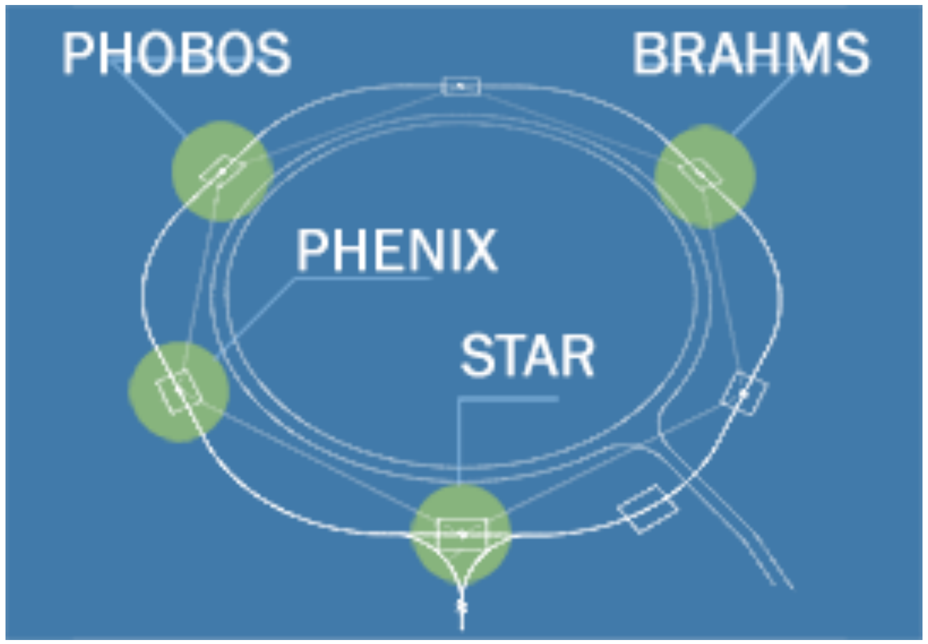
Freeze-out parameters from continuum extrapolation

Szabolcs Borsanyi
Wuppertal
for the Wuppertal-Budapest collaboration

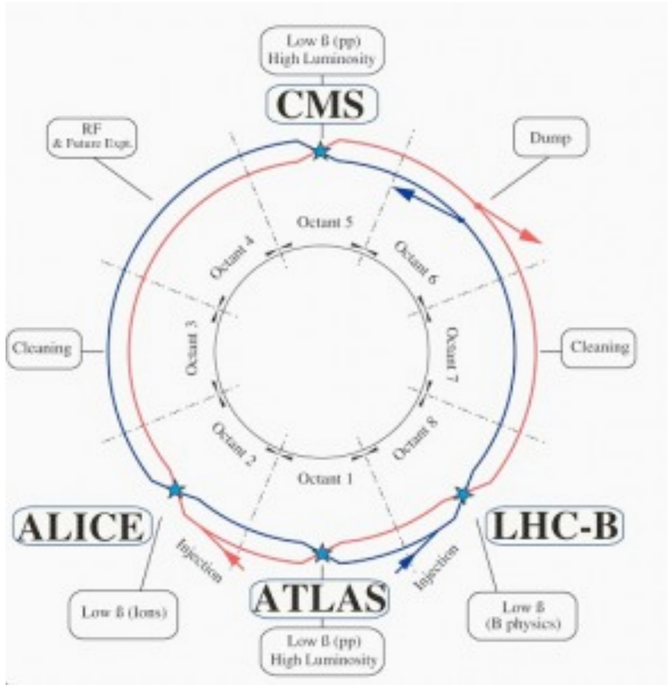
Lattice13, Mainz
29 July 2013

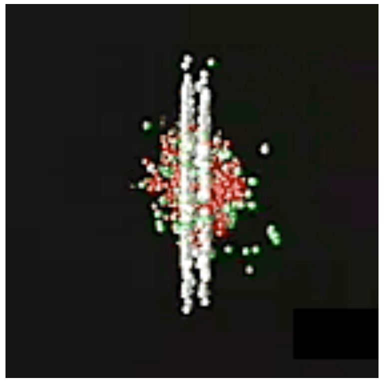
Heavy Ion Physics

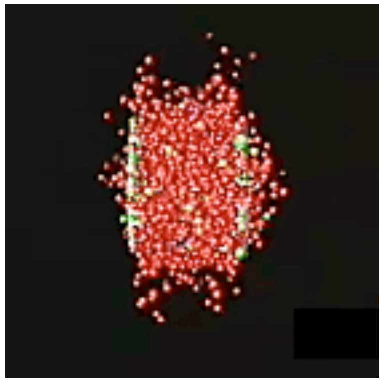
RHIC at Brookhaven

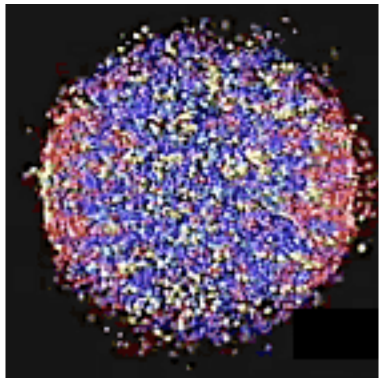


LHC at CERN





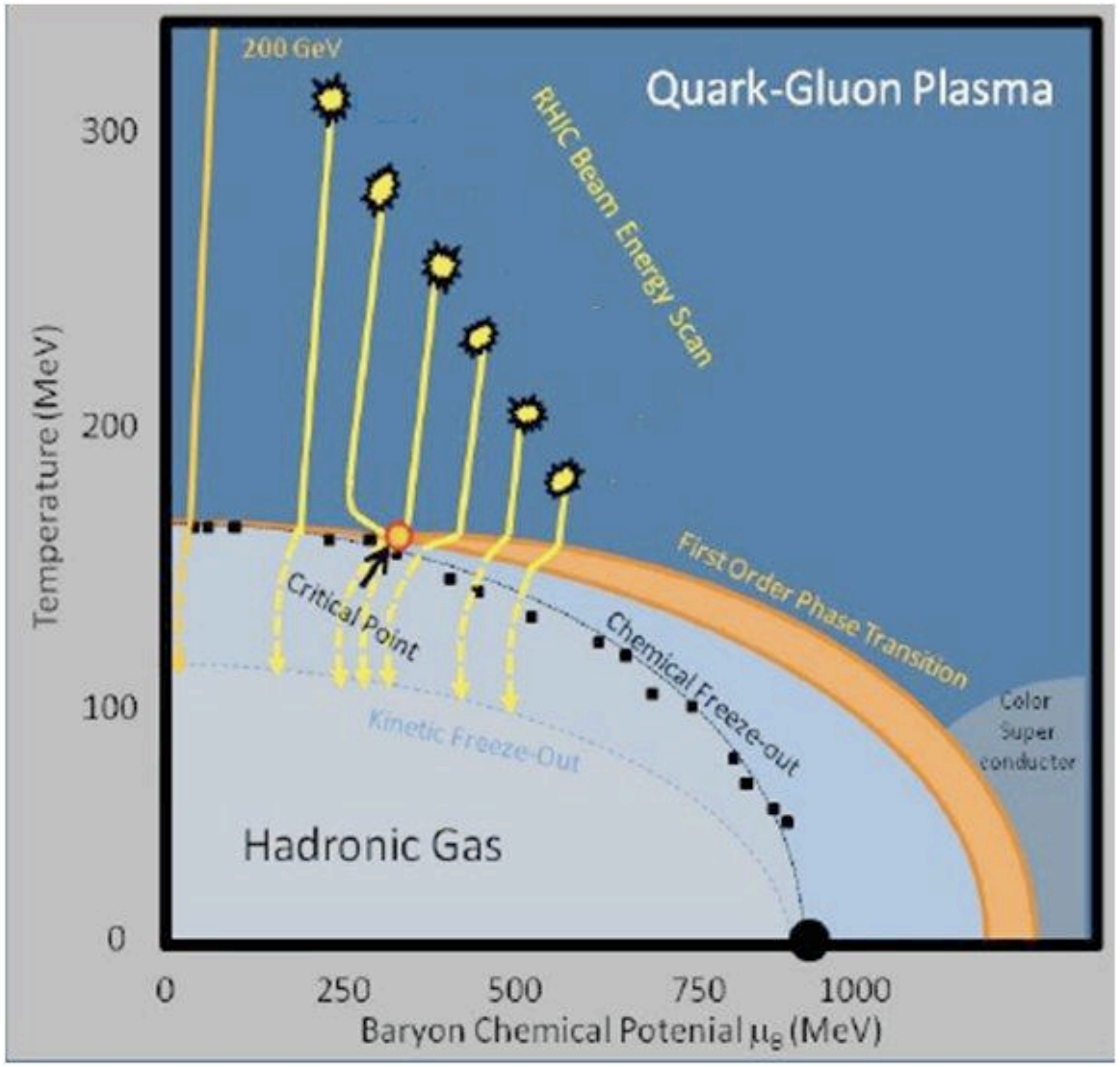




High energy (LHC): most baryons fly through → small baryon density (μ=0 physics)

Lower energy (RHIC): baryons are stopped → large baryon density (μ>0 physics)

Beam energy scan program



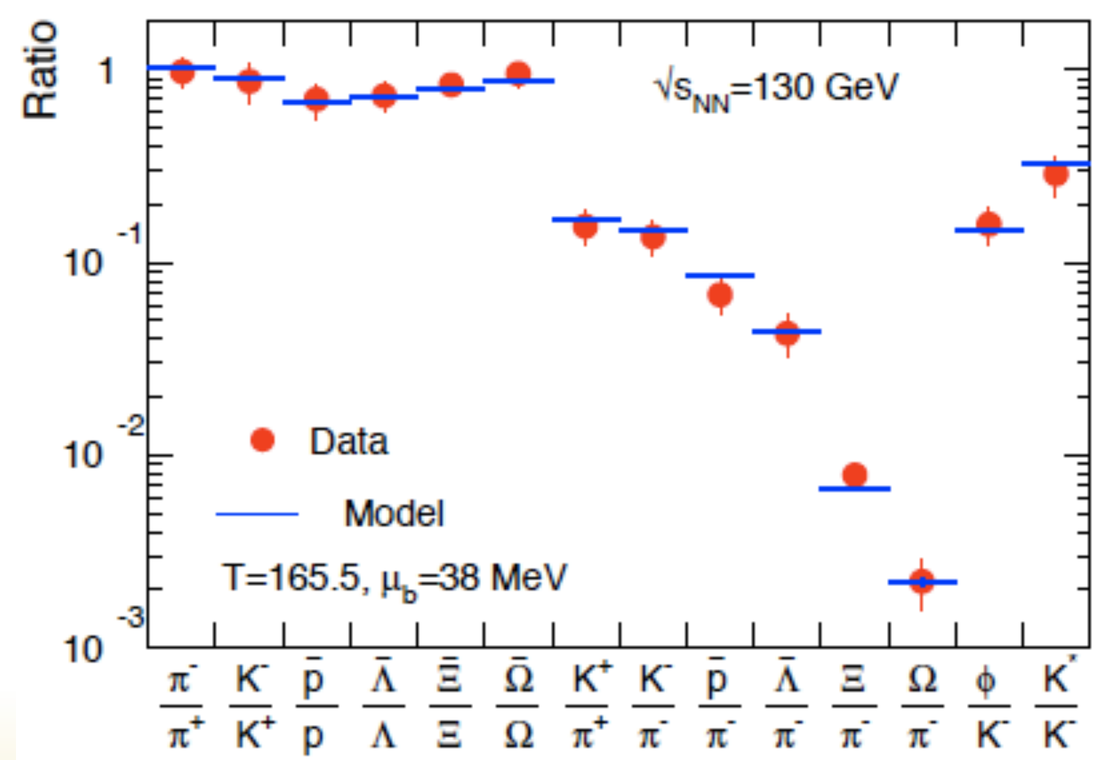
Lattice transition line:

[Aoki et al hep-lat/0609068]
[Endrodi et al 1102.1356]
[Kaczmarek et al 1011.3130]

Freeze-out curve:

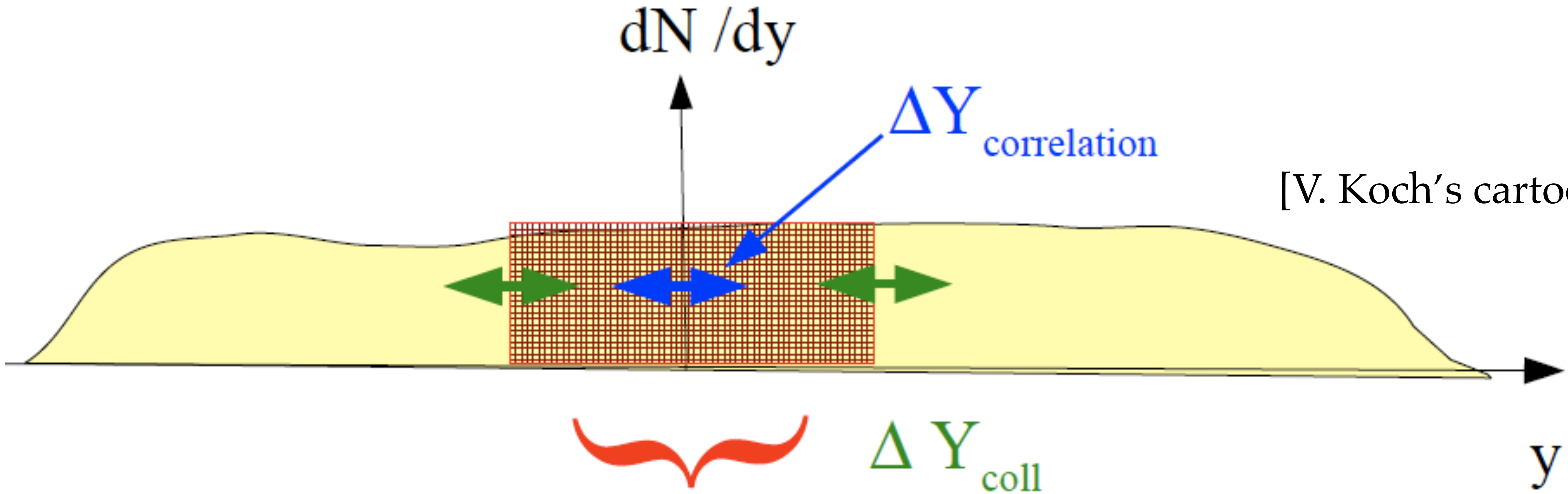
A model of free hadrons (with T=0 masses) is fitted on the ratios of particel yields.

This (grand canonical) thermal model fits well and gives a (T,μ) pair for each collision energy.



Thermal models: [Andronic et al nucl-th/0511071]
[Cleymans et al hep-ph/0511094]

Fluctuations of the net charge



[V. Koch’s cartoon]

A conserved quantum number is counted for a subsystem. ***(e.g. net electric charge)***
For each event (individual collision) this net quantum number reflects the instant when the hadrons were formed.

The acceptance condititon must be sufficiently
- narrow, to allow to approximate a grand canonical ensemble
- wide, to allow all correlation lengths characteristic to a ***grand canonical ensmeble***

Experiment measures the distribution of the net charge (mean, variance, skewness, kurtosis...)
Lattice calculates these moments in a grand canonical ensemble.

O(V) cancellation

$$\langle N_X^2 \rangle - \langle N_X \rangle^2 = \frac{\partial^2 \log Z}{(\partial \mu_X / T)^2}$$

Fluctuations on the lattice

A derivative of log Z acts on the fermion determinant inside of the effective gauge action:

$$\partial_i \log Z = \frac{1}{Z}\int \mathcal{D}U\ \partial_i e^{-S_{\text{eff}}} = \langle A_i \rangle$$

Through further derivatives disconnected diagrams appear alongside with higher operators.

$$\begin{aligned}\partial_j \langle X \rangle &= -\langle X \rangle (\partial_j \log Z) + \left\langle X \partial_j e^{-S_{\text{eff}}} \right\rangle + \langle \partial_j X \rangle \\ &= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \langle \partial_j X \rangle \ .\end{aligned}$$

(X is one of A,B,C ...)

Disconnected: *noisy*

Connected: sensitive to taste breaking

$$\begin{aligned}
A_j &= \tfrac{d}{d\mu_j} \log(\det M_j)^{1/4} = && \tilde{\text{tr}} M_j^{-1} M_j' \,, \\
B_j &= \tfrac{d^2}{(d\mu_j)^2} \log(\det M_j)^{1/4} = && \tilde{\text{tr}} \left(M_j'' M_j^{-1} - M_j' M_j^{-1} M_j' M_j^{-1} \right) \,, \\
C_j &= \tfrac{d^3}{(d\mu_j)^3} \log(\det M_j)^{1/4} = && \tilde{\text{tr}} \left(M_j' M_j^{-1} - 3 M_j'' M_j^{-1} M_j' M_j^{-1} \right. \\
& && \left. + 2 M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} \right) \,, \\
D_j &= \tfrac{d^4}{(d\mu_j)^4} \log(\det M_j)^{1/4} = && \tilde{\text{tr}} \left(M_j'' M_j^{-1} - 4 M_j' M_j^{-1} M_j' M_j^{-1} \right. \\
& && - 3 M_j'' M_j^{-1} M_j'' M_j^{-1} + 12 M_j'' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} \\
& && \left. - 6 M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} M_j' M_j^{-1} \right) \,,
\end{aligned}$$

These traces are evaluated with several (N=O(1000)) random sources, for each gauge configuration:

$$A = \frac{1}{N} \sum_i \chi_i^+ M^{-1} M' \chi_i$$

a conjugate gradient solver

Fluctuations on the lattice

A derivative of log Z acts on the fermion determinant inside of the effective gauge action:

$$\partial_i \log Z = \frac{1}{Z}\int \mathcal{D}U\ \partial_i e^{-S_{\rm eff}} = \langle A_i \rangle$$

Through further derivatives disconnected diagrams appear alongside with higher operators.

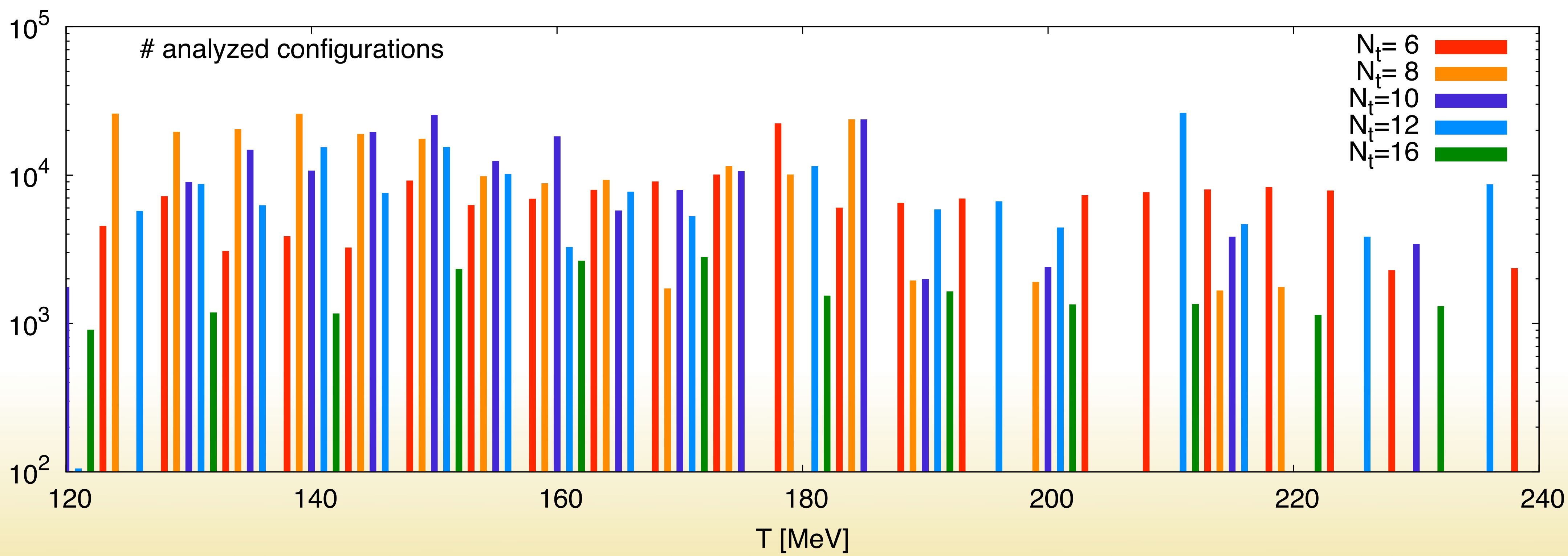
$$\begin{aligned}\partial_j \langle X \rangle &= -\langle X \rangle (\partial_j \log Z) + \left\langle X \partial_j e^{-S_{\rm eff}} \right\rangle + \langle \partial_j X \rangle \\ &= \langle X A_j \rangle - \langle X \rangle \langle A_j \rangle + \langle \partial_j X \rangle \ .\end{aligned}$$

(X is one of A,B,C ...)

Disconnected: *noisy*

Connected: sensitive to taste breaking

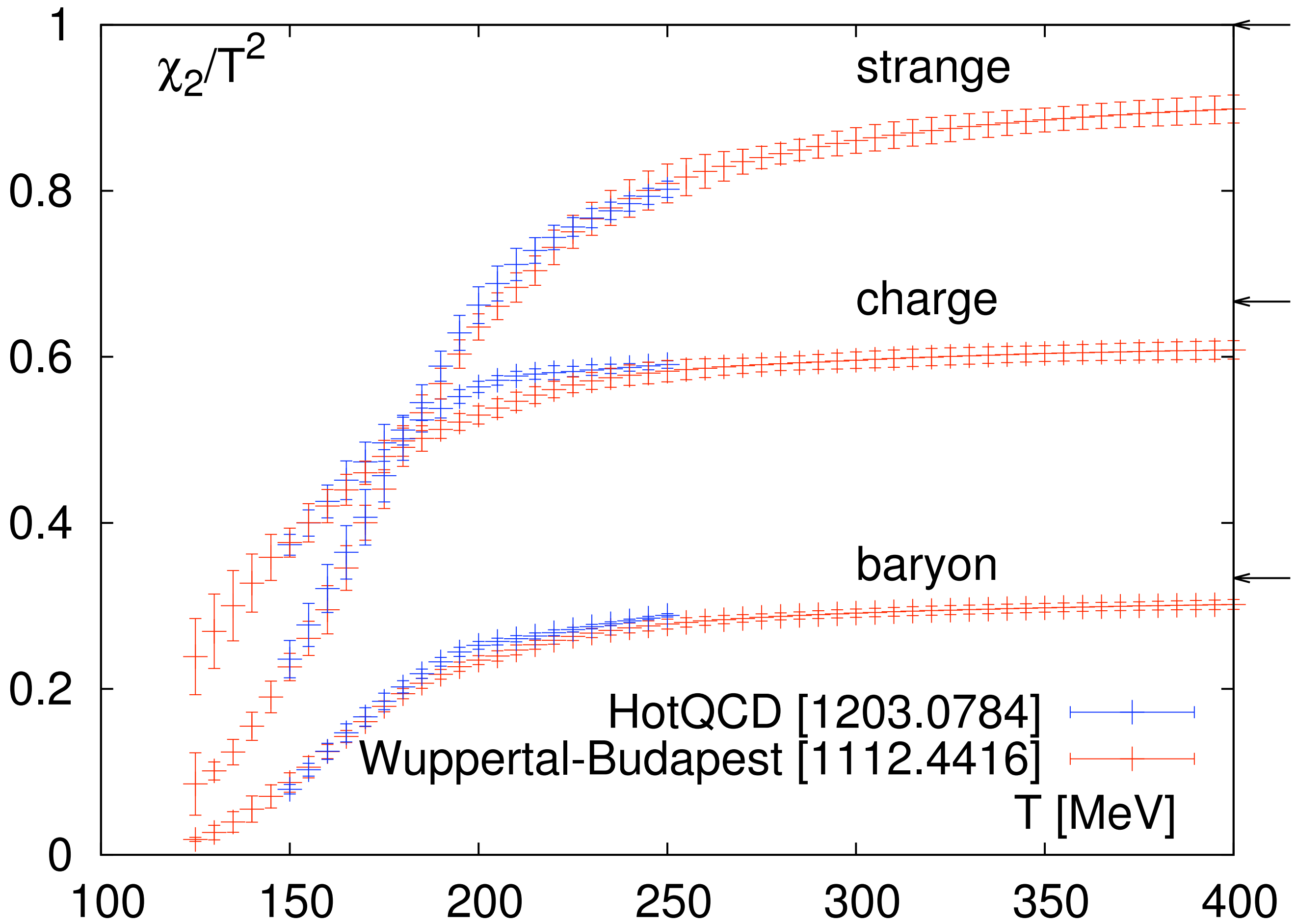
T>0 simulations with 2-stout staggered fermions:



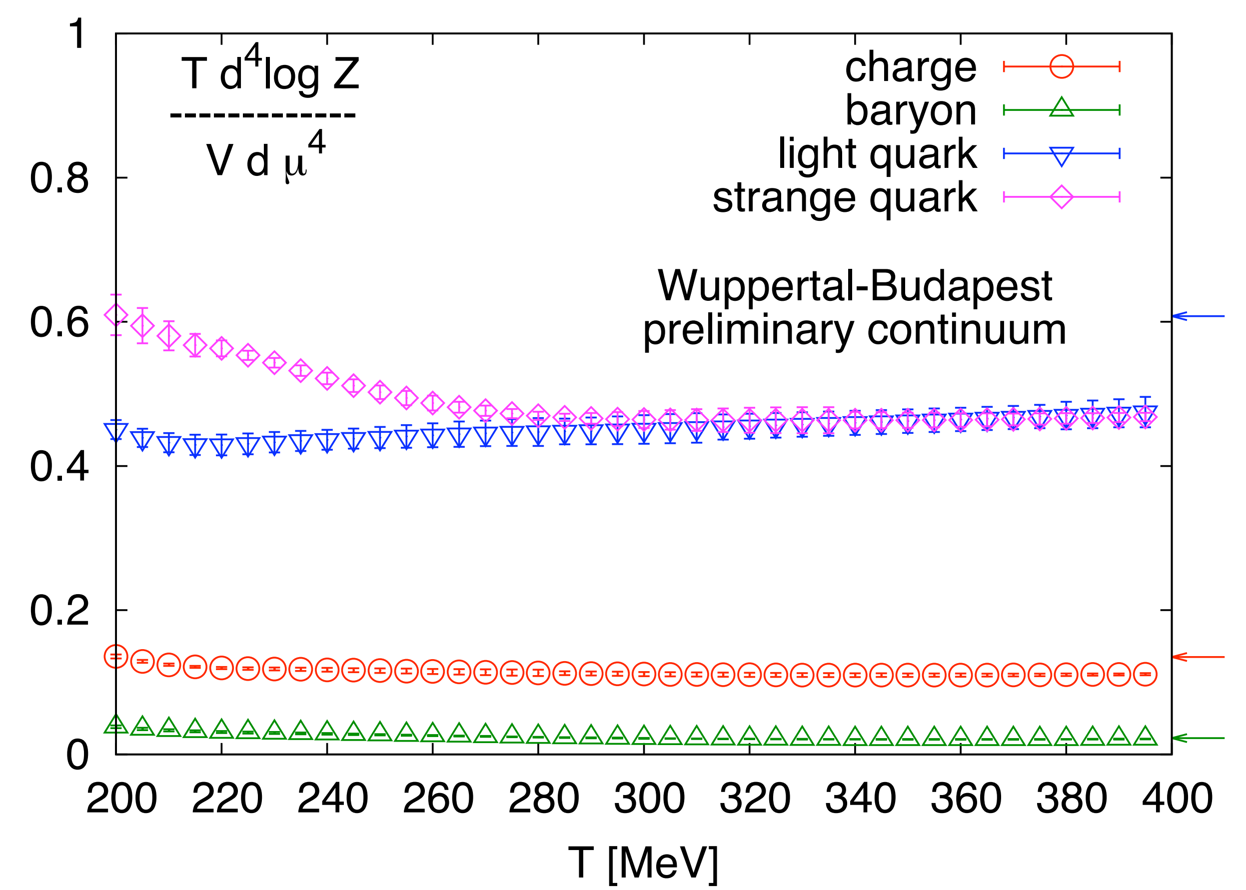
6

Results in the continuum

Comparison of the published continuum results:



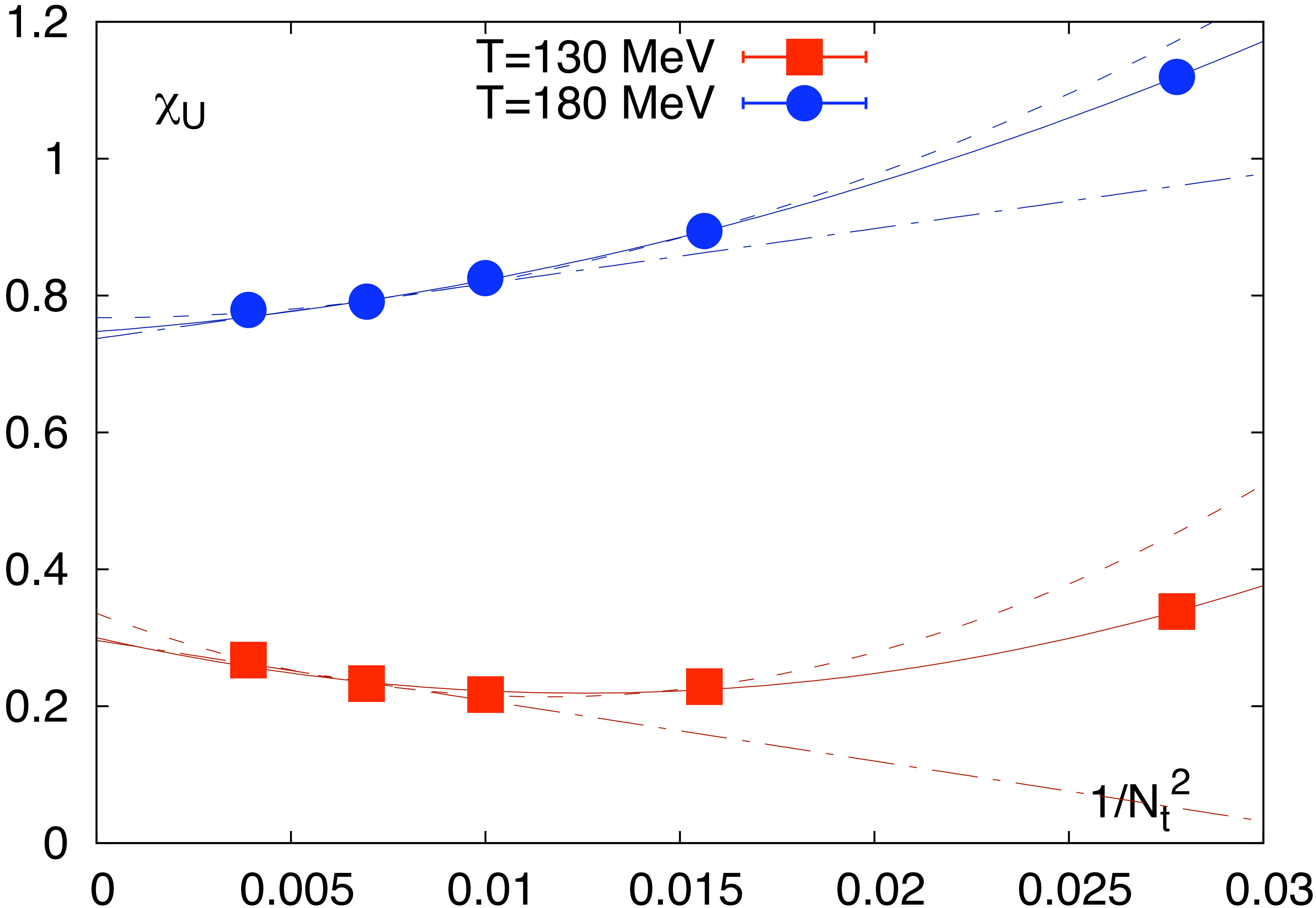
Fourth moment at high T



[preliminary WB data: 1210.6901]

[discussion at hight T see Peterczky Mon 15.00]

Continuum extrapolation



8

Looking for a thermometer

Assumption:

Before freeze-out the system is described with a time-dependent temperature and baryo, charge and strange chemical potentials.
After freeze-out the net bayron, charge and strangeness reflects the system an equilibrium well-defined freeze-out tempearture.

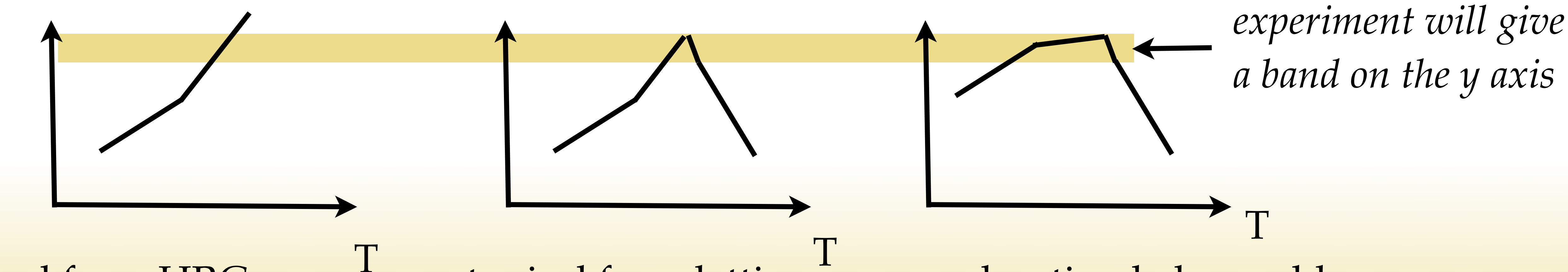
This picture was supported by the statistical models.

$$\hat{\chi}_4^X = \frac{1}{VT^3}\frac{\partial^4 \log Z}{(\partial \mu_X/T)^4} \qquad \hat{\chi}_2^X = \frac{1}{VT^3}\frac{\partial^2 \log Z}{(\partial \mu_X/T)^2}$$

Thermometer:

- experimentally accessible (ratios to cancel the volume factor)
- monotonic in T
- known from theory

[Karsch 1202.4173]



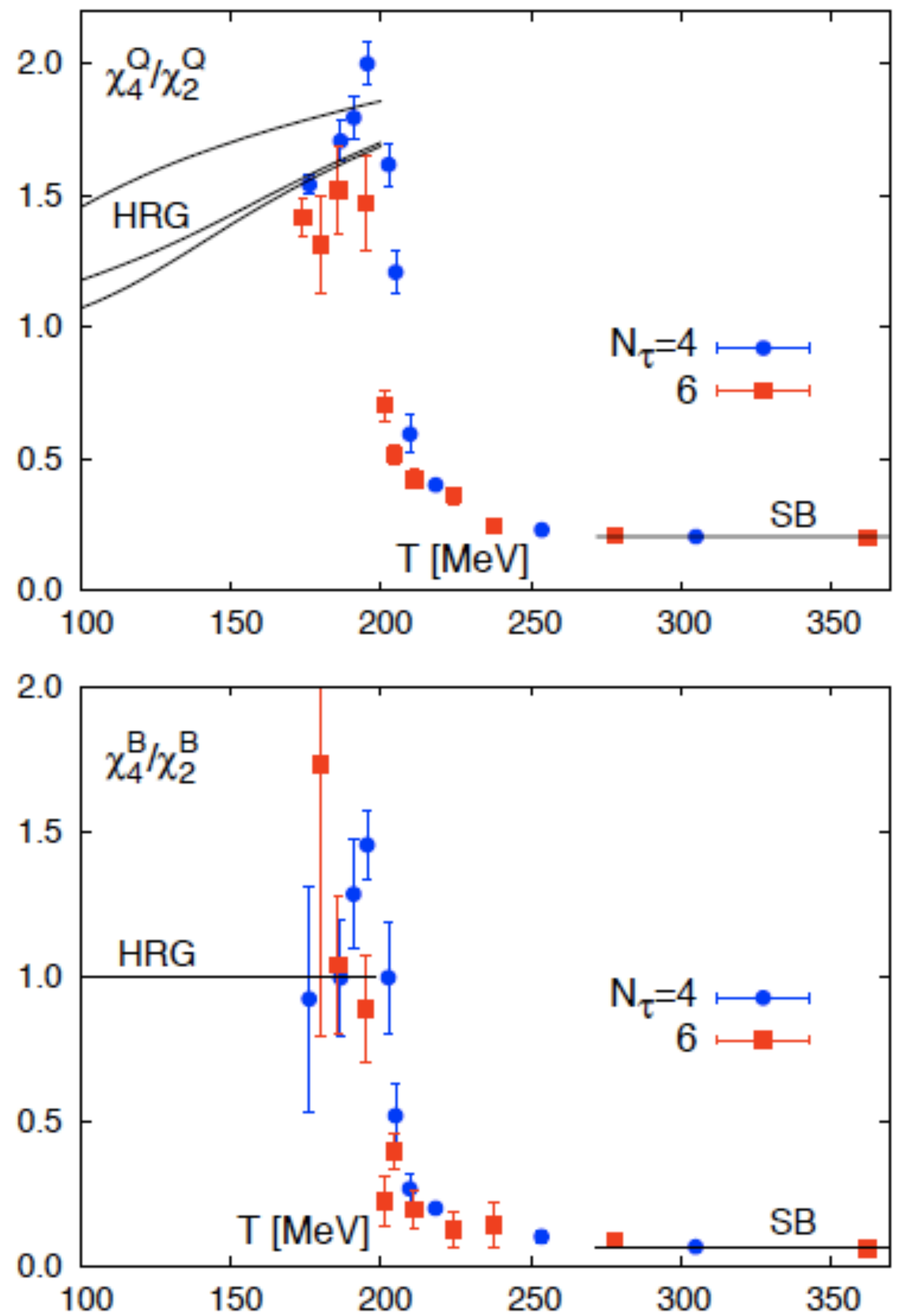
typical from HRG | typical from lattice | suboptimal observable

Charge and baryon kurtosis

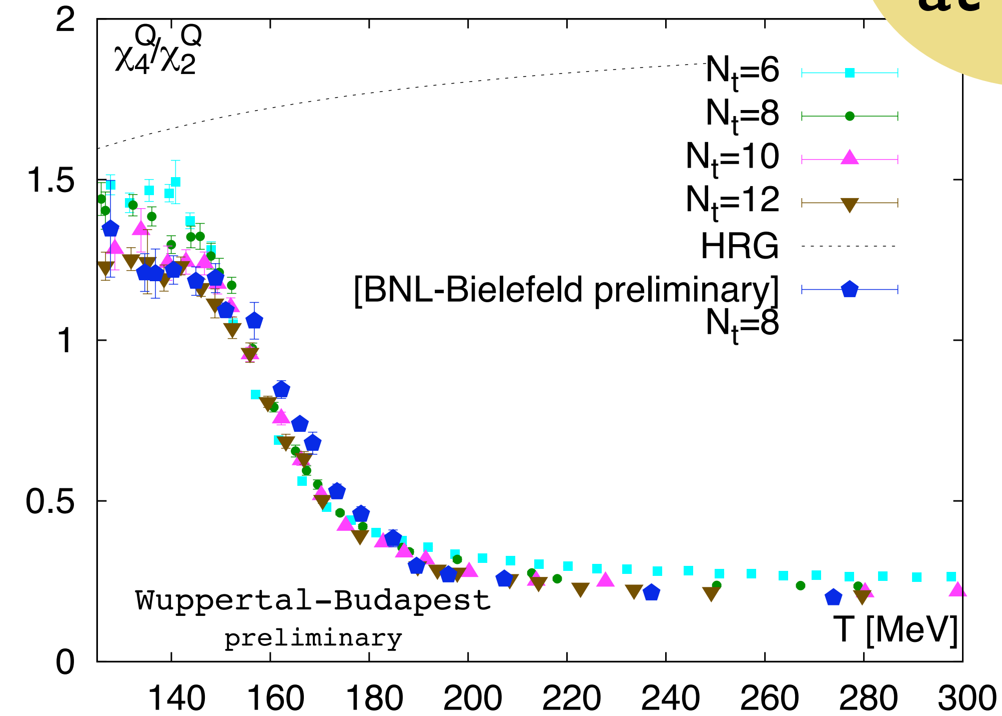
Useful at LHC

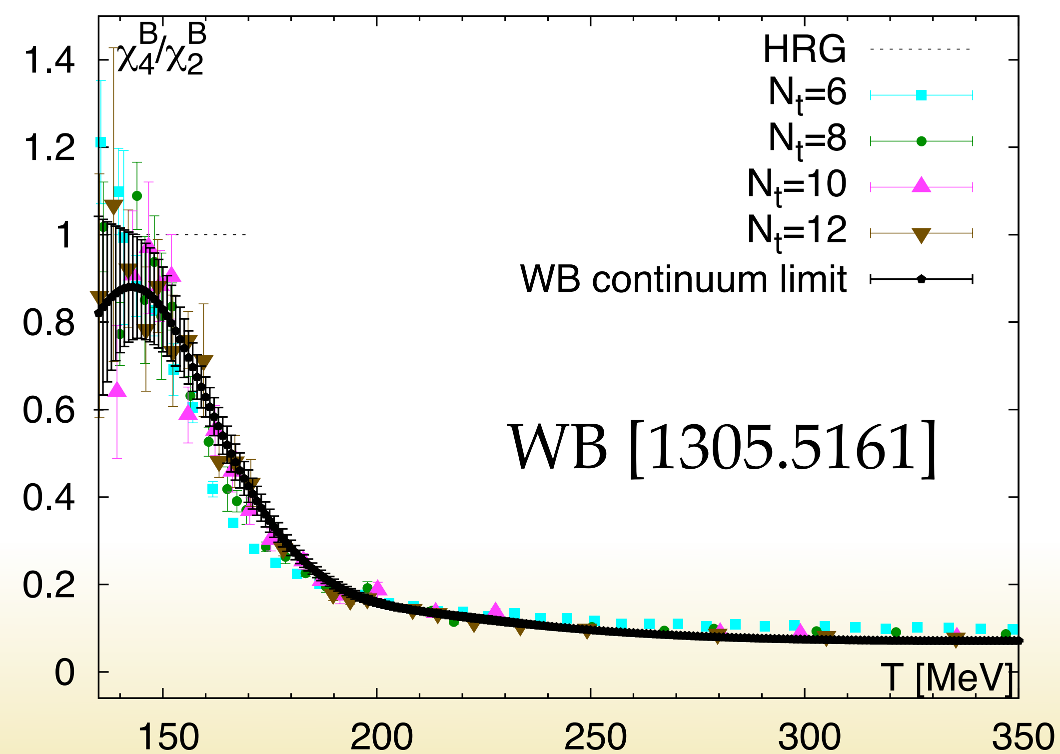
old data (2008)

RBC-Bielefeld [0811.1006]



new data (2012)





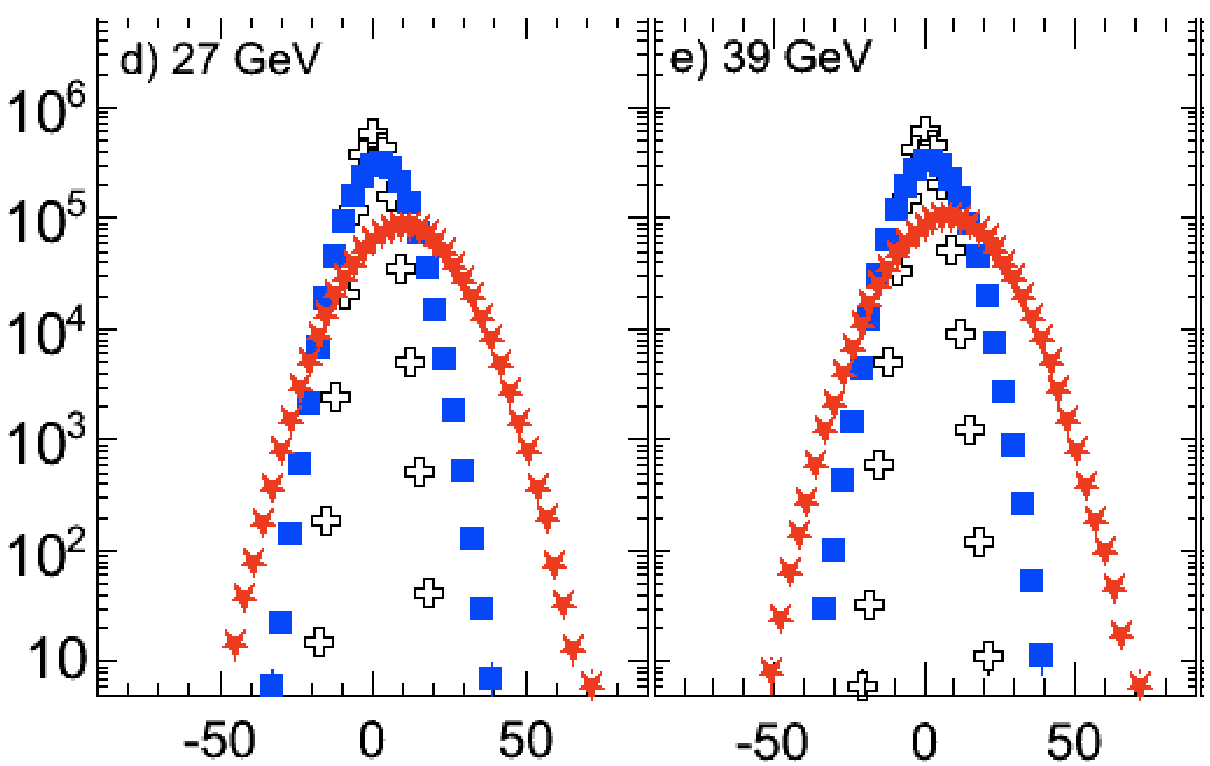
Charge skewness at RHIC

RHIC:

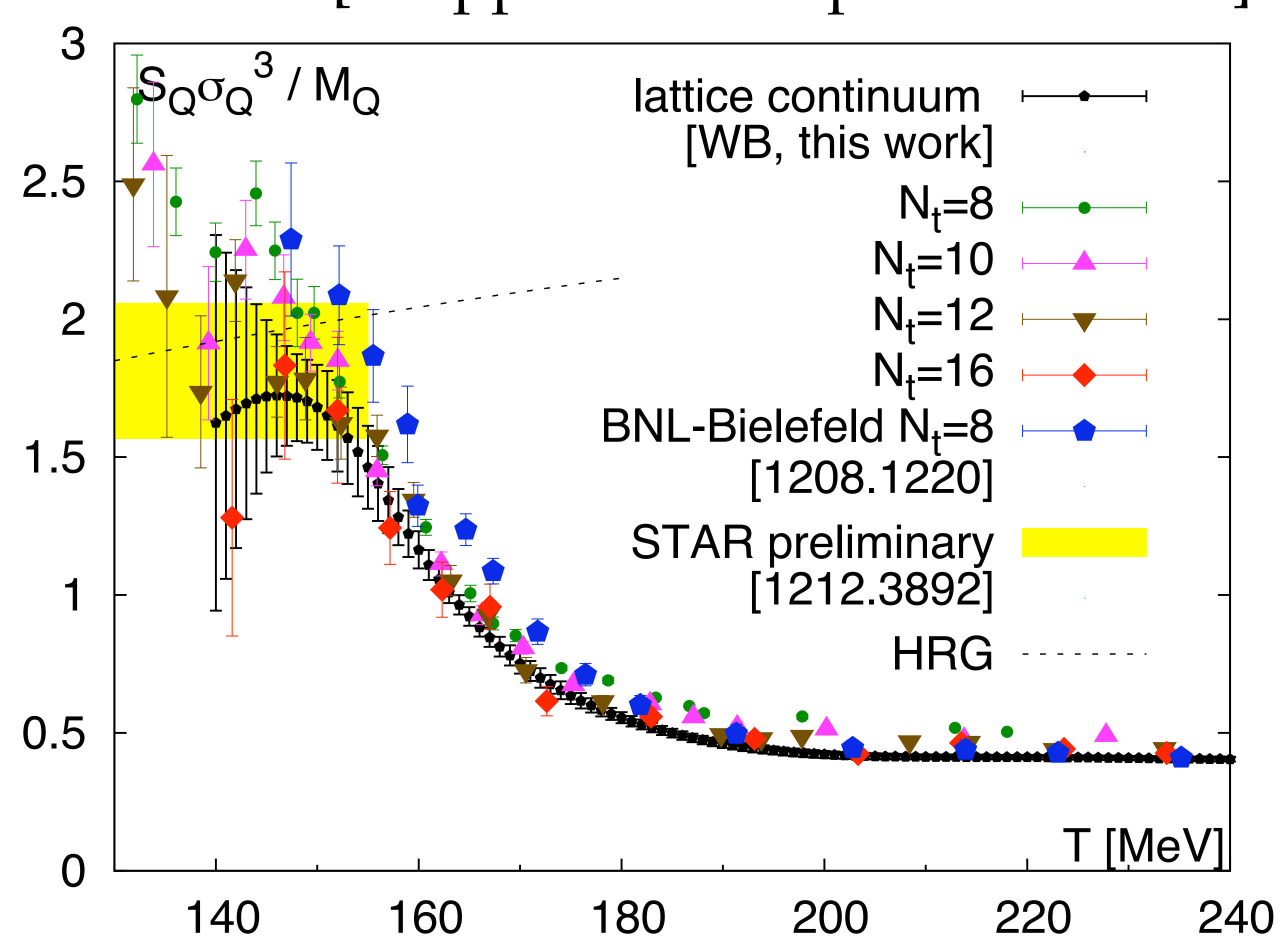
- Lattice data must be extrapolated to μ>0 keeping the onstraints: **<S>=0 and <Q>=Z/A <B>**

\+ **Mean** and **skewness** are zero at LHC (μ=0) but **nonzero** here → they define a thermometer.

[STAR 1212.3892]
net charge distribution
(most central in red)



Lattice: [Wuppertal-Budapest 1305.5161]

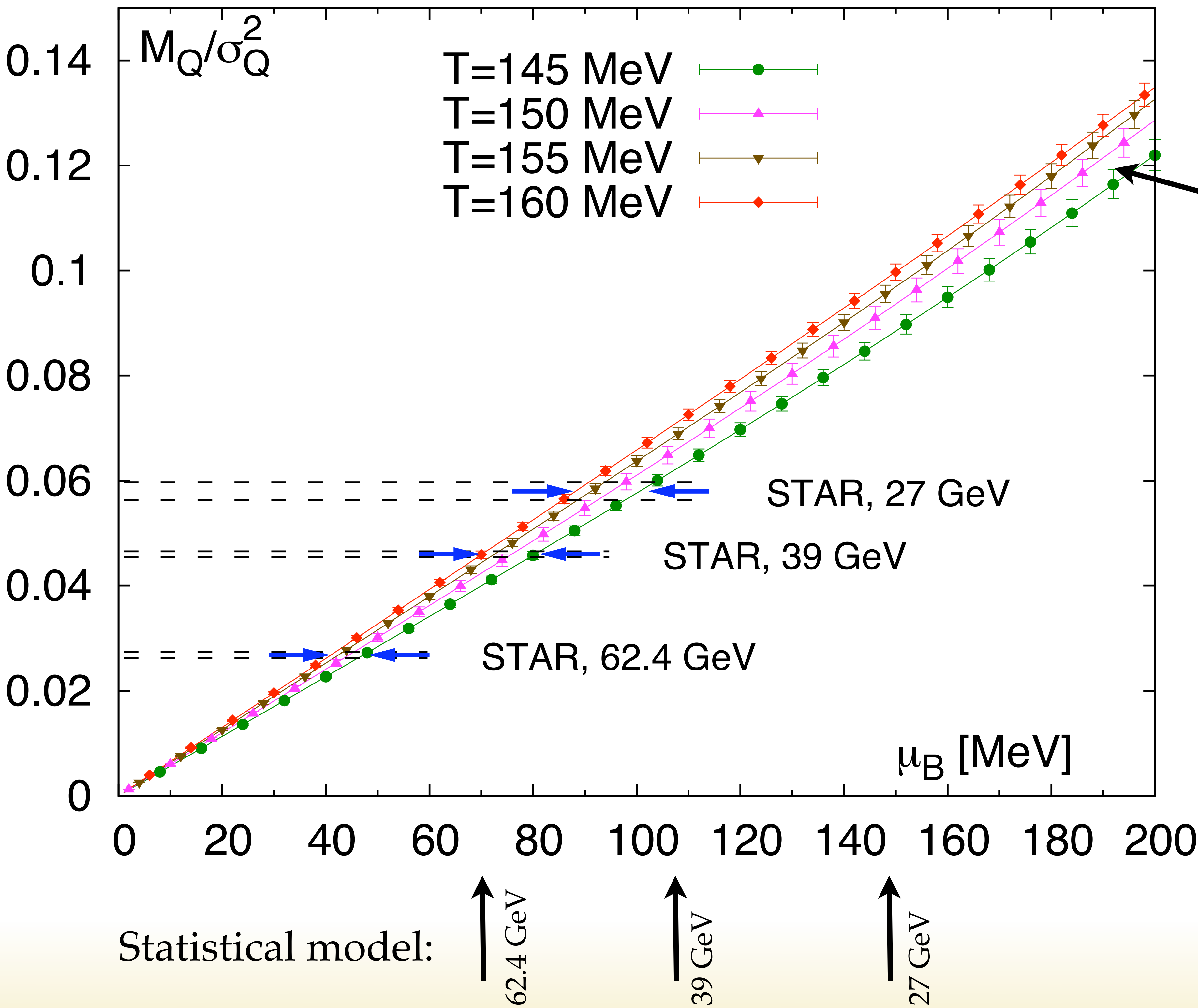


suggested and calculated in
[BNL-Bielefeld 1208.1220]

Result: T < 157 MeV

[continuum limit and WB data: 1305.5161]

Baryometer of the electric charge



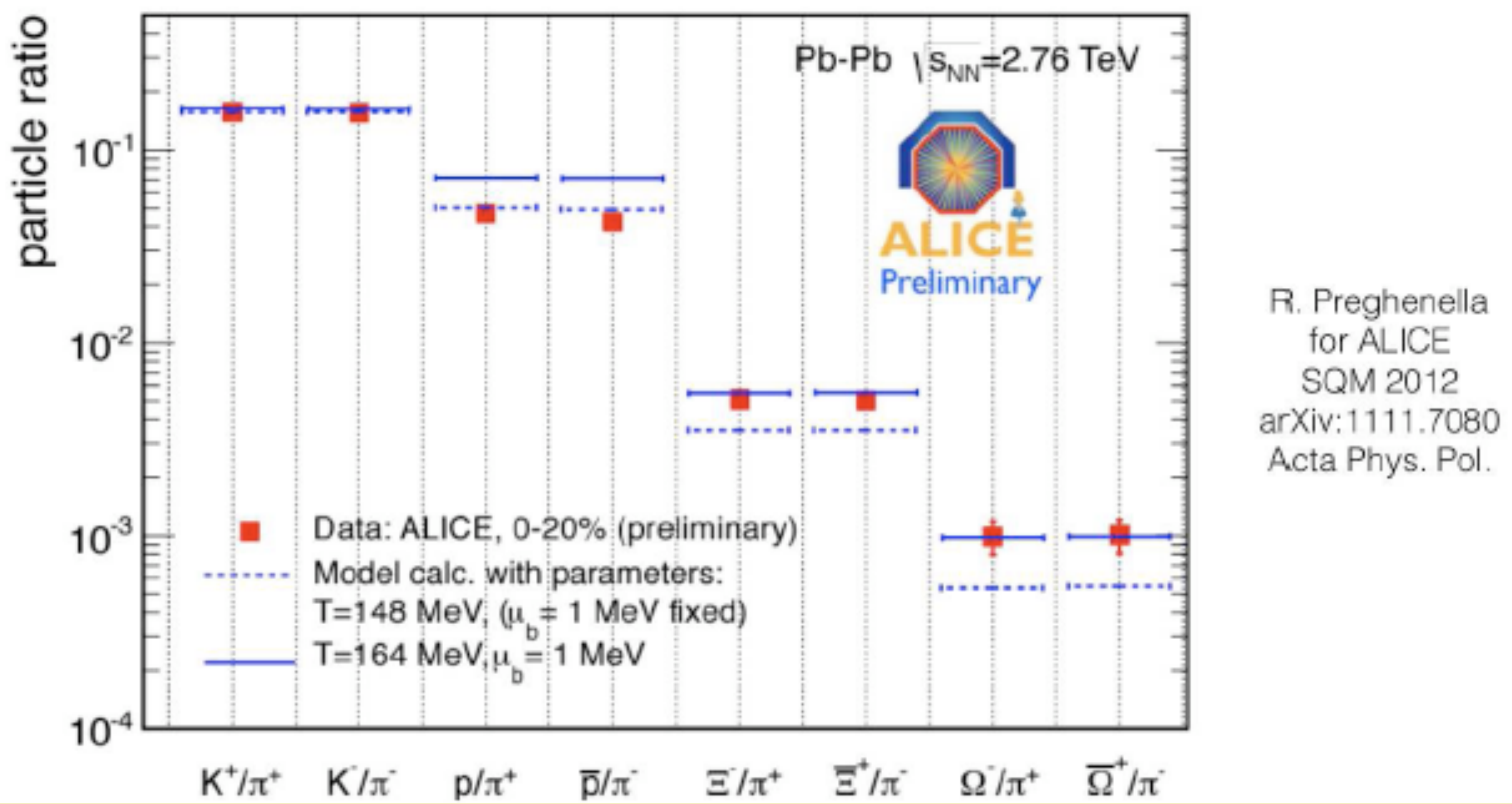
Continuum extrapolated lattice data from Wuppretal - Budapest [1305.5161]

$\sqrt{s}[GeV]$	μ_B^f [MeV]
62.4	44(3)(1)(2)
39	75(5)(1)(2)
27	95(6)(1)(5)
	$()_{\delta T}()_{\text{lat}}()_{\text{exp}}$

[based on the proposal in 1202.4173 & 1208.1220 of the BNL-Bielefeld group]

Limitations of this picture

- Experiments (Phenix vs. Star) do not yet agree on fluctuations, thus the freeze-out result is not final yet. *(Final state interactions must be modelled)*
- The results from baryon and charge fluctuations are inconsistent (not baryon fluctuations but proton fluctuations are measured, the protons do not really form a grand canonical ensemble) [see Wagner Mon 15.20]
- We assumed that all degrees of freedom in the quark gluon plasma is turned into hadronic matter at a unique temperature. (The heavier strange might freeze out earlier)



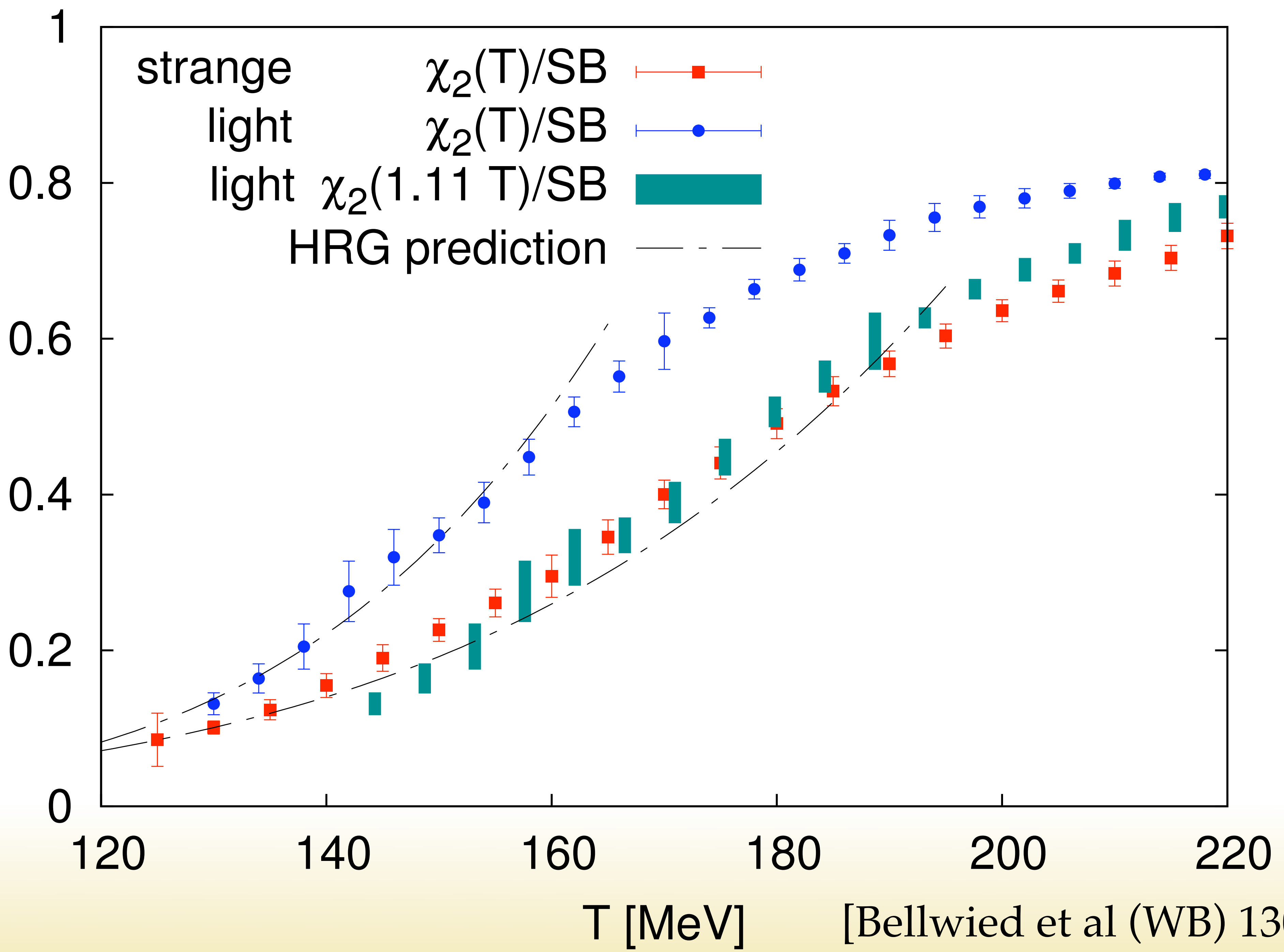
R. Preghenella for ALICE SQM 2012 arXiv:1111.7080 Acta Phys. Pol.

Statistical model fit (gas of free hadrons) show inconistent results for strange / non-strange yield ratios at LHC.

13

Flavor sensitivity in the fluctuation

Strange susceptibility vs light quark susceptibility:
about 16 MeV difference in the characteristic point.

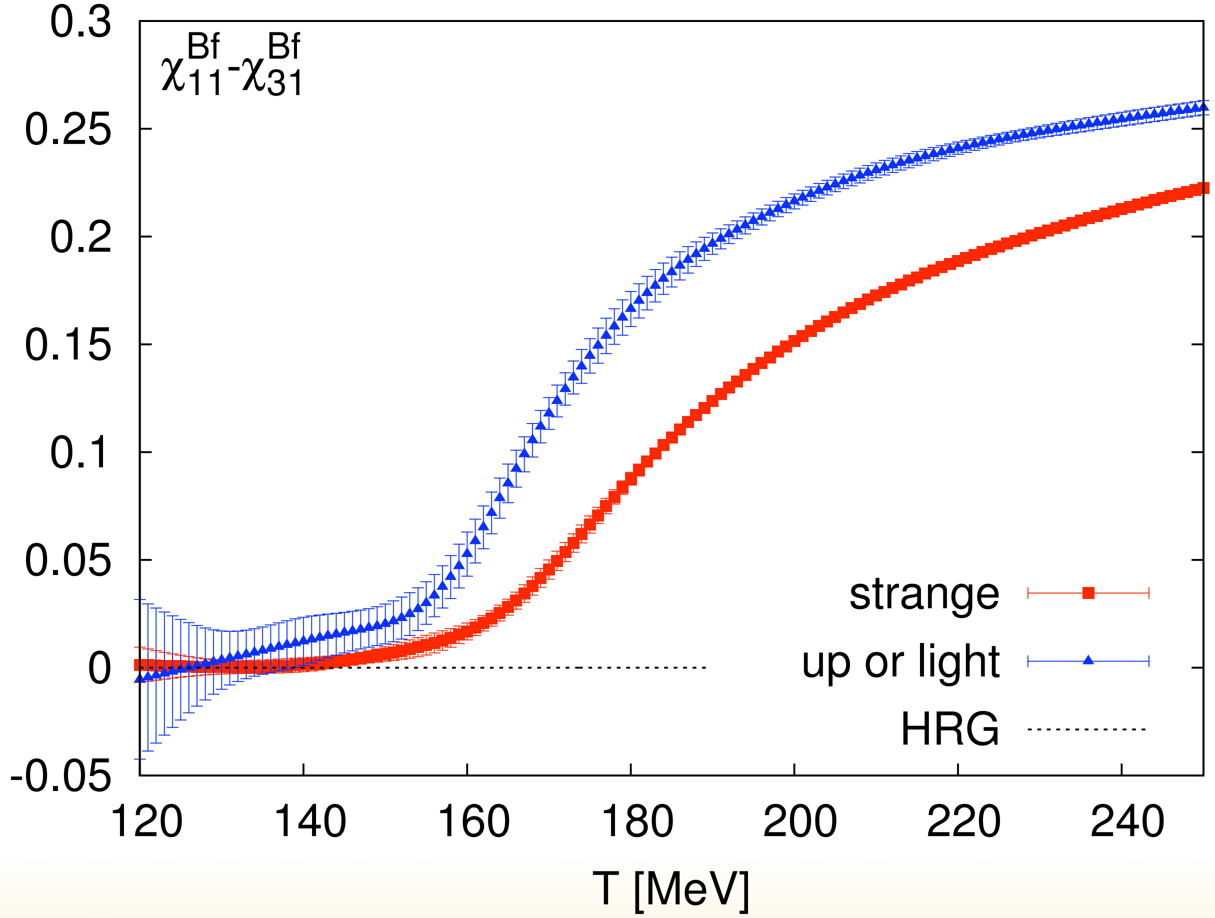


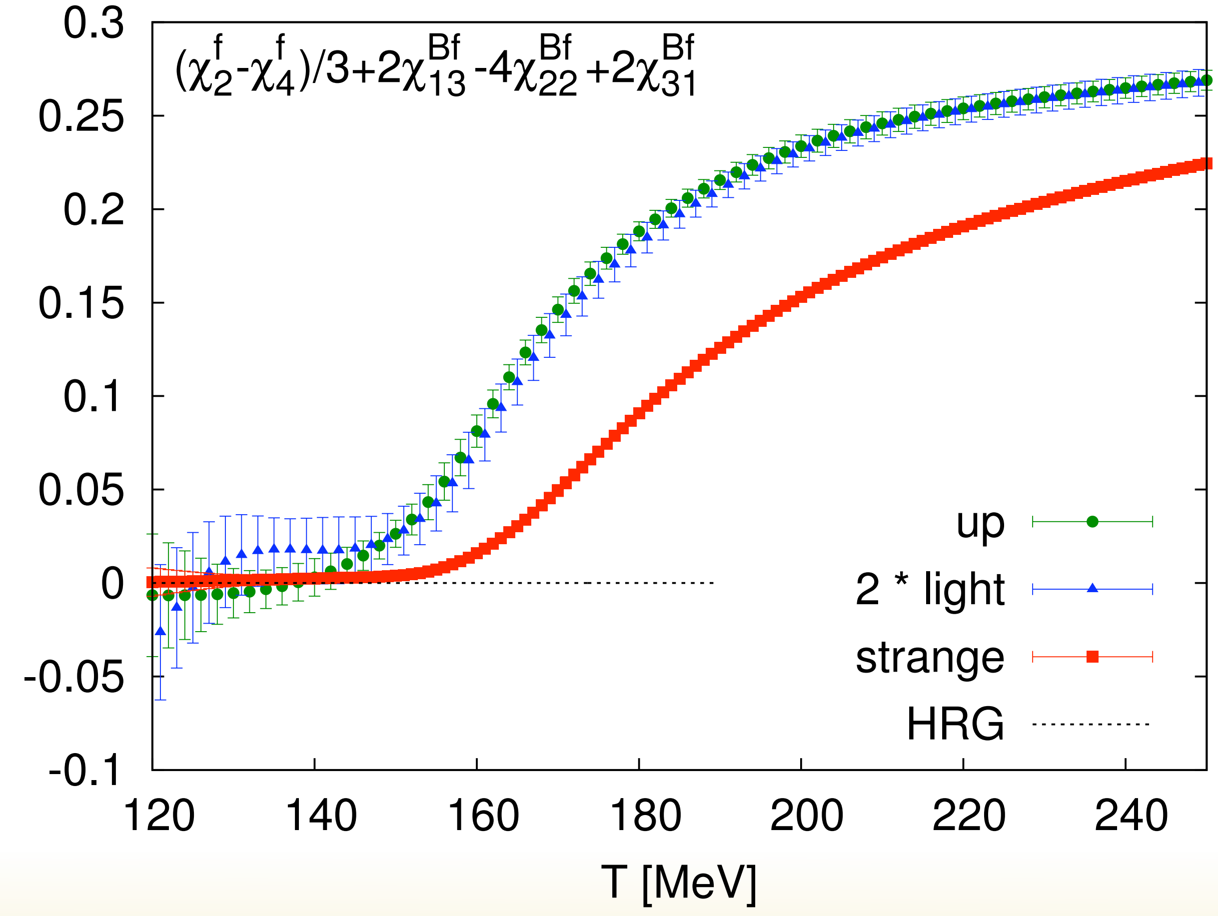
[Bellwied et al (WB) 1305.6297]

Two indicators of deconfinement

In [1304.7220] the BNL-Bielefeld group has suggested two combinations of fluctuations that are nonzero only if a non-hadronic strange degree of freedom is excited.
We extended this to the light flavors and calculated the continuum limit.

These combinations are constructed such that a free gas of baryons or mesons give zero, but a free quark would give a non-zero contribution. [Schmidt Tue 15.00]

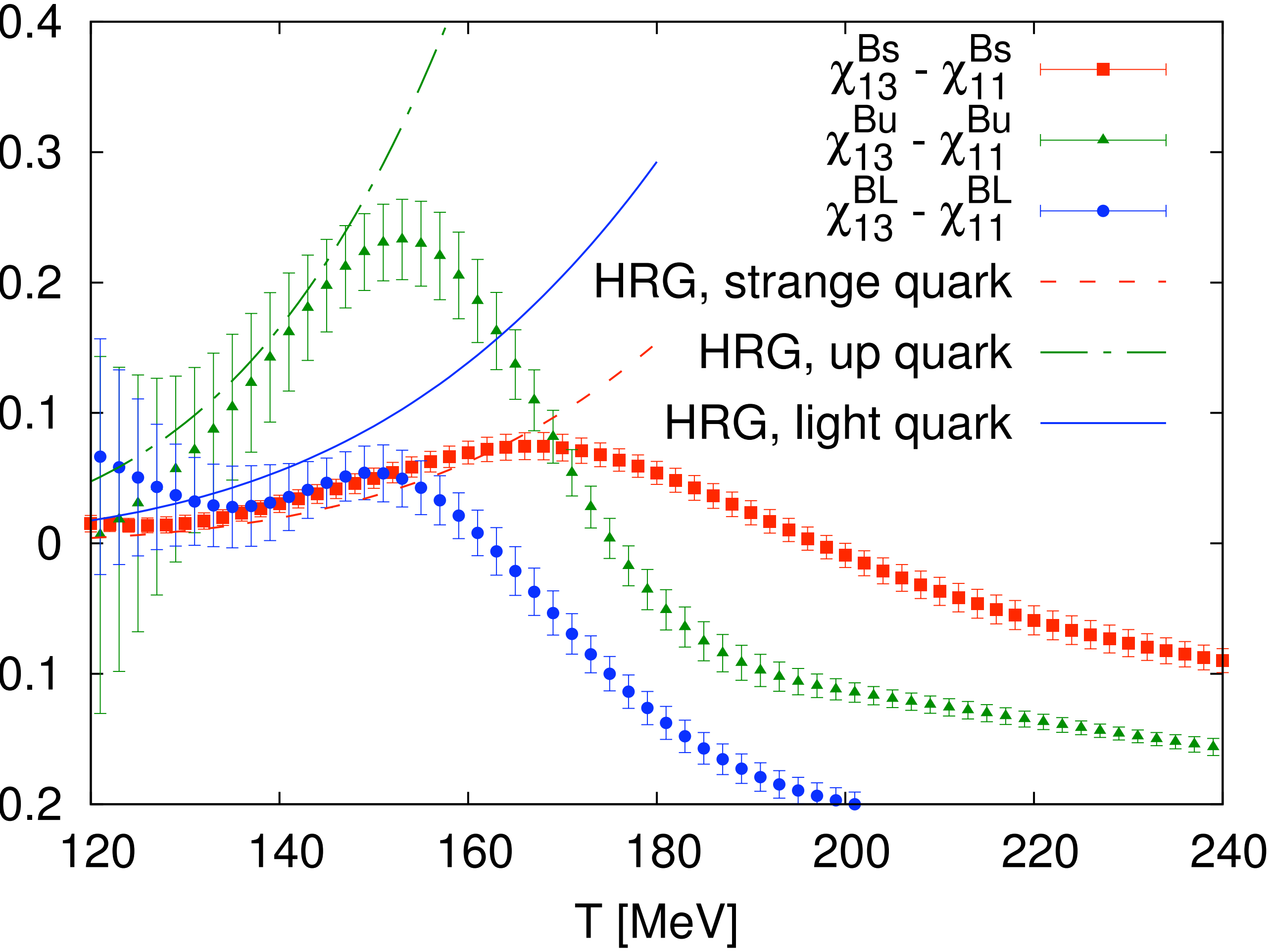


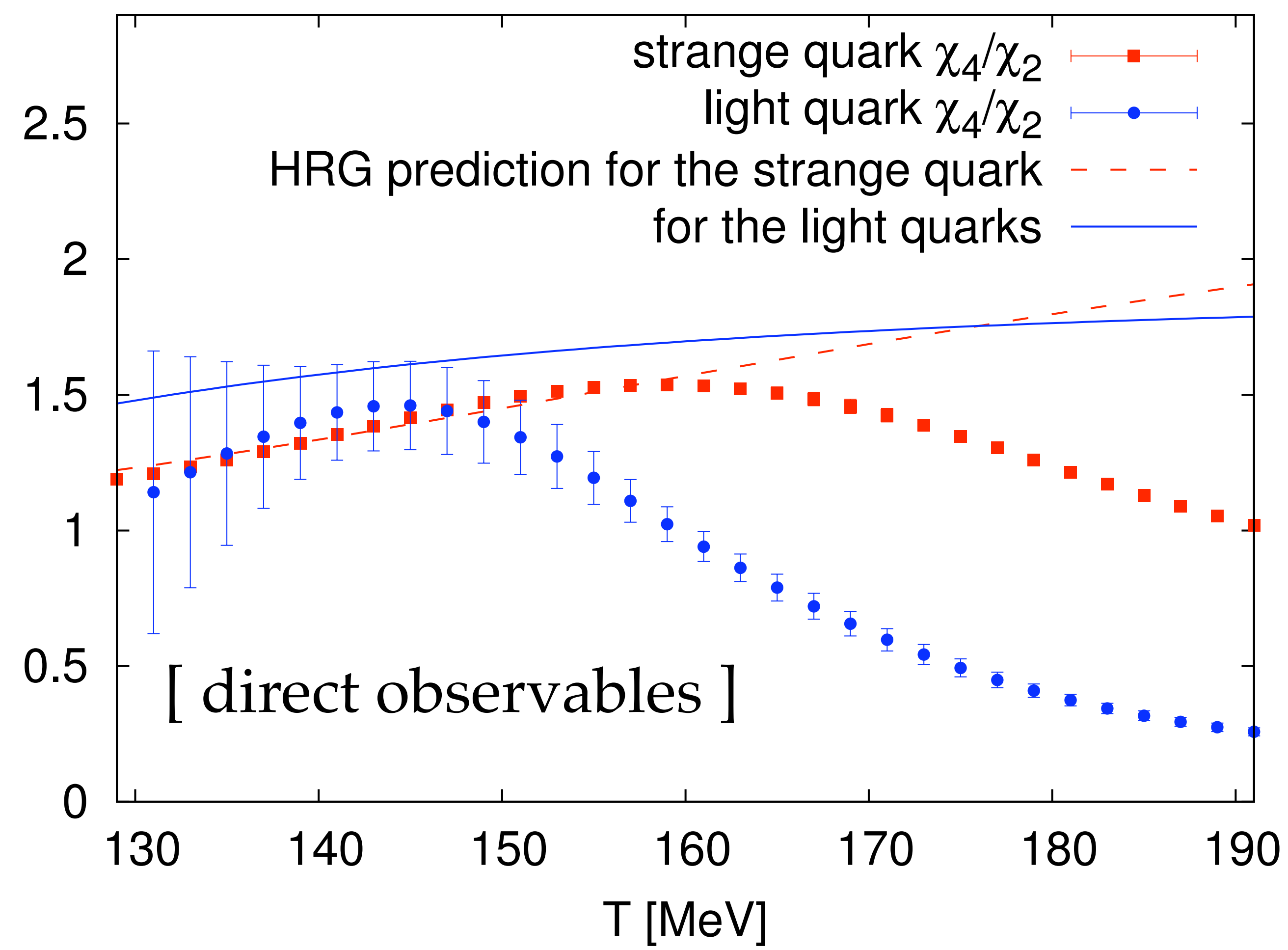


[Bellwied et al (WB) 1305.6297]

Multi-strange observables

It is possible to construct other observables that are dominated by the multi-strange hadrons. Here the flavor separation is the strongest.





The Hadron Resonance Gas prediction is non-trivial here.

Observation: **the lattice data has a kink where it departs from the HRG result**.

A possible scenario

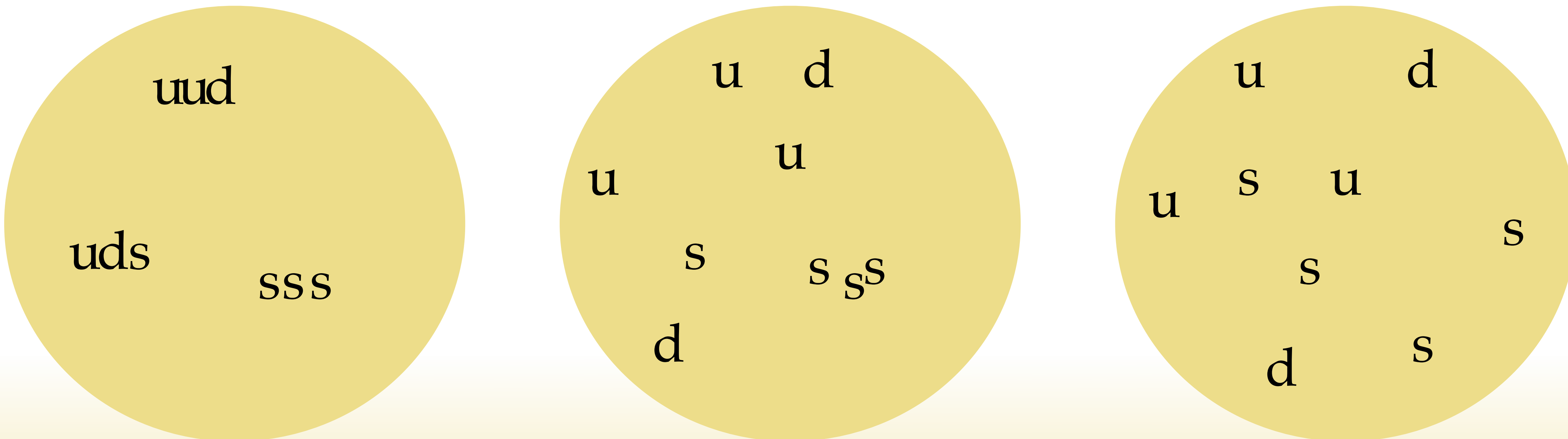
Evidence:

- Lattice observables are sensitive to quark mass (multi-strange: stronger sensitivity)
- Last point of agreement to Hadron Resonance Gas is flavor dependent
- Fit to yields shows a preference to flavor dependent freeze-out

No evidence:

- lattice cannot say anything about freeze-out
- Tc is not necessary the peak/inflection point of some susceptibility curve

A picture:



T < 150 MeV   150 MeV < T < 165 MeV   T > 165 MeV

Summary

- The **width of net charge/strangeness/baryon** distribution is predicted in the continuum limit by both the HotQCD and Budapest-Wuppertal collaborations.

- Higher cumulants (**curtosis**) have been presented, comparison to experiment is now possible

- Ratios of fluctuations have been used to define freeze-out thermometers and bayrometers.

- In several cases preliminary data shows significant deviations from the simplest HRG result, even below T_c.

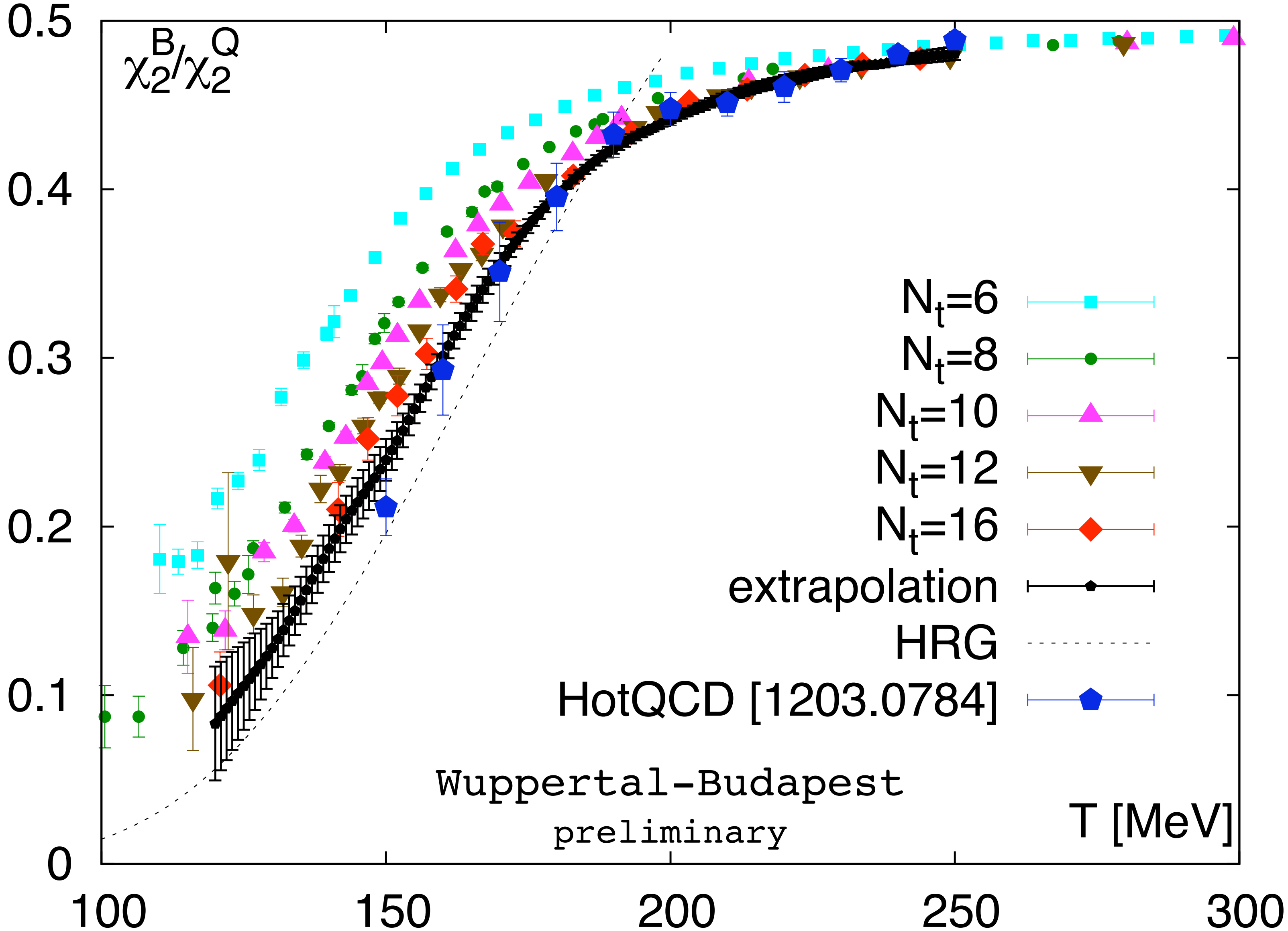
- Experimentally accessible flavor sensitive observables have been calculated.
  Is there a flavor hierarchy in the deconfinement transition of QCD?

All data: based on continuum extrapolated lattice resuts from the Wuppertal-Budapest collaboration.

[1112.4416] [1204.6710] [1210.6901] [1305.5161] [1305.6297]

Spare slides

Baryon/charge ratio

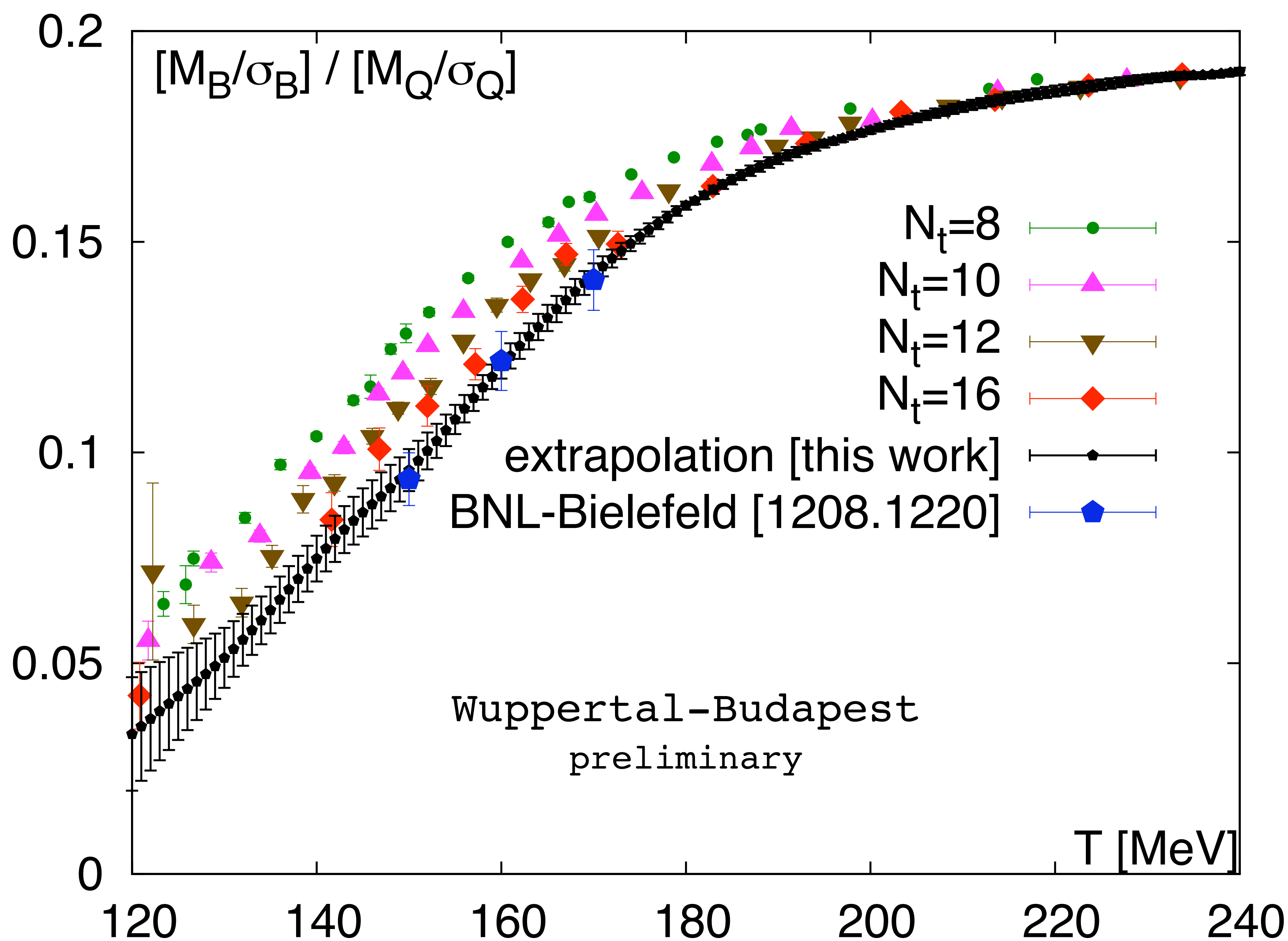


Steep T-dependence throughout the relevant temperature range.

It is based on 2nd order derivatives (continuum limit is feasible), precise in experiment.

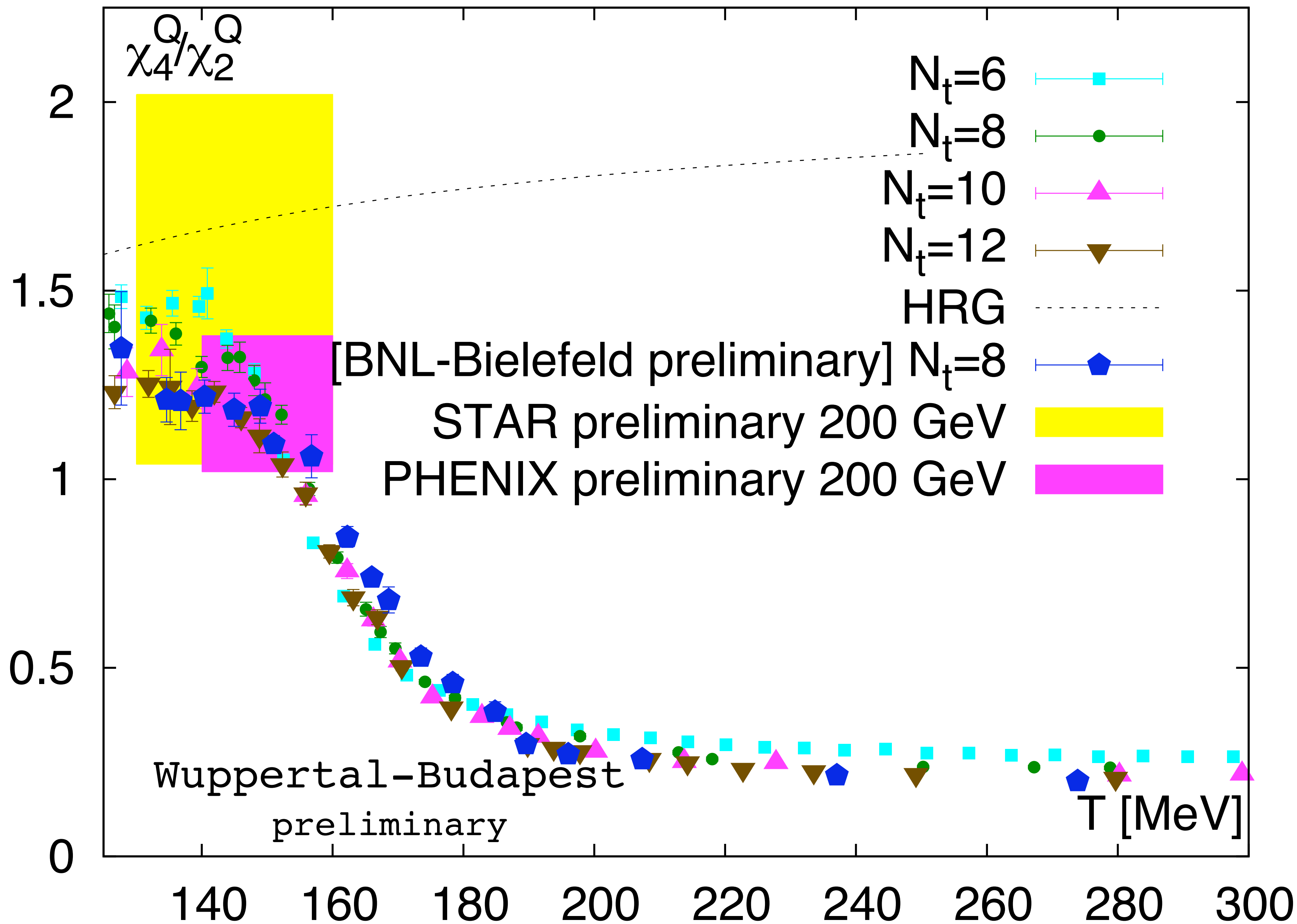
Mixes the baryon and charge systematics of the experiment.

Ratio of ratios as a thermometer



If the baryon number could be measured reliably, a thermometer could be constructed based on 2nd order fluctuations: "almost trivial" both for lattice and for experiment.

Charge thermometer?



[PHENIX: J Mitchell QM12] [BNL-Bielefeld: C Schmidt 1212.4278]

[STAR: N R Sohoo 1212.3892] [preliminary WB data: 1210.6901]

Hadron Resonance Gas

The hadron resonance gas (HRG) model discribes a mixture of free hadrons:
all mesons and baryons and their excited states you find in the particle data book.

[Dashen,Ma,Bernstein 1969]

$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{VT^3} \sum_{i \in\, mesons} \log \mathcal{Z}^M(T, V, \mu_{X^a}, m_i) + \frac{1}{VT^3} \sum_{i \in\, baryons} \log \mathcal{Z}^B(T, V, \mu_{X^a}, m_i)$$

$$\begin{aligned} \ln \mathcal{Z}^{M/B}_{M_i} &= \mp \frac{V d_i}{2\pi^2} \int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \\ &= \frac{VT^3}{2\pi^2} d_i \left(\frac{M_i}{T}\right)^2 \sum_{k=1}^{\infty} (\pm 1)^{k+1} \frac{z_i^k}{k^2} K_2(kM_i/T) \end{aligned} \qquad \varepsilon_i = \sqrt{k^2 + m_i^2}$$

Degeneracy factor d_i: spin, etc, chemical potentials enter through $z_i = \exp\left((\sum_a X_i^a \mu_{X^a})/T \right)$

This model is a good approximation to QCD in the hadronic phase:

low T: mostly pions, they interact very weakly in QCD and chiPT

[Grasser&Leutwyler 1984] [Greber&Leutwyler 1989]

higher T: interactions are included through the growing number of resonances.

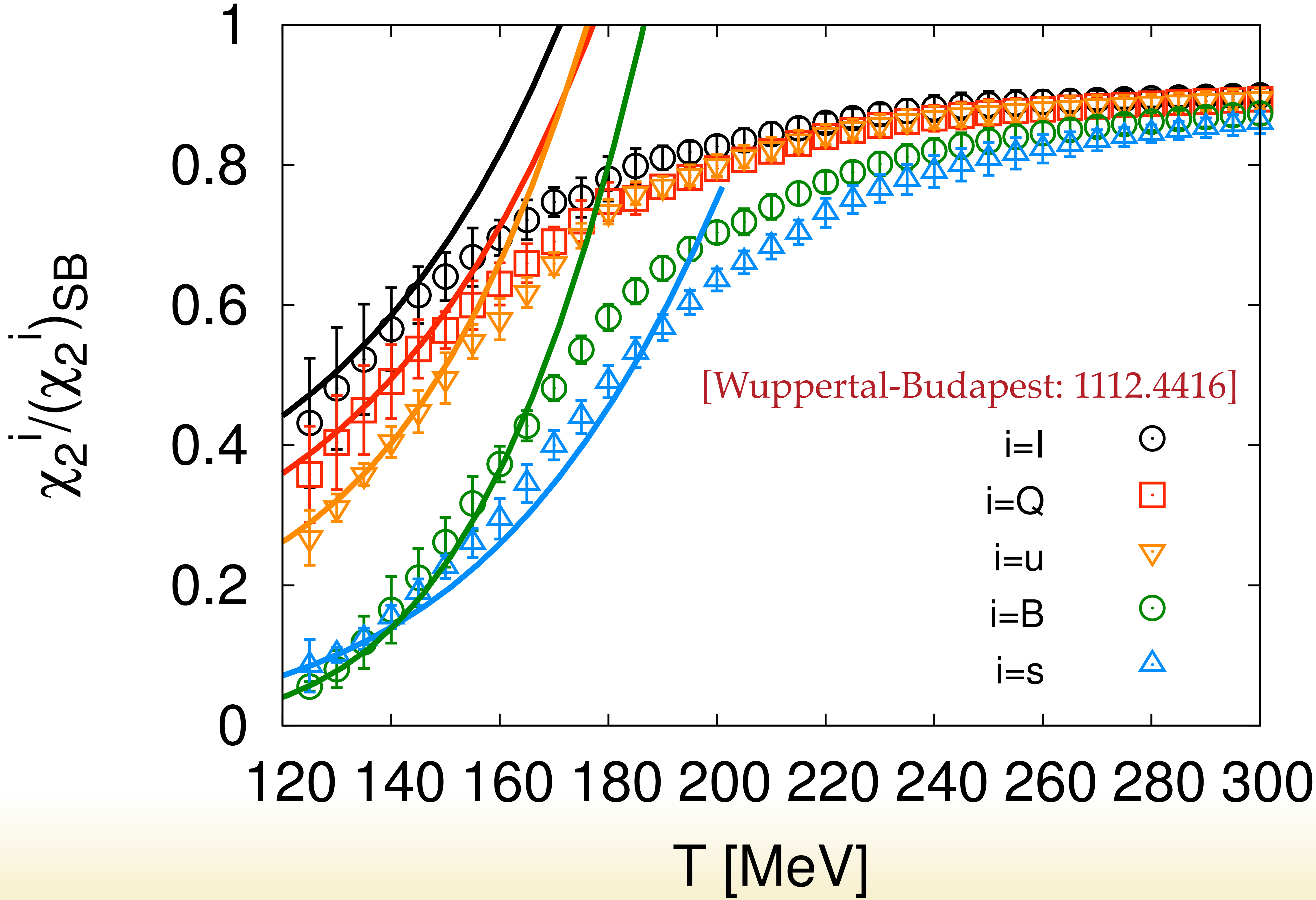
In the strong coupling expansion the partition function reproduces HRG.

[Langelage&Philipsen 1002.1507]

HRG has been tested against lattice in a heavy pion world.

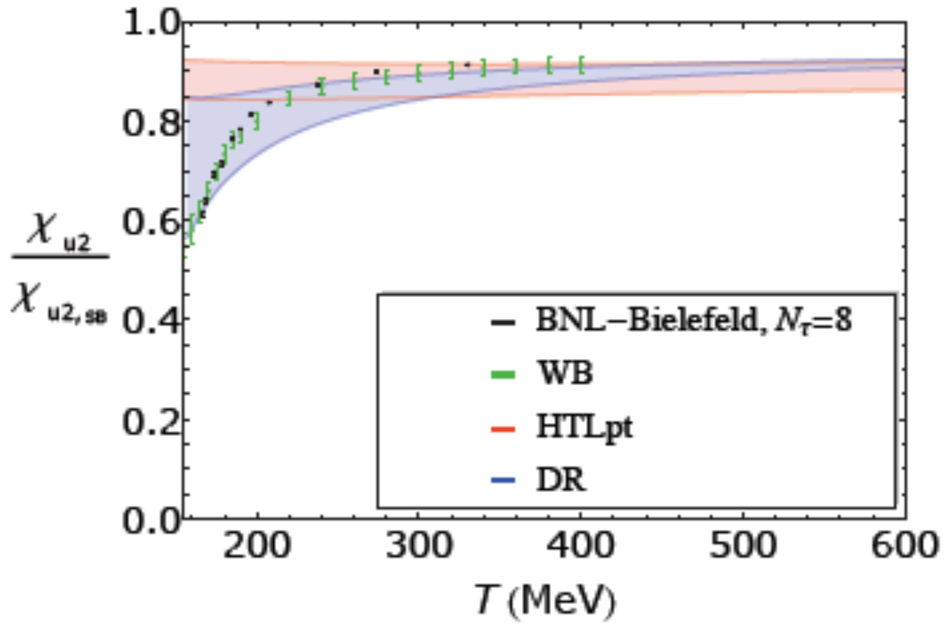
[Karsch et al 0303108] [Petreczky&Huovinen 0912.2541,1005.0324,1106.6227]

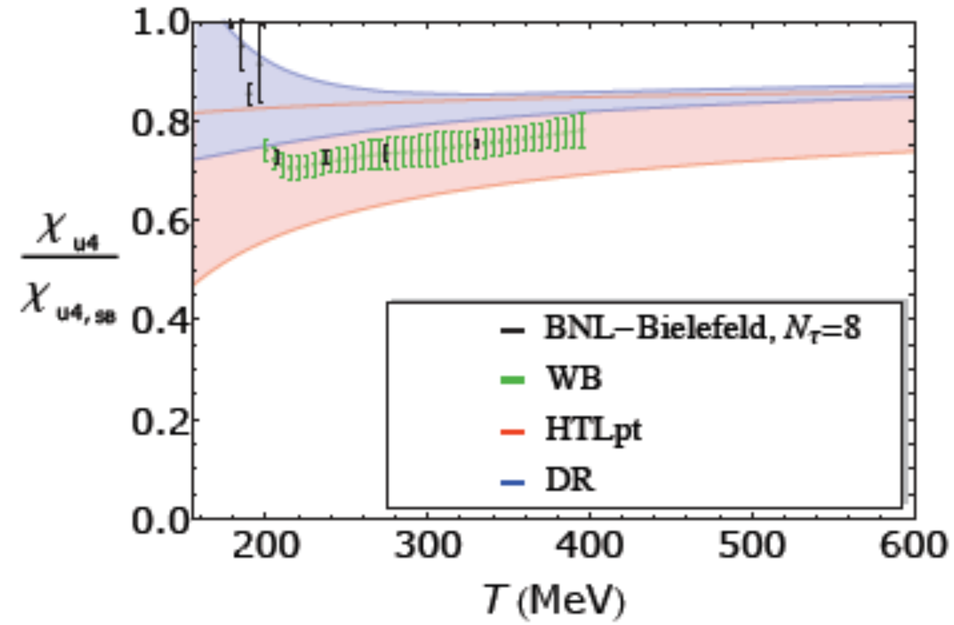
All diagonal fluctuations



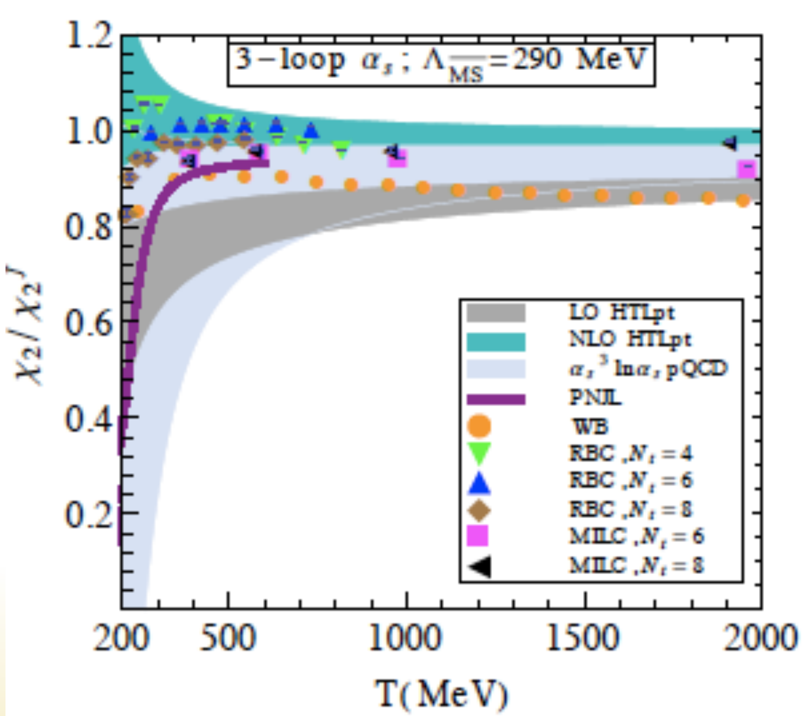
24

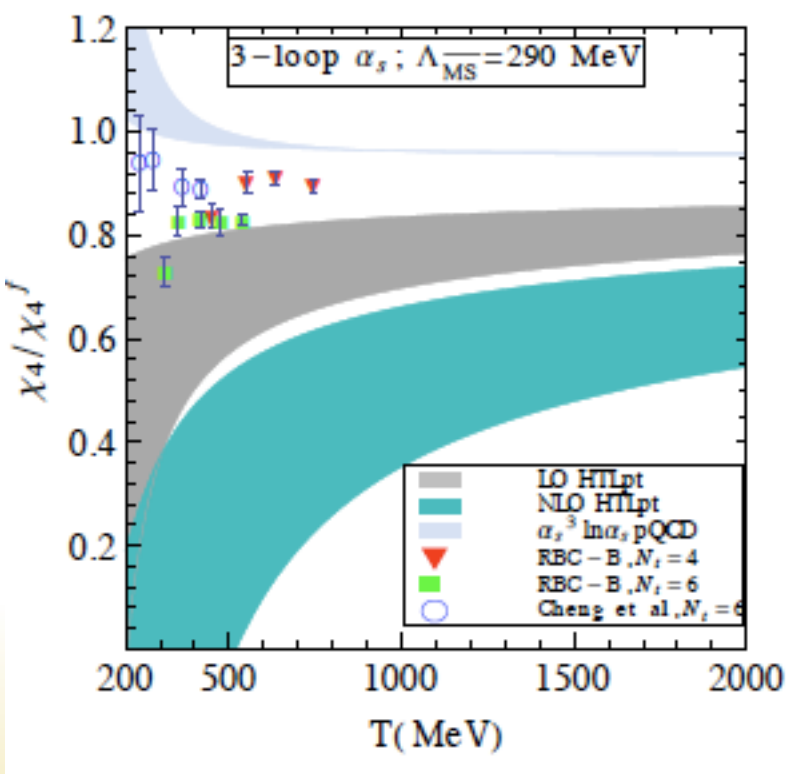
Comparison to HTL expansion





[Andersen et al 1210.0912]

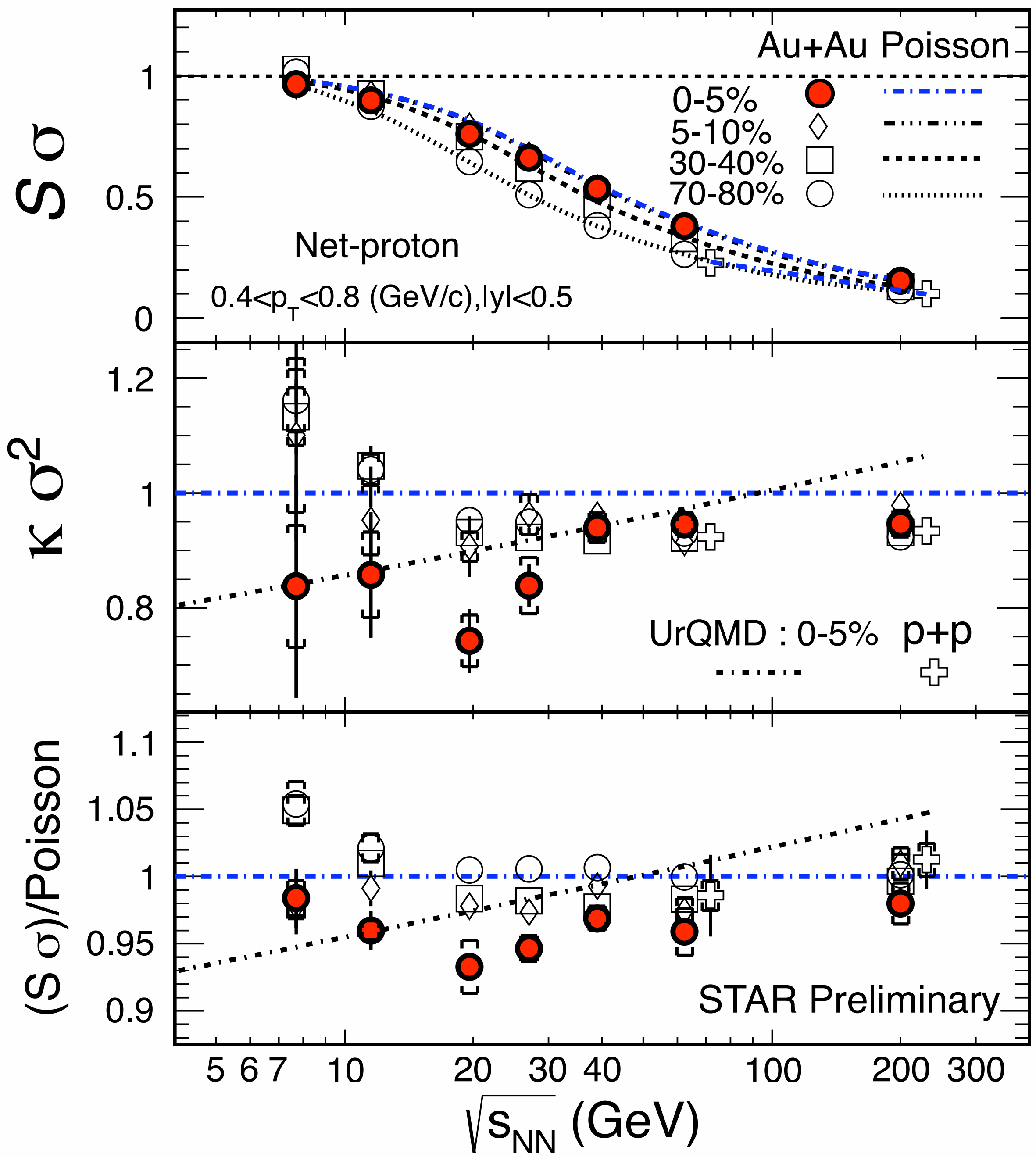




[Haque&Mustafa&Strickland 1302.3228] 25

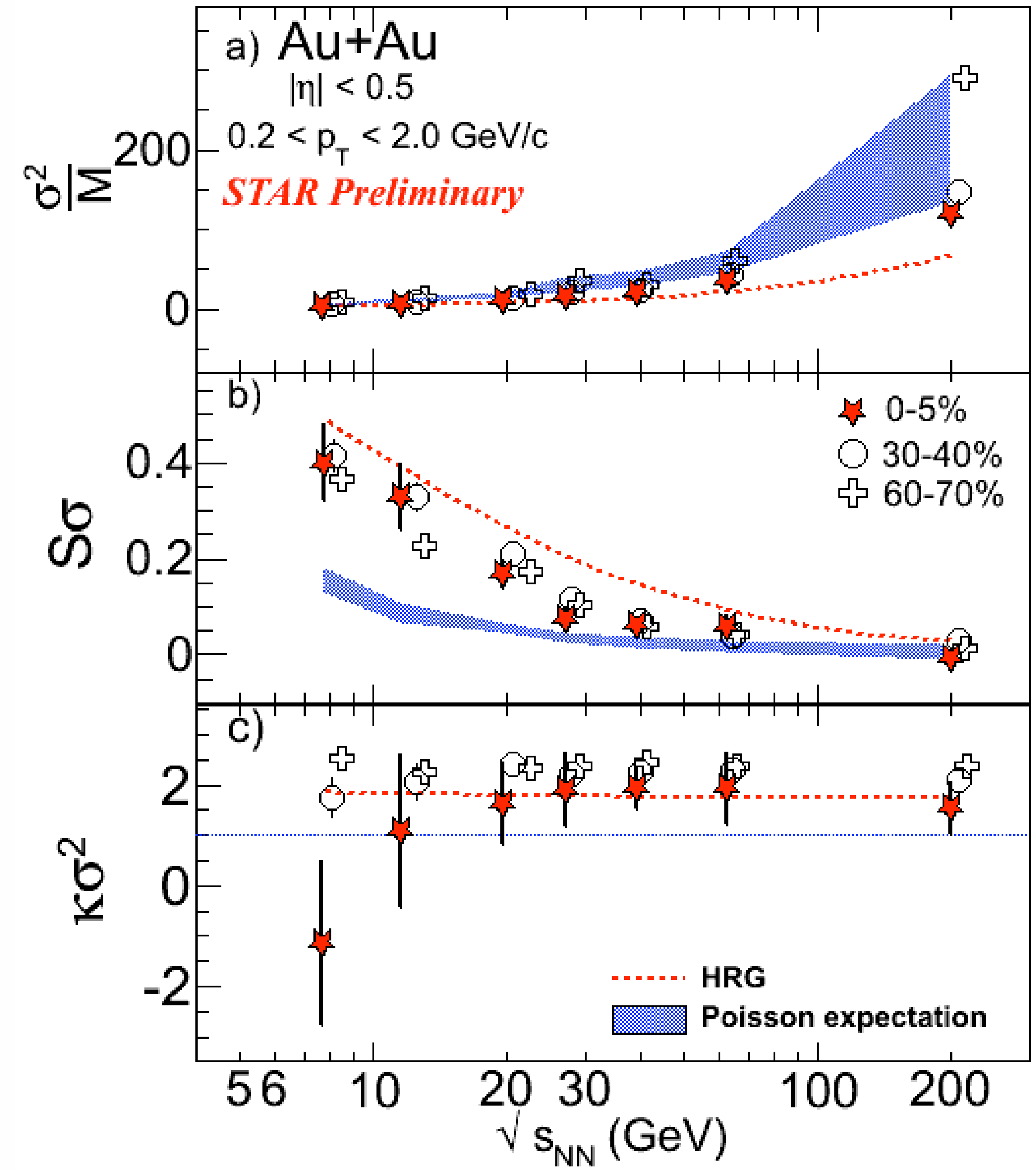
Status of experiment

proton



[X. Luo (STAR) 1210.5573]

electric charge



[N R Sahoo (STAR) 1212.3892]
[McDonald (STAR) 1210.7023]

At finite chemical potential

$\sqrt{s}$ [GeV]	μ_B
200	22.4
62.4	69
39	107.2
27	149

[Andronic et al 0812.1186]

Lower energies (RHIC): higher chemical potential

From lattice QCD we can calculate

$$\chi_{AB\ldots} = \frac{1}{VT^3}\left[\frac{\partial}{\partial\mu_A/T}\frac{\partial}{\partial\mu_B/T}\ldots\right]\log Z$$

at zero chemical potential.

$$\begin{aligned}
\chi_Q|_{\mu_B} &= \chi_{QB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right) + \frac{1}{6}\,\chi_{QBBB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^3 + \ldots \\
\chi_{QQ}|_{\mu_B} &= \chi_{QQ}|_{\mu_B=0} + \frac{1}{2}\,\chi_{QQBB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots \\
\chi_{QQQ}|_{\mu_B} &= \chi_{QQQB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right) + \frac{1}{6}\,\chi_{QQQBBB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^3 + \ldots \\
\chi_{QQQQ}|_{\mu_B} &= \chi_{QQQQ}|_{\mu_B=0} + \frac{1}{2}\,\chi_{QQQQBB}|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots
\end{aligned}$$

Odd/even ratios: $\chi_Q/\chi_{QQ} = M/\sigma^2 = \chi_{QB}/\chi_{QQ}\cdot\mu_B/T + \mathcal{O}\mu_B^3$

Odd/odd ratios: $\chi_{QQQ}/\chi_Q = S\sigma^3/M = \chi_{QQQB}/\chi_{QB} + \mathcal{O}(\mu_B{}^2)$

Even/even ratios: $\chi_{QQQQ}/\chi_{QQ} = \kappa\sigma^2 = \chi_{QQQQ}/\chi_{QQ}|_{\mu_B=0} + \mathcal{O}(\mu_B{}^2)$ [Karsch 1202.4173]

Line of constant net "M" ratios

Input matter content: two colliding nuclei

$$M_Q = M_B \frac{Z}{A} \qquad r = \frac{Z}{A} = \frac{82}{207} \approx 0.4$$

$$M_S = 0$$

(lead)

To leading order:

$$\begin{aligned} M_B &\sim \chi_2^B(T)\mu_B + \chi_{11}^{BQ}(T)\mu_Q + \chi_{11}^{BS}(T)\mu_S \\ M_Q &\sim \chi_{11}^{BQ}(T)\mu_B + \chi_2^Q(T)\mu_Q + \chi_{11}^{QS}(T)\mu_S \\ M_S &\sim \chi_{11}^{BS}(T)\mu_B + \chi_{11}^{QS}(T)\mu_Q + \chi_2^S(T)\mu_S \end{aligned}$$

The total matter content constrains the chemical potentials onto a 1D manifold (line), which we conveniently parametrize through μ_B:

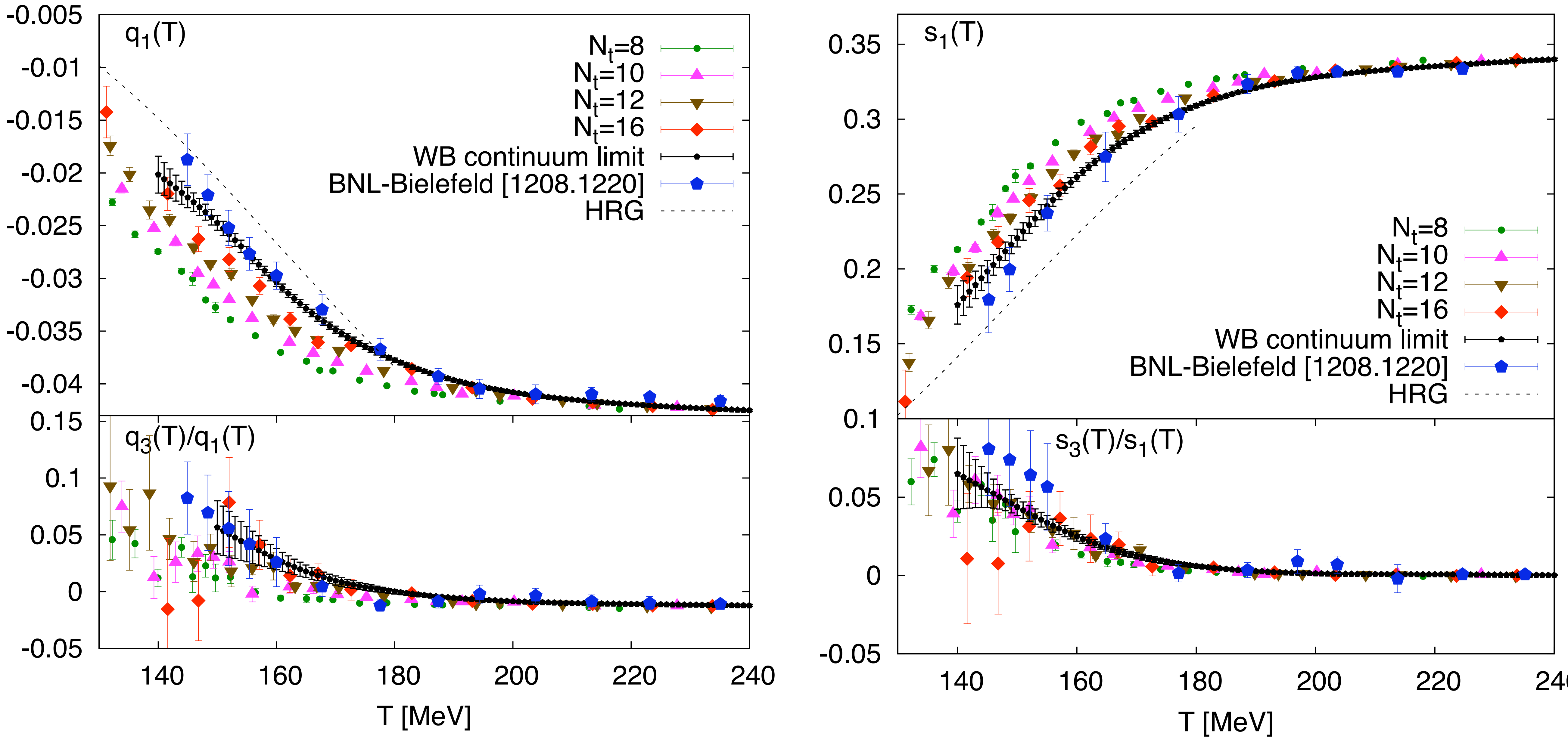
$$\mu_Q(T,\mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + \ldots$$
$$\mu_S(T,\mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + \ldots$$

$$\begin{aligned} q_1 &= \frac{(r\chi_{11}^{BS} - \chi_{11}^{QS})\chi_{11}^{BS} - (r\chi_2^B - \chi_{11}^{BQ})\chi_2^S}{(r\chi_{11}^{BQ} - \chi_2^Q)\chi_2^S - (r\chi_{11}^{BS} - \chi_{11}^{QS})\chi_{11}^{QS}} \\ s_1 &= -(\chi_{11}^{QS} q_1 + \chi_{11}^{BS})\chi_2^S \end{aligned}$$

The NLO coefficients (q_3 and s_3) contain 4th derivatives.

These have been first calculated in [BNL-Bielfeld1208.1220]

Line of constant net "*M*" ratios



[WB data: 1305.5161]

Example use: $$\frac{d}{d\mu_B}X(T) = \frac{\partial}{\partial\mu_B}X(T) + q_1(T)\frac{\partial}{\partial\mu_Q}X(T) + s_1(T)\frac{\partial}{\partial\mu_S}X(T)$$

Baryon skewness as thermometer