Mass anomalous dimension from large N twisted volume reduction

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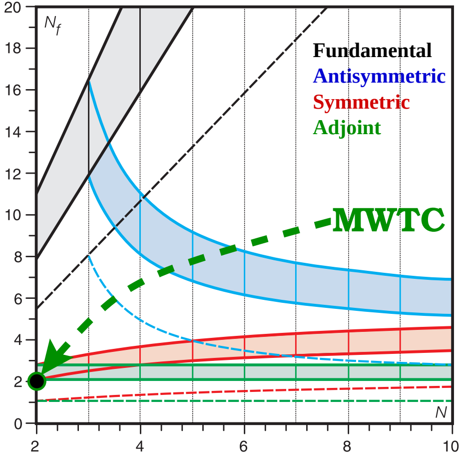
Goal

Measure the **mass anomalous dimension** of the SU(N) gauge theory with two adjoint Dirac fermions, using the **Dirac operator mode number** method, on a single site lattice using **large N twisted volume reduction**.

Why large N?



[arXiv:hep-ph/0611341]

- Expect it to behave similarly to the SU(2) gauge theory, MWT
- 2–loop perturbation theory predicts γ_* is independent of N
- Large N volume independence: can simulate on a single site lattice

Why the Mass Anomalous Dimension?

Size of quark mass terms in the effective action depend on the value of the anomalous mass dimension γ.

Quark Masses

$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}}{\Lambda_{ETC}^2}\overline{\psi}\psi$$

Power Enhancement

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma}\langle\overline{\Psi}\Psi\rangle_{TC}$$

- Need $\gamma \simeq 1$ to generate large enough quark masses.
- Important quantity to measure in TC models.

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of $\mathrm{U}(N_c)$ Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} = S_{EK} \equiv N_c b \sum_{\mu<\nu} Tr\left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c.\right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Conditions

...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Twisted Eguchi–Kawai

Gonzalez–Arroyo Okawa '83

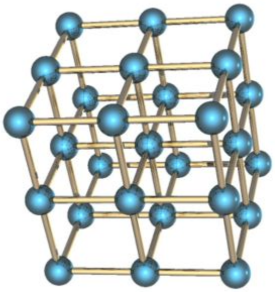
Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

$$S_{TEK} = N_c b \sum_{\mu<\nu} Tr \left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right), \quad z_{\mu\nu} = exp\{2\pi i k/\sqrt{N}\}$$

- Center symmetry is preserved for all N by scaling the twist k with N

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

Lattice Field Theory



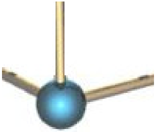
Formulate field theory on a discrete set of space–time points:

- $\hat{L}^4$ points, lattice spacing a
- Physical volume $L^4 = (\hat{L}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/L$

Twisted Reduction



Twisted reduction: $\hat{L} \to \sqrt{N}$

- Single site lattice, lattice spacing a
- Physical volume $L^4 = (\sqrt{N}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/\sqrt{N}$

Mode Number Method

In the infinite volume, chiral limit, and for small eigenvalues,

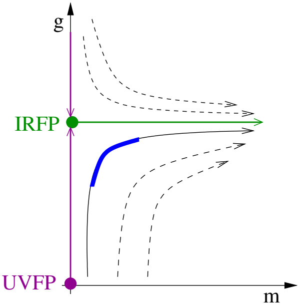
Spectral density of the Dirac Operator

$$\lim_{m\to 0} \lim_{V\to\infty} \rho(\omega) \propto \omega^{\frac{3-\gamma_*}{1+\gamma_*}} + \dots$$

- Integral of this is the mode number, which is just counting the number of eigenvalues of the Dirac Operator on the lattice.
- Fitting this to the above form can give a precise value for γ, as done recently for MWT by Agostino Patella.

DeGrand [arXiv:0906.4543], Del Debbio et. al. [arXiv:1005.2371], Patella [arXiv:1204.4432], Hasenfratz et. al. [arXiv:1303.7129]

Mode Number Fit Range



RG flows in mass–deformed CFT:

- Flow from UV (high eigenvalues) to IR (low eigenvalues)
- Finite mass drives us away from FP in the IR
- Interested in intermediate blue region
- $\frac{1}{\sqrt{N}} \ll m \ll \Omega_{IR} < \Omega < \Omega_{UV} \ll \frac{1}{a}$

Fit Function

Split low and high eigenvalue contributions to the mode number:

$$\nu(\Omega) = \int_0^{\sqrt{\Omega_{IR}^2 - m^2}} \rho(\omega) d\omega + \int_{\sqrt{\Omega_{IR}^2 - m^2}}^{\sqrt{\Omega^2 - m^2}} \rho(\omega) d\omega$$

Inserting $\rho(\omega) \sim \omega^{\frac{3-\gamma_*}{1+\gamma_*}}$ in the second term:

Mode number fit function

$$\nu(\Omega) = \nu(\Omega_{IR}) + A\left[\left(\Omega^2 - m^2\right)^{\frac{2}{1+\gamma_*}} - \left(\Omega_{IR}^2 - m^2\right)^{\frac{2}{1+\gamma_*}}\right]$$

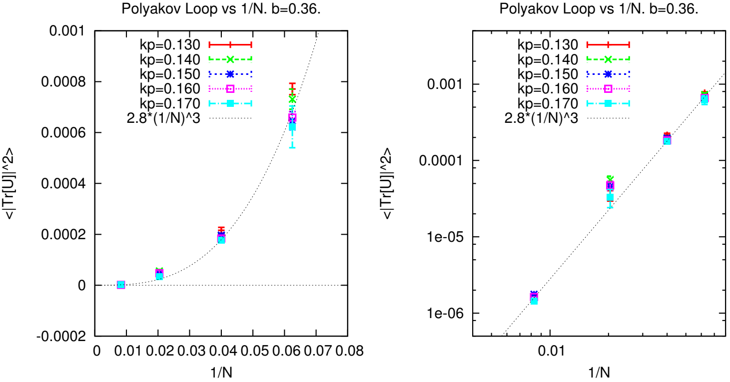
- Fit in range $\Omega_{IR} < \Omega < \Omega_{UV}$.
- 3 fit parameters: A, m and γ_*.

Simulation Details

- Simulate large N version of MWT.
  - SU(N) gauge theory with 2 light adjoint Dirac fermions with periodic boundary conditions.
- Use single site 1^4 lattices with N up to 289.
  - $V_{eff} = N^2$, so equivalent to $L^4 = 17^4$.
- Measure up to 2000 lowest eigenvalues of the Dirac operator.
- Choose bare lattice coupling $b = 1/\lambda = 0.35, 0.36$.
  - Need to stay in weak coupling phase.
  - But want fairly strong coupling to minimise $1/N$ effects.

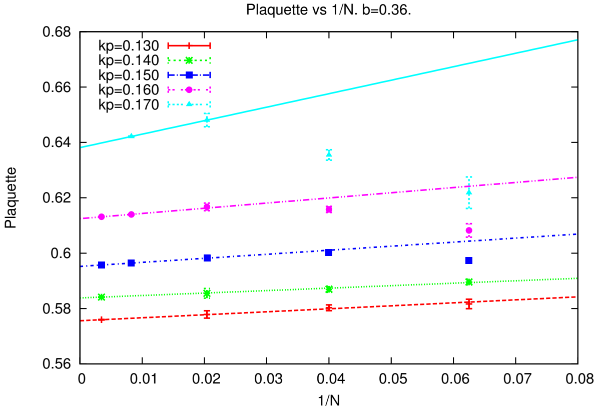
Polyakov Loop

Polyakov loop is zero up to $1/N$ corrections, so reduction holds.



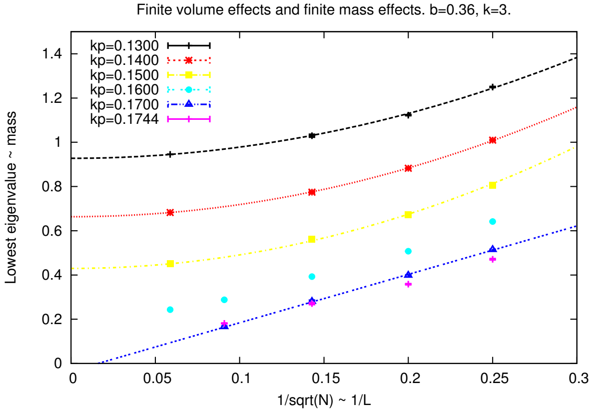
Plaquette vs 1/N

Plaquette: see larger finite–N effects for lighter masses.



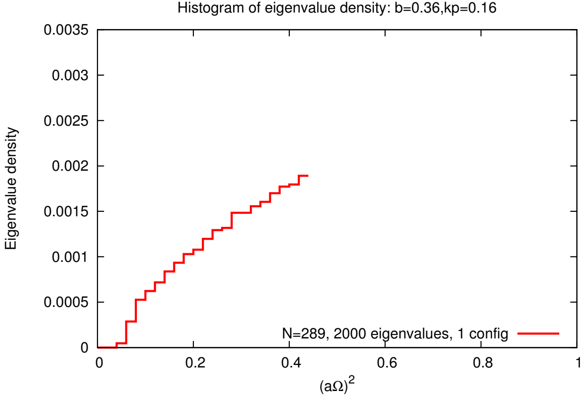
Lowest Dirac Eigenvalue vs 1/N

Lowest eigenvalue has two distinct regimes.



Eigenvalue density histogram

Histogram shows change between the two regimes as the volume is increased.



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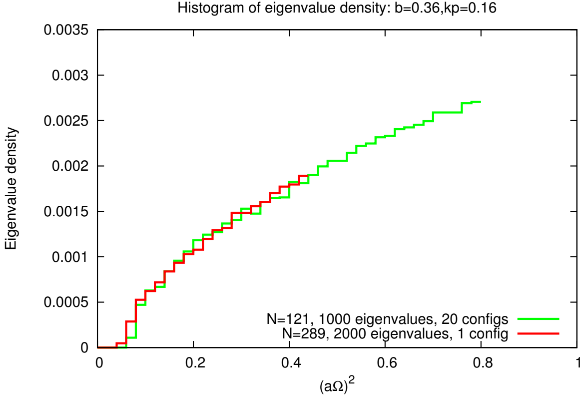

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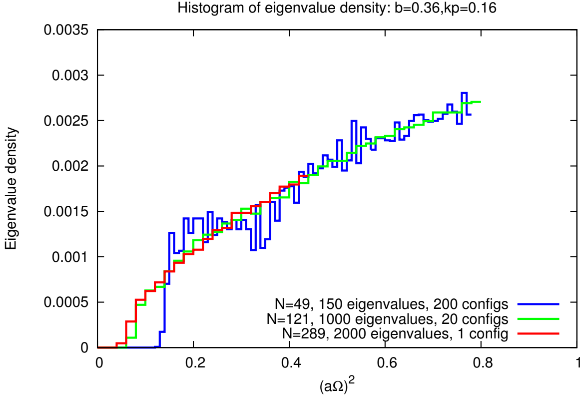

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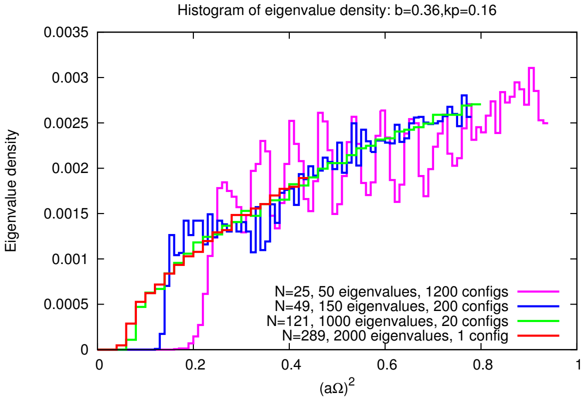


Large volume vs small volume

- Large volume regime (p–regime)
  - $mL \gg 1$
  - $\lambda = m + c/N$
  - Can perform mode number fit
- Small volume regime (ϵ–regime)
  - $mL \ll 1$
  - $\lambda \sim 1/L$
  - Can also perform mode number fit (if affected eigenvalues are excluded from the fit region)
  - Comparison to chiral random matrix theory?

Method

Fit data to the function

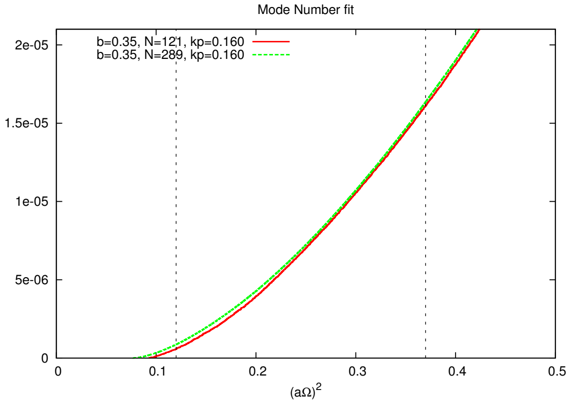
$$\nu(\Omega) = \nu(\Omega_{IR}) + A\left[\left(\Omega^2 - m^2\right)^{\frac{2}{1+\gamma_*}} - \left(\Omega_{IR}^2 - m^2\right)^{\frac{2}{1+\gamma_*}}\right]$$

in some intermediate range $\Omega_{IR} < \Omega < \Omega_{UV}$ where

- $\nu(\Omega)$ is the number of eigenvalues of M below Ω^2 divided by the volume
- m is a fitted parameter (physical mass)
- A is a fitted parameter
- γ_* is the mass anomalous dimension

Mode Number Example Fit $b = 0.35, \kappa = 0.16$

$N = 289$: $A = 1.16 \times 10^{-4}, (am)^2 = 0.068, \gamma = 0.258$
$N = 121$: $A = 1.04 \times 10^{-4}, (am)^2 = 0.108, \gamma = 0.417$



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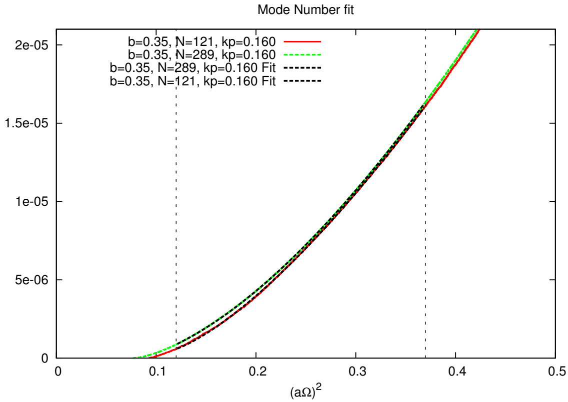
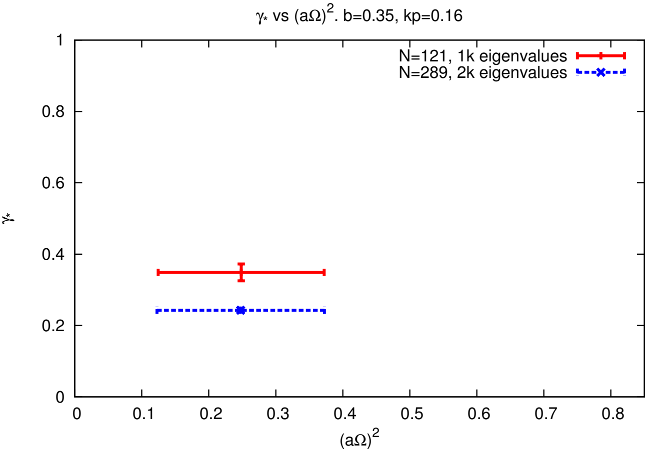


Mass anomalous dimension results [preliminary]



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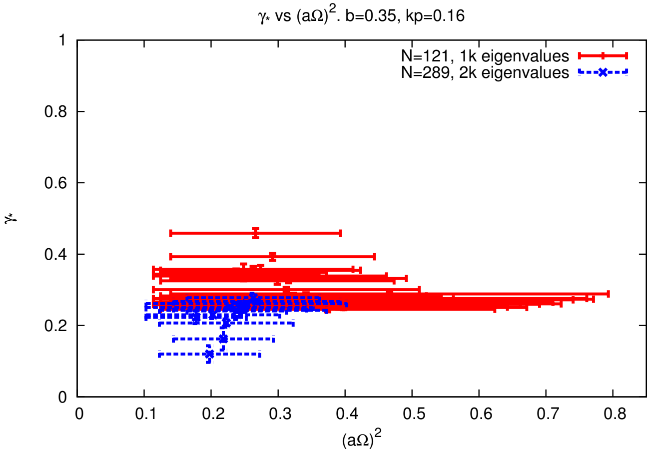
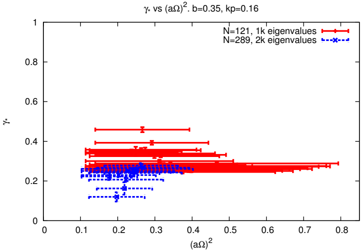
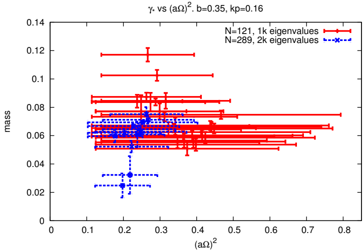


Conclusion and Future Work

- Promising initial results.
    - Volume reduction seems to work
    - Finite volume and finite mass effects understood
    - Preliminary results give $\gamma \sim 0.25$
      (smaller than the string tension determination)
- Would like to investigate lighter masses
  (larger volumes required)
- As well as different bare couplings
- Would also be very interesting to compare with $n_f = 1$

Correlation between γ_* and m





2–param fit: fixed *am*

2–parameter fit: determine *am* by extrapolating lowest eigenvalue in $1/N$, then fit in γ_*, A with $(am)^2 = 0.0697$ fixed.