Single site model of large N gauge theories coupled to adjoint fermions

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work done in collaboration with
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Large N gauge theories coupled to adjoint fermions

- Connection to gravity and string theories.
- Possibility of using a matrix model to study continuum gauge theories.
- Possibility to work with real number of fermion flavors.
- Understand the transition from conformal to confining field theories.

Two main questions pertaining to the matrix model

1) What is the range of fermion flavors for which the single-site massless theory can be expected to reproduce the infinite-volume continuum theory?

2) Can we reproduce the infinite-volume continuum theory with massive fermions?

We will provide an answer to both these questions using Wilson fermions and overlap fermions

THE SINGLE-SITE MODEL

$$Z = \int \prod_{\mu} dU_\mu e^{S_g + f S_f} \qquad S_g = bN \sum_{\mu,\nu=1}^{d} \mathrm{Tr}\left[U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger - 1\right] \qquad S_f = \ln\det H_{w,o}$$

b = $1/(g^2N)$ is the inverse ‘t Hooft coupling

$$H_w(m_w) = \begin{pmatrix} 4 + m_w - \frac{1}{2}\sum_\mu \left(A_\mu + A_\mu^t\right) & \frac{1}{2}\sum_\mu \sigma_\mu \left(A_\mu - A_\mu^t\right) \\ -\frac{1}{2}\sum_\mu \sigma_\mu^\dagger \left(A_\mu - A_\mu^t\right) & -4 - m_w + \frac{1}{2}\sum_\mu \left(A_\mu + A_\mu^t\right) \end{pmatrix} \qquad H_o(m_o) = \frac{1}{2}\left[(1+m_o)\gamma_5 + (1-m_o)\,\epsilon\left[H_w(m_w)\right]\right]$$

$$A_\mu^{ab} = \frac{1}{2}\mathrm{Tr}\left[T^a U_\mu T^b U_\mu^\dagger\right] \qquad \mathrm{Tr} T^a T^b = 2\delta^{ab} \qquad [T^a, T^b] = \sum_c i f_c^{ab} T^c \qquad \sum_a T_{ij}^a T_{kl}^a = 2\delta_{il}\delta_{jk} - \frac{2}{N}\delta_{ij}\delta_{kl}$$

Wilson fermions: $m_w \geq 0$

overlap fermions: $m_o \in [0,1] \quad m_w < 0$

Gauge symmetry: $U_\mu \rightarrow g U_\mu g^\dagger$

Z_N^d symmetry: $U_\mu \rightarrow e^{i\alpha_\mu} U_\mu \quad \alpha_\mu = \frac{2\pi k_\mu}{N} \quad 0 \leq k_\mu < N$

Rajamani Narayanan
Aug 2, 2013

How do we figure out if we can reproduce continuum infinite volume result at the level of perturbation theory?

We set $U_\mu = V_\mu D_\mu V_\mu^\dagger ; \quad D_\mu^{jk} = e^{i\theta_\mu^j}\delta^{jk}$

We replace the integral over U_μ by an integral over V_μ and θ^j_μ

We write $V_\mu = e^{ia_\mu}; \quad a_\mu^\dagger = a_\mu; \quad a_\mu^{ii} = 0 \quad \forall \ i$

Expand in powers of a_μ

Show that the integral over θ^j_μ is dominated in the large N limit such that continuum infinite volume perturbation theory is reproduced order by order

We will ask if this is indeed the case at lowest order in perturbation theory

Density function

We define a joint distribution, ρ(θ), as N → ∞

$$\rho(\theta) = \frac{1}{N}\sum_i \prod_\mu \delta(\theta_\mu - \theta^i_\mu)\,; \qquad \int \prod_\mu d\theta_\mu \rho(\theta) = 1\,, \qquad \int_{-\pi}^{\pi} d\theta\delta(\theta) = 1.$$

At the lowest order

$$S^0_{g,f} = N^2 \int d^4\theta d^4\phi\, \rho(\theta) S_{g,f}(\theta - \phi)\rho(\phi)$$

If $\rho(\theta) = \frac{1}{(2\pi)^4}$

and if $S_g(\theta)$, $S_f(\theta)$ has the correct infinite volume momentum dependence

then the single site model reproduces the infinite volume continuum theory at this order.

We will need to verify this order by order in perturbation theory

The formulas at the lowest order

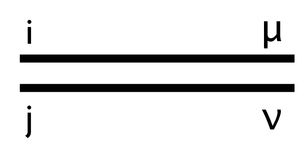
$$S^0_{g,f} = N^2 \fint d^4\theta d^4\phi \, \rho(\theta) S_{g,f}(\theta - \phi)\rho(\phi)\,;$$

$$S_g(\theta) = -\ln \hat{p}\,; \qquad \hat{p} = \sum_\mu 4 \sin^2 \frac{\theta_\mu}{2}\,;$$

$$S_f(\theta) = 2 \ln \gamma_{w,o}(m_{w,o})\,;$$

$$\gamma_w(m_w) = \left(m_w + \frac{\hat{p}}{2}\right)^2 + \bar{p}\,; \qquad \bar{p} = \sum_\mu \sin^2 \theta_\mu\,;$$

$$\gamma_o(m_o, m_w) = \frac{1 + m_o^2}{2} + \frac{1 - m_o^2}{2} \frac{m_w + \frac{\hat{p}}{2}}{\sqrt{\gamma_w(m_w)}}\,.$$

Gauge propagator in axial gauge: 

$$\frac{1}{2bN} \frac{1}{\hat{p}} \left(\frac{1}{\hat{p}_1} + \frac{\delta_{\mu\nu}}{\hat{p}_\mu} \right)$$

Density function

- Assume that only one $\rho(\theta)$ dominates in the large N limit.
- Only allow non-negative distributions to contribute.
- Assume that the dominating distribution is a smooth function.
- Note that the action is invariant under $\rho(\theta) \to \rho(\theta+\alpha)$

Owing to the periodic and symmetric nature of $S_{g,f}(\theta)$, it follows that

$$\int_{-\pi}^{\pi} \prod_{\nu} \frac{d\phi_\nu}{2\pi} S_{g,f}(\theta-\phi)\, e^{i\sum_\mu k_\mu \phi_\mu} = \lambda_k^{(g,f)}\, e^{i\sum_\mu k_\mu \theta_\mu}\,;$$

$$\lambda_k^{(g,f)} = \int_0^{\pi} \prod_{\nu} \frac{d\phi_\nu}{\pi} S_{g,f}(\phi) \prod_{\mu} \cos(k_\mu \phi_\mu)\,.$$

Therefore, Fourier expanding

$$\rho(\theta) = \frac{1}{(2\pi)^4} \sum_k c_k e^{i\sum_\mu k_\mu \theta_\mu} \qquad \text{with} \qquad c_{-k} = c_k^*\,, \quad c_0 = 1$$

results in

$$S_{g,f}^0 = N^2 \sum_k c_k c_k^* \lambda_k^{(g,f)}\,,$$

Condition for domination by the constant mode

$$S^0_{g,f} = N^2 \int d^4\theta d^4\phi \, \rho(\theta) S_{g,f}(\theta - \phi)\rho(\phi) \qquad \rho(\theta) = \frac{1}{(2\pi)^4} \sum_k c_k e^{i\sum_\mu k_\mu \theta_\mu}$$

$$S^0_{g,f} = N^2 \sum_k c_k c_k^* \lambda_k^{(g,f)}$$

$$\lambda_k = \lambda_k^{(g)} + f\lambda_k^{(f)}$$

- If all λ_k are less than zero for all $k \neq 0$ then the constant mode will dominate and we will reproduce the continuum infinite volume perturbation theory at the lowest order.
- If one or more λ_k is greater than zero, then we can make the corresponding coefficient in the Fourier expansion as large as we want as long as $\rho(\theta)$ remains non-negative everywhere.
  - The maximum will be obtained at the boundary of the allowed values of c_k.
  - The density function will be zero at least in one point in the four dimensional Brillouin zero.
  - The single site model will not reproduce infinite volume continuum physics.

We will numerically compute the eigenvalues from the gauge and fermion parts separately by dividing the four dimensional space of θ_μ into M^4 equally spaced points.

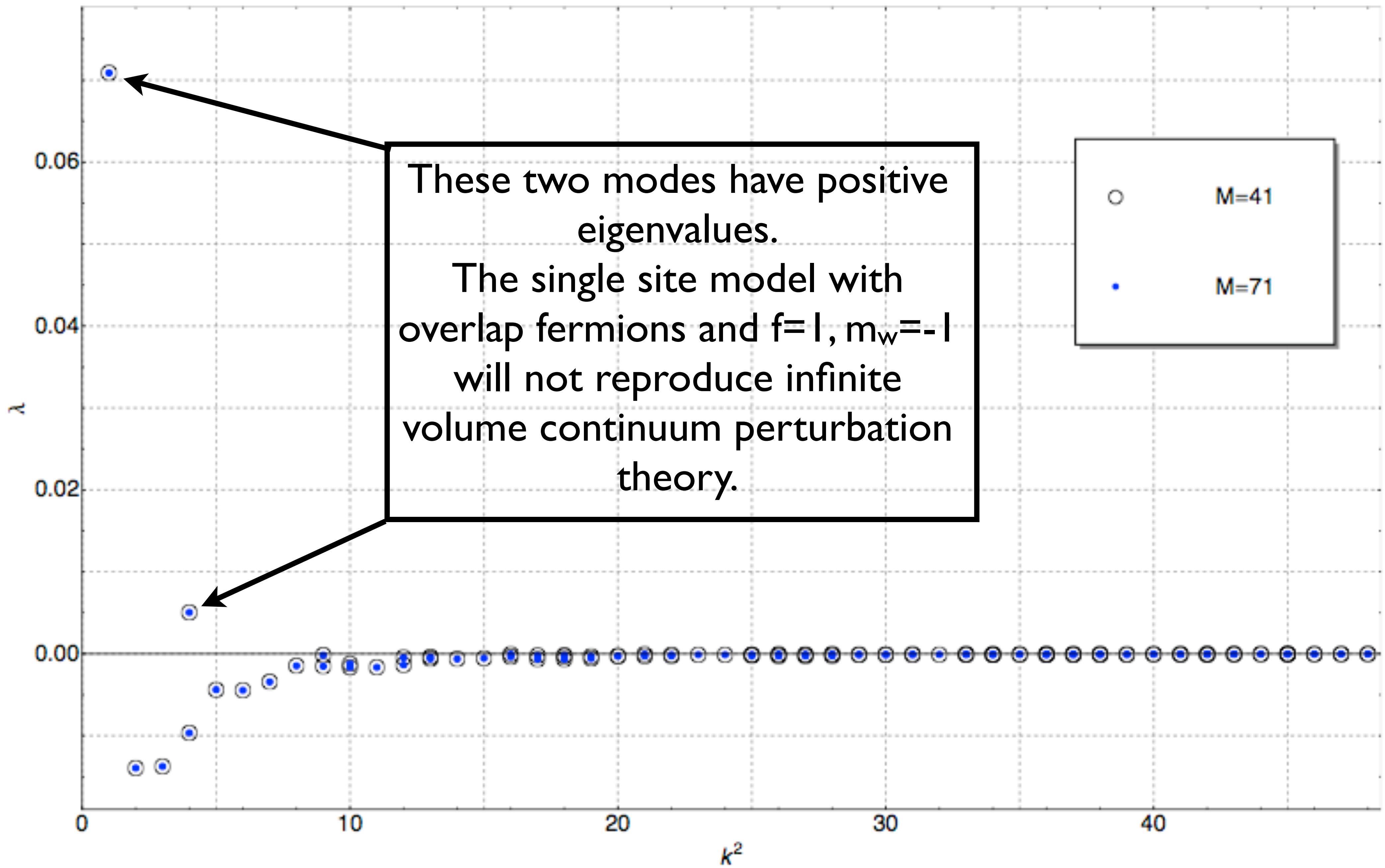


FIG. 1: Eigenvalues $\lambda_k = \lambda_k^{(g)} + f\lambda_k^{(o)}$ as a function of k^2 for the massless overlap Dirac operator with $f = 1$ and $m_w = -1$ obtained using numerical integration with M^4 equally spaced points in the four-dimensional integration space.

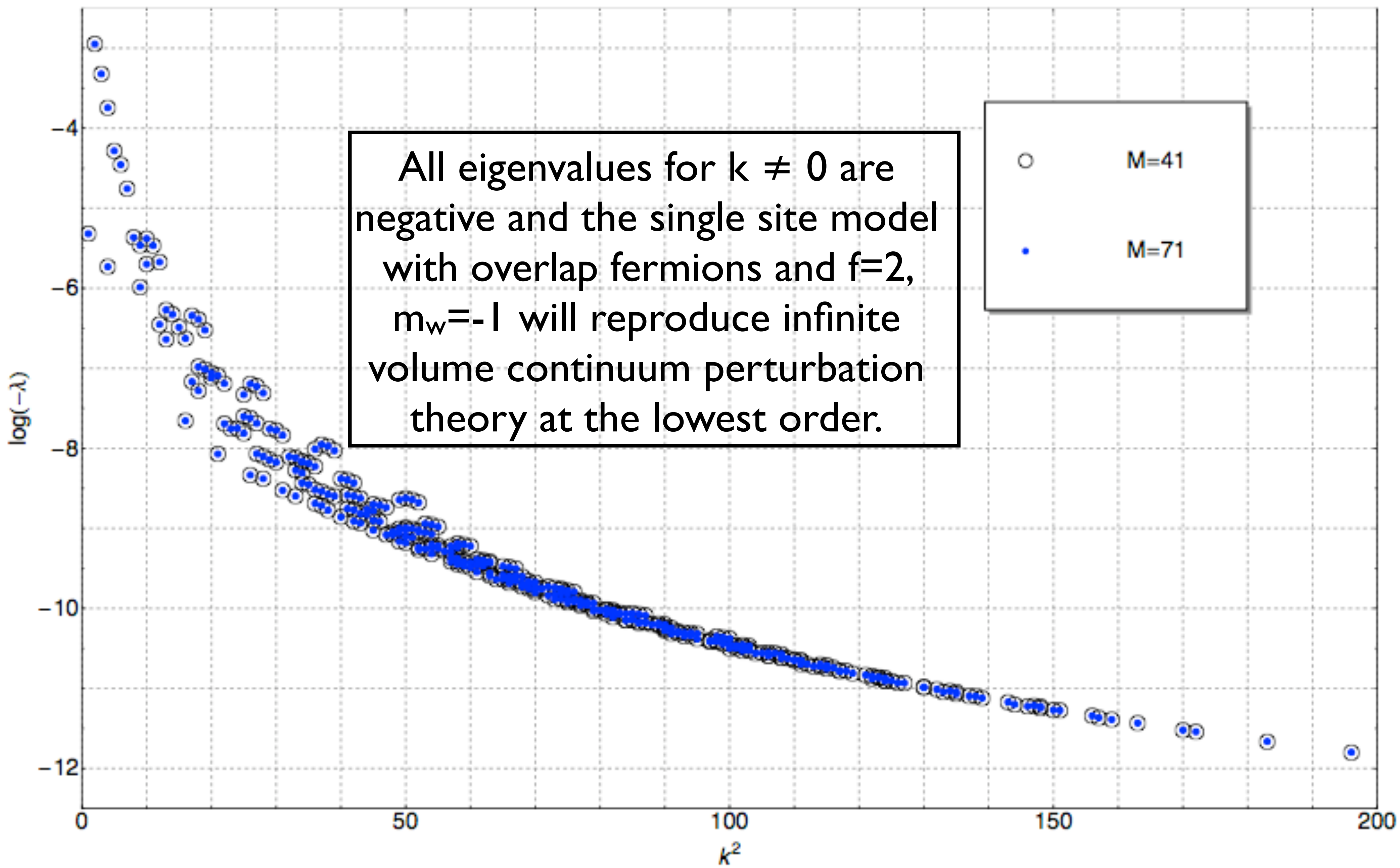


FIG. 2: Logarithm of the eigenvalues for the massless overlap Dirac operator with $f = 2$ and $m_w = -1$ obtained using numerical integration with M^4 equally spaced points in the four-dimensional integration space.

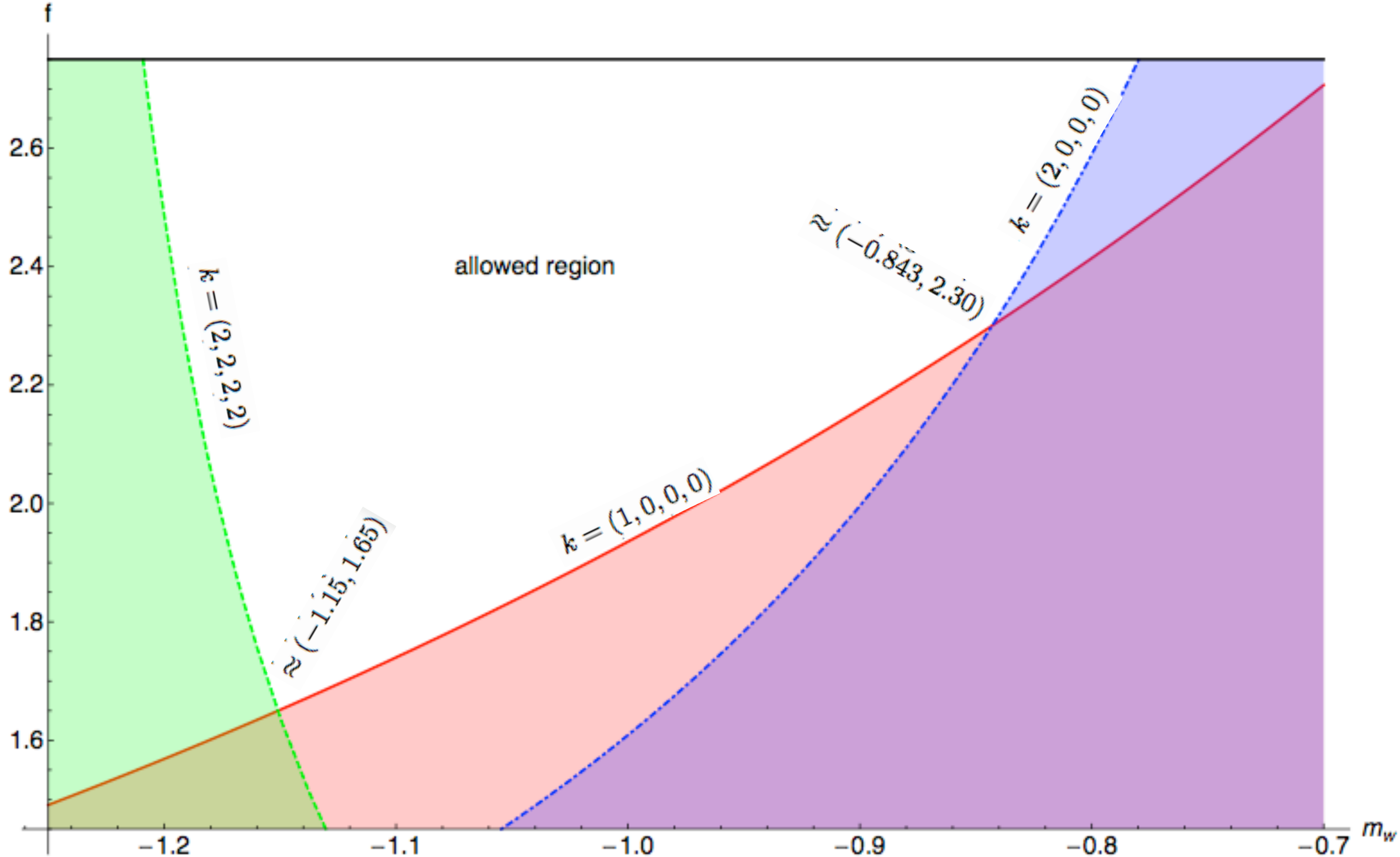
Behavior of eigenvalues as a function of k

- All gauge modes are larger than zero.
  - This implies that single site large N pure gauge theory will not reproduce infinite volume continuum theory. This is nothing but the statement that Eguchi-Kawai reduction does not work for pure gauge theory in four dimensions.

- For $\lambda_k = \lambda_k^{(g)} + f\lambda_k^{(f)} < 0$ for all k
  - $\lambda_k^{(o)}(m_o, m_w) < 0$ for all k,
  - $f > \max_k \left\{ -\lambda_k^{(g)}/\lambda_k^{(o)} \right\}$.

Massless overlap fermions

$$-\lambda_k^{(g)}/\lambda_k^{(o)} \to \tfrac{1}{2} \text{ as } k \to \infty \text{ for all } m_w < 0.$$



massless overlap fermions

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Why the single site model does not work for massive fermions

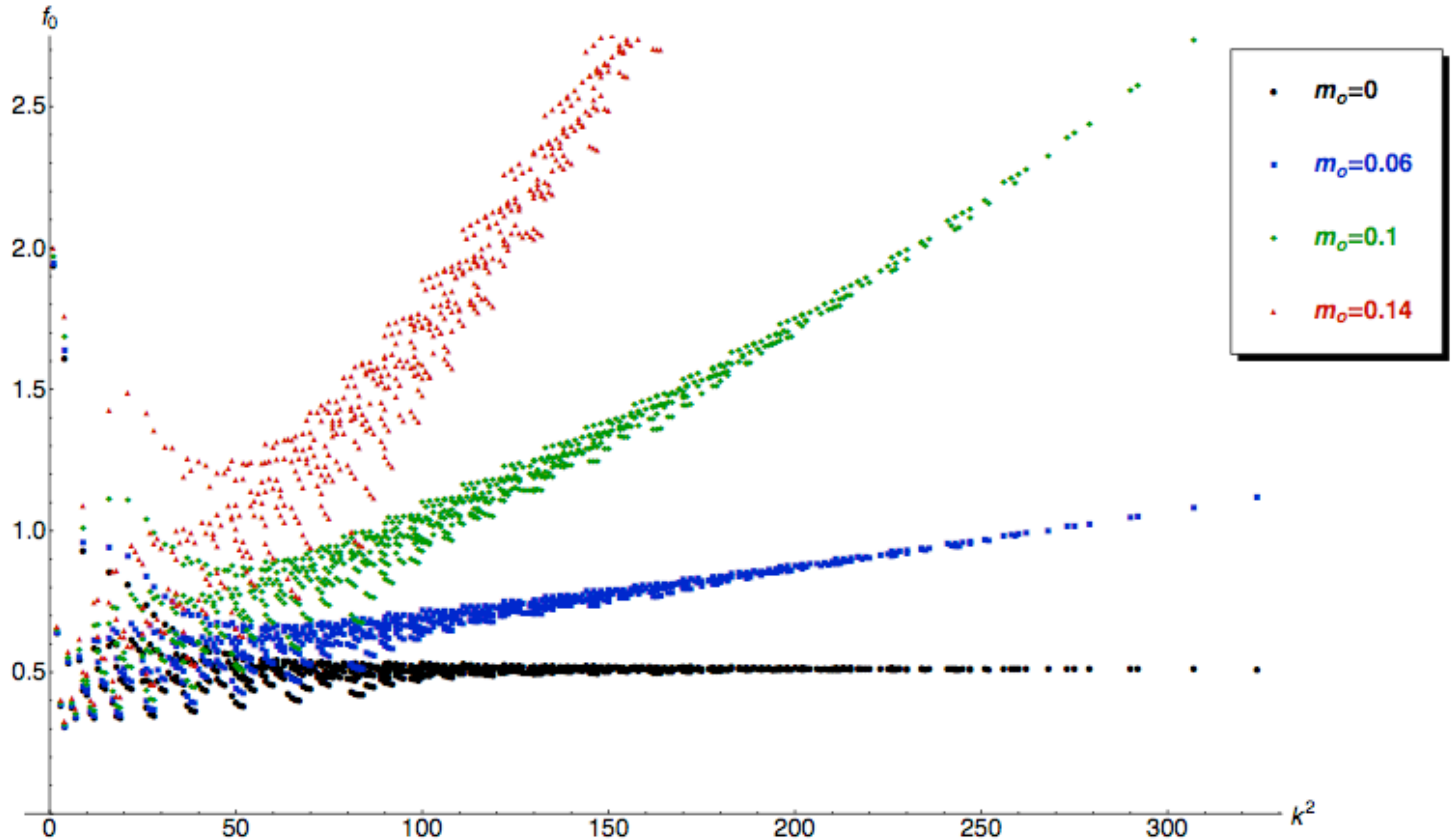

FIG. 3: Plots of $f_0 \equiv -\lambda_k^{(g)}/\lambda_k^{(o)}$ (for $k_\mu \leq 9$) at $m_w = -1$ and different choices for m_o. For $m_o = 0$, $f_0 \to 0.5$ as $k^2 \to \infty$; for all $m_o > 0$, $f_0 \to \infty$ as $k^2 \to \infty$. The boundary of the allowed region is determined by $\max_k f_0(k)$.

Wilson fermions needs to be massless and f > 2.39 in order to reproduce infinite volume perturbation theory

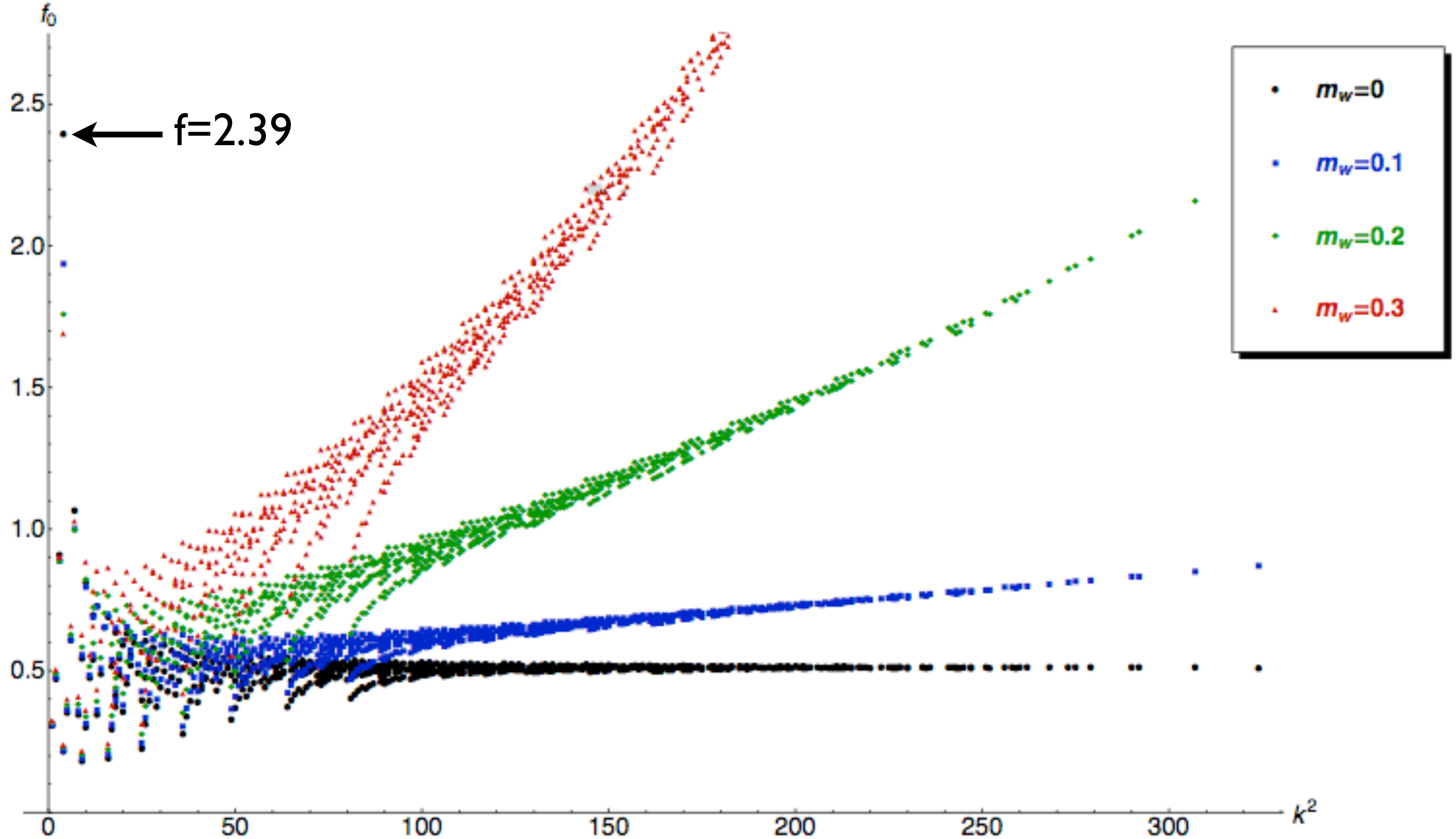

FIG. 6: Plots of $f_0 \equiv -\lambda_k^{(g)}/\lambda_k^{(w)}$ (for $k_\mu \leq 9$) for different choices of m_w. For $m_w = 0$, $f_0 \to 0.5$ as $k^2 \to \infty$; for $m_w > 0$, $f_0 \to \infty$ as $k^2 \to \infty$.

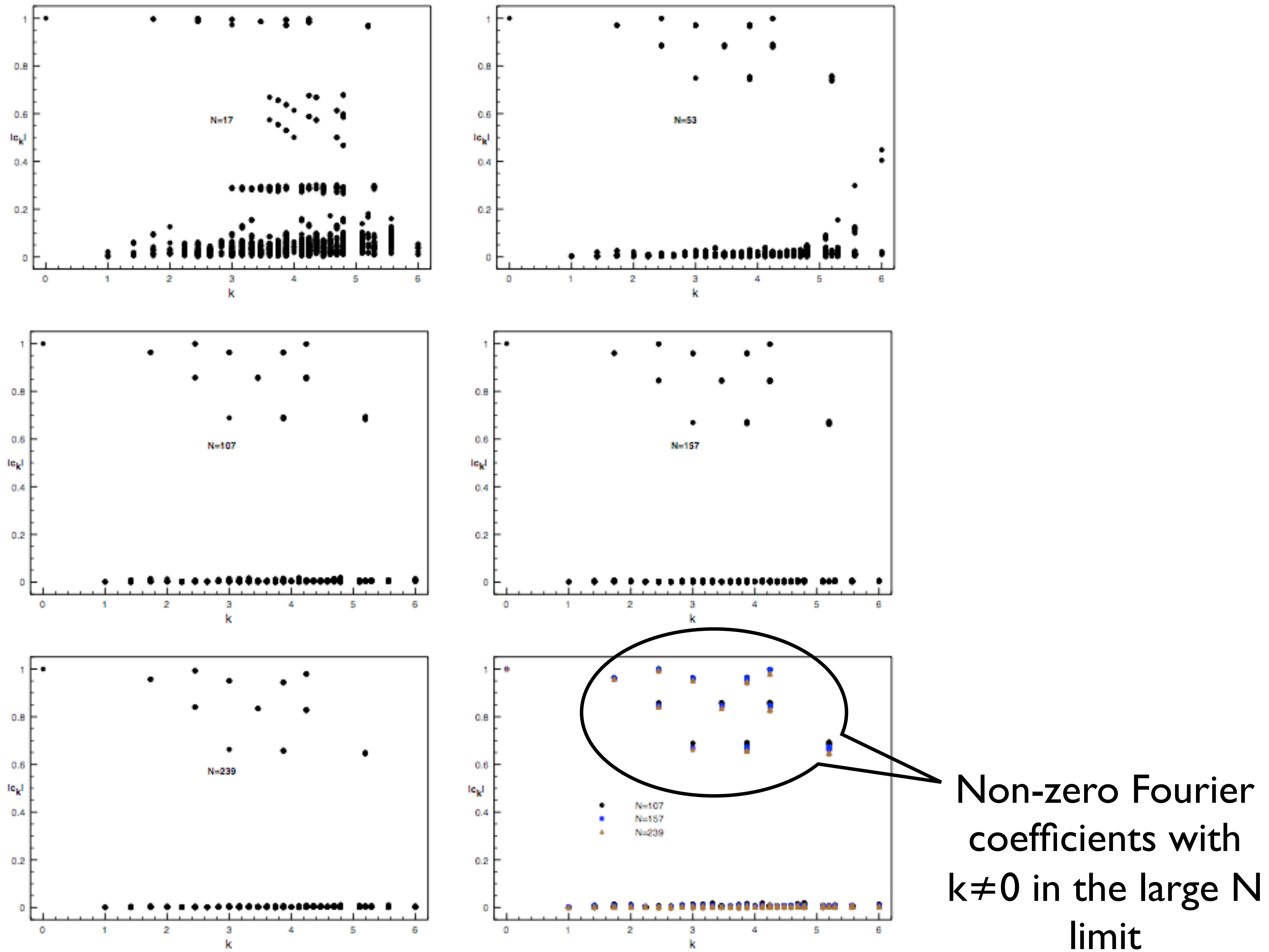

FIG. 10: The Fourier coefficients, $|c_k|$, of $\rho(\theta)$ as a function of $k = \sqrt{k^2}$ for several values of N with $f = 1$ flavors of massless overlap fermions and the Wilson mass parameter set to $m_w = -4$.

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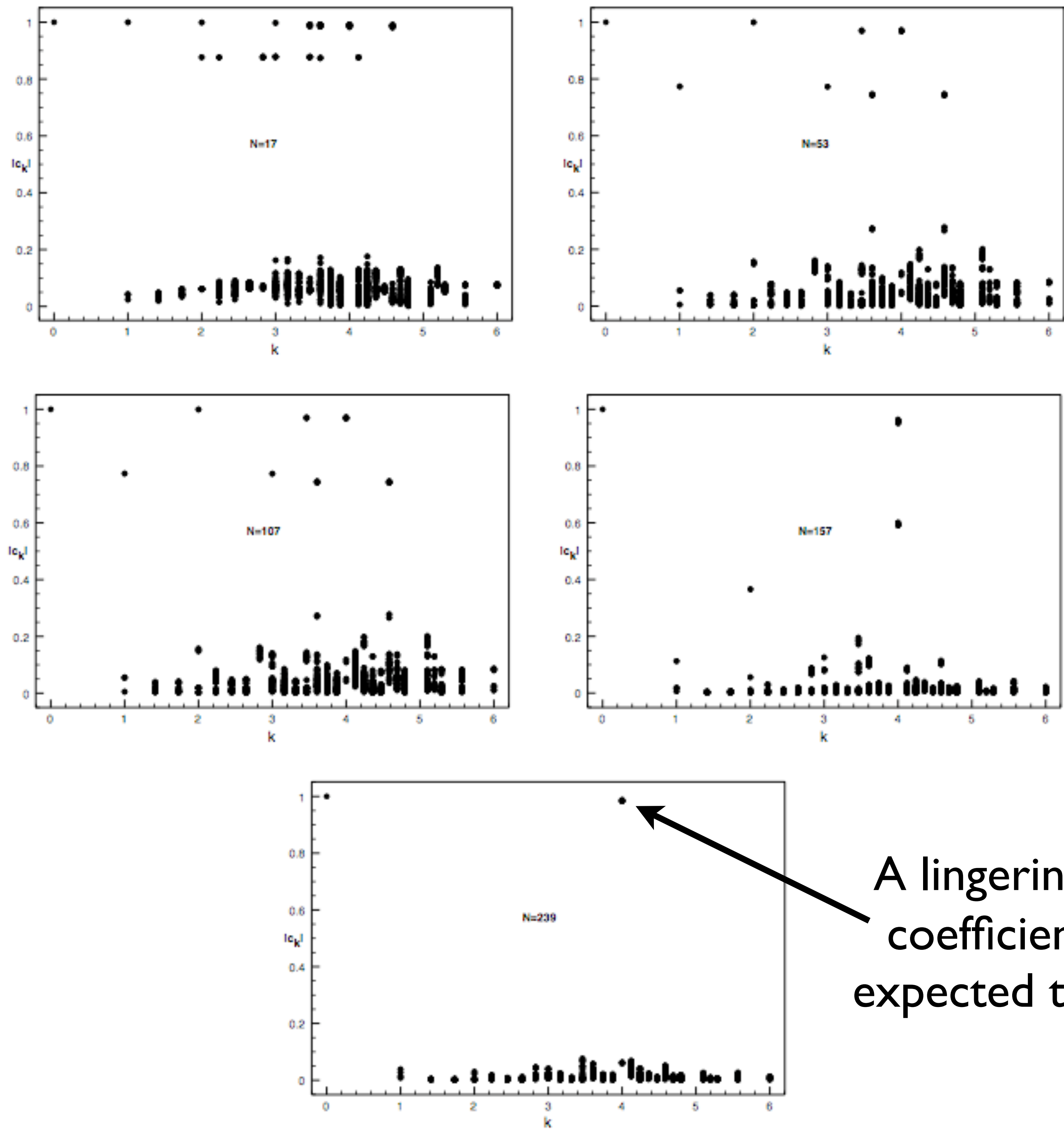

A lingering non-zero Fourier coefficient with k≠0 that is expected to vanish in the large N limit

FIG. 9: The Fourier coefficients, $|c_k|$, of $\rho(\theta)$ as a function of $k = \sqrt{k^2}$ for several values of N with $f = 2$ flavors of massless overlap fermions and the Wilson mass parameter set to $m_w = -1$.

Discussion of previous numerical work

The Fourier coefficients are nothing but

$$c_k(b) = \mathrm{Tr}\, U_1^{k_1} U_2^{k_2} U_3^{k_3} U_4^{k_4}$$

We tried to look at all of them in the weak coupling limit, $b \to \infty$

Previous numerical work has mainly focussed on $k_\mu=(1,0,0,0)$ and its permutations. We have seen here that this can lead to incorrect conclusions about the validity of the single site model.

Since some coefficients with small k could be accidentally small, even looking at $k_\mu=(1,-1,0,0)$ might not be sufficient to check if the single site model can reproduce the infinite volume continuum theory

Much of the previous numerical work has been done at finite values of the lattice coupling. Even if there is some evidence for a infinite volume limit at finite lattice coupling, the results here show that one cannot take the weak coupling limit. This also applies to numerical work done with massive fermions.