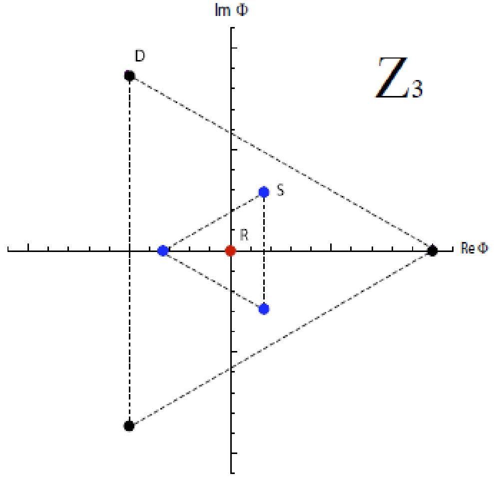


Phase diagram





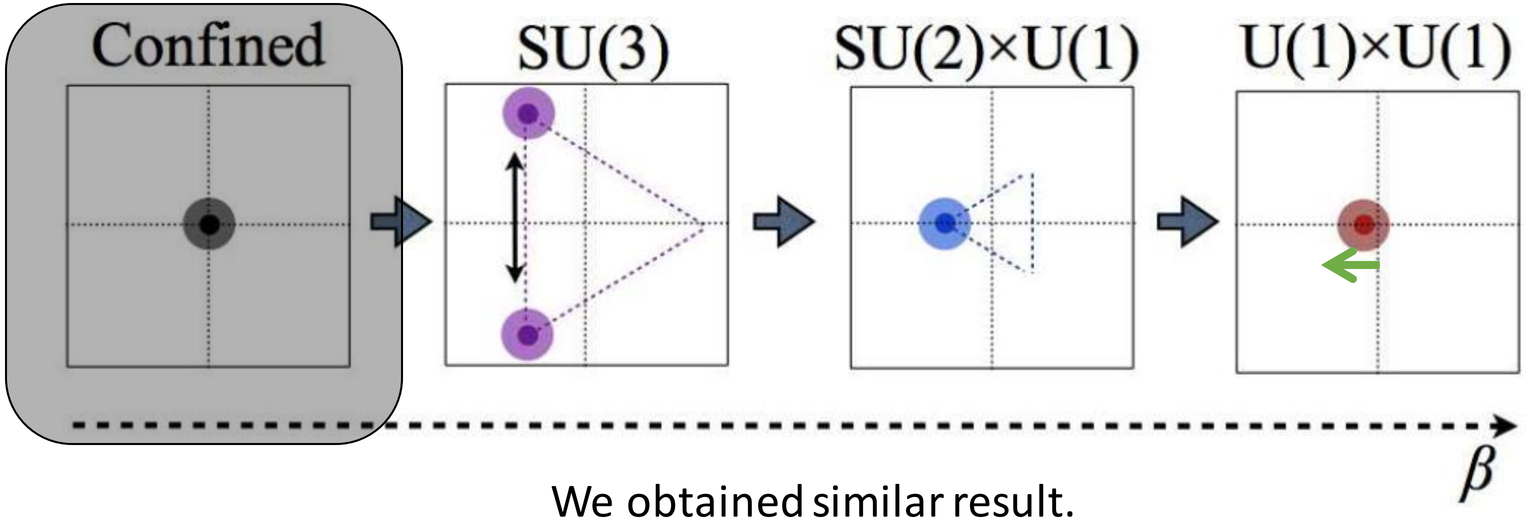
These phases can be understood from Hosotani mechanism!

Effect of fundamental fermion

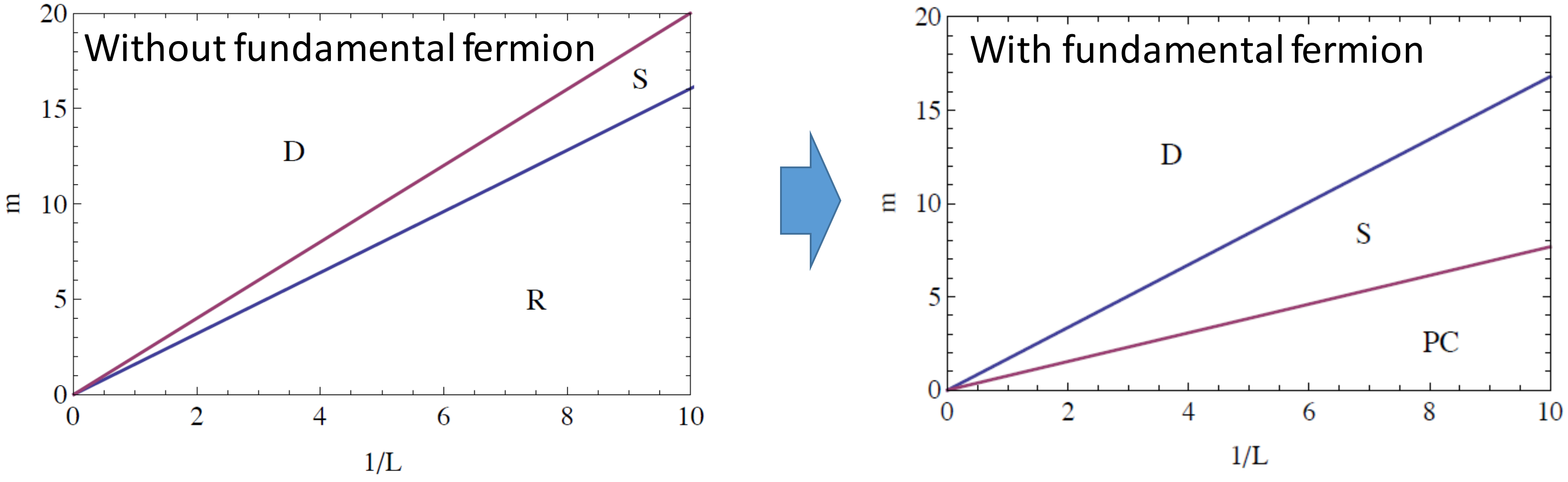
K.K. and T. Misumi, JHEP 05 (2013) 042

+ Fundamental fermion

Fundamental fermion breaks the center symmetry explicitly.



We obtained similar result.



Fundamental fermion with flavor dependent boundary condition

We can get the gauge symmetry breaking by using fundamental quark if we consider the **flavor twisted boundary condition**.

H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro, Phys. Rev .D 88 (2013) 016002.

Flavor twisted boundary condition

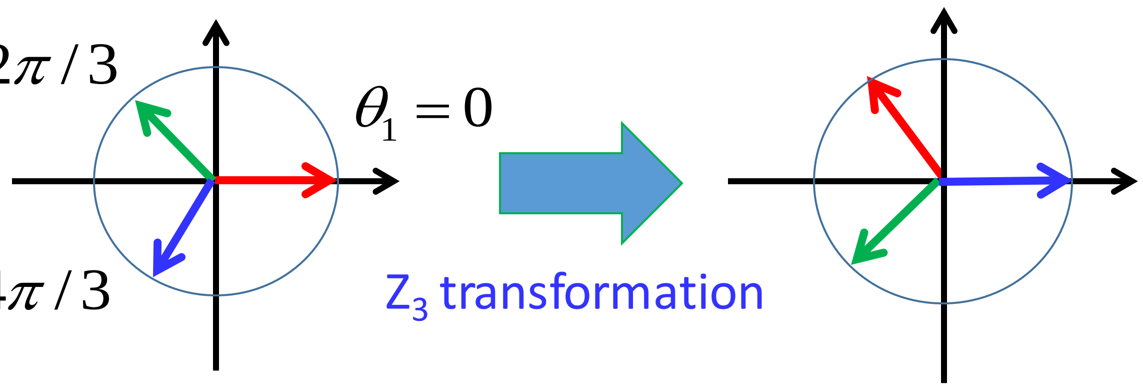
Number of flavor and color should be same.

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f] q_f(x, 0)$$

$\theta_2 = 2\pi/3$

$\theta_1 = 0$

$\theta_3 = 4\pi/3$



Z_3 transformation

$$\boxed{(q_1, q_2, q_3)_{x,y+L} = (q_1, e^{2\pi i/3} q_2, e^{4\pi i/3} q_3)_{x,y}}$$

cf.) Flavored chemical potential

$\downarrow$ Z_3 transformation

$$(e^{2\pi i/3} q_1, e^{4\pi i/3} q_2, q_3)_{x,y}$$

Relabeling

$\boxed{Z_3 \text{ center is preserved by use of } Z_3 \text{ of flavor SU(3).}}$

FTB fundamental fermion

$$q_{if} = q_i + (f-1)/3$$

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2} \underbrace{\sum_{i}^{3} \sum_{f}^{3}}_{} \sum_{n=1}^{\infty} \frac{\cos[2\pi n q_{if}]}{n^4}$$

This part can be treated as the adjoint trace

$\updownarrow$

Adjoint fermion

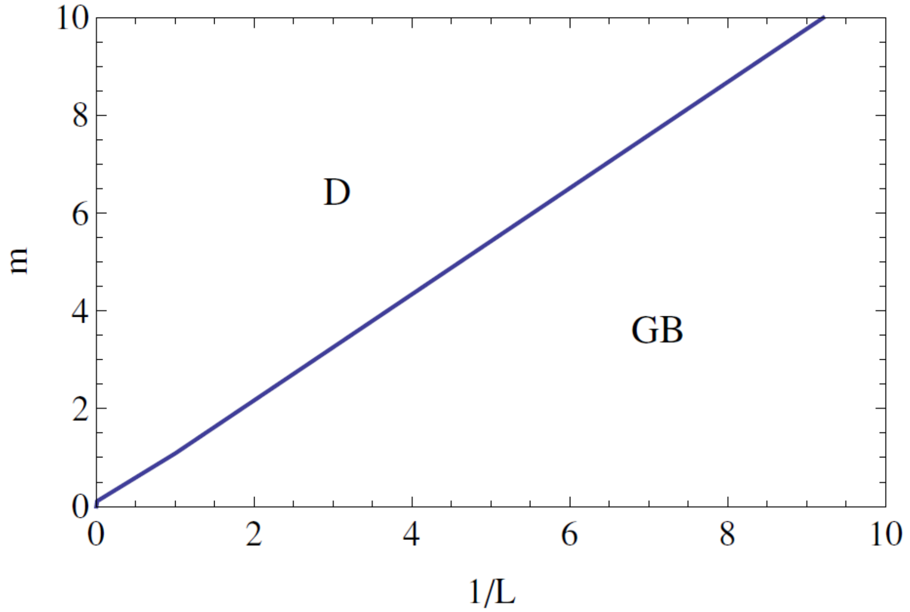
$$\mathcal{V}_a = +\frac{4}{L^4\pi^2} \sum_{i,j=1}^{3} \sum_{n=1}^{\infty} \left(1 - \frac{1}{3}\delta_{ij}\right) \frac{\cos[2\pi n q_{ij}]}{n^4}$$

Phase diagram and scatter plot

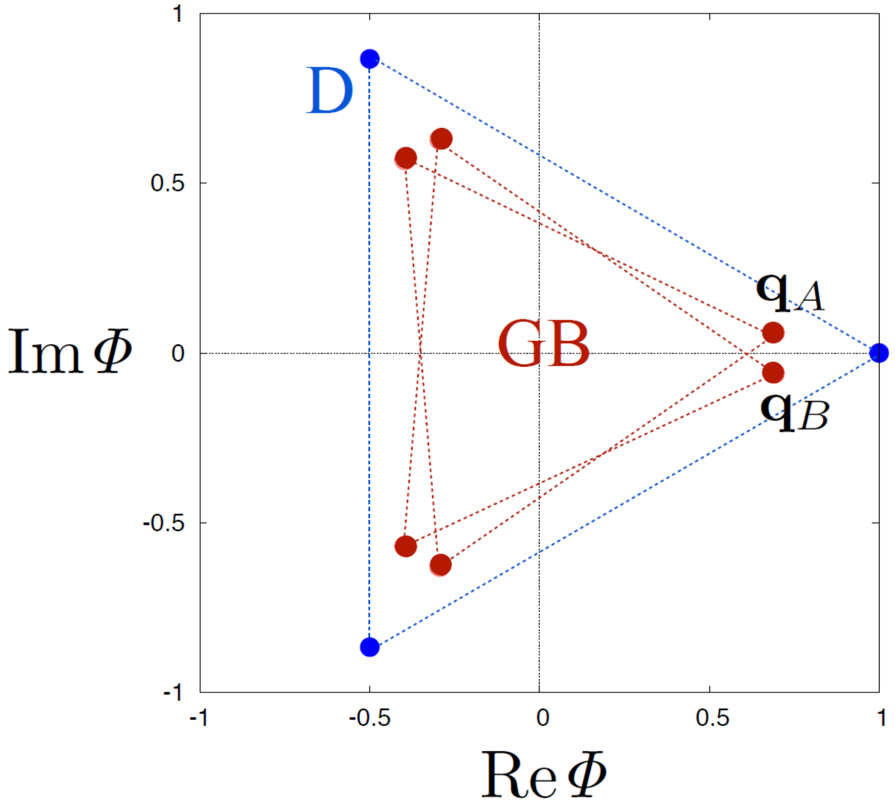
H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro,
Phys. Rev .D 88 (2013) 016002.

We get the gauge-symmetry breaking by using fundamental fermion if we consider the **flavor twisted boundary condition**.

Phase diagram



Polyakov-loop



These results are obtained in 4+1dimensional system.

There is the gauge symmetry breaking (SU(2) × U(1)) phase.

Z_3 symmetry is not explicitly broken.

Charge conjugation pairs are appeared in the gauge symmetry broken phase.

Summary

The spontaneous gauge symmetry breaking is discussed by using the adjoint and fundamental fermions.

Effects of the boundary condition are investigated. (periodic, anti-periodic and flavor twisted boundary conditions)

The boundary angle can be treated as the imaginary chemical potential,
and thus we can use some knowledge obtained in the investigation of QCD phase diagram.

By using the perturbative one-loop effective, we can explain the lattice QCD data from Hosotani mechanism.

By using the flavor twisted boundary condition, we can construct the center symmetric effective potential,
and it shows the spontaneous gauge symmetry breaking.

In this theory, there is clear center symmetry breaking and thus,
we can investigate the correlation between the chiral and deconfinement transition of QCD at finite T.

The lattice simulation of this case is interesting!

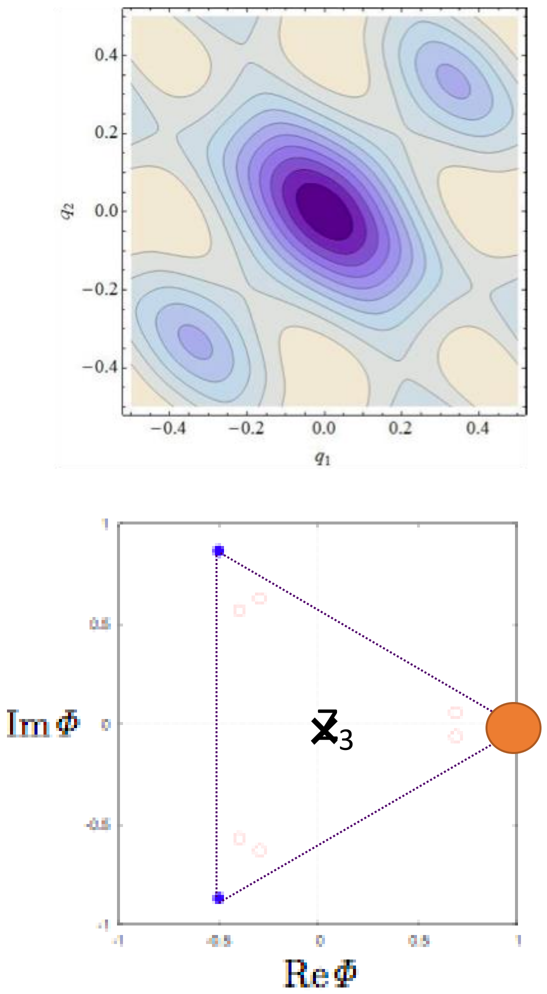
Backup

Contour plot

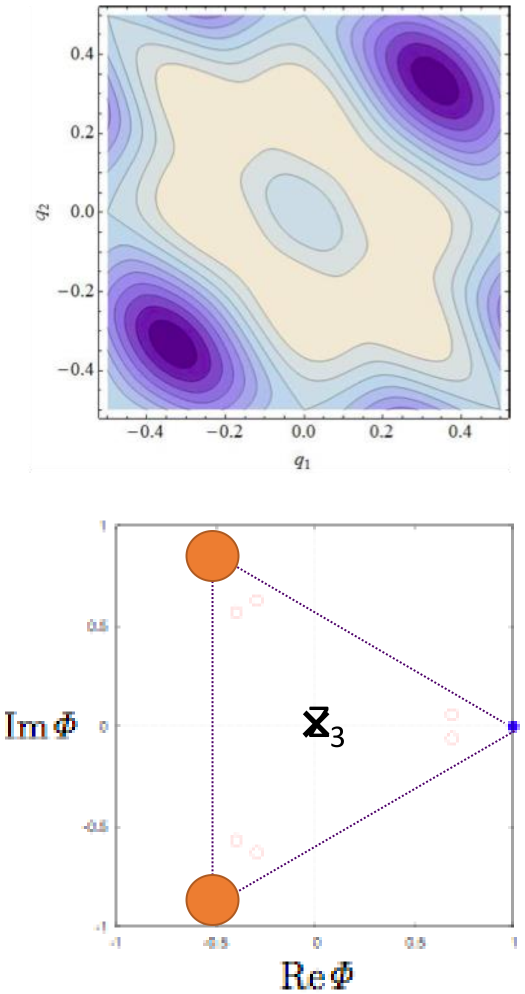
K.K. and T. Misumi, JHEP 05 (2013) 042

Contour plot $q_1 + q_2 + q_3 = 0 \pmod 1$ In this cases, there are no gauge symmetry breaking.

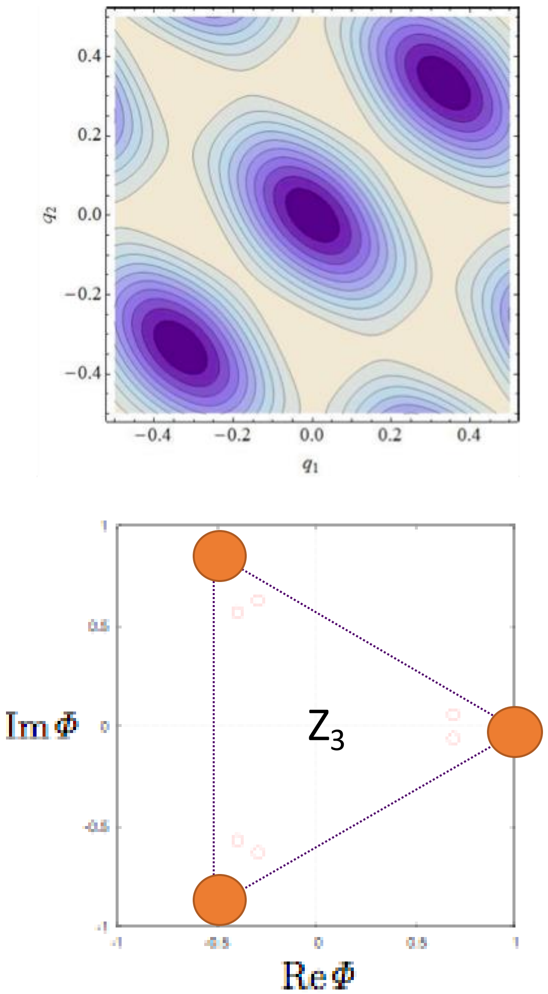
(1) aPBC fund.



(2) PBC fund.



(3) aPBC adjoint

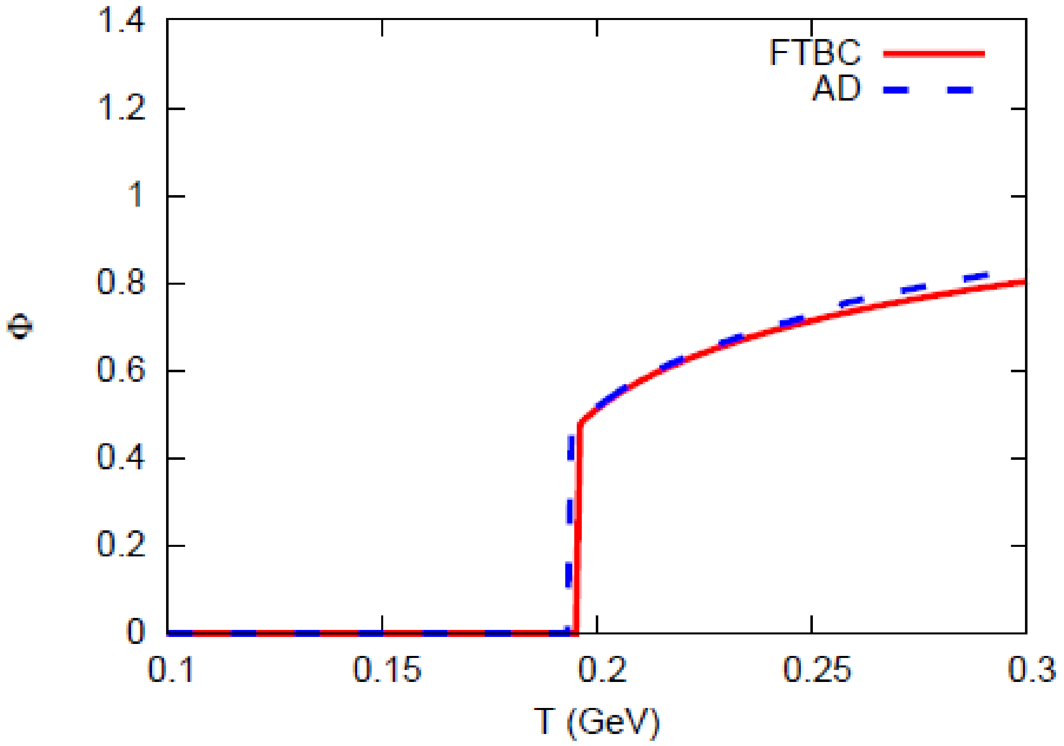


Fundamental fermion with flavor dependent boundary condition

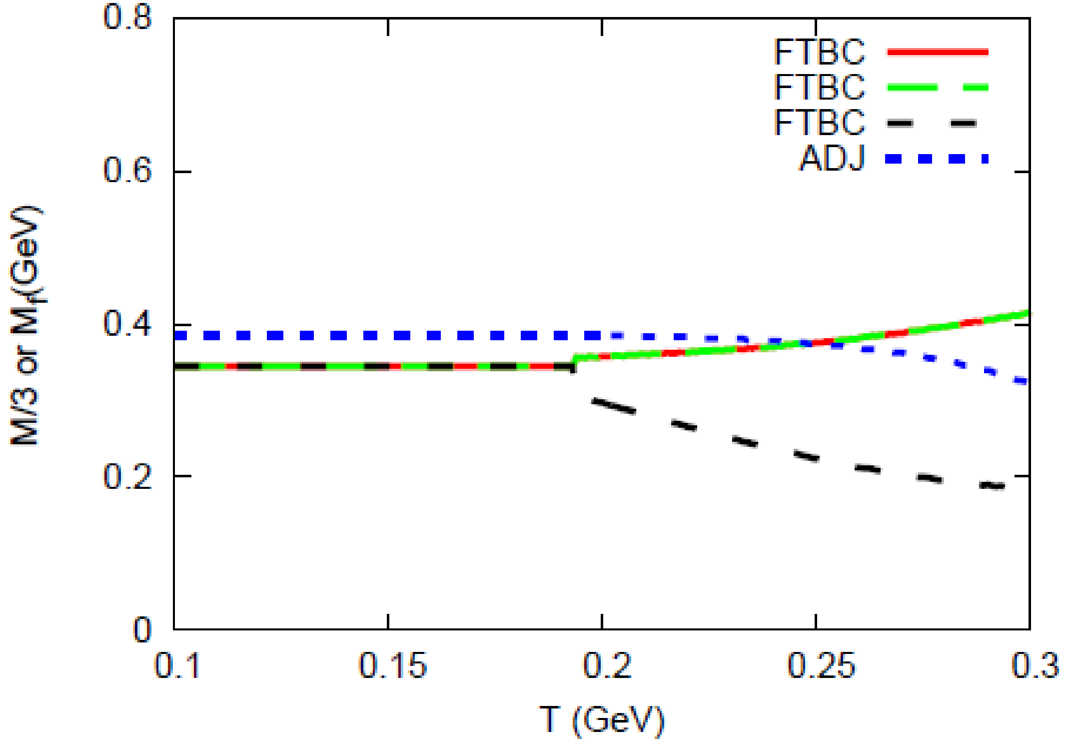
H. Kouno, T. Misumi, **K.K.**, T. Makiyama, T. Sasaki, M. Yahiro,
Phys. Rev .D 88 (2013) 016002.

4D, PNJL model

Polyakov-loop



Constituent quark mass



The adjoint quark and fundamental quark with flavor twisted boundary condition shows similar behavior of the Polyakov-loop.

The constituent quark mass behave differently because the flavor symmetry breaking is happen in the fundamental fermion with FTBC.

$$\mathcal{V}_f^{FT} = +\frac{4}{L^4\pi^2}\sum_i^3\sum_f^3\sum_{n=1}^{\infty}\frac{\cos[2\pi n q_{if}]}{n^4} \longleftarrow$$

Flavor and color space is strongly entangled in the case!

K.K. and T. Misumi, JHEP 05 (2013) 042

To describe the chiral symmetry breaking and restoration,
we use the Polyakov-loop extended Nambu—Jona-Lasinio type model.

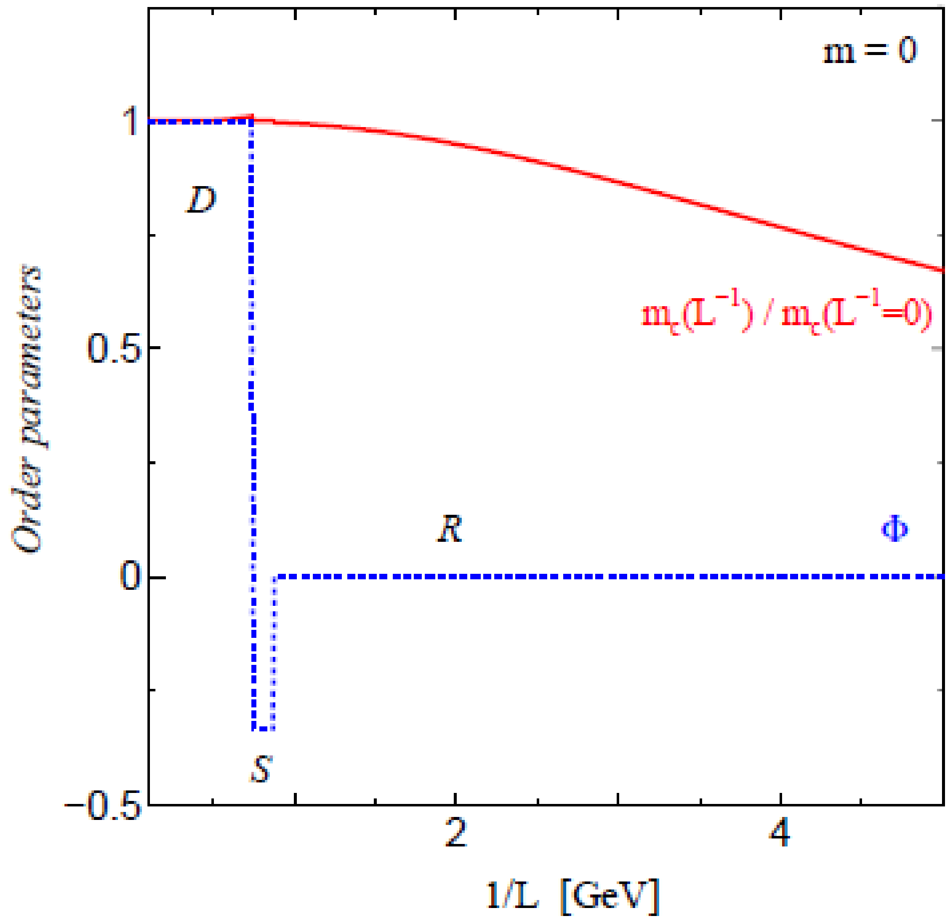
K. Fukushima, Phys. Lett. B591 (2004) 277.

H. Nishimura and M. Ogilvie, Phys. Rev.D81 (2010) 014018.

4Fermi interaction:

$$(g_S)_f[(\bar{\psi}_f\psi_f)^2+(\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f)^2]+(g_S)_a[(\bar{\psi}_a\psi_a)^2+(\bar{\psi}_a i\gamma_5\vec{\tau}\psi_a)^2]$$
$$+(g_S)_{fa}[\{(\bar{\psi}_f\psi_f)^2+(\bar{\psi}_f i\gamma_5\vec{\tau}\psi_f)^2\}^2\{(\bar{\psi}_a\psi_a)^2+(\bar{\psi}_a i\gamma_5\vec{\tau}\psi_a)^2\}^2]$$

With adjoint fermion



With adjoint and fundamental fermion