Freeze-out parameters from continuum extrapolation

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Heavy Ion Physics

RHIC at Brookhaven

LHC at CERN

High energy (LHC): most baryons fly through → small baryon density (μ=0 physics)
Lower energy (RHIC): baryons are stopped → large baryon density (μ>0 physics)
Beam energy scan program

Freeze-out curve:
A model of free hadrons (with T=0 masses) is fitted on the ratios of particle yields.
This (grand canonical) thermal model fits well and gives a (T,μ) pair for each collision energy.

Lattice transition line:
[Aoki et al hep-lat/0609068]
[Endrodi et al 1102.1356]
[Kaczmarek et al 1011.3130]

Thermal models:
[Andronic et al nucl-th/0511071]
[Cleymans et al hep-ph/0511094]
Fluctuations of the net charge

A conserved quantum number is counted for a subsystem. *(e.g. net electric charge)*

*For each event (individual collision) this net quantum number reflects the instant when the hadrons were formed.*

The acceptance condition must be sufficiently
- narrow, to allow to approximate a grand canonical ensemble
- wide, to allow all correlation lengths characteristic to a *grand canonical ensemble*

**Experiment** measures the distribution of the net charge (mean, variance, skewness, kurtosis...)

**Lattice** calculates these moments in a grand canonical ensemble.

\[
\langle N^2_X \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}
\]

*O(V) cancellation*
Fluctuations on the lattice

A derivative of $\log Z$ acts on the fermion determinant inside of the effective gauge action:

$$\partial_i \log Z = \frac{1}{Z} \int DU \partial_i e^{-S_{\text{eff}}} = \langle A_i \rangle$$

Through further derivatives disconnected diagrams appear alongside with higher operators.

$$\partial_j \langle X \rangle = -\langle X \rangle (\partial_j \log Z) + \langle X \partial_j e^{-S_{\text{eff}}} \rangle + \langle \partial_j X \rangle$$

$$= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \langle \partial_j X \rangle .$$

(X is one of $A,B,C \ldots$)

Disconnected: noisy

Connected: sensitive to taste breaking

These traces are evaluated with several ($N=O(1000)$) random sources, for each gauge configuration:

$$A = \frac{1}{N} \sum_i \chi^+_i M^{-1} M' \chi_i$$

a conjugate gradient solver
Fluctuations on the lattice

A derivative of log $Z$ acts on the fermion determinant inside of the effective gauge action:

$$\partial_i \log Z = \frac{1}{Z} \int DU \, \partial_i e^{-S_{\text{eff}}} = \langle A_i \rangle$$

Through further derivatives disconnected diagrams appear alongside with higher operators.

$$\partial_j \langle X \rangle = -\langle X \rangle (\partial_j \log Z) + \langle X \partial_j e^{-S_{\text{eff}}} \rangle + \langle \partial_j X \rangle$$

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(X is one of $A,B,C$ ...)

Disconnected: noisy

Connected: sensitive to taste breaking

T>0 simulations with 2-stout staggered fermions:
Results in the continuum

Comparison of the published continuum results:

HotQCD [1203.0784]
Wuppertal-Budapest [1112.4416]

[preliminary WB data: 1210.6901]

[discussion at high T see Peterczky Mon 15.00]
Continuum extrapolation

The graph shows the dependence of $\chi_U$ on $1/N_t^2$ for two different temperatures, $T=130$ MeV (red squares) and $T=180$ MeV (blue circles). The data points are connected by solid lines, with dashed lines for the extrapolation.
Looking for a thermometer

**Assumption:**
Before freeze-out the system is described with a time-dependent temperature and baryo, charge and strange chemical potentials.
After freeze-out the net bayron, charge and strangeness reflects the system an equilibrium well-defined freeze-out temperature.

*This picture was supported by the statistical models.*

**Thermometer:**
- experimentally accessible (ratios to cancel the volume factor)
- monotonic in $T$
- known from theory

\[
\hat{\chi}_4^X = \frac{1}{VT^3} \frac{\partial^4 \log Z}{(\partial \mu_X / T)^4}
\]

\[
\hat{\chi}_2^X = \frac{1}{VT^3} \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}
\]

[Karsch 1202.4173]

*experiment will give a band on the y axis*

Typical from HRG $T$

Typical from lattice $T$

Suboptimal observable $T$
Charge and baryon kurtosis

old data (2008)
RBC-Bielefeld [0811.1006]

new data (2012)

Useful at LHC
Charge skewness at RHIC

RHIC:
- Lattice data must be extrapolated to $\mu>0$ keeping the constraints: $<S>=0$ and $<Q>=Z/A <B>$
+ Mean and skewness are zero at LHC ($\mu=0$) but nonzero here $\rightarrow$ they define a thermometer.

[STAR 1212.3892]

net charge distribution
(most central in red)

Result: $T < 157$ MeV

Lattice: [Wuppertal-Budapest 1305.5161]

suggested and calculated in
[BNL-Bielefeld 1208.1220]
[continuum limit and WB data: 1305.5161]
Baryometer of the electric charge

Statistical model:

- 62.4 GeV
- 39 GeV
- 27 GeV

Continuum extrapolated lattice data from Wupprechtal - Budapest [1305.5161]

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>$\mu_B^f$ [MeV]</th>
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<tbody>
<tr>
<td>62.4</td>
<td>44(3)(1)(2)</td>
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<tr>
<td>39</td>
<td>75(5)(1)(2)</td>
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<tr>
<td>27</td>
<td>95(6)(1)(5)</td>
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</table>

[Based on the proposal in 1202.4173 & 1208.1220 of the BNL-Bielefeld group]
Limitations of this picture

- Experiments (Phenix vs. Star) do not yet agree on fluctuations, thus the freeze-out result is not final yet. (*Final state interactions must be modelled*)
- The results from baryon and charge fluctuations are inconsistent (not baryon fluctuations but proton fluctuations are measured, the protons do not really form a grand canonical ensemble)
- We assumed that all degrees of freedom in the quark gluon plasma is turned into hadronic matter at a unique temperature. (The heavier strange might freeze out earlier)

Statistical model fit (gas of free hadrons) show inconsistent results for strange / non-strange yield ratios at LHC.
Flavor sensitivity in the fluctuation

Strange susceptibility vs light quark susceptibility: about 16 MeV difference in the characteristic point.

Strange susceptibility $\chi_2(T)/SB$

Light susceptibility $\chi_2(T)/SB$

Light susceptibility $\chi_2(1.11 T)/SB$

HRG prediction

[Bellwied et al (WB) 1305.6297]
Two indicators of deconfinement

In [1304.7220] the BNL-Bielefeld group has suggested two combinations of fluctuations that are nonzero only if a non-hadronic strange degree of freedom is excited. We extended this to the light flavors and calculated the continuum limit.

These combinations are constructed such that a free gas of baryons or mesons give zero, but a free quark would give a non-zero contribution.  

[Schmidt Tue 15.00]  

[Bellwied et al (WB) 1305.6297]
Multi-strange observables

It is possible to construct other observables that are dominated by the multi-strange hadrons. Here the flavor separation is the strongest.

The Hadron Resonance Gas prediction is non-trivial here. Observation: the lattice data has a kink where it departs from the HRG result.
A possible scenario

Evidence:
- Lattice observables are sensitive to quark mass (multi-strange: stronger sensitivity)
- Last point of agreement to Hadron Resonance Gas is flavor dependent
- Fit to yields shows a preference to flavor dependent freeze-out

No evidence:
- Lattice cannot say anything about freeze-out
- $T_c$ is not necessary the peak/inflection point of some susceptibility curve

A picture:

![Diagram showing quark states at different temperatures](image)

- $T < 150$ MeV
- $150$ MeV $< T < 165$ MeV
- $T > 165$ MeV
The **width of net charge/strangeness/baryon** distribution is predicted in the continuum limit by both the HotQCD and Budapest-Wuppertal collaborations.

Higher cumulants (**curtosis**) have been presented, comparison to experiment is now possible.

Ratios of fluctuations have been used to define **freeze-out thermometers and bayrometers**.

In several cases preliminary data shows significant deviations from the simplest HRG result, even below $T_c$.

Experimentally accessible flavor sensitive observables have been calculated. **Is there a flavor hierarchy in the deconfinement transition of QCD?**

All data: based on continuum extrapolated lattice results from the Wuppertal-Budapest collaboration.

[1112.4416] [1204.6710] [1210.6901] [1305.5161] [1305.6297]
Spare slides
Steep $T$-dependence throughout the relevant temperature range.

It is based on 2nd order derivatives (continuum limit is feasible), precise in experiment.

Mixes the baryon and charge systematics of the experiment.
If the baryon number could be measured reliably, a thermometer could be constructed based on 2nd order fluctuations: “almost trivial” both for lattice and for experiment.
Charge thermometer?

![Graph showing temperature vs. Q/Q_2 ratio with data points and curves labeled for different groups like Wuppertal-Budapest, STAR preliminary, PHENIX preliminary, and BNL-Bielefeld preliminary. Each group has a specific temperature range indicated.]

- Wuppertal-Budapest preliminary
- STAR preliminary 200 GeV
- PHENIX preliminary 200 GeV
- BNL-Bielefeld preliminary

[PHENIX: J Mitchell QM12]  [BNL-Bielefeld: C Schmidt 1212.4278 ]
[STAR: N R Sohoo 1212.3892]  [preliminary WB data: 1210.6901]
The hadron resonance gas (HRG) model describes a mixture of free hadrons: all mesons and baryons and their excited states you find in the particle data book.

\[ \frac{p_{\text{HRG}}}{T^4} = \frac{1}{VT^3} \sum_{i \in \text{mesons}} \log Z^M(T, V, \mu_{X^a}, m_i) + \frac{1}{VT^3} \sum_{i \in \text{baryons}} \log Z^B(T, V, \mu_{X^a}, m_i) \]

\[ \ln Z_{M/B}^{M/B} = \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\epsilon_i/T}) \]

\[ = \frac{VT^3}{2\pi^2} d_i \left( \frac{M_i}{T} \right)^2 \sum_{k=1}^\infty (\pm 1)^{k+1} \frac{z_i^k}{k^2} K_2(kM_i/T) \]

Degeneracy factor \(d_i\): spin, etc, chemical potentials enter through

This model is a good approximation to QCD in the hadronic phase:
- low \(T\): mostly pions, they interact very weakly in QCD and chiPT
  - [Grasser&Leutwyler 1984] [Greber&Leutwyler 1989]
- higher \(T\): interactions are included through the growing number of resonances.
  - In the strong coupling expansion the partition function reproduces HRG.
    - [Langelage&Philipsen 1002.1507]

HRG has been tested against lattice in a heavy pion world.
- [Karsch et al 0303108] [Petreczky&Huovinen 0912.2541, 1005.0324, 1106.6227]
All diagonal fluctuations

\[ \frac{\chi^i_2}{(\chi^i_2)_{SB}} \]

[T [MeV]]

[Wuppertal-Budapest: 1112.4416]
Comparison to HTL expansion

[Andersen et al 1210.0912]

[Haque&Mustafa&Strickland 1302.3228]
Status of experiment

proton

\[ S \sigma \]

\[ k \sigma^2 \]

\[ (S \sigma)/\text{Poisson} \]

\[ \sqrt{s_{NN}} \ (\text{GeV}) \]

[STAR Preliminary]

[McDonald (STAR) 1210.7023]

electrical charge

\[ \sigma^2 \]

\[ \sigma \]

\[ k\sigma^2 \]

\[ \sqrt{s_{NN}} \ (\text{GeV}) \]

[STAR Preliminary]

[McDonald (STAR) 1210.7023]

[X. Luo (STAR) 1210.5573]

[N R Sahoo (STAR) 1212.3892]
At finite chemical potential

Lower energies (RHIC): higher chemical potential

From lattice QCD we can calculate

\[
\chi_{AB\ldots} = \frac{1}{VT^3} \left[ \frac{\partial}{\partial \mu_A/T} \frac{\partial}{\partial \mu_B/T} \ldots \right] \log Z
\]

at zero chemical potential.

\[
\begin{align*}
\chi_Q|_{\mu_B} &= \chi_{QB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right) + \frac{1}{6} \chi_{QBB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right)^3 + \ldots \\
\chi_{QQ}|_{\mu_B} &= \chi_{QQ}|_{\mu_B=0} + \frac{1}{2} \chi_{QQBB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right)^2 + \ldots \\
\chi_{QQQ}|_{\mu_B} &= \chi_{QQQB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right) + \frac{1}{6} \chi_{QQQBBB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right)^3 + \ldots \\
\chi_{QQQQ}|_{\mu_B} &= \chi_{QQQQ}|_{\mu_B=0} + \frac{1}{2} \chi_{QQQQBB}|_{\mu_B=0} \left( \frac{\mu_B}{T} \right)^2 + \ldots 
\end{align*}
\]

Odd/even ratios:
\[
\frac{\chi_Q}{\chi_{QQ}} = \frac{M}{\sigma^2} = \frac{\chi_{QB}}{\chi_{QQ}} \cdot \frac{\mu_B}{T} \cdot \mathcal{O}(\mu_B^3)
\]

Odd/odd ratios:
\[
\frac{\chi_{QQQ}}{\chi_Q} = \frac{S\sigma^3}{M} = \frac{\chi_{QQQB}}{\chi_{QB}} + \mathcal{O}(\mu_B^2)
\]

Even/even ratios:
\[
\frac{\chi_{QQQQ}}{\chi_{QQ}} = \frac{\kappa\sigma^2}{\chi_{QQ}|_{\mu_B=0}} = \frac{\chi_{QQQQ}}{\chi_{QQ}|_{\mu_B=0}} + \mathcal{O}(\mu_B^2)
\]

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<th>$\sqrt{s}$ [GeV]</th>
<th>$\mu_B$</th>
</tr>
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<tbody>
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<td>200</td>
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<tr>
<td>62.4</td>
<td>69</td>
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<tr>
<td>39</td>
<td>107.2</td>
</tr>
<tr>
<td>27</td>
<td>149</td>
</tr>
</tbody>
</table>

[Andronic et al 0812.1186]

[Karsch 1202.4173]
Line of constant net “$M$” ratios

Input matter content: two colliding nuclei

$$M_Q = M_B \frac{Z}{A} \quad r = \frac{Z}{A} = \frac{82}{207} \approx 0.4$$

$$M_S = 0$$

(lead)

To leading order:

$$M_B \sim \chi_2^B(T)\mu_B + \chi_{11}^{BQ}(T)\mu_Q + \chi_{11}^{BS}(T)\mu_S$$

$$M_Q \sim \chi_{11}^{BQ}(T)\mu_B + \chi_2^Q(T)\mu_Q + \chi_{11}^{QS}(T)\mu_S$$

$$M_S \sim \chi_{11}^{BS}(T)\mu_B + \chi_{11}^{QS}(T)\mu_Q + \chi_2^S(T)\mu_S$$

The total matter content constrains the chemical potentials onto a 1D manifold (line), which we conveniently parametrize through $\mu_B$:

$$\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + \ldots$$

$$\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + \ldots$$

$$q_1 = \frac{(r\chi_{11}^{BS} - \chi_{11}^{QS})\chi_{11}^{BS} - (r\chi_2^B - \chi_{11}^{BQ})\chi_2^S}{(r\chi_{11}^{BQ} - \chi_2^Q)\chi_2^S - (r\chi_{11}^{BS} - \chi_{11}^{QS})\chi_{11}^{QS}}$$

$$s_1 = -(\chi_{11}^{QS}q_1 + \chi_{11}^{BS})\chi_2^S$$

The NLO coefficients ($q_3$ and $s_3$) contain 4th derivatives.

These have been first calculated in [BNL-Bielfeld1208.1220]
Line of constant net \( "M" \) ratios

Example use:

\[
\frac{d}{d\mu_B} X(T) = \frac{\partial}{\partial \mu_B} X(T) + q_1(T) \frac{\partial}{\partial \mu_Q} X(T) + s_1(T) \frac{\partial}{\partial \mu_S} X(T)
\]
Baryon skewness as thermometer

\[ S_B \sigma_B^3 / M_B \]

- \( N_t=6 \)
- \( N_t=8 \)
- \( N_t=10 \)
- \( N_t=12 \)

WB continuum limit

\( T \text{ [MeV]} \)