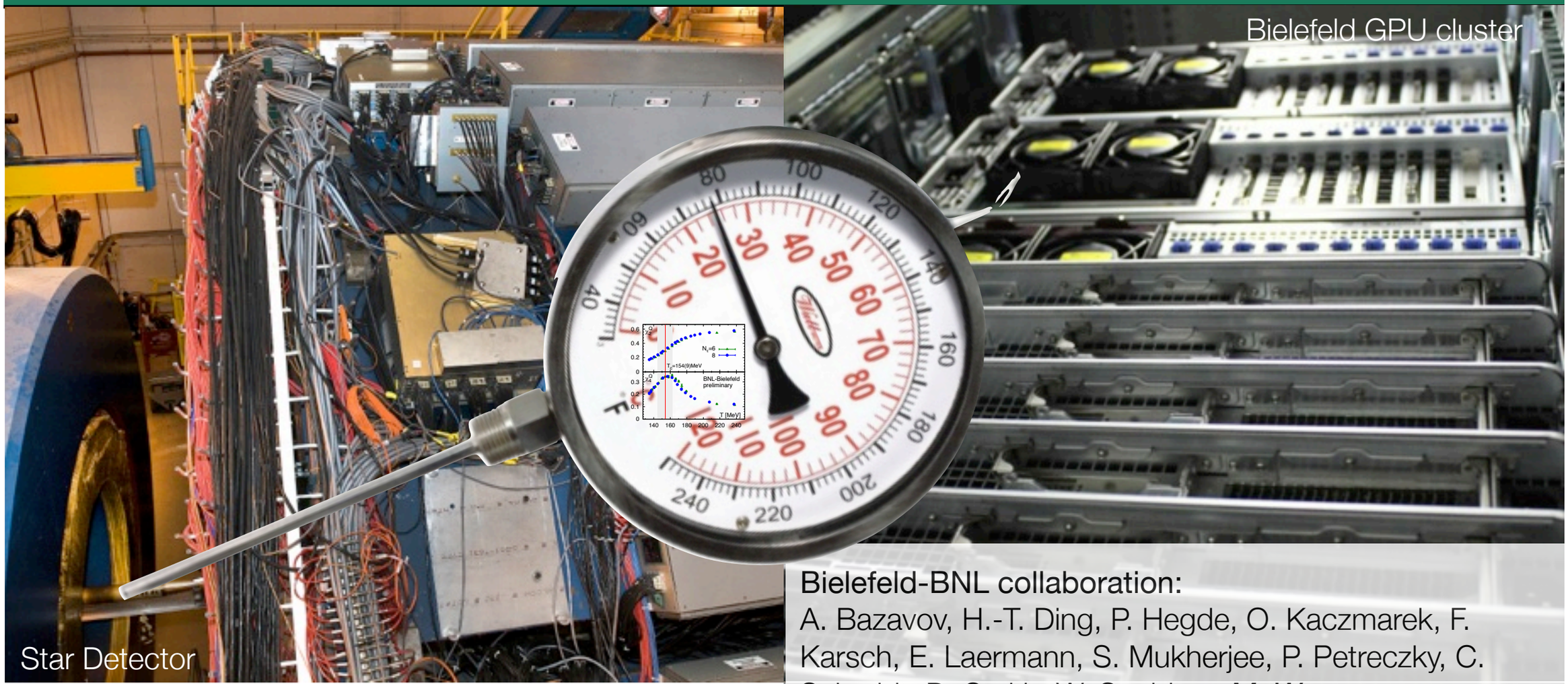
Charge Fluctuations as Thermometer for Heavy-Ion Collisions



Bielefeld-BNL collaboration:
A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, C. Schmidt, D. Smith, W. Soeldner, *M. Wagner*

Outline

- Status of HISQ fluctuations data
- Freeze-out conditions from QCD thermodynamics
- Comparison with experimental data
  - electric charge fluctuations (STAR, PHENIX)
  - proton fluctuations (STAR)
- Summary

Fluctuations from Lattice QCD

- expansion of the pressure in

$$\frac{p}{T^4} = \sum_{i,j,k}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- B,Q,S conserved charges (baryon number, electric charge, strangeness)

Fluctuations from Lattice QCD

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- B,Q,S conserved charges (baryon number, electric charge, strangeness)

- generalized susceptibilities

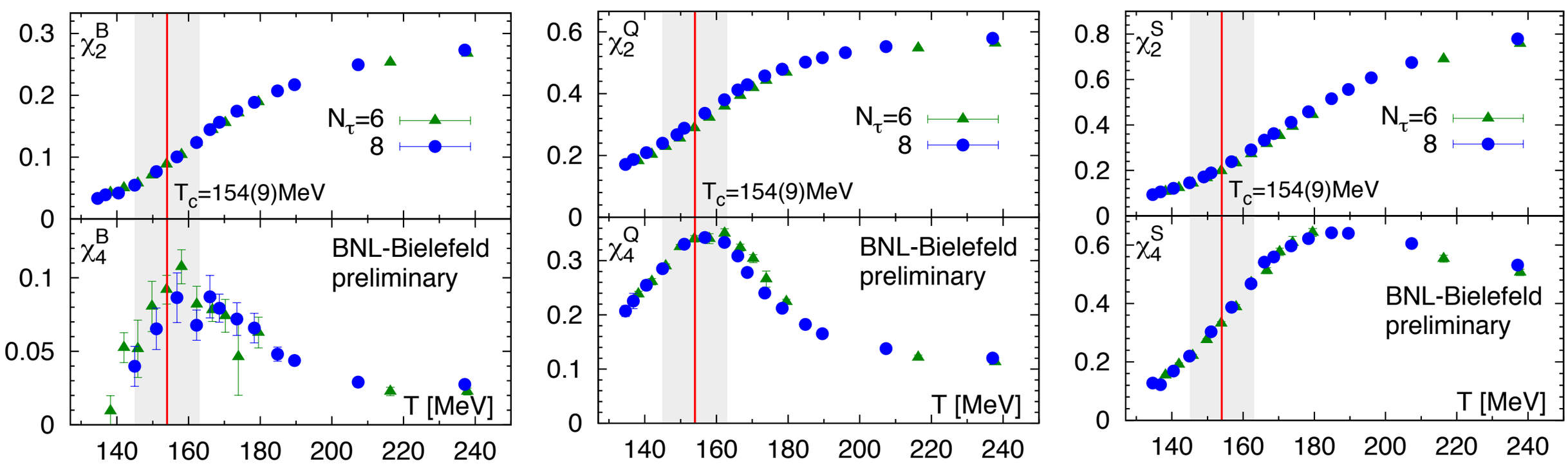
$$\chi_{ijk,\mu}^{BQS} = \frac{1}{VT^3} \frac{\partial^{i+j+k} \ln Z(\mu,T)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \qquad \hat{\mu}_X = \frac{\mu_X}{T}$$

- related to cumulants of net charge fluctuations, e.g.

$$VT^3 \chi_2^B = \langle (\delta N_B)^2 \rangle = \left\langle N_B^2 - 2N_B \langle N_B \rangle + \langle N_B \rangle^2 \right\rangle$$

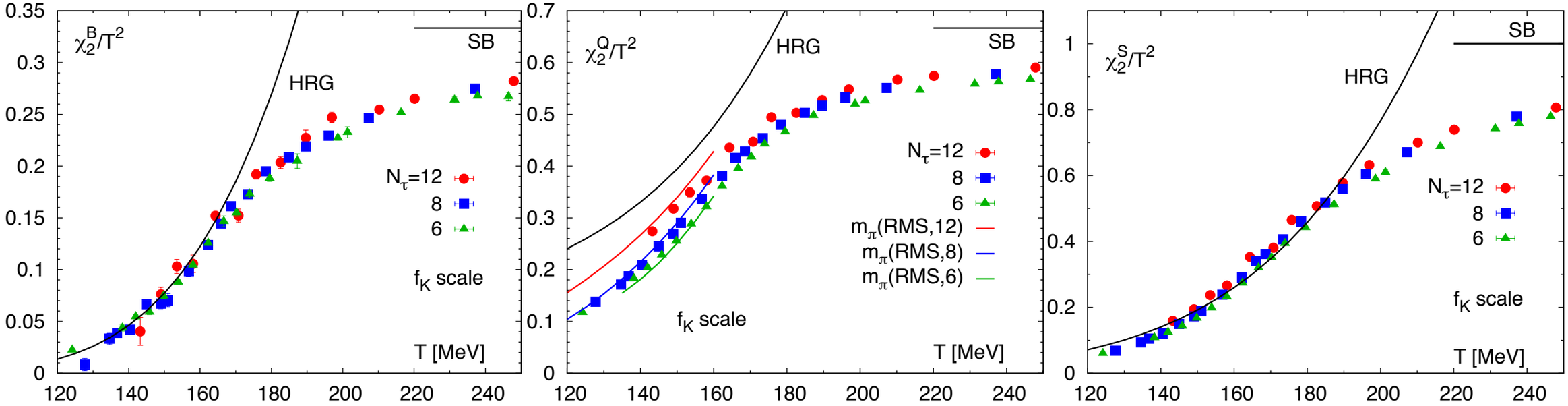
Status of lattice data

- highly-improved staggered quarks HISQ, close to physical mass ($m_l/m_s = 1/20$)
- up to 12000 configurations for each beta
- measured with 500 up to 1500 (around T_c) random sources



Comparison with HRG results

HotQCD, PRD86 (2012) 034509



- low temperature described by hadron resonance gas

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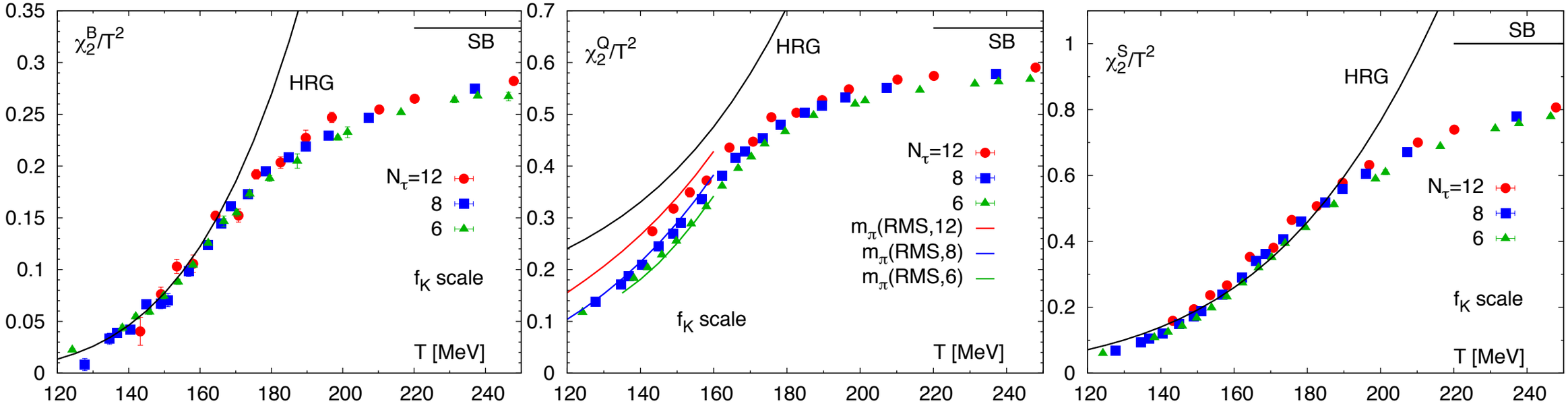
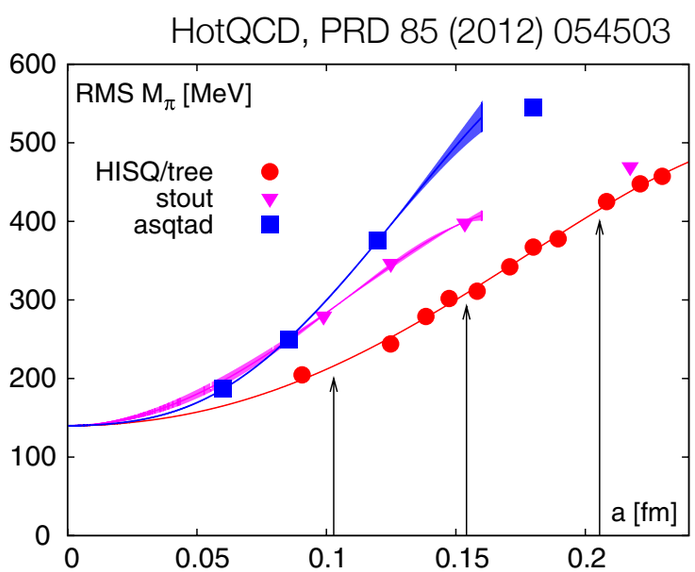


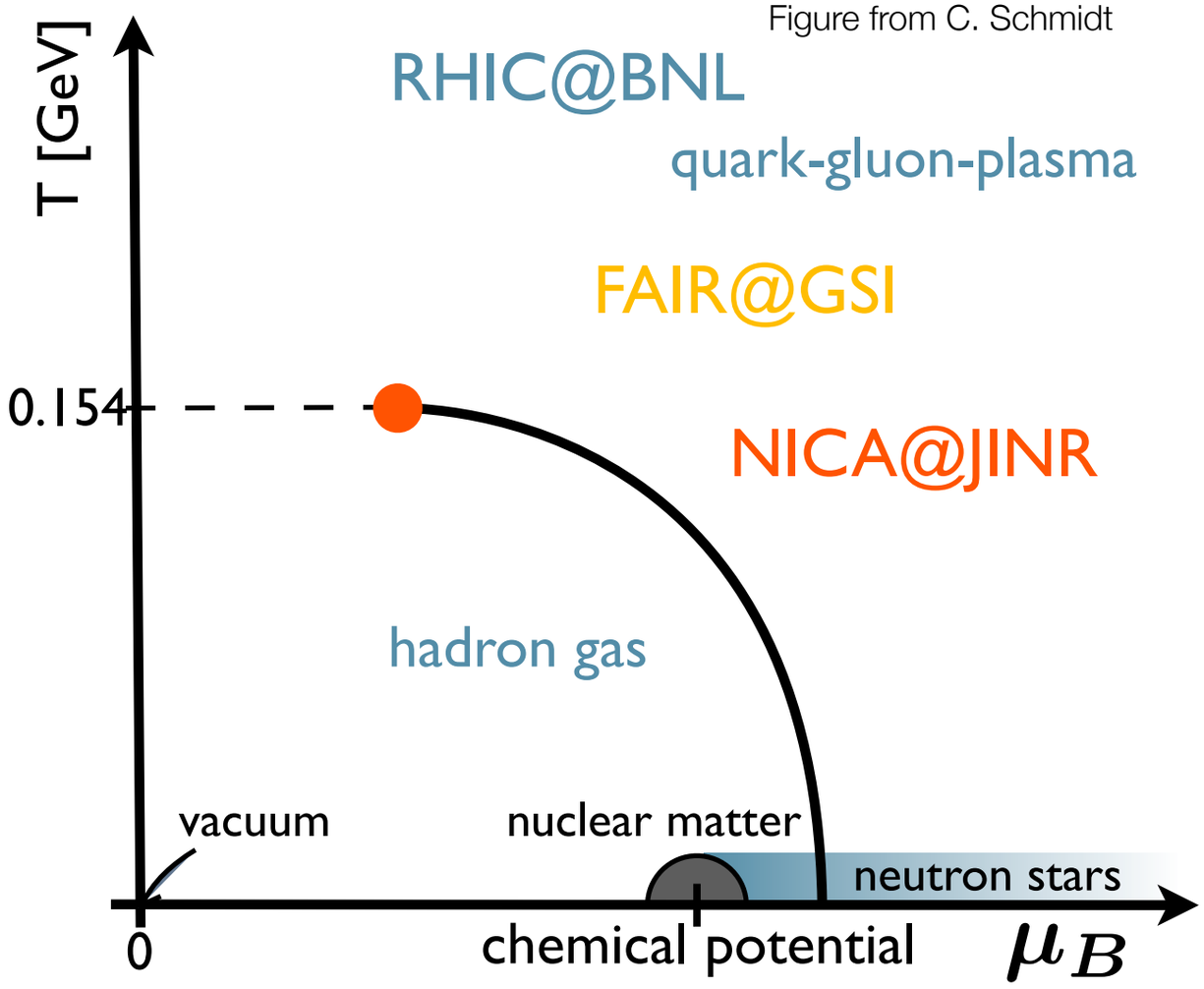
- low temperature described by hadron resonance gas
- taste violations visible in charge sector
- use action with reduced taste splitting

HotQCD, PRD 85 (2012) 054503



Freeze-out curve from heavy-ion collision

Figure from C. Schmidt



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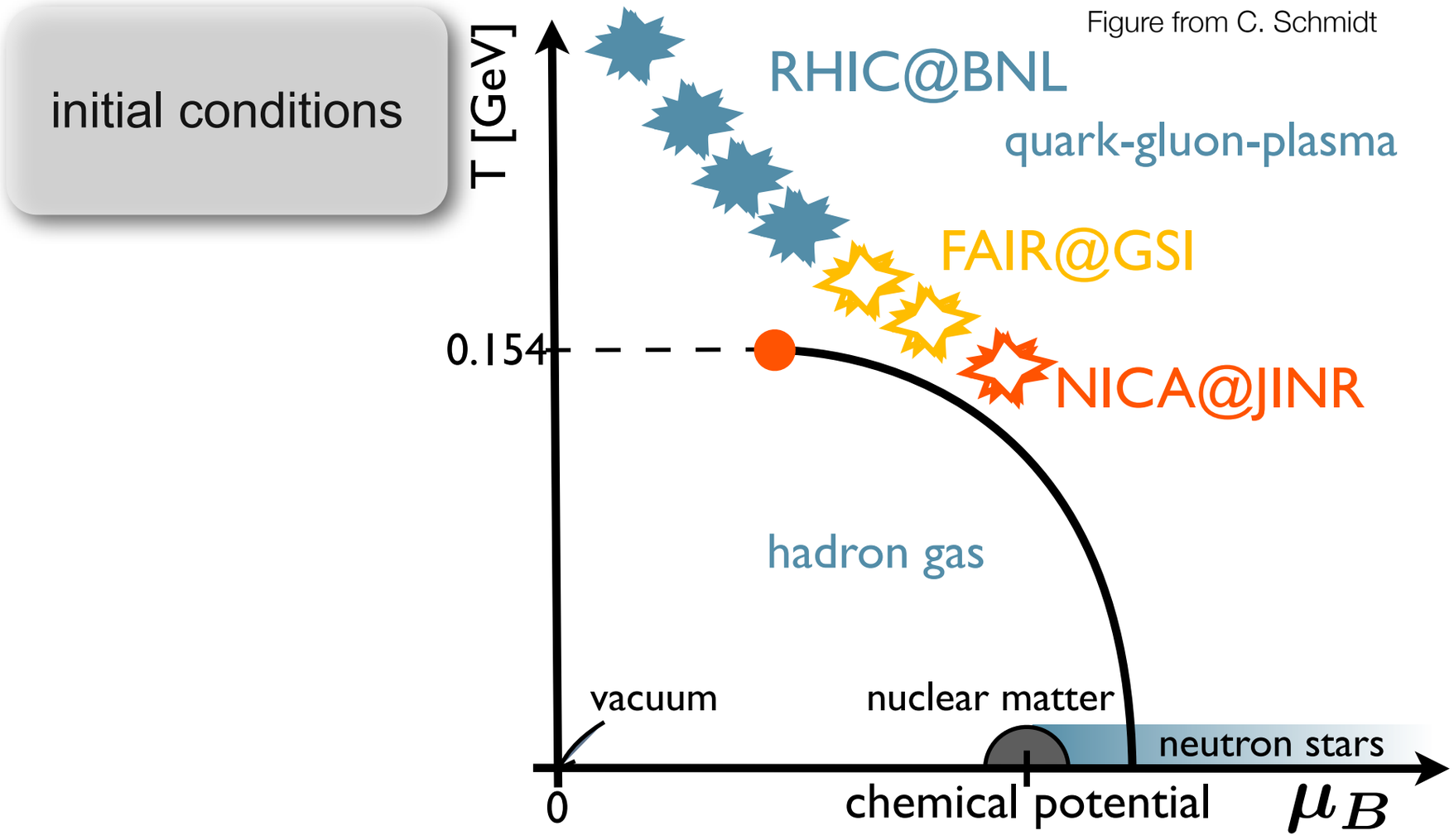

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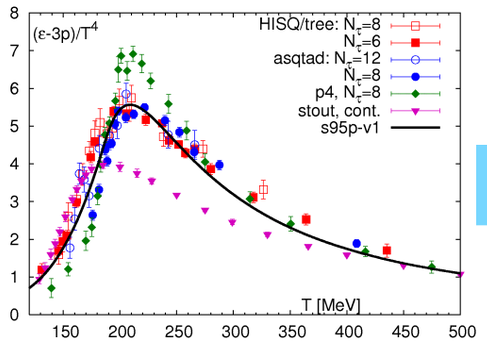
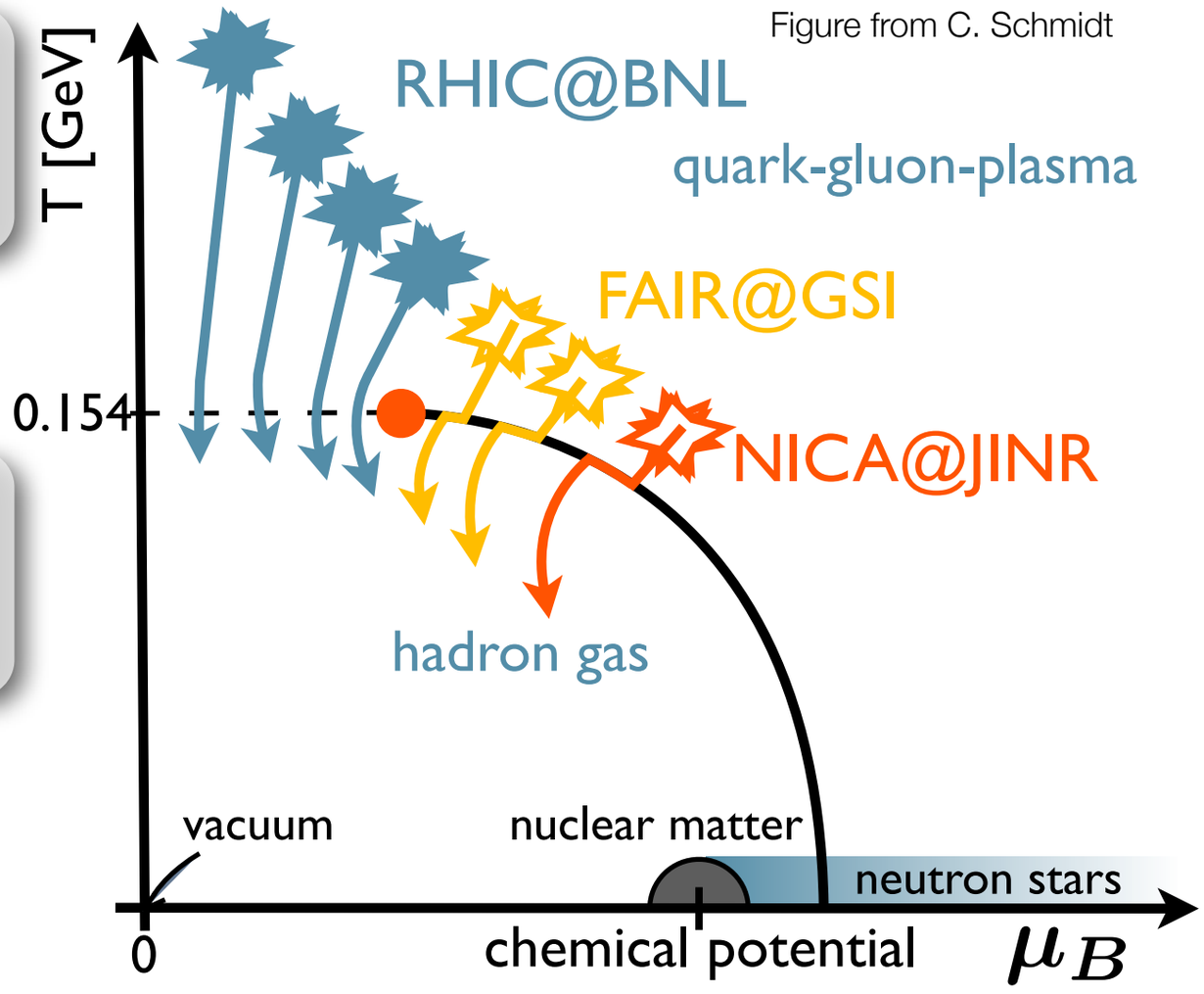
Lattice EoS



initial conditions

evolution



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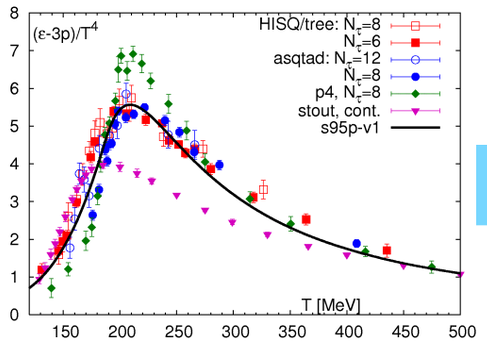
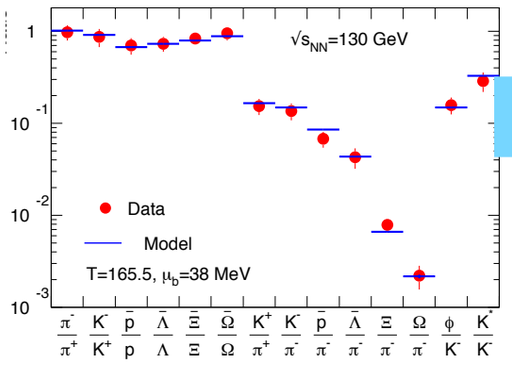


Hadron abundances from HRG model

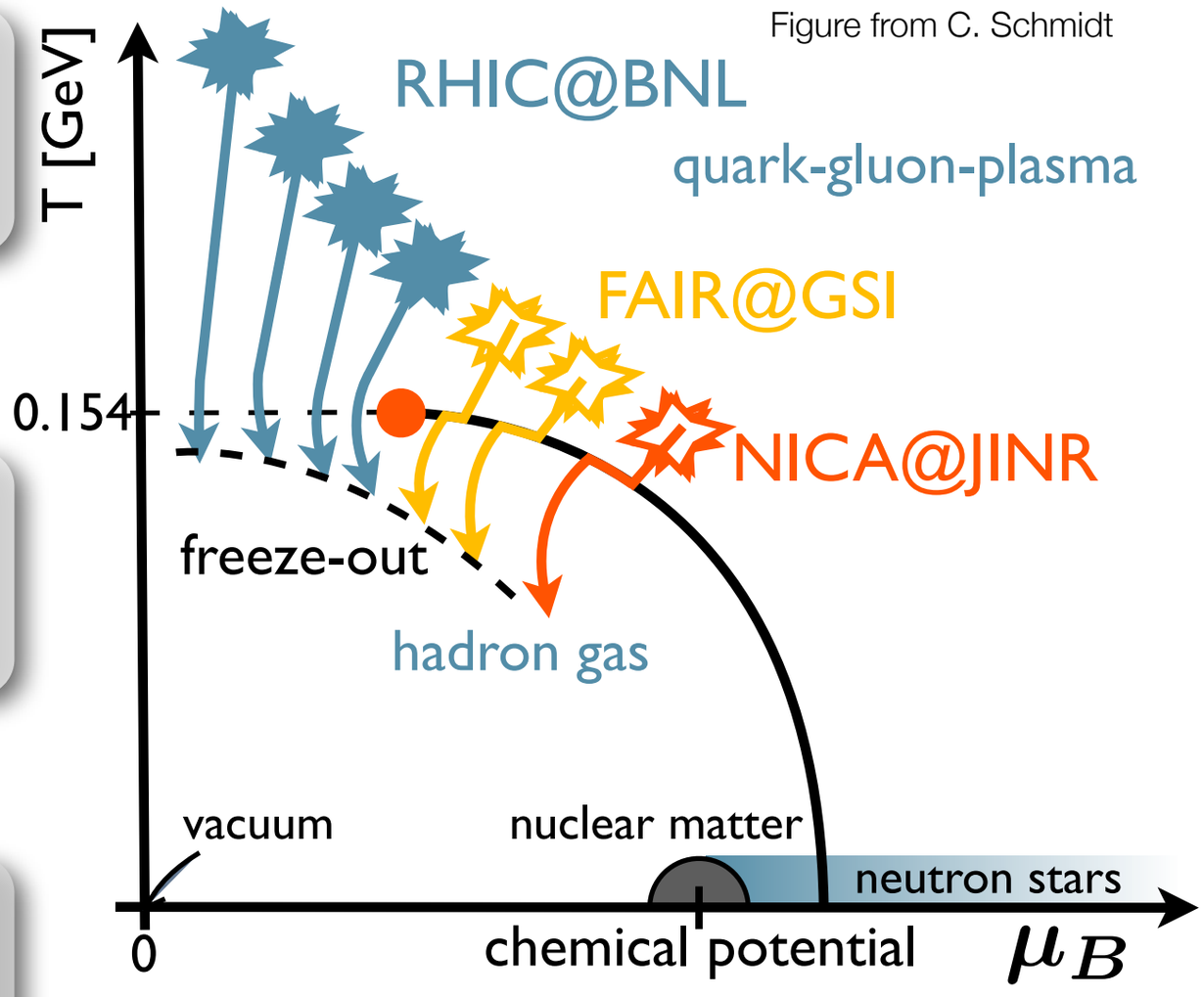


see e.g. Braun-Munzinger, Cleymans

initial conditions

evolution

freeze-out



$$T_f(\sqrt{s}), \mu_f(\sqrt{s})$$

Freeze-out curve from heavy-ion collision

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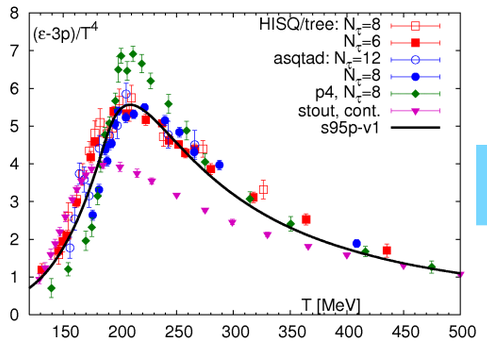
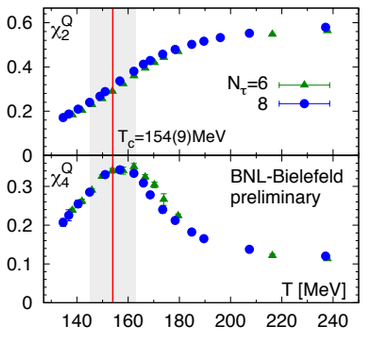


Fluctuations from Lattice QCD



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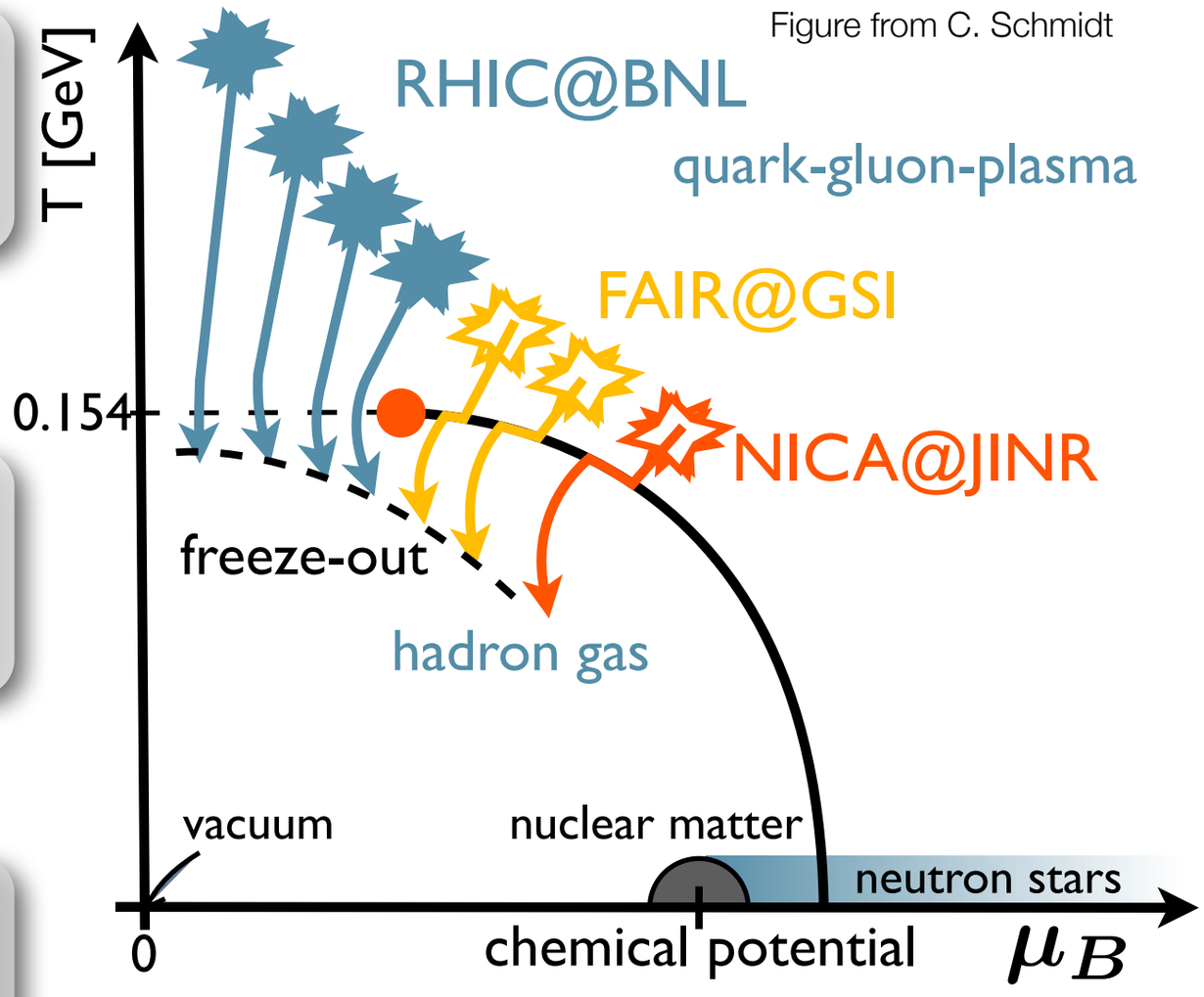

$$T_f(\sqrt{s}), \mu_f(\sqrt{s})$$

Constraining isospin and strangeness

Assume: homogenous model in thermal equilibrium $\left(T^f,\, \mu_B^f, \mu_Q^f, \mu_S^f\right)$

Exploit: initial conditions

- strangeness neutrality: $\langle N_S \rangle = 0$
- isospin asymmetry: $\langle N_Q \rangle = r \, \langle N_B \rangle$

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expand in powers of μ_B, μ_Q, μ_S

solve for μ_Q, μ_S

$$\hat{\mu}_Q(T,\mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$$
$$\hat{\mu}_S(T,\mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$$

LO NLO

$$\hat{\mu}_X = \frac{\mu_X}{T}$$

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LO NLO

$$\hat{\mu}_X = \frac{\mu_X}{T}$$

$$\mu_Q^f \equiv \mu_Q(T^f, \mu_B^f)$$
$$\mu_S^f \equiv \mu_S(T^f, \mu_B^f)$$

two independent parameters
$T^f,\, \mu_B^f$

Constraining isospin and strangeness

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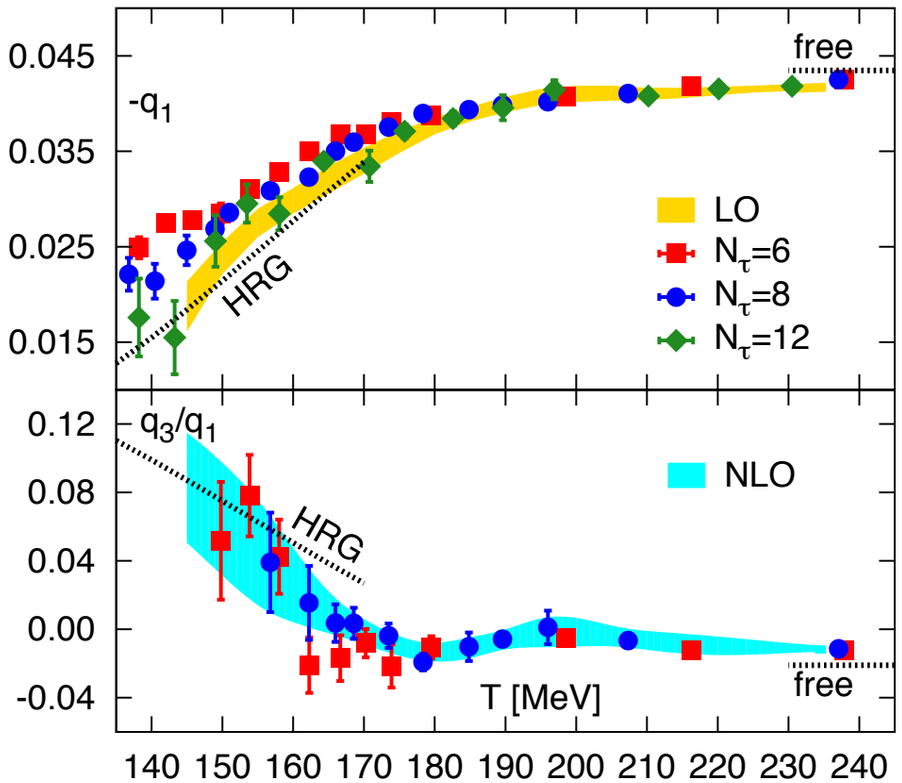
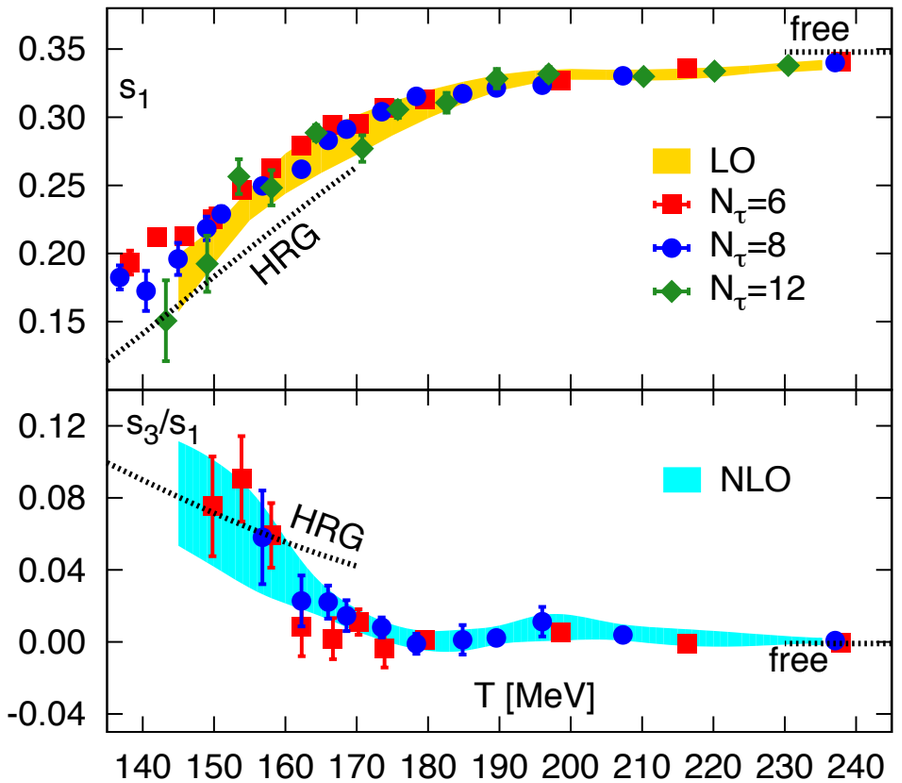
BNL-BI, PRL 109 (2012) 192302





LO: continuum extrapolated based on N_t=6,8,12

NLO: small cut-off dependence, continuum estimate based on N_t=6,8

Constraining isospin and strangeness

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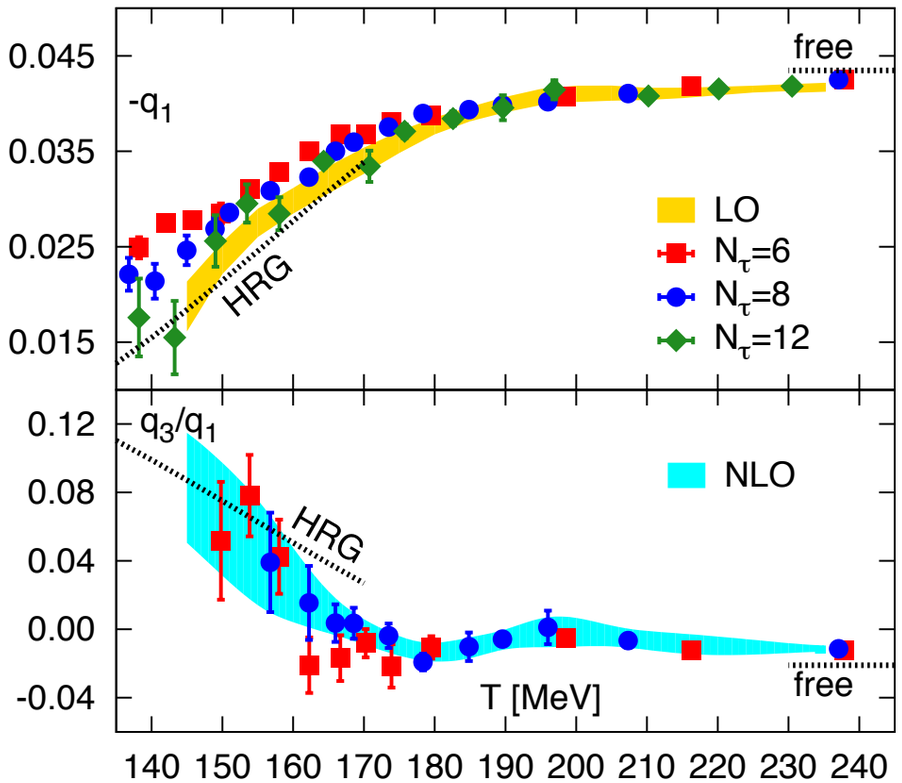

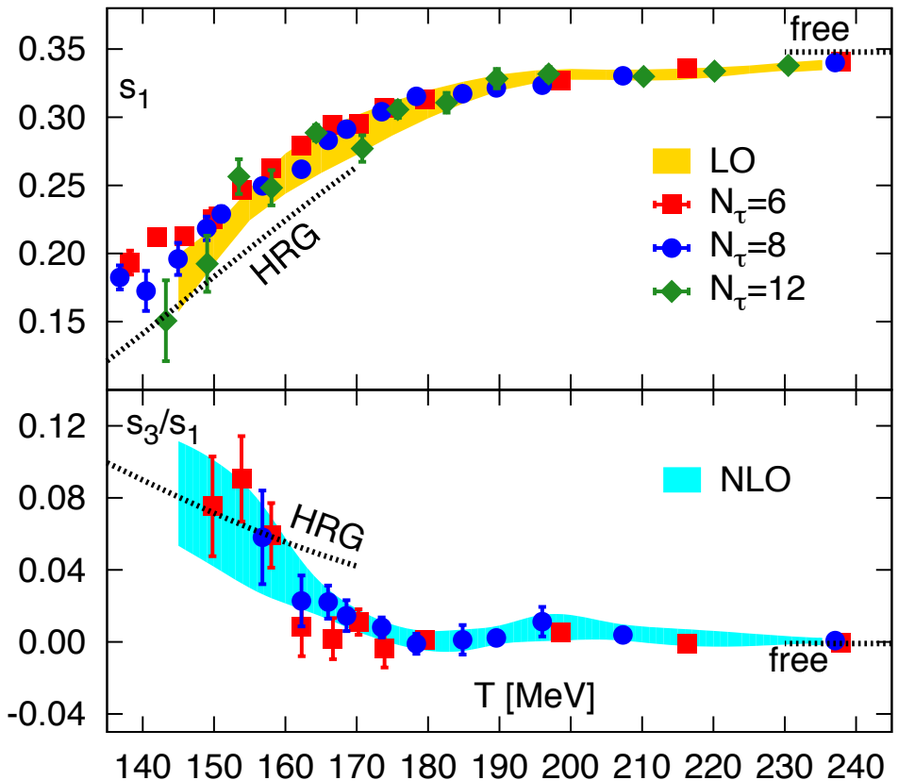

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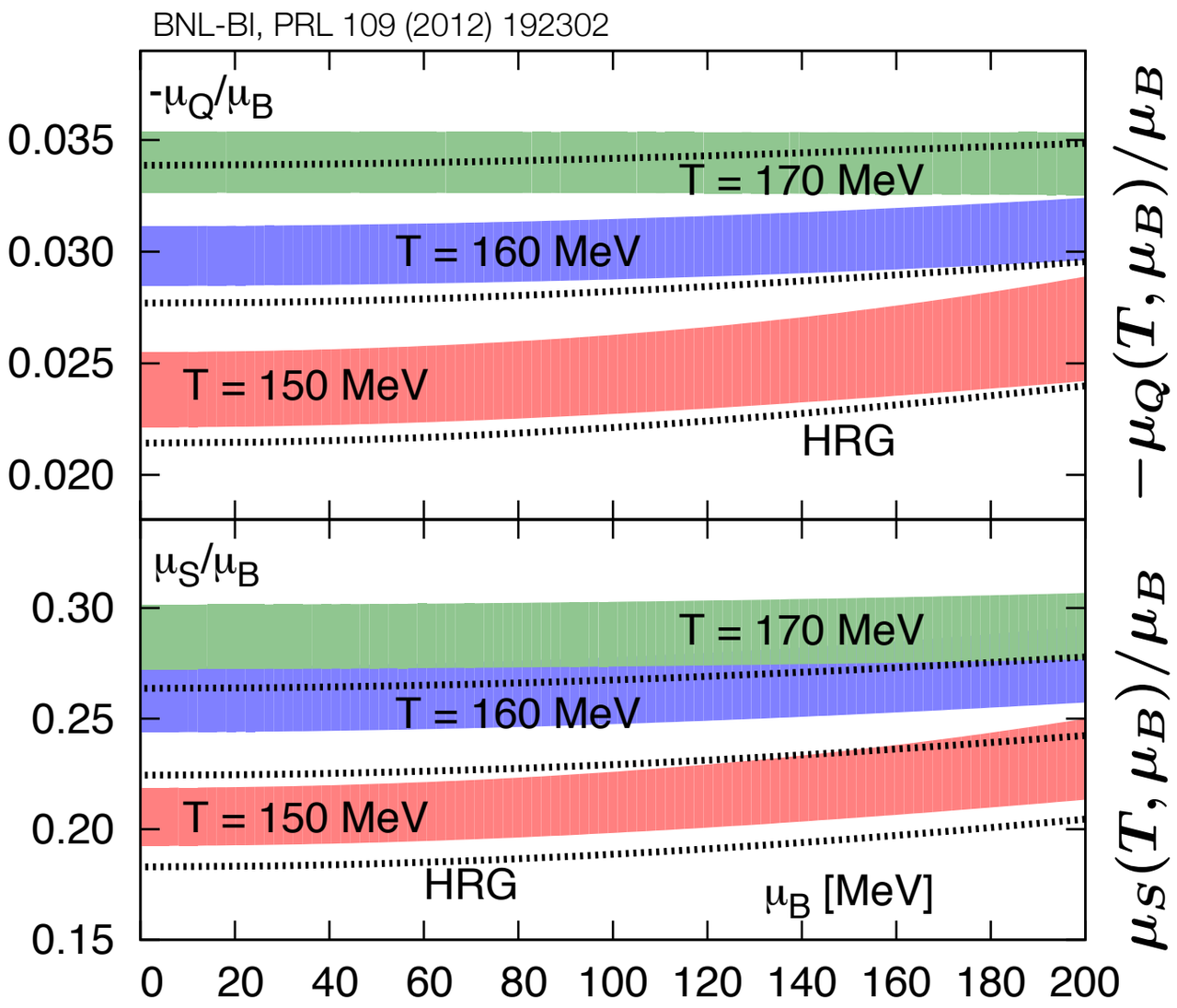
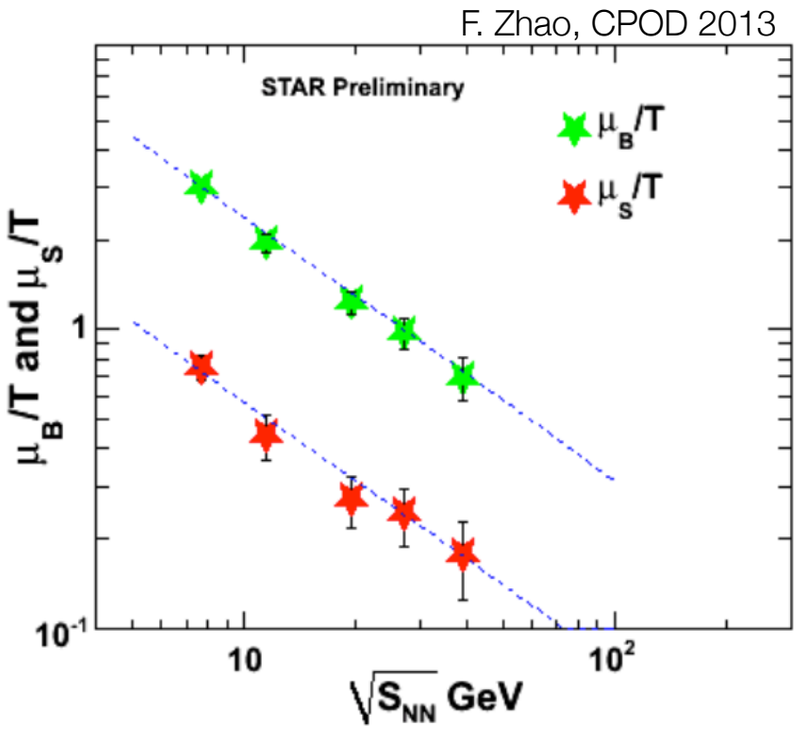
NLO corrections <10% for $\mu_B/T \lesssim 1.3$

Isospin and strangeness constrained



- consistent with STAR data

$$\mu_S/\mu_B \approx 0.24$$



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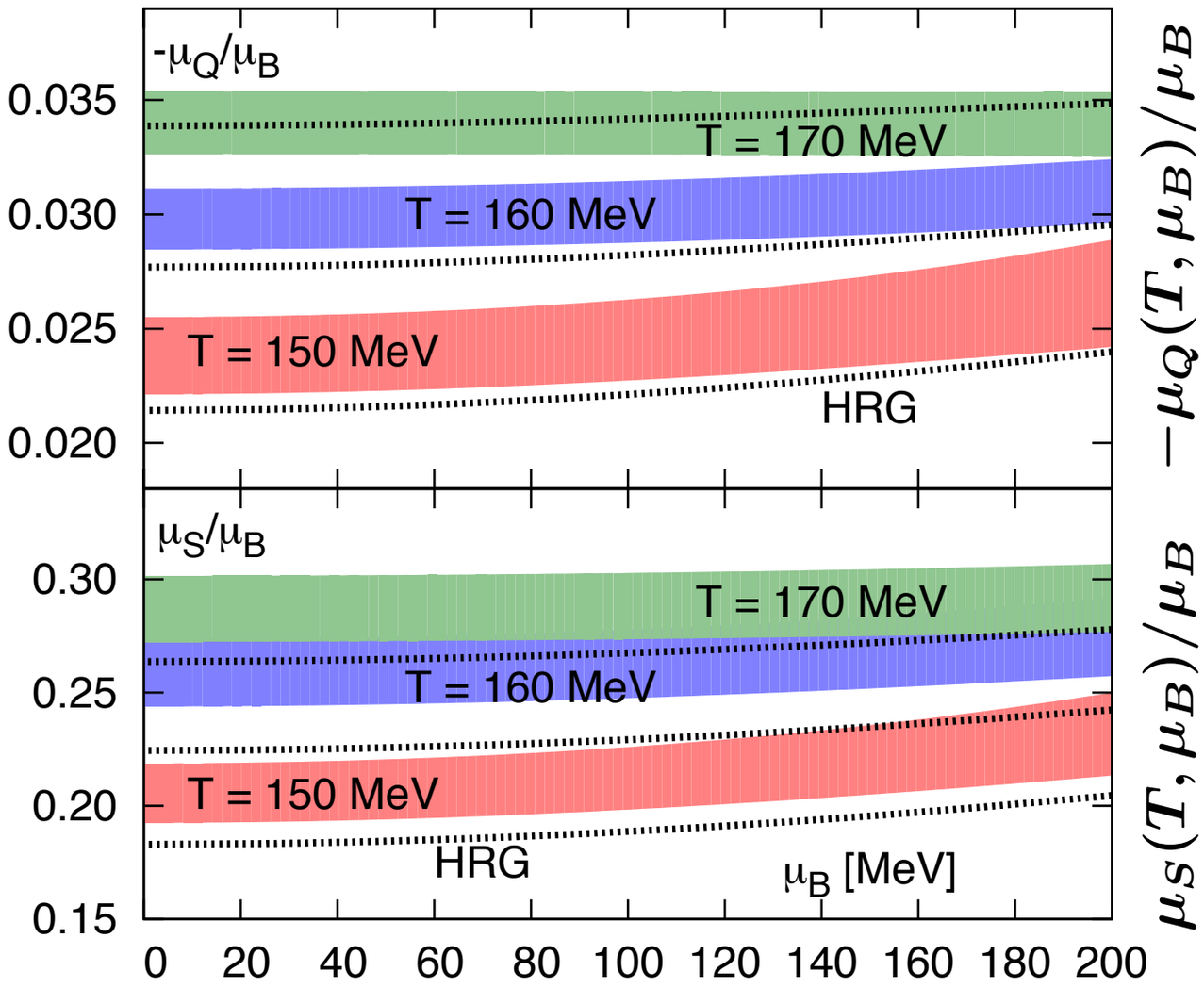

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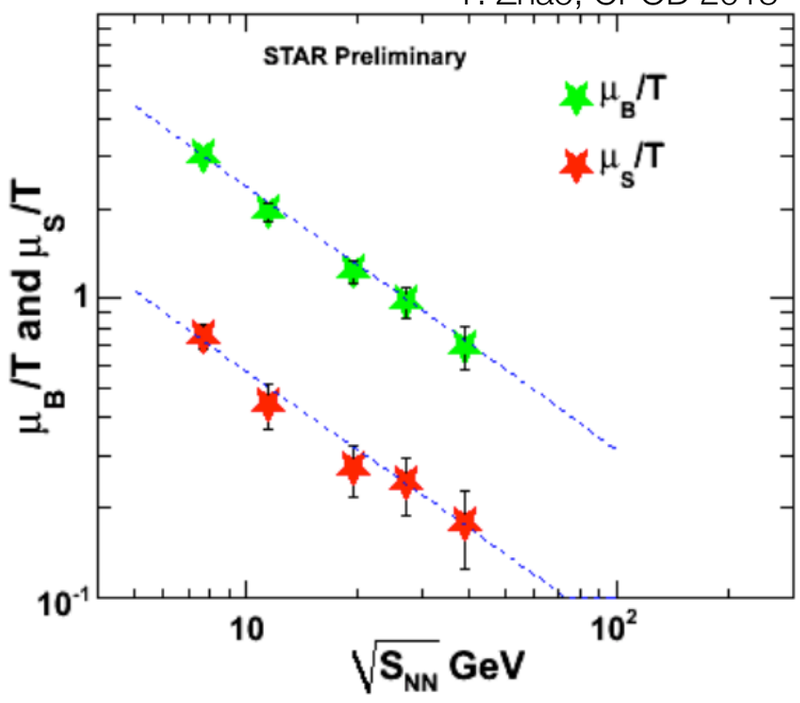
F. Zhao, CPOD 2013



deviations from HRG < 5 - 15 %

Pinning down the freeze-out parameters

- need two experimental quantities to determine (T^f, μ_B^f)
  - ratios eliminate (unknown) volume factor
- only proton (not baryon!) number fluctuations are experimentally available
  - use electric charge fluctuations

$$\frac{M_Q(\sqrt{s})}{\sigma_Q^2(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = R_{12}^{Q,1}\hat{\mu}_B + R_{12}^{Q,3}\hat{\mu}_B^3 + \cdots = R_{12}^Q(T,\mu_B)$$

LO linear in μ_B, fixes μ_B^f

$$\frac{S_Q(\sqrt{s})\sigma_Q^3(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi_3^Q(T,\mu_B)}{\chi_1^Q(T,\mu_B)} = R_{31}^{Q,0} + R_{31}^{Q,2}\hat{\mu}_B^2 + \cdots = R_{31}^Q(T,\mu_B)$$

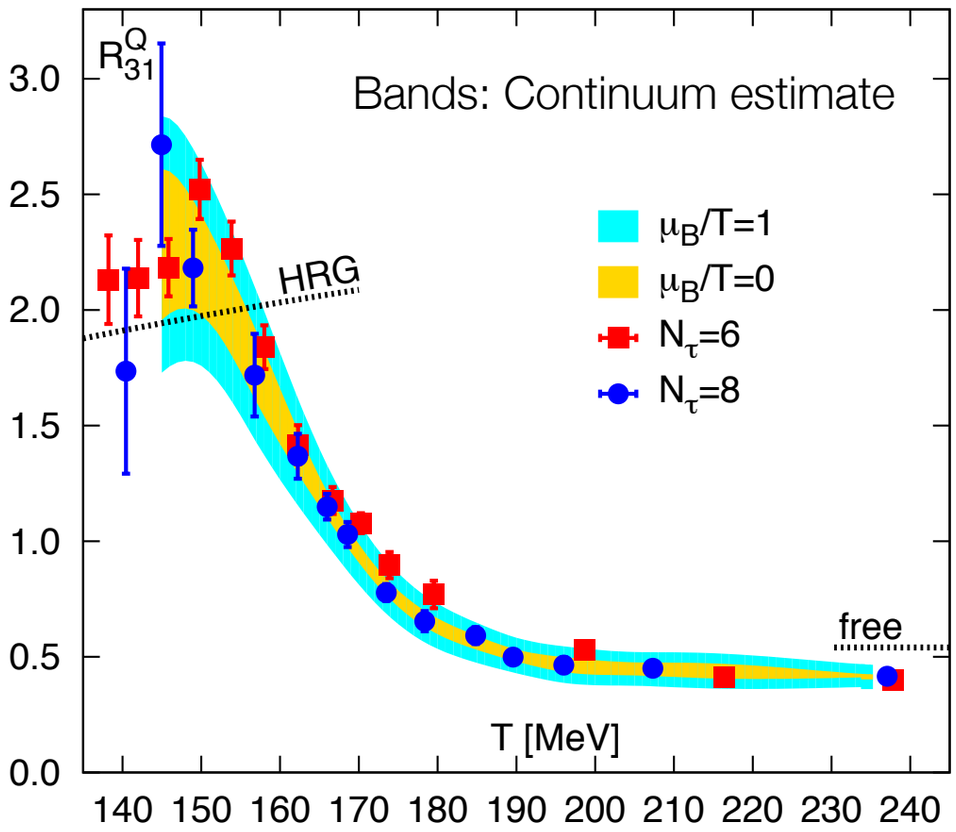
LO independent of μ_B, fixes T^f

M: mean
σ: variance
S: skewness

Determination of freeze-out temperature

$$R_{31}^{Q}(T,\mu_B) = R_{31}^{Q,0} + R_{31}^{Q,2}\hat{\mu}_B^2$$

BNL-BI, PRL 109 (2012) 192302



$S_Q\sigma_Q^3/M_Q$	$T^f\,[MeV]$
$\gtrsim 2$	$\lesssim 155$
~ 1.5	~ 160
$\lesssim 1$	$\gtrsim 165$

- small cutoff effects

- small NLO corrections (<10%) for $\mu/T < 1.3$

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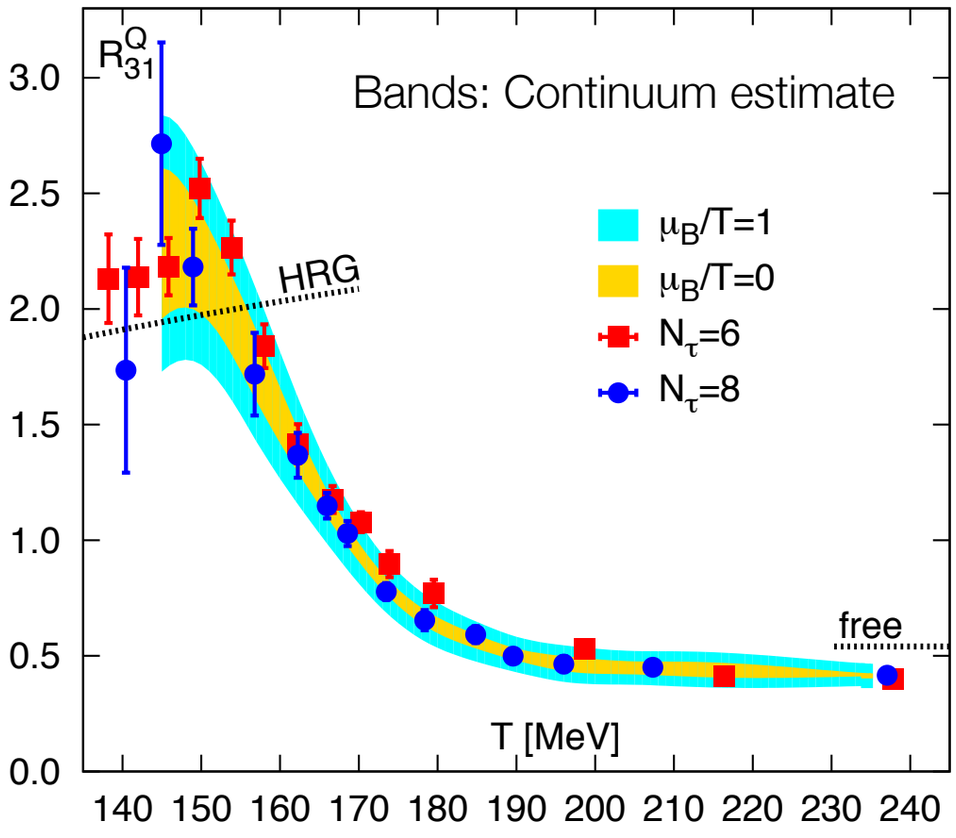
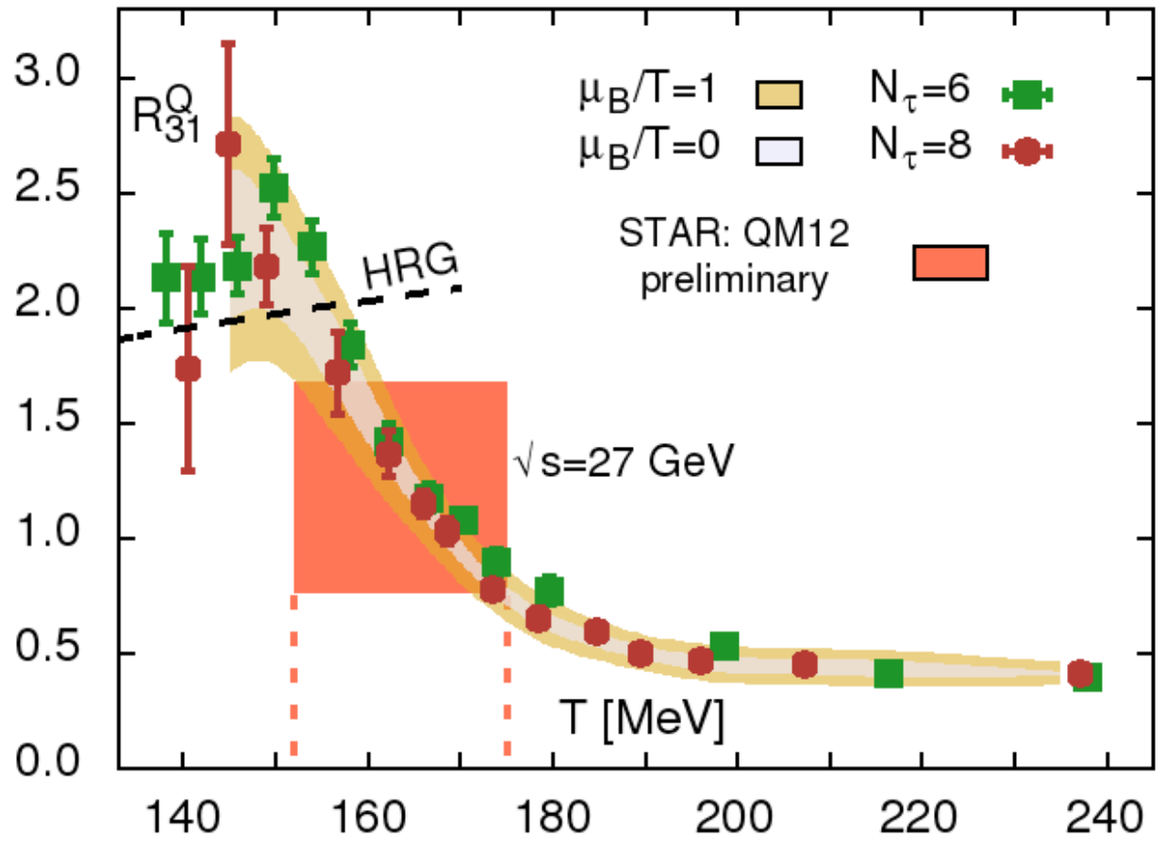


S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255



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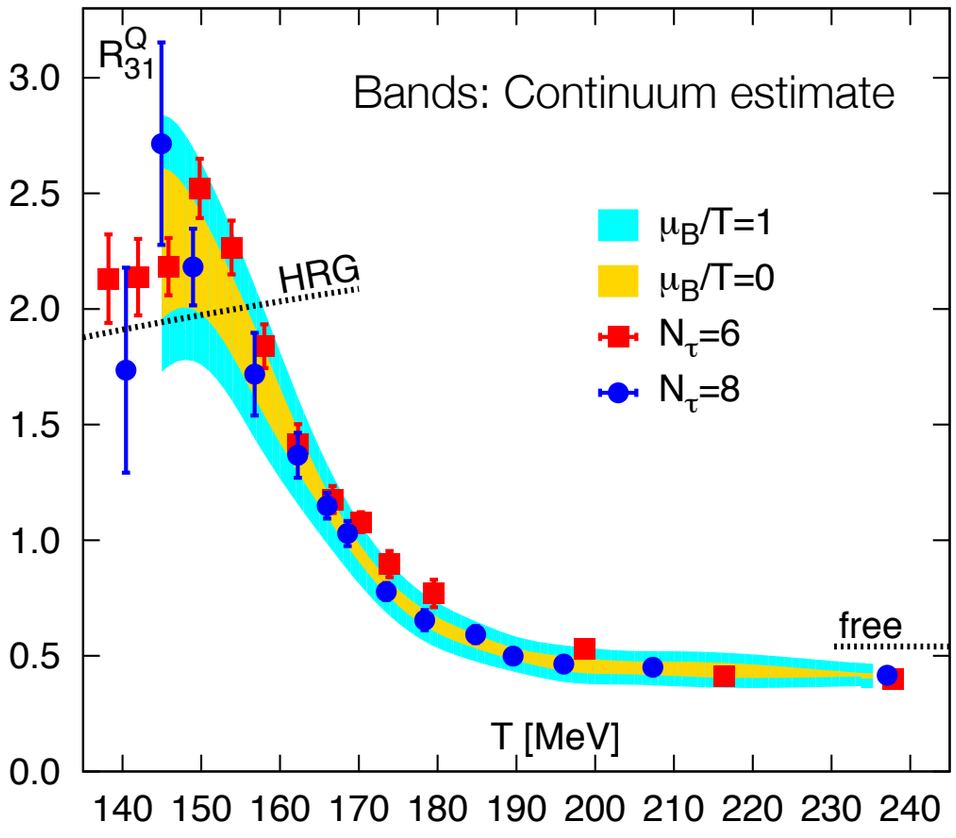

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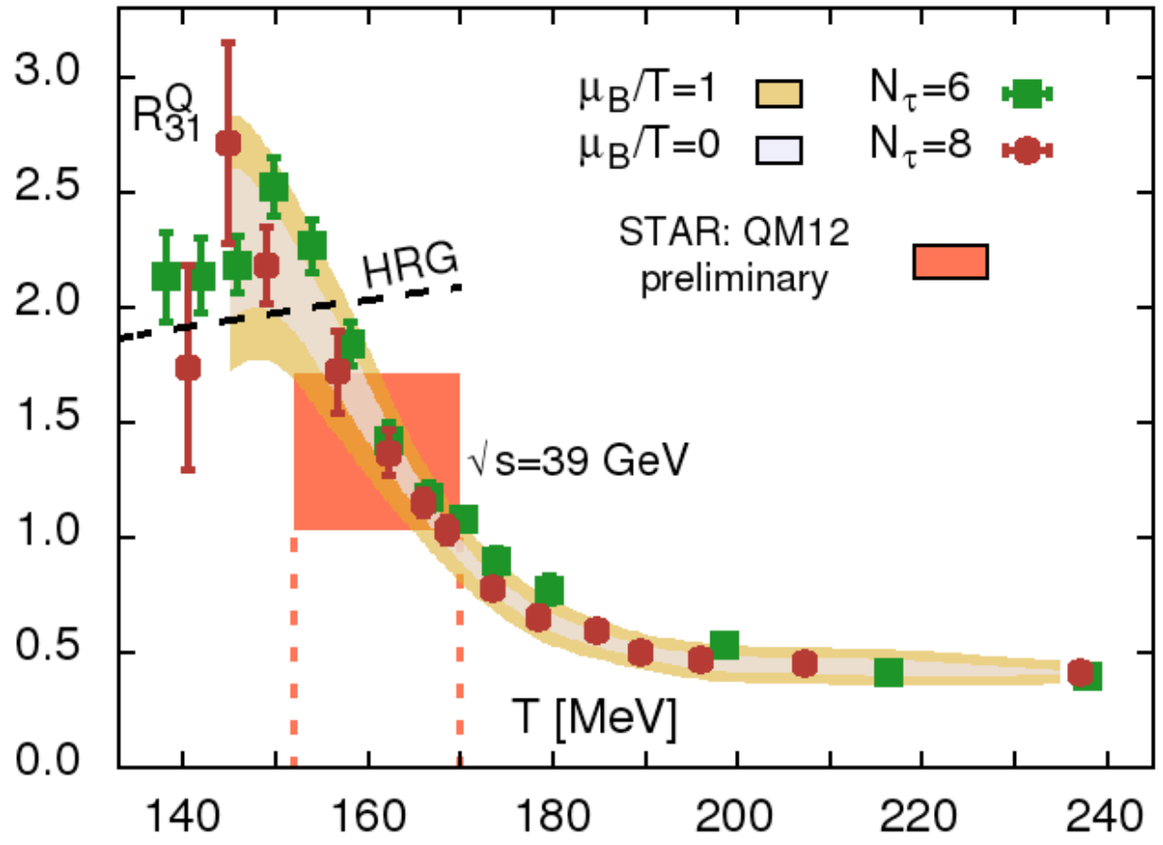

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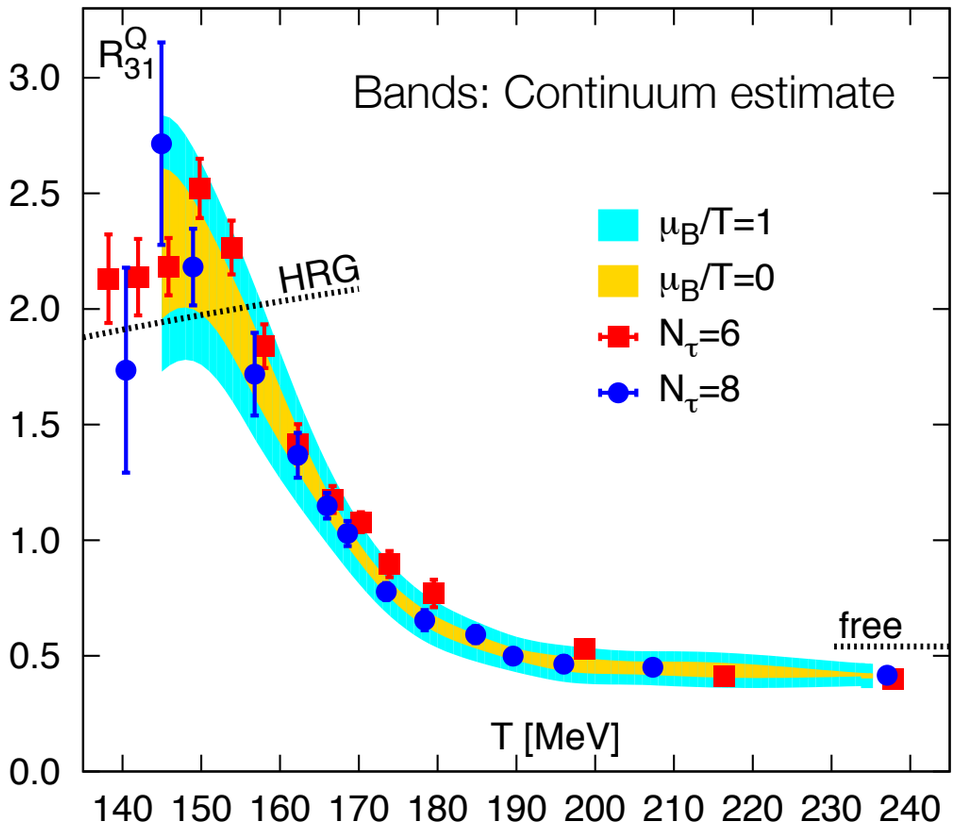

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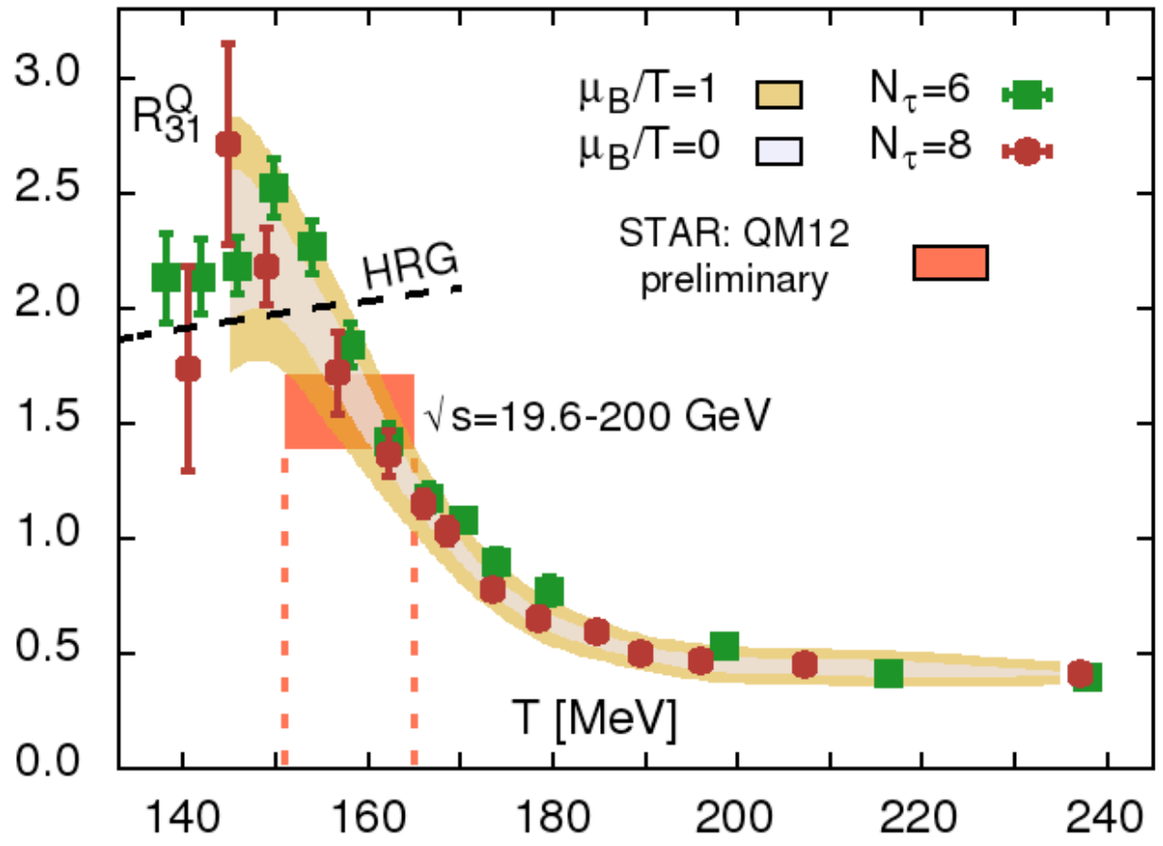

- small cutoff effects

Extraction of energy dependence of T not possible from preliminary data !

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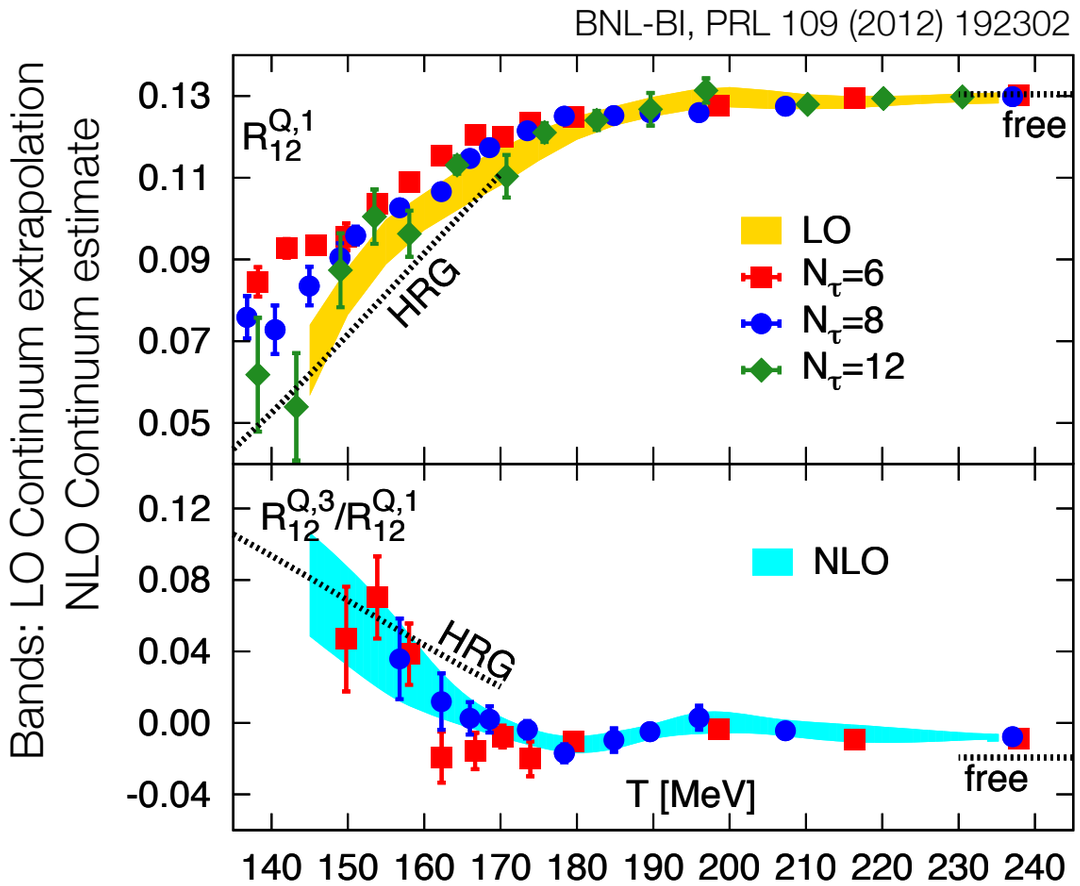
Determination of freeze-out chemical potential

$$R_{12}^{Q}(T,\mu_B) = R_{12}^{Q,1}\hat{\mu}_B + R_{12}^{Q,3}\hat{\mu}_B^3$$



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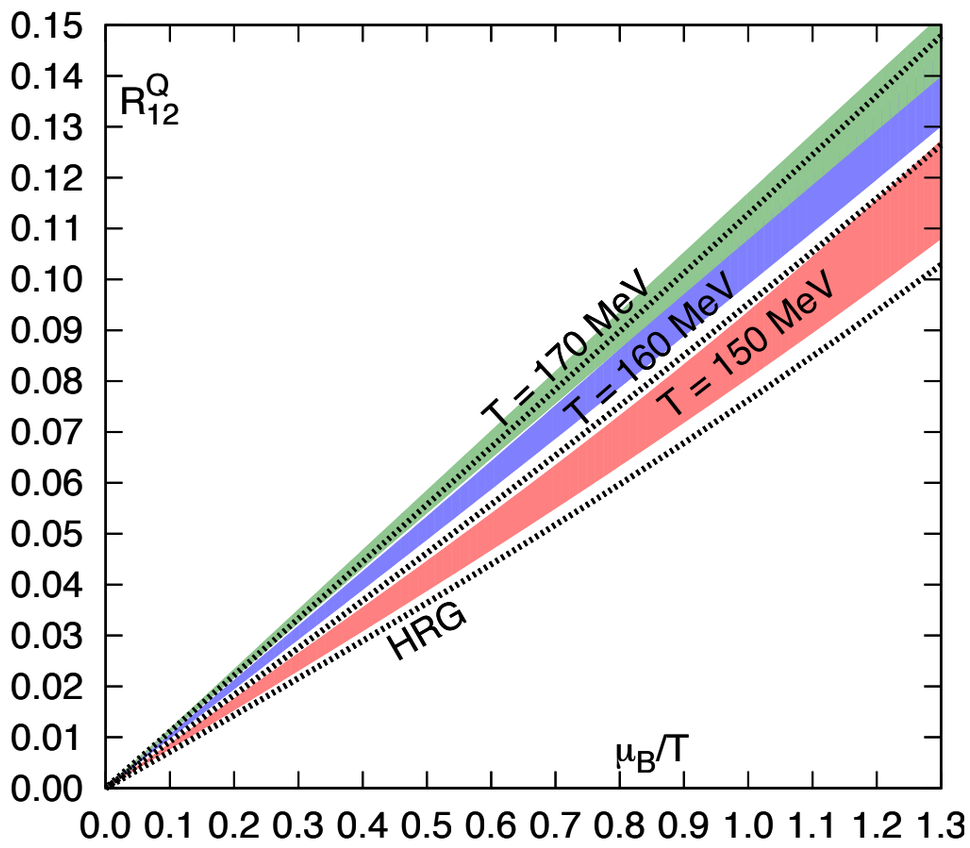

M_Q/σ_Q^2	μ_B^f/T^f
0.01-0.02	0.1-0.2
0.03-0.04	0.3-0.4
0.05-0.08	0.5-0.7

(for $T^f \sim 160\ MeV$)

- small cutoff effects at NLO

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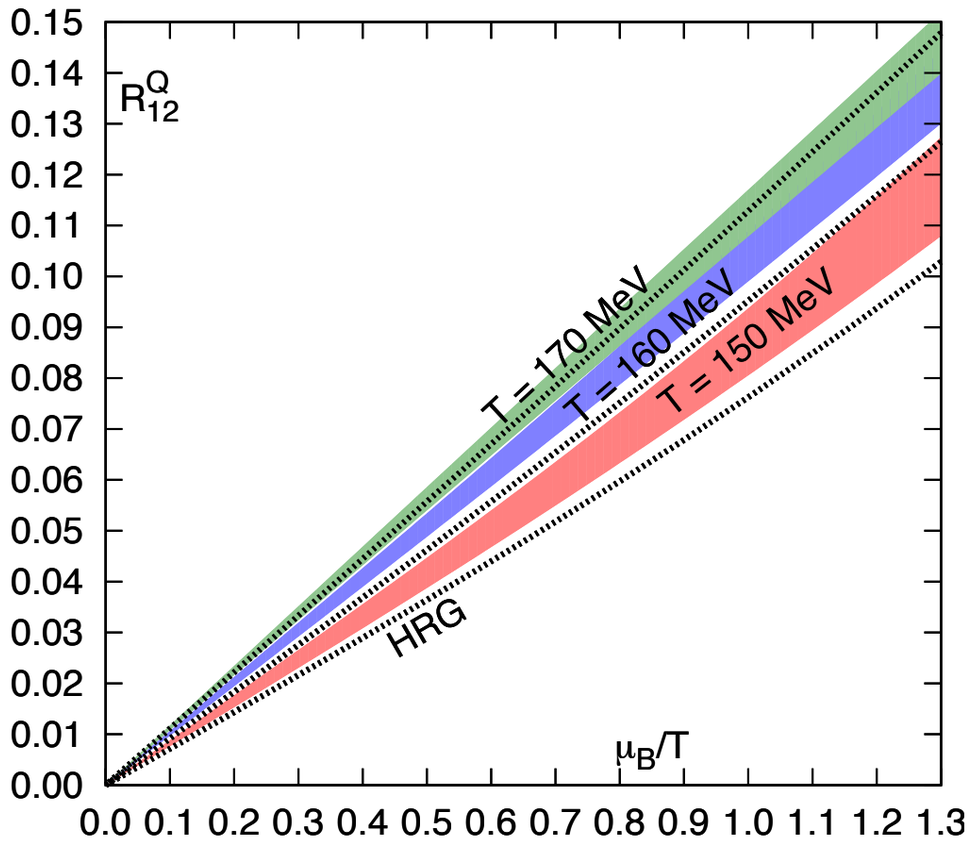

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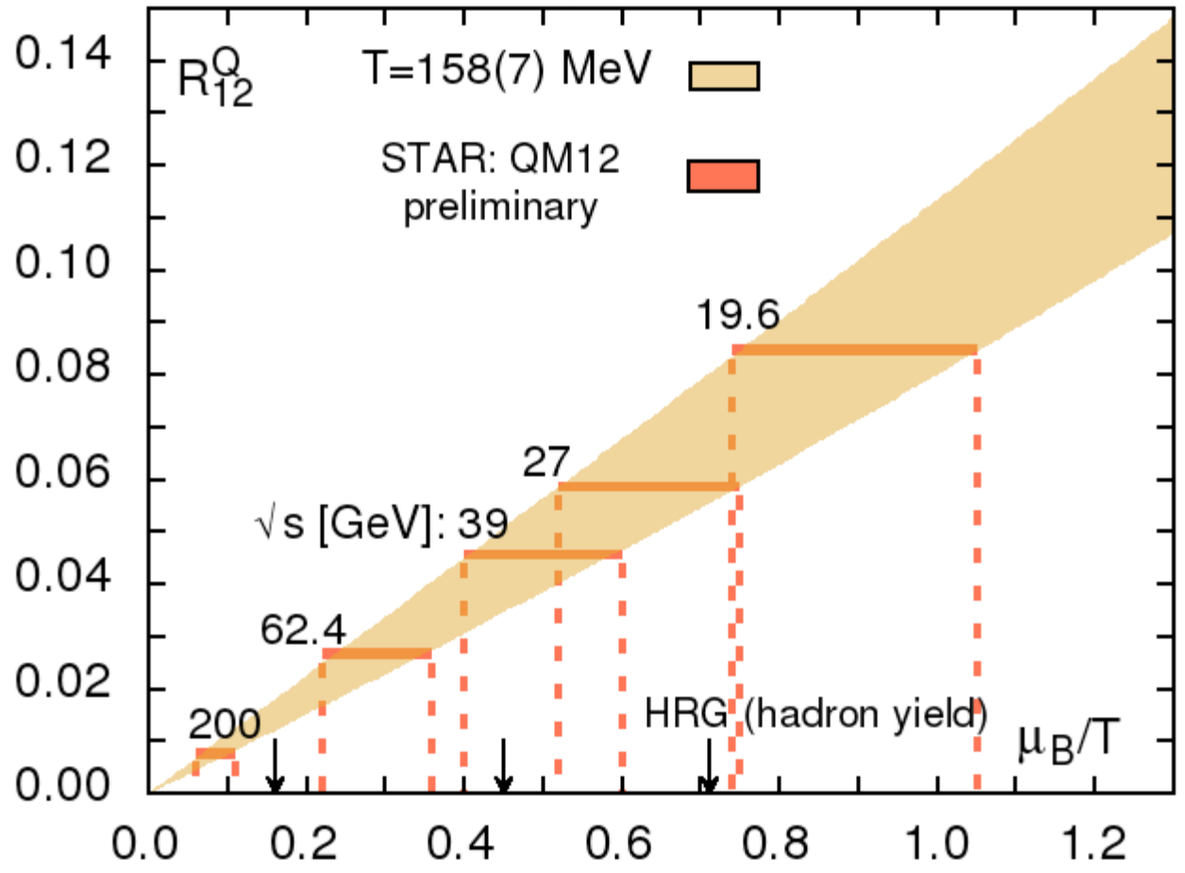

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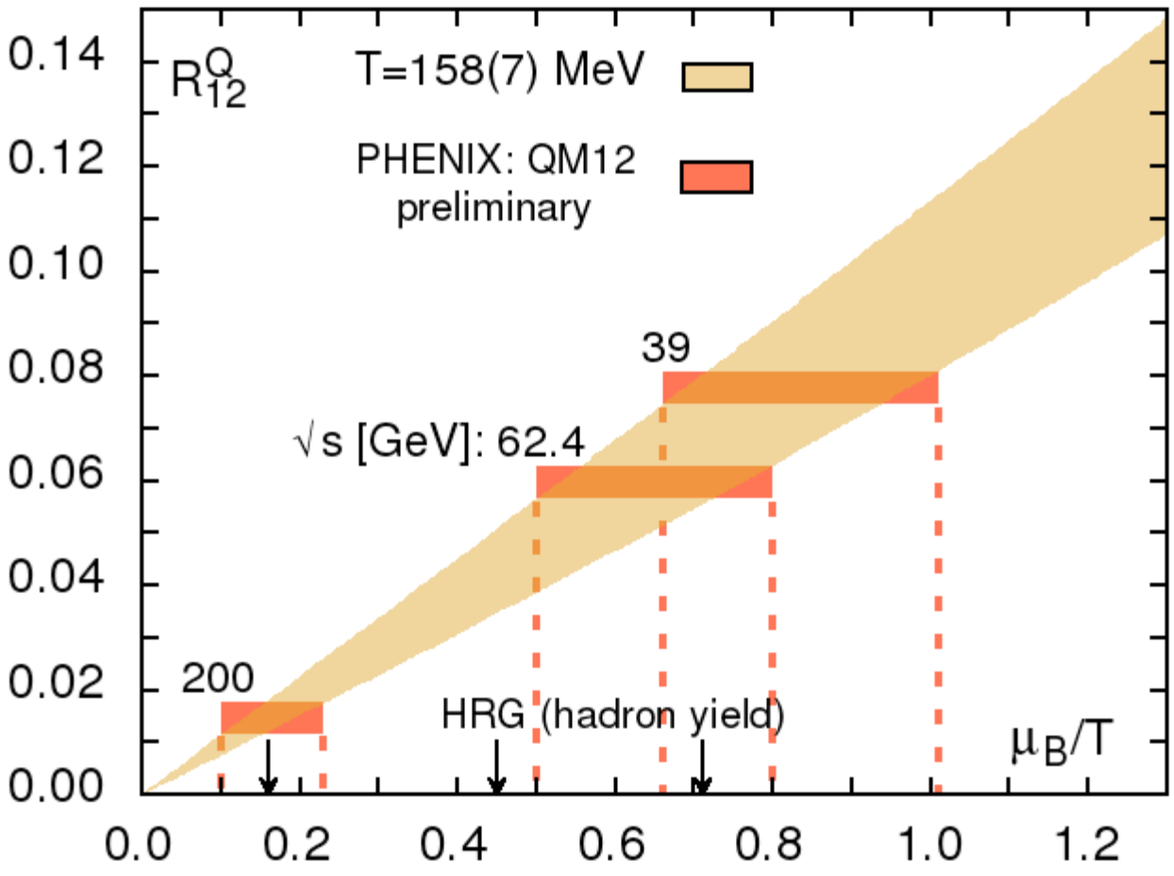
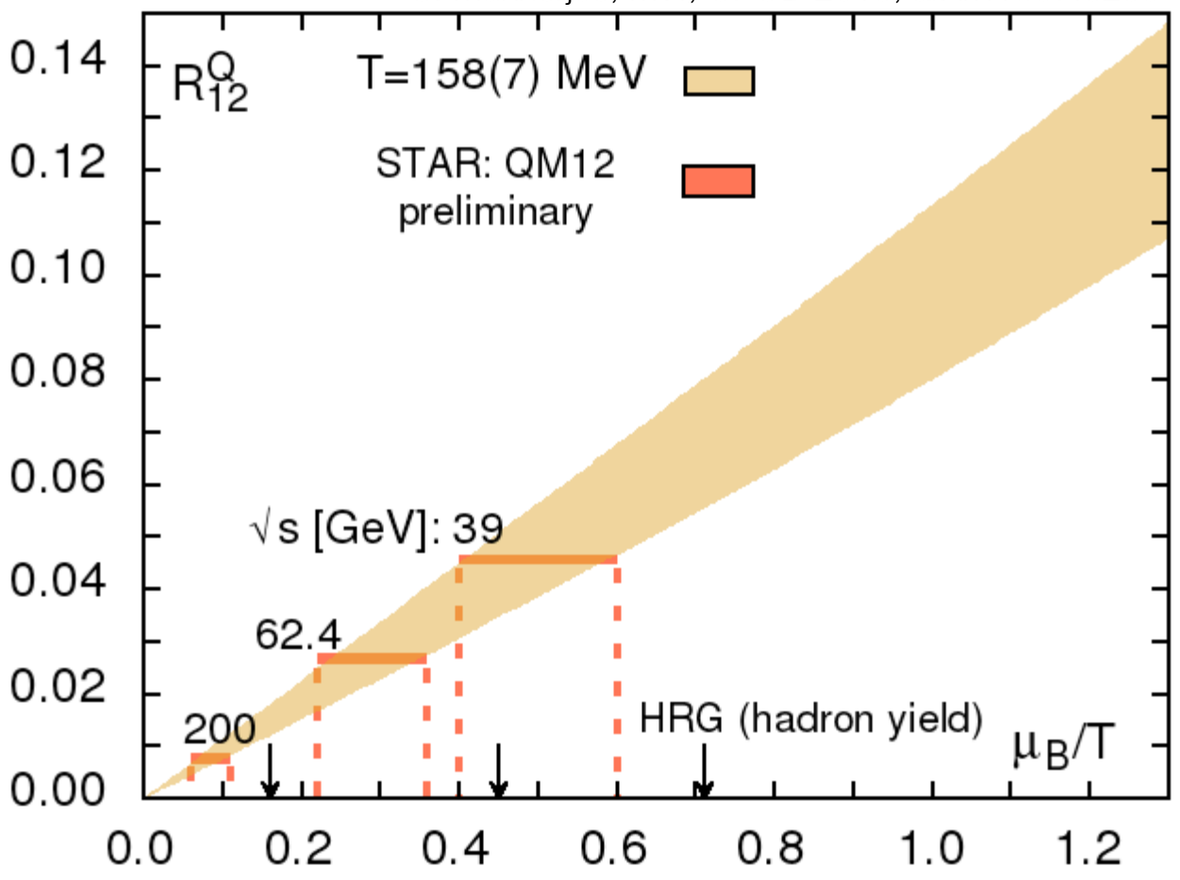
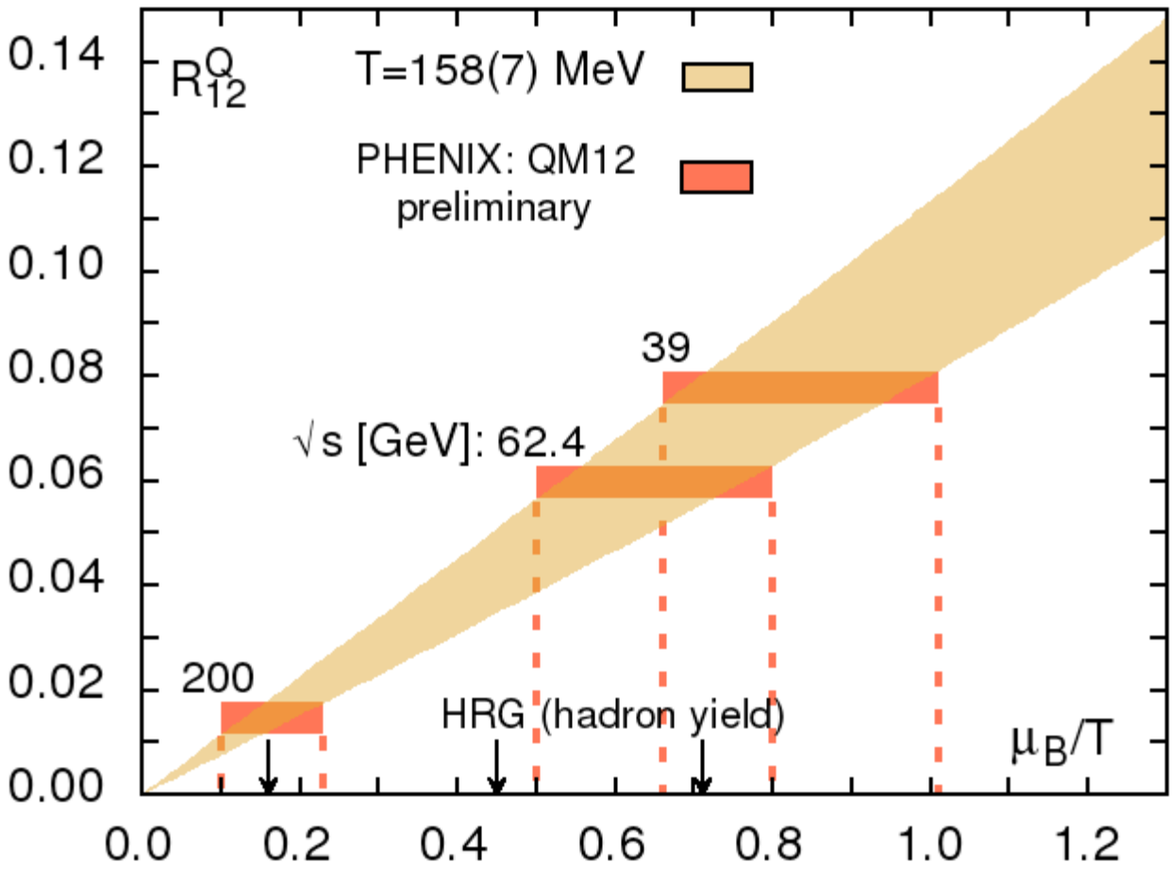


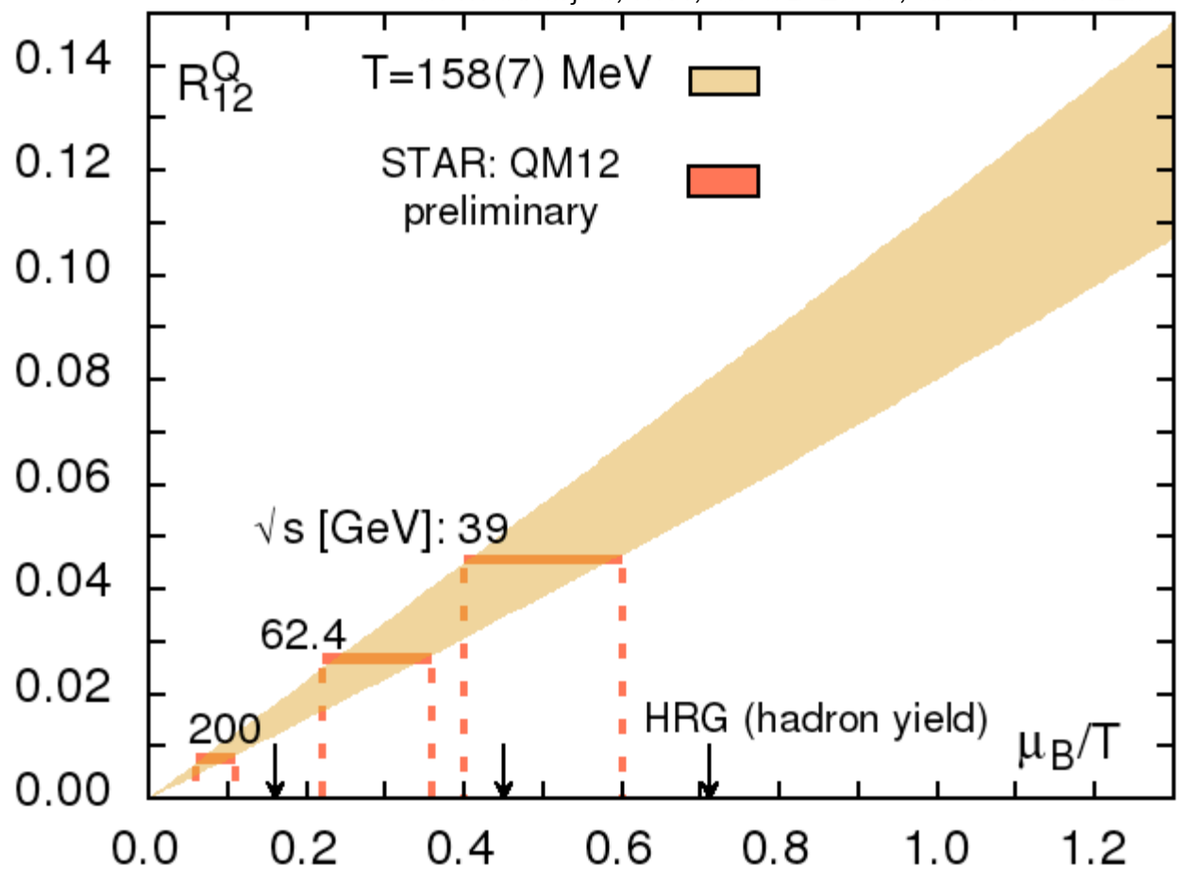


Determination of freeze-out chemical potential

$$R_{12}^{Q}(T,\mu_B) = R_{12}^{Q,1}\hat{\mu}_B + R_{12}^{Q,3}\hat{\mu}_B^3$$

S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255



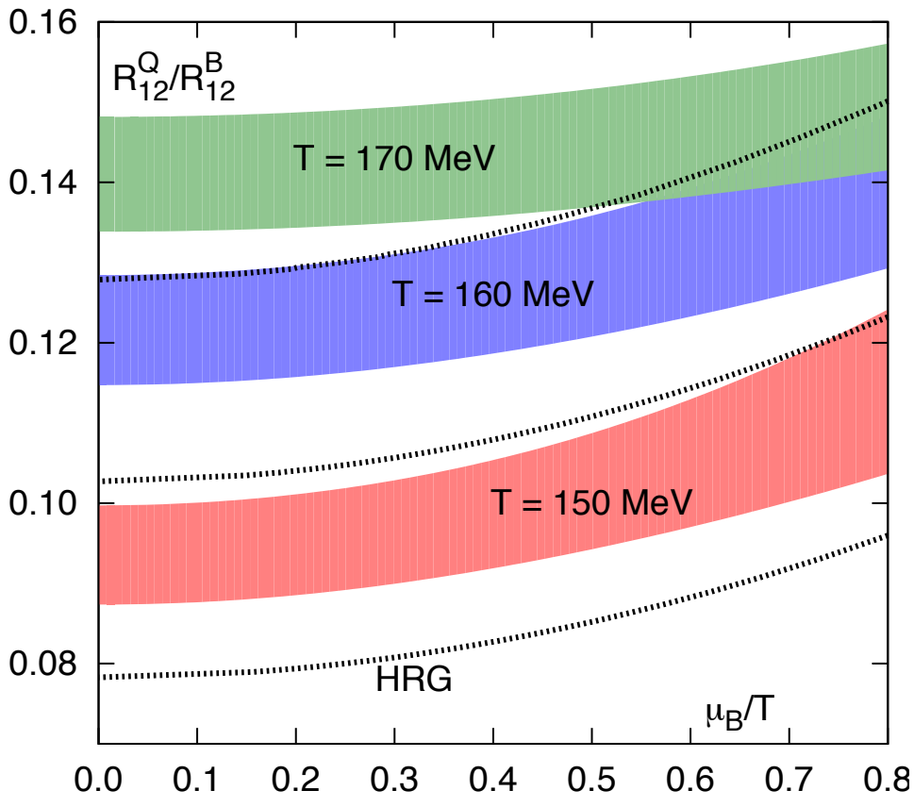


Preliminary data from PHENIX and STAR are not in agreement !

With preliminary data LQCD and HRG do not agree !

Thermodynamic consistency

- cross-check: compare with measurements of proton fluctuations



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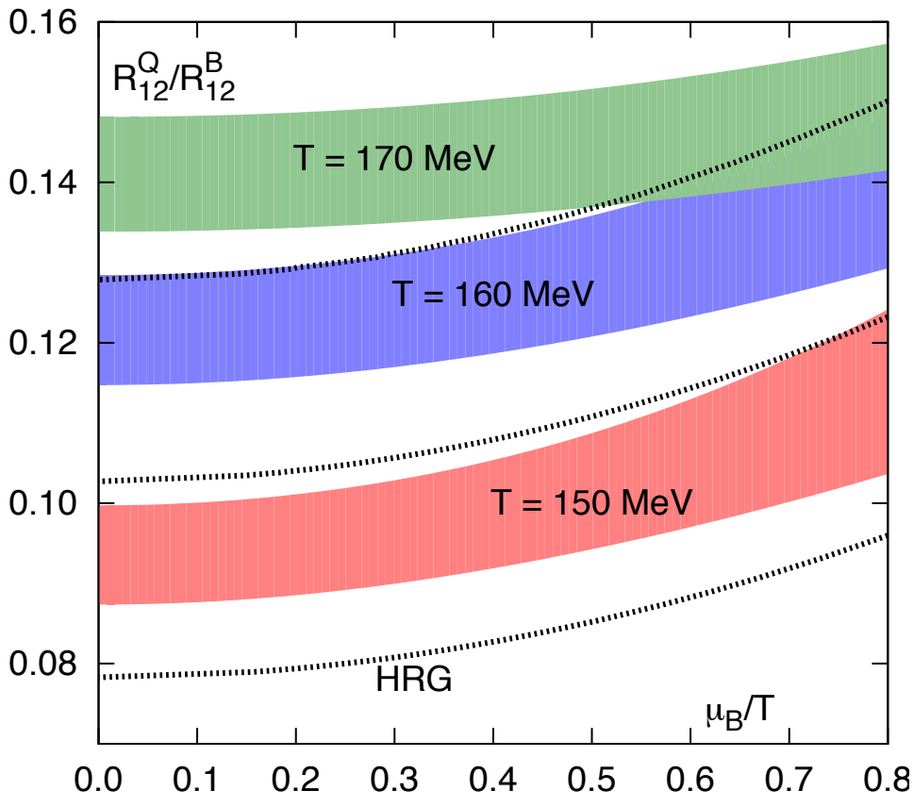
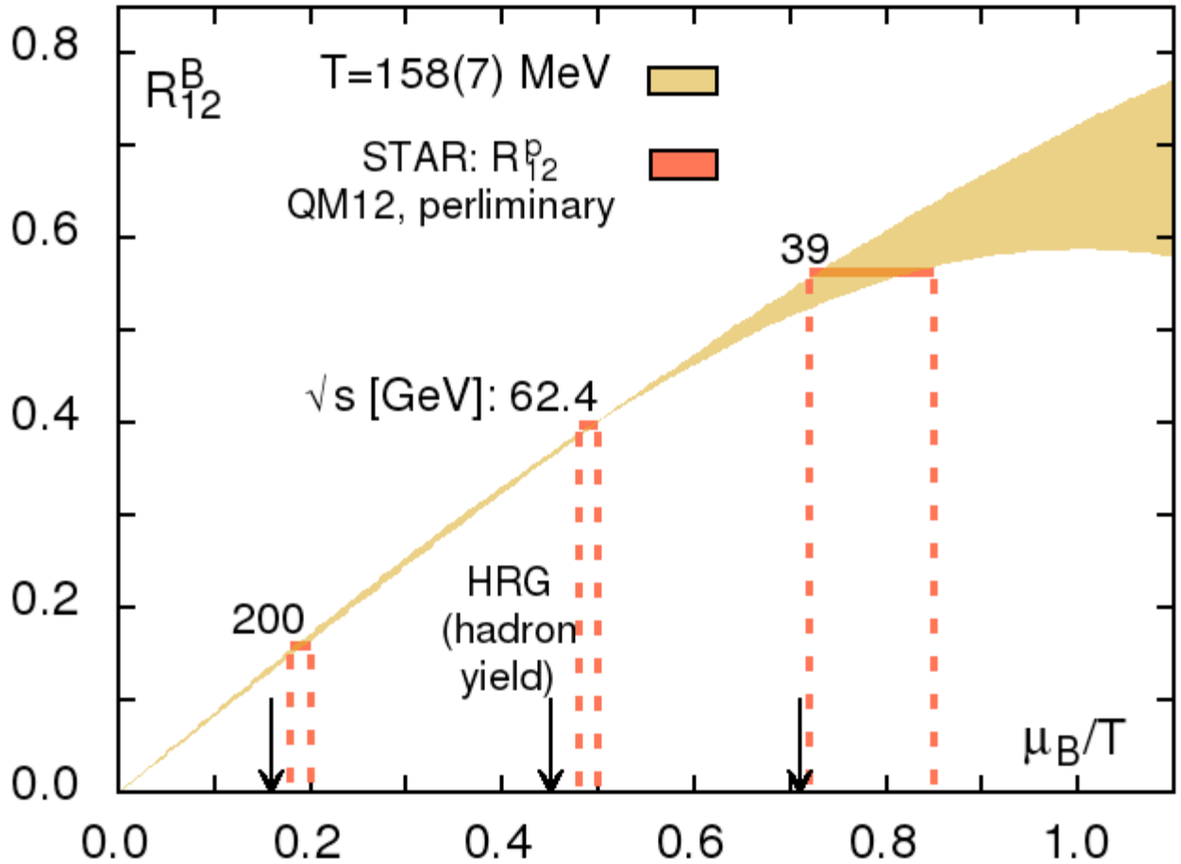
S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255





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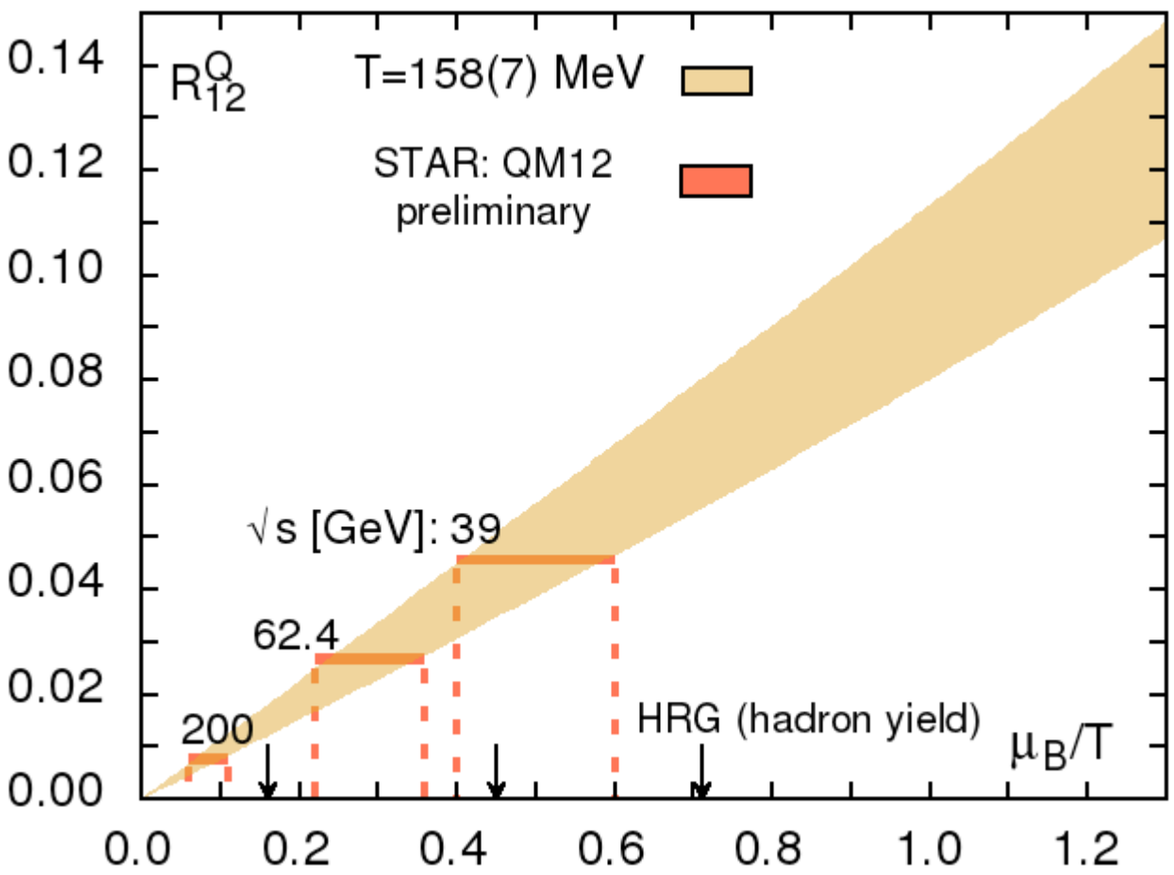
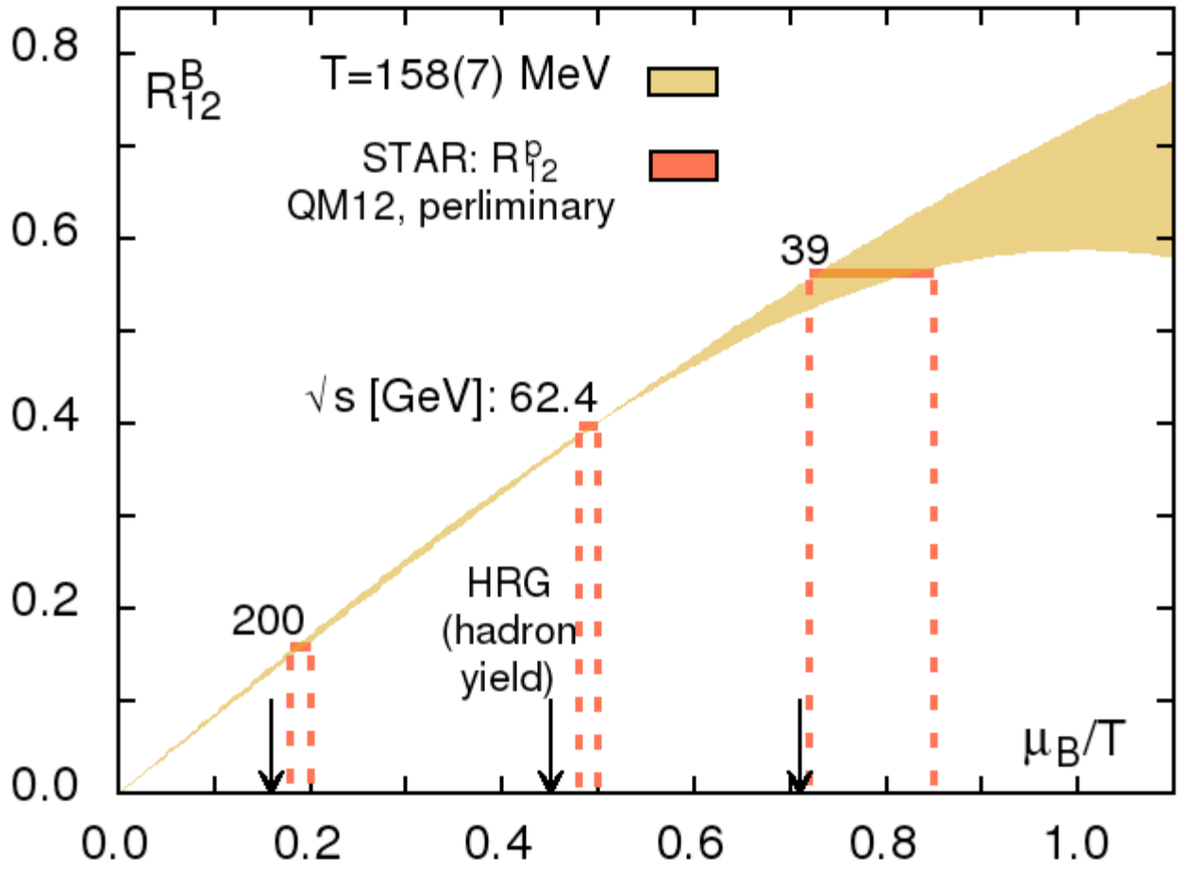
S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255





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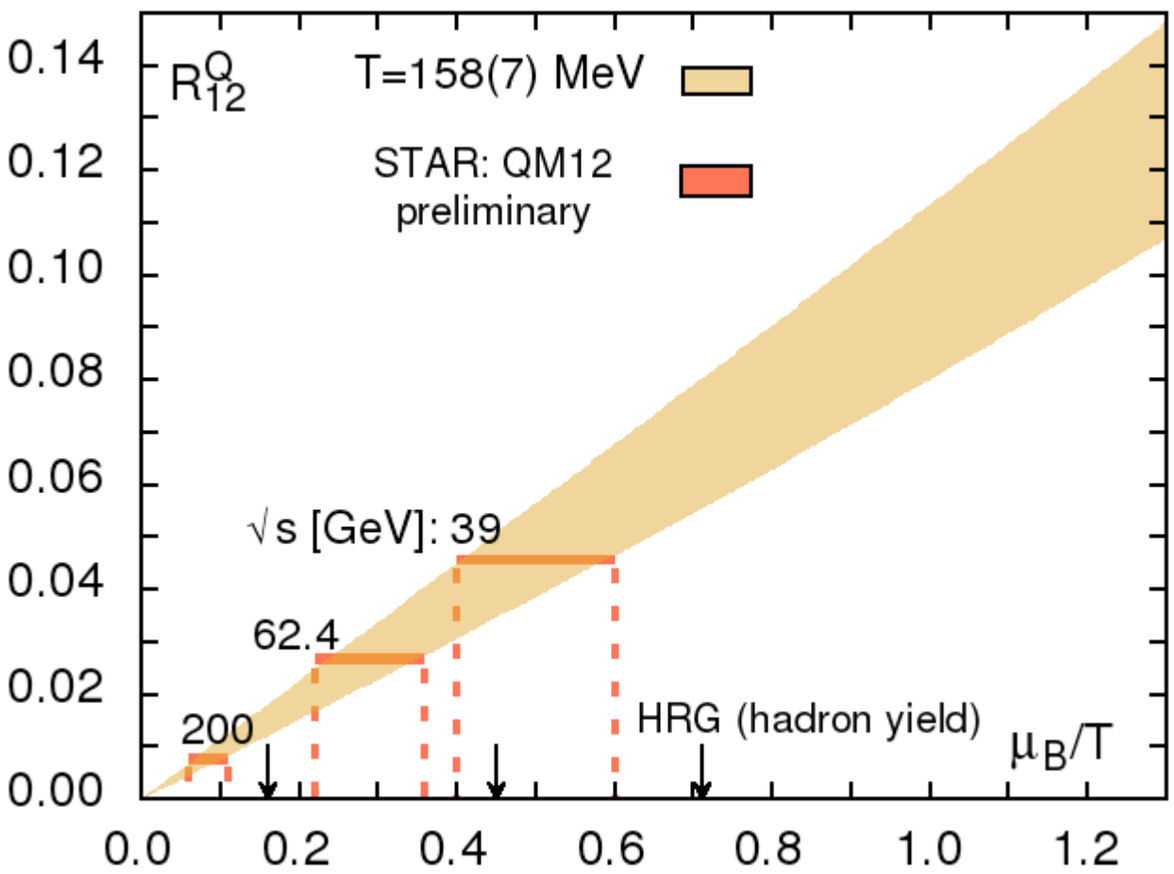
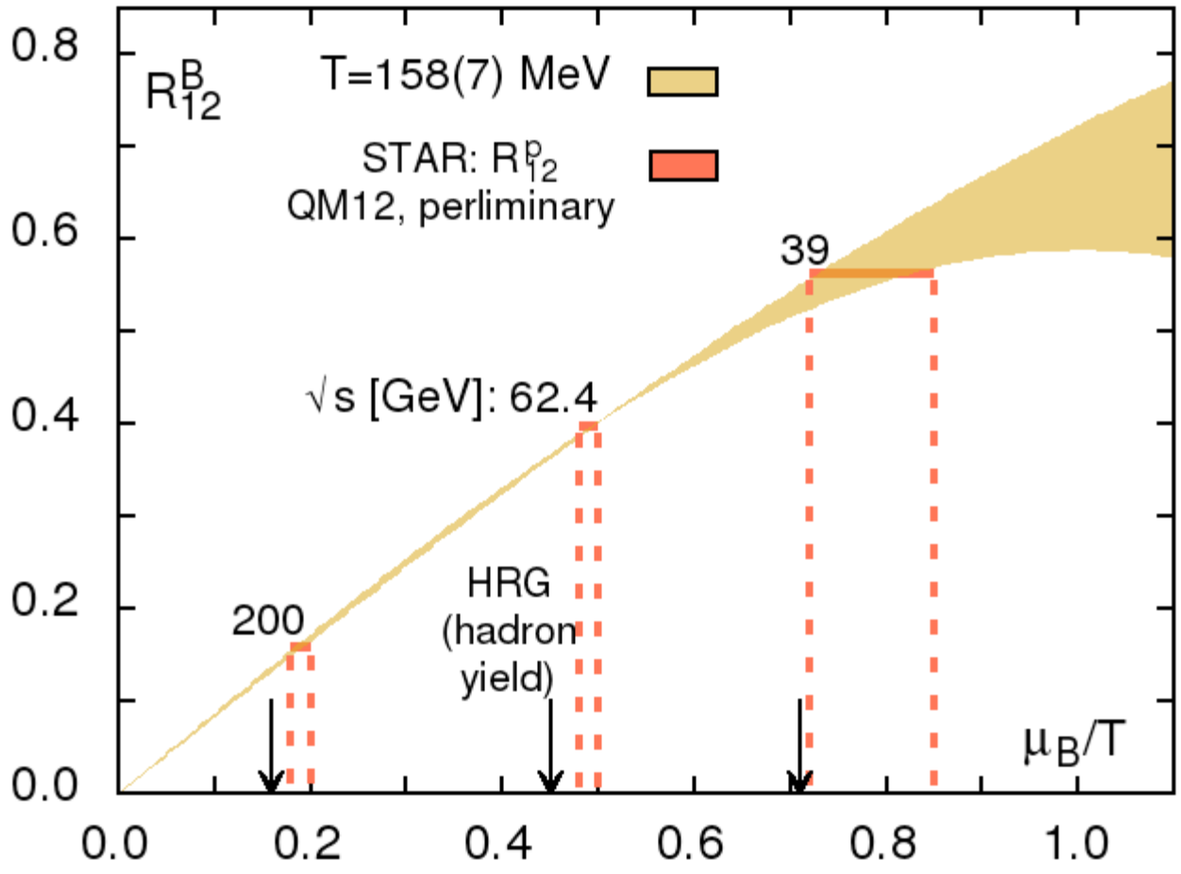


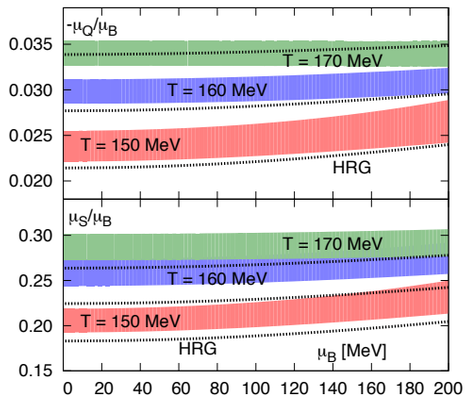


inconsistent! But proton number fluctuations ≠ baryon number fluctuations

(see Asakawa-Kitazawa; Bzdak-Koch-Skokov)

Summary

- Introduction of model-independent determination of freeze-out parameters
  - based on fluctuations of conserved charges up to 4th order
- Fix isospin and strangeness chemical potential to initial conditions



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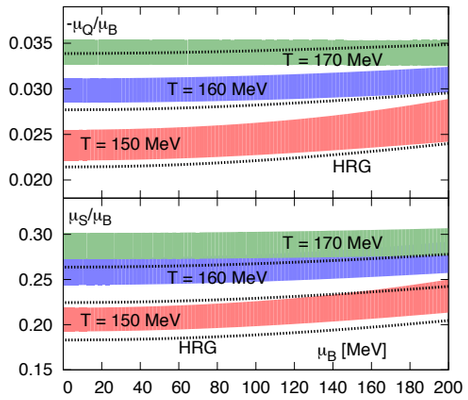
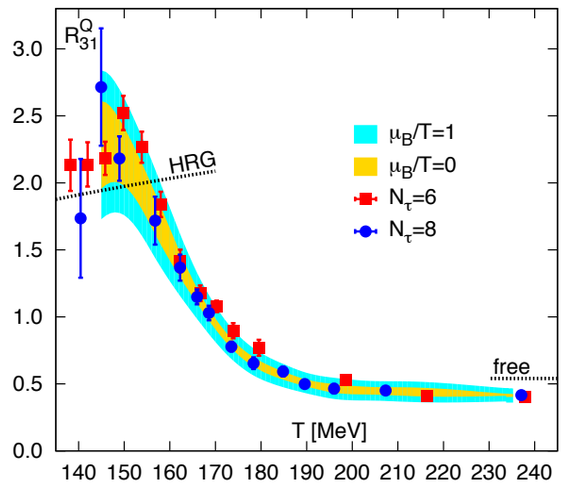


- Thermometer

$$R^Q_{31}(T,\mu_B) = R^{Q,0}_{31} + R^{Q,2}_{31}\hat{\mu}^2_B$$



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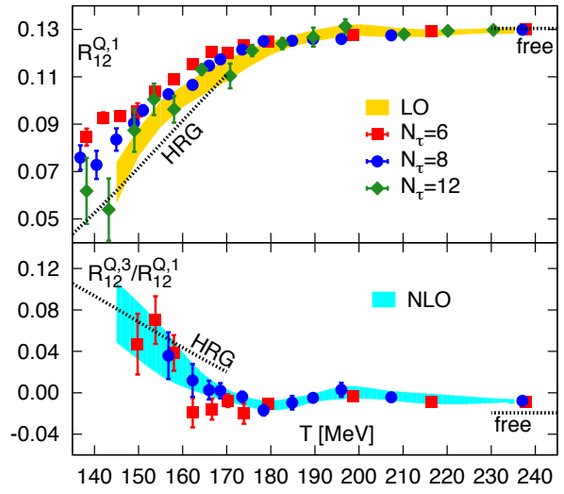

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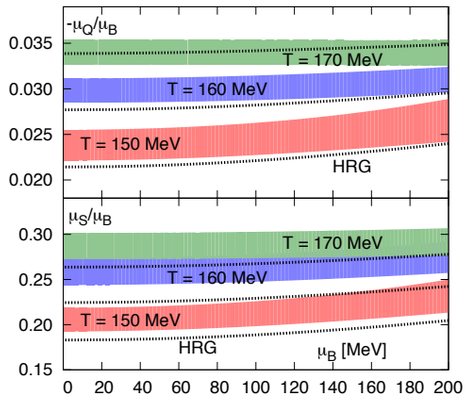

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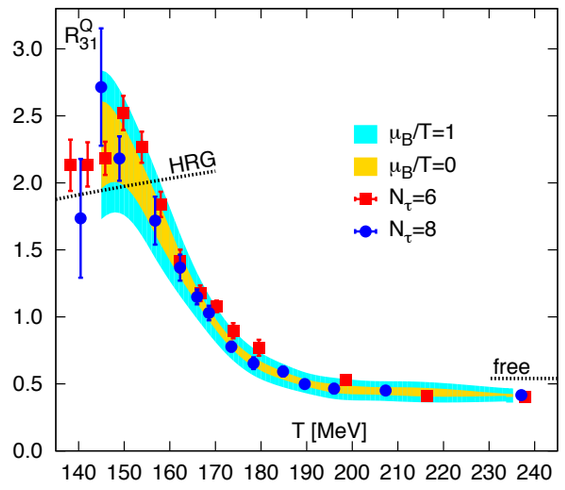

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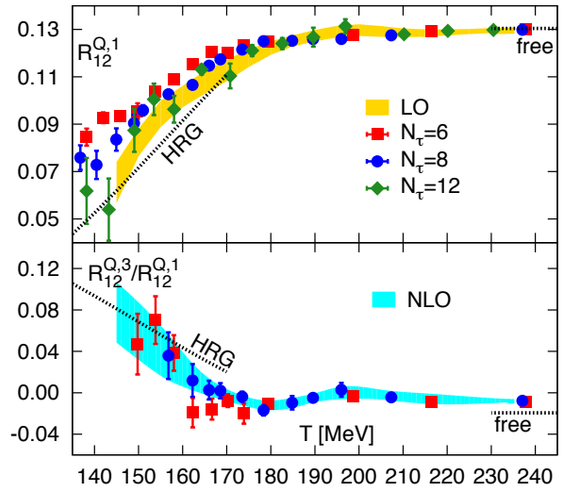
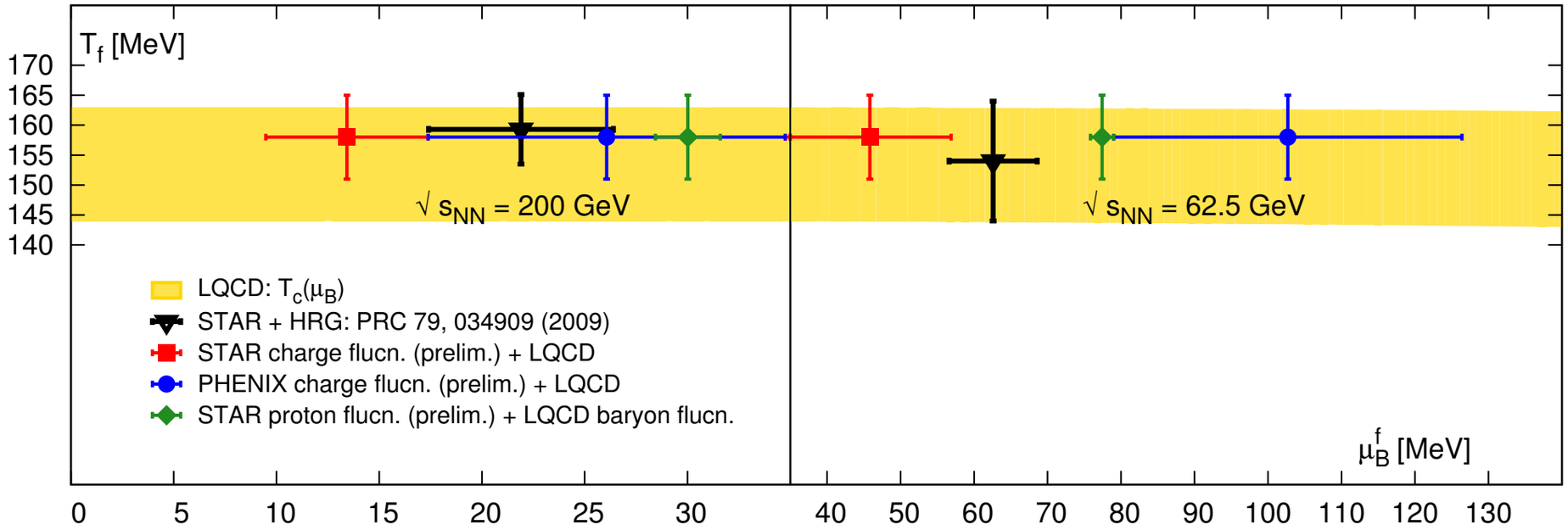


- controlled Taylor expansion up to $\mu_B \lesssim 200\,\mathrm{MeV}$ or $\sqrt{S_{NN}} \gtrsim 19.6\,\mathrm{MeV}$
- check of consistency from baryon number fluctuations and/or higher-order ratios

Summary: Freeze-out parameters



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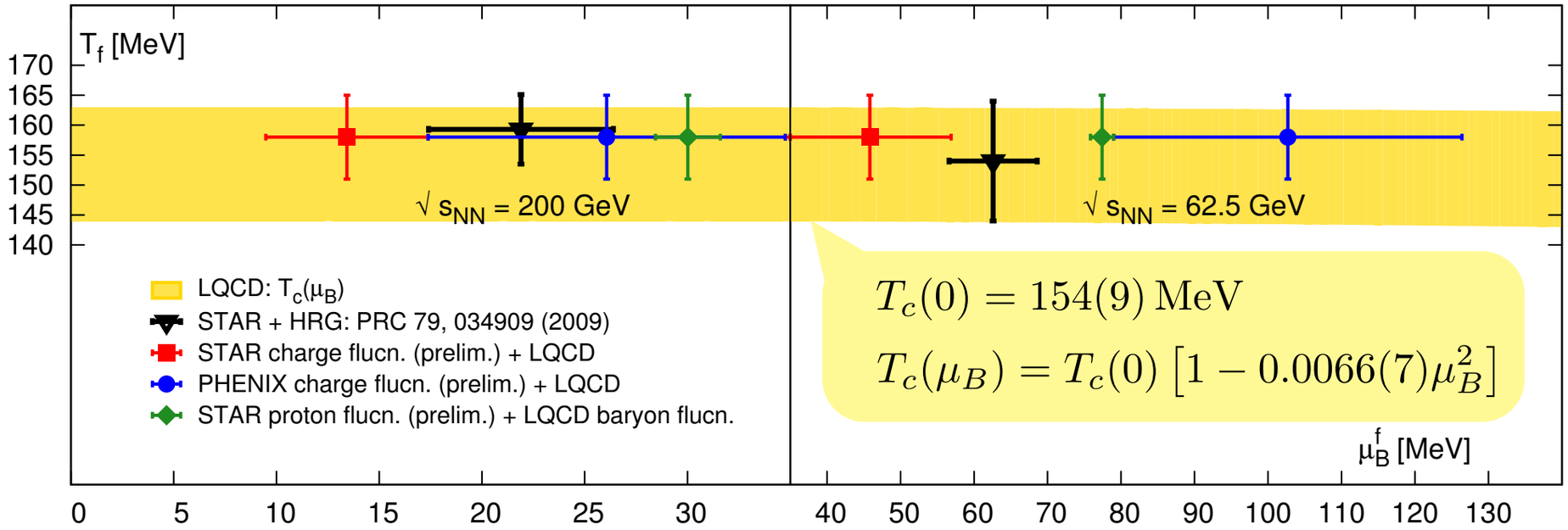

- all observed freeze-out temperatures fall in the crossover region of QCD

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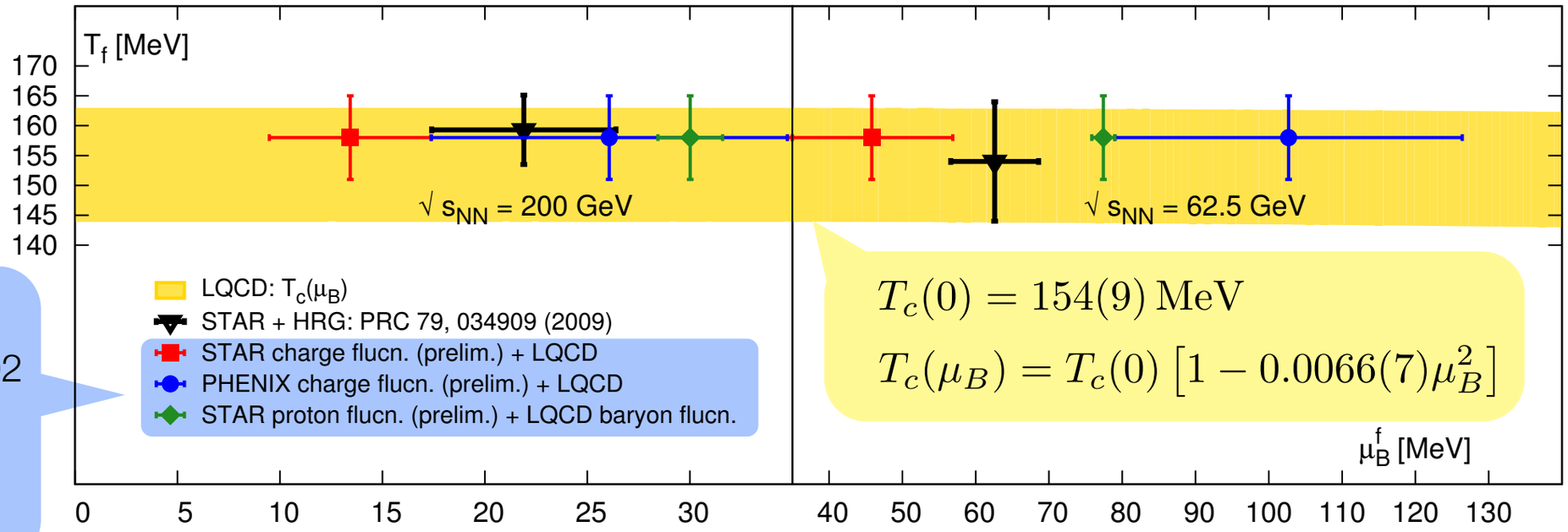

Bielefeld-BNL
PRL 109 (2012) 192302
&
CPOD March 2013
(arXiv:1307.6255)

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$$T_c(\mu_B) = T_c(0)\left[1 - 0.0066(7)\mu_B^2\right]$$

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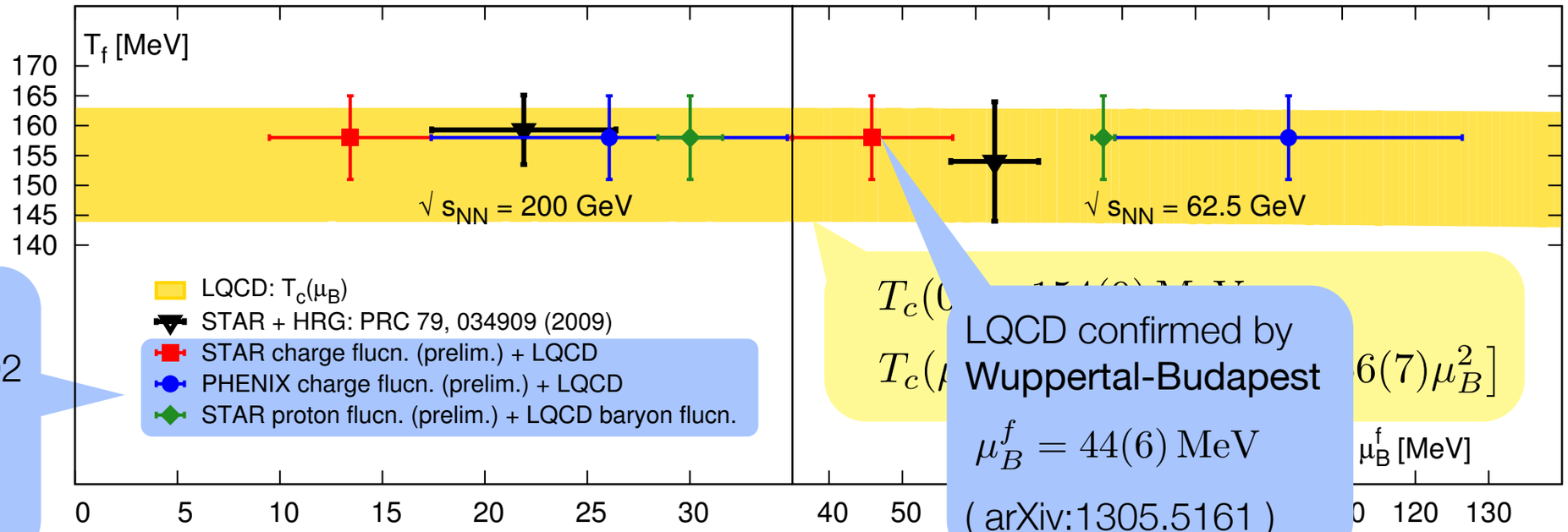

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$T_c(\mu_B) = 154(9)\,\mathrm{MeV}\left[1 - 0.0056(7)\mu_B^2\right]$

LQCD confirmed by
Wuppertal-Budapest
$\mu_B^f = 44(6)\,\mathrm{MeV}$
(arXiv:1305.5161)

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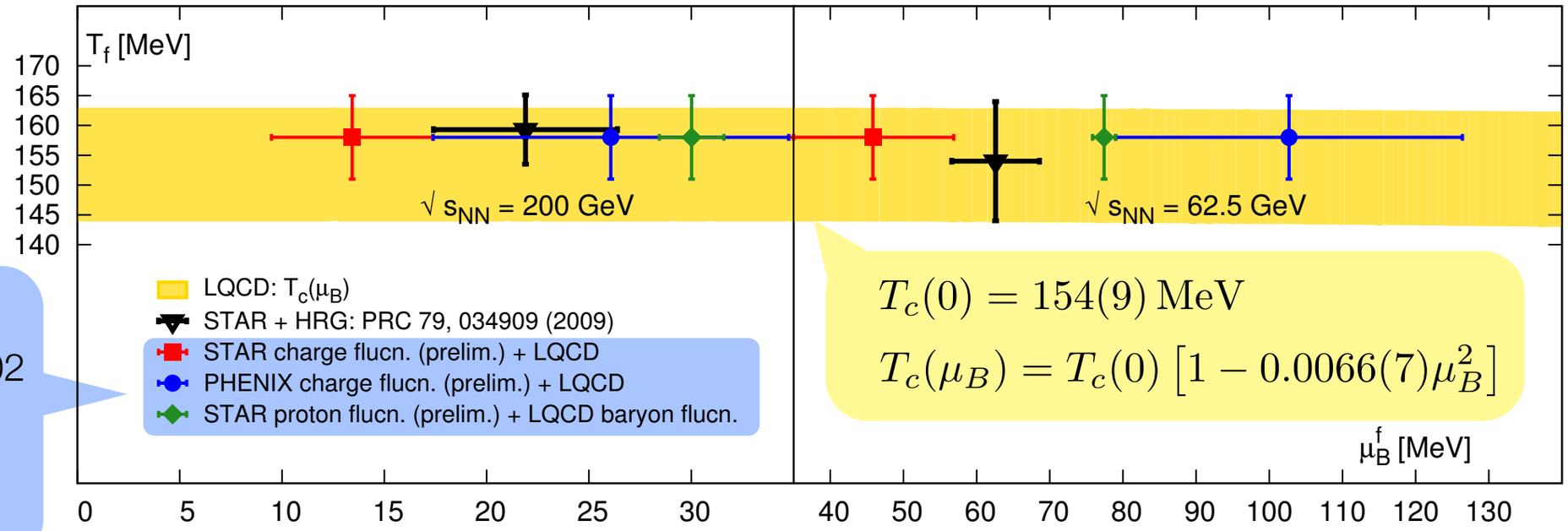

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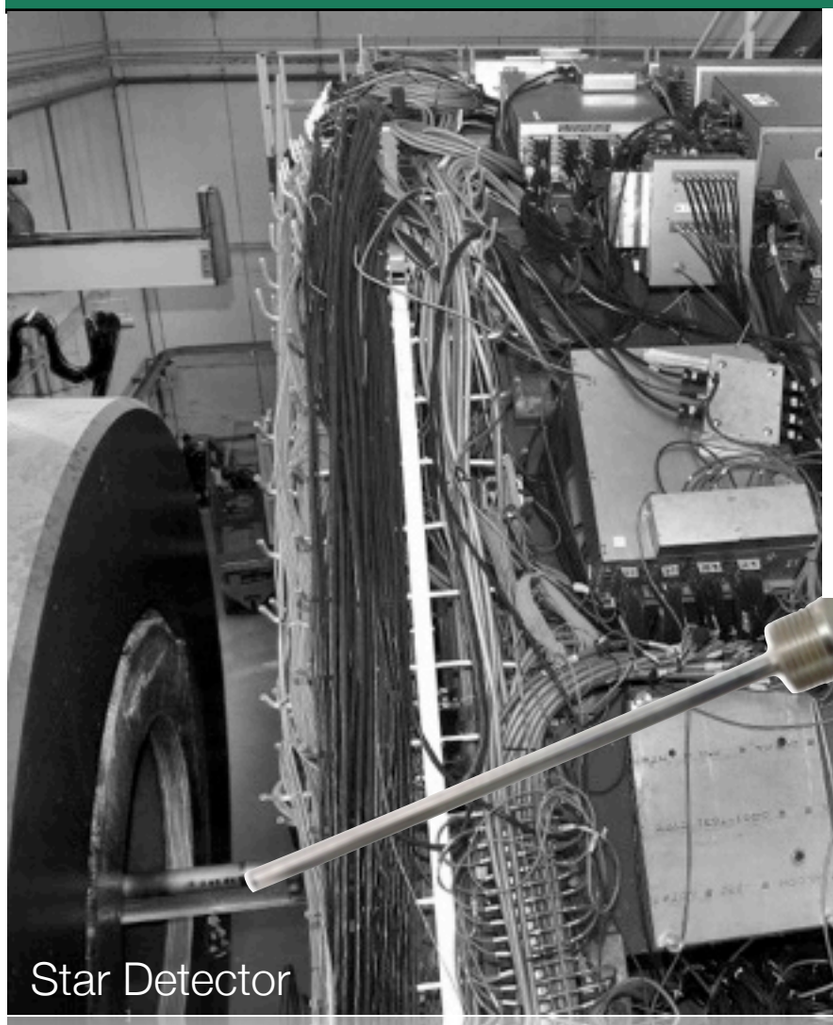
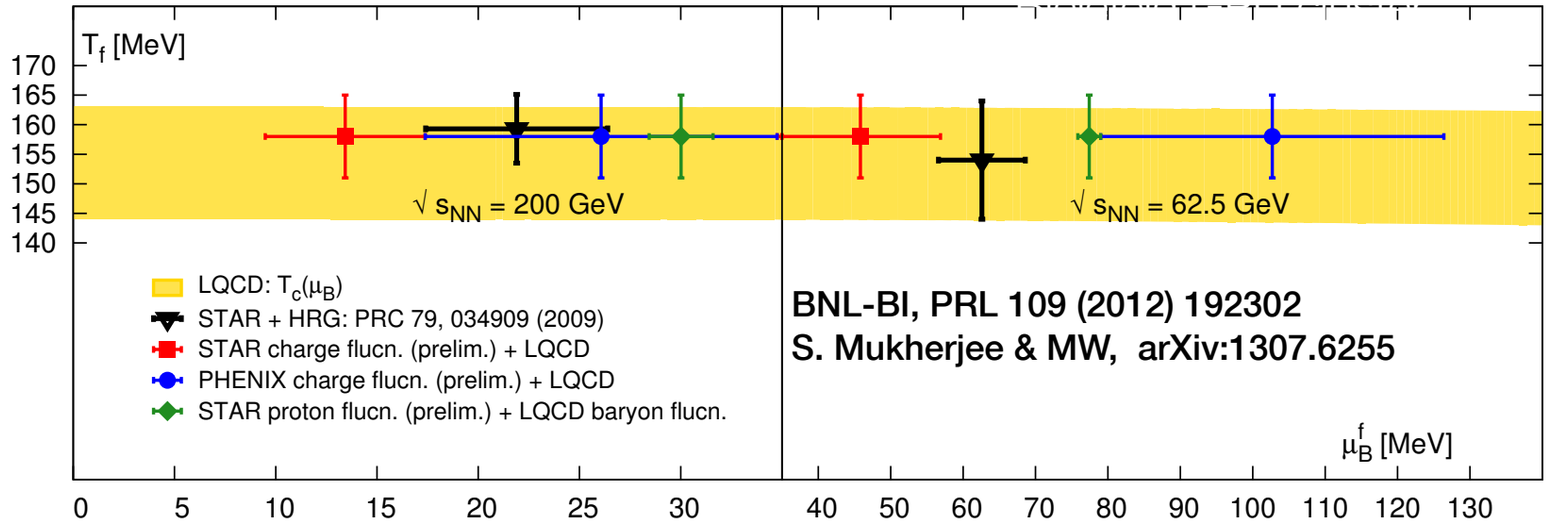
- all observed freeze-out temperatures fall in the crossover region of QCD
- preliminary data from STAR (charge, proton) and PHENIX (charge) are not in agreement
  - need to check grand canonical approach (finite size effects, acceptance cuts)
  - addressed in the experimental analysis

Charge Fluctuations as Thermometer for Heavy-Ion Collisions

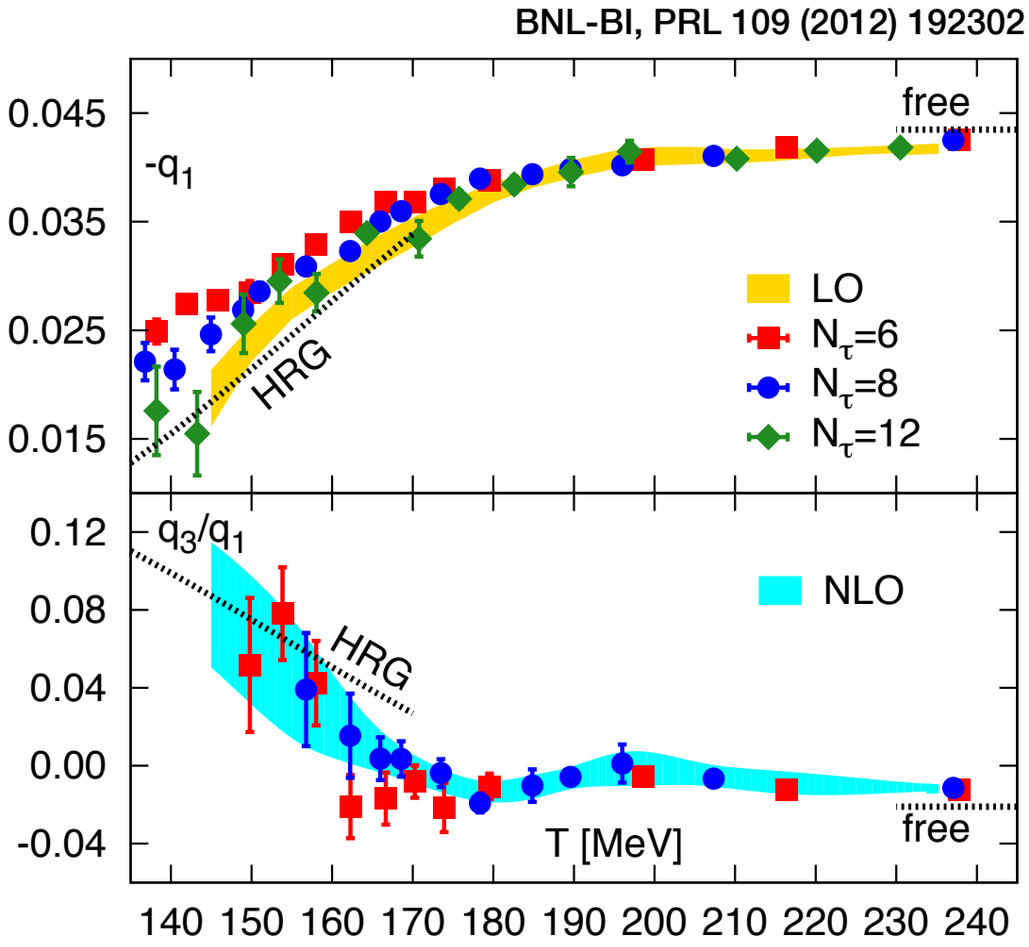




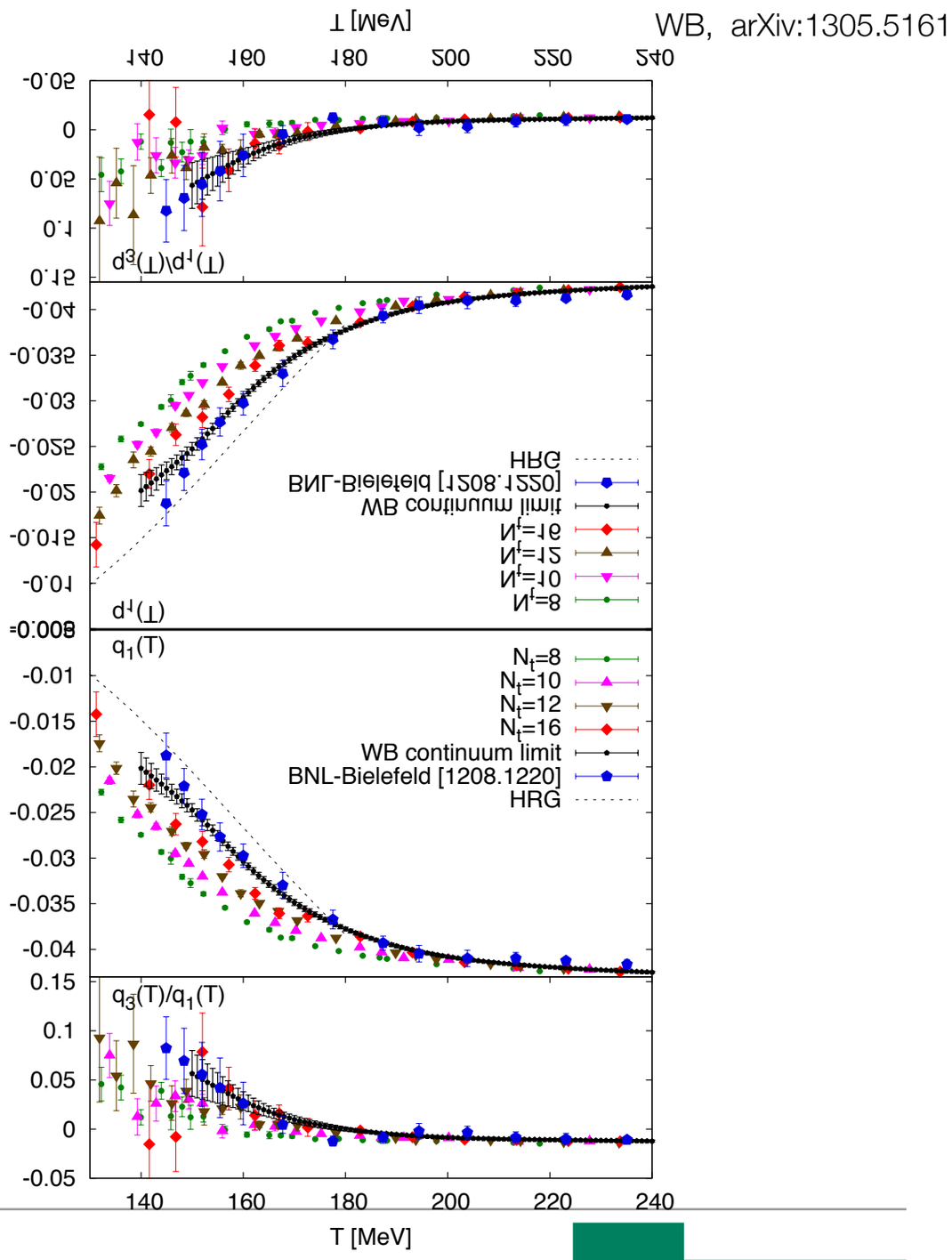
Bielefeld-BNL collaboration:
A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee, P. Petreczky, C. Schmidt, D. Smith, W. Soeldner, *M. Wagner*

Universität Bielefeld

Charge chemical potential

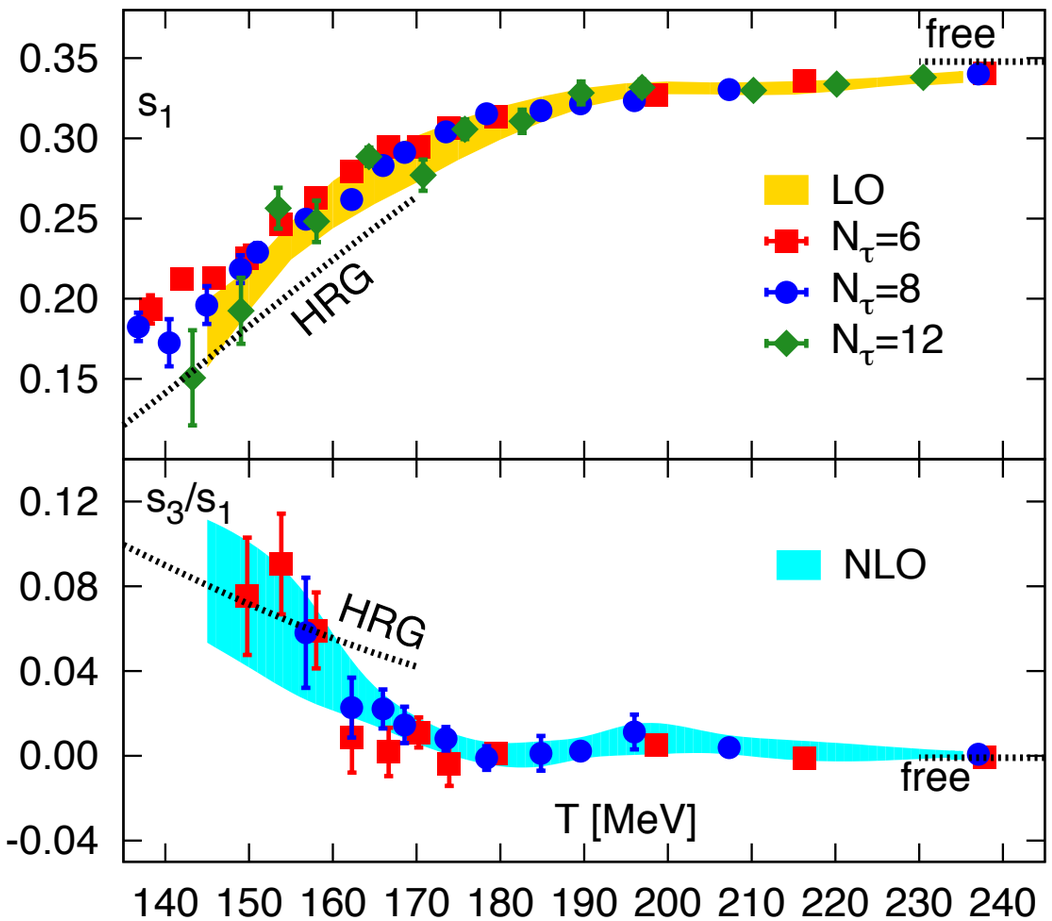


WB, arXiv:1305.5161



Strangeness

BNL-BI, PRL 109 (2012) 192302



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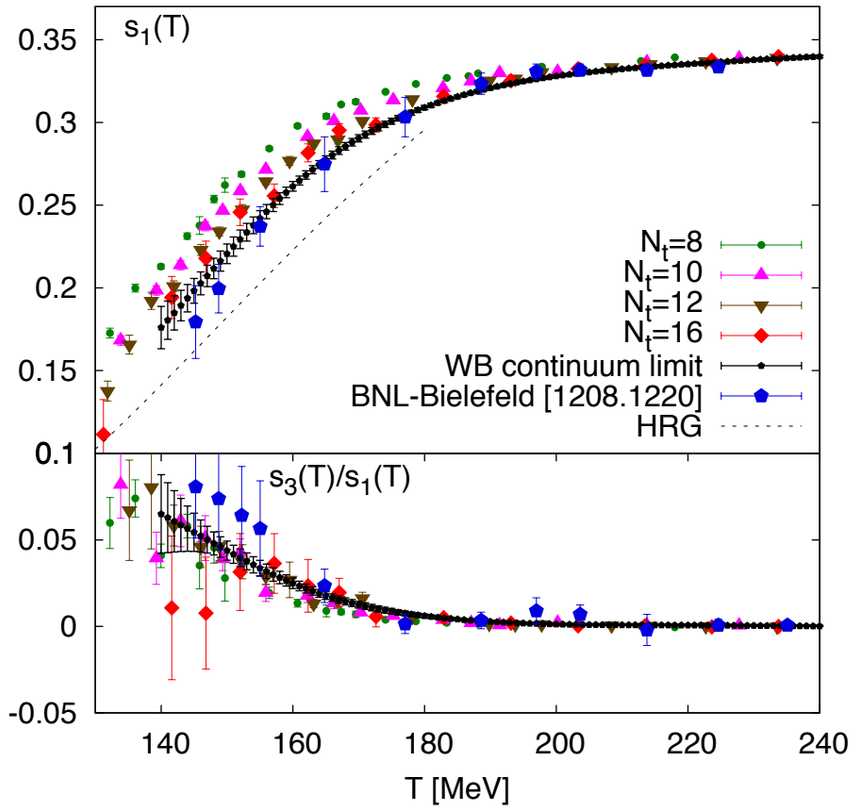
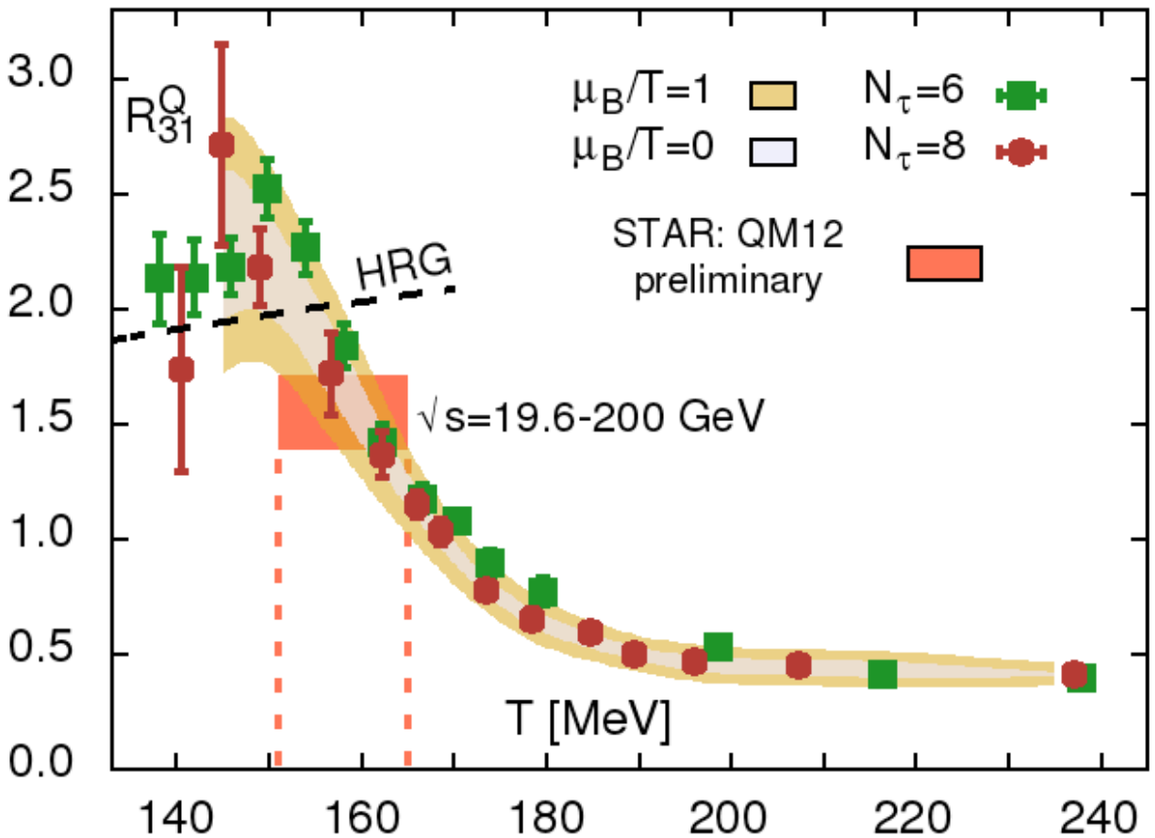


Freeze-out temperature

BNL-BI, PRL 109 (2012) 192302
S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255



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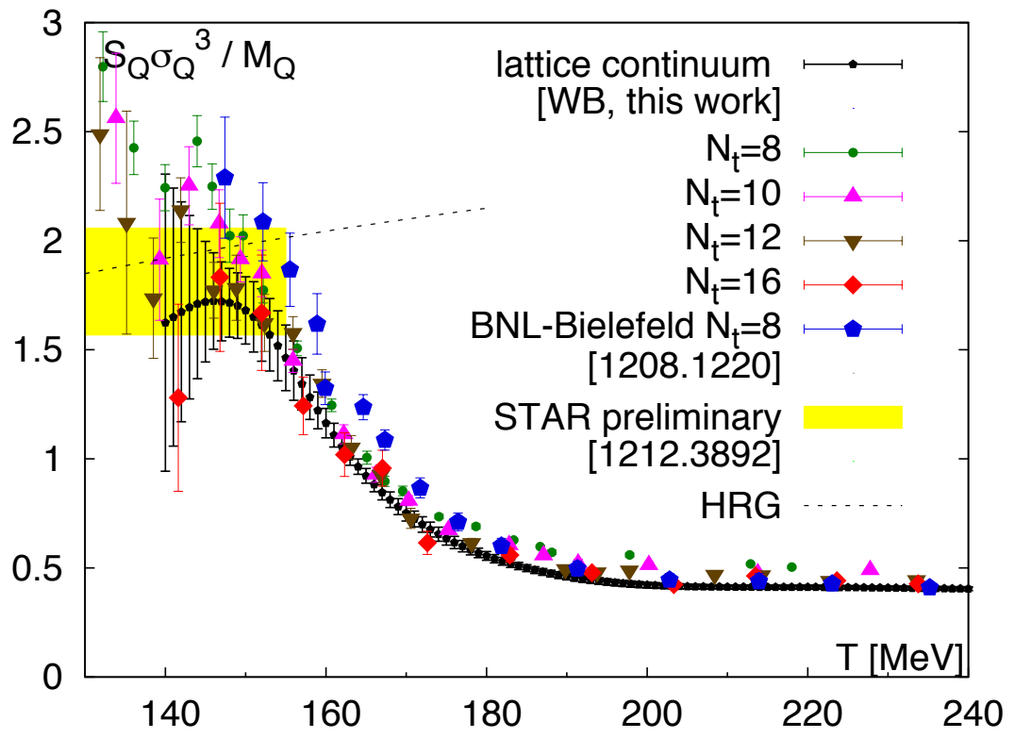
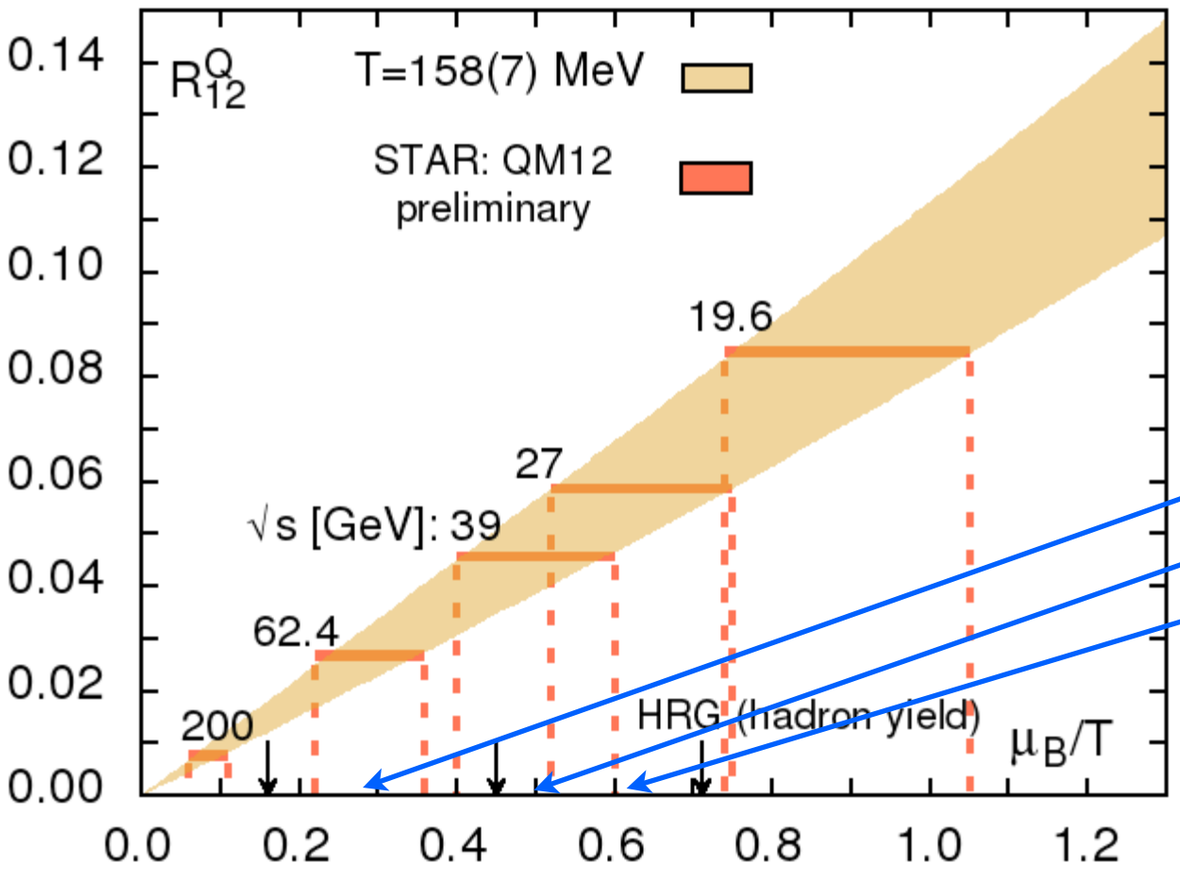


Freeze-out baryon chemical potential

S. Mukherjee, MW, CPOD 2013, arXiv:1307.6255



WB, arXiv:1305.5161

$$T \approx 145 - 160\,\mathrm{MeV}$$

$\sqrt{s}[GeV]$	μ_B^f [MeV]
62.4	$44(3)(1)(2)$
39	$75(5)(1)(2)$
27	$95(6)(1)(5)$
	$()_{\delta T}()_{\mathrm{lat}}()_{\mathrm{exp}}$