On the N_f-dependence of gluonic observables

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ALPHA Collaboration

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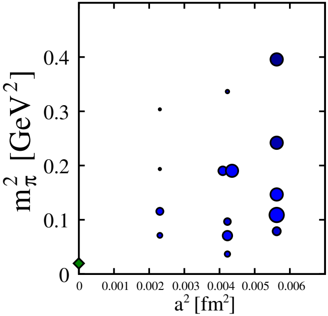
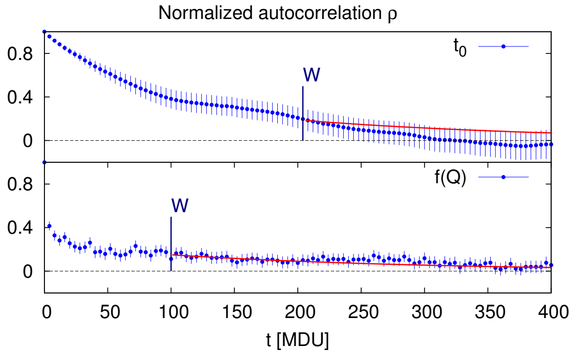
CLS Ensembles

β	ID	m_π [MeV]	MDU	stat/τ_{exp}	$m_\pi L$
5.2	A2	630	8000	121	7.7
	A3	490	8032	121	6.0
	A4	380	8096	122	4.6
	A5	330	4004	163	4.0
	B6	280	1272	52	5.1
5.3	E4	580	2784	10	6.1
	E5f	430	16000	60	4.6
	E5g	430	16000	120	4.6
	F6	310	4800	36	4.9
	F7	260	9616	72	4.2
	G8	190	1114	23	4.0
5.5	N4	550	6552	7	6.5
	N5	440	6208	7	5.2
	N6	340	8040	40	4.0
	O7	260	3920	20	4.2



Errors and Autocorrelations

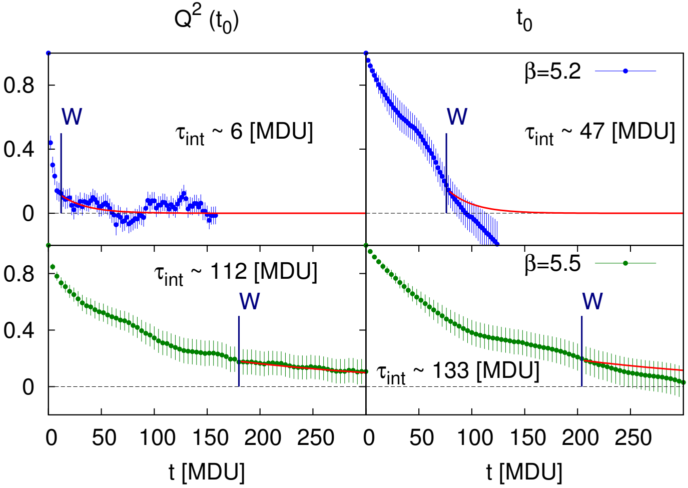
$$\sigma^2 = 2\frac{\tau_{int}}{N}\Gamma(0)\,, \quad \tau_{int} = \left(\frac{1}{2} + \sum_{t=1}^{W-1}\rho(t)\right) + \tau_{exp}\rho(W)$$



- $m_\pi = 340$ MeV
- $L = 2.3$ fm
- MDU ≈ 8000
- $stat/\tau_{exp} \approx 40$
- $f(Q)$: $\tau_{exp}\rho(W) \approx 50\%$
- t_0: $\tau_{exp}\rho(W) \approx 30\%$

Definition of t_0: $t_0^2\langle E(t_0)\rangle = 0.3$, Wilson flow [Lüscher, '10]

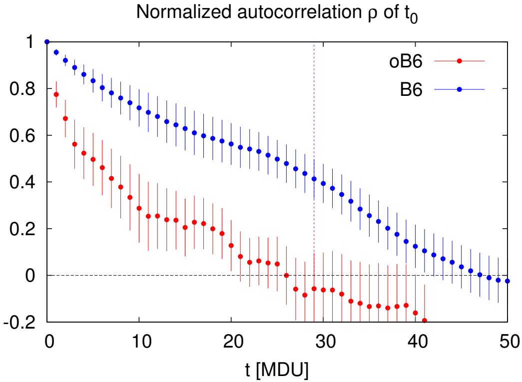
Autocorrelations



$\frac{m_\pi}{\text{MeV}}$	$\frac{stat}{\tau_{exp}}$
280	50
340	40

Both in Q^2 and t_0: tail contribution to τ_{int} is at most $\approx 30\%$

Open boundary conditions

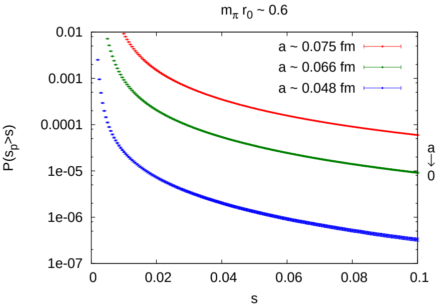


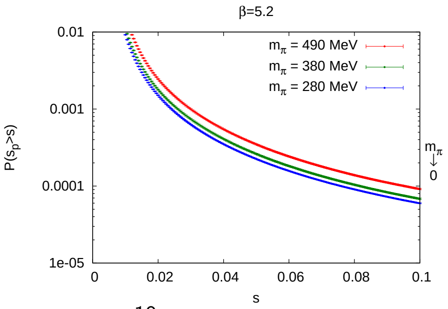
	B6	oB6
β	5.2	5.2
κ	0.13597	0.13597
MDU	1272	1000
L	48	48
T	96	192
plateau	1...96	40...156
t_0	3.3292(80)	3.3317(54)

τ_{int} reduced by factor ≈ 2

Separation between sectors

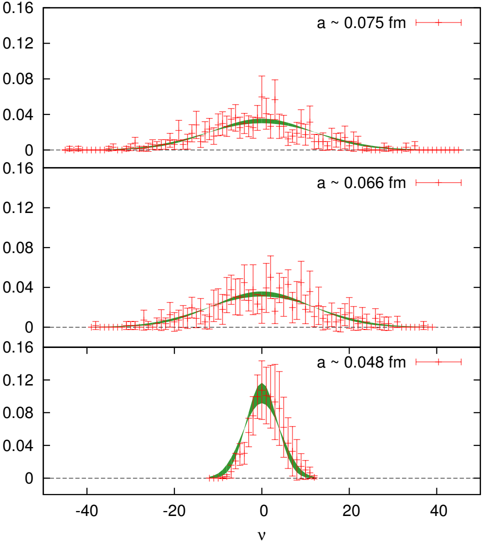
- Defining $h = max\ s_p$, with s_p value of plaquette p at t_0
- configurations "between the sectors" characterized by $h > 0.067$ [Lüscher, '10]
- occur less when reducing a or m_π:





With dynamical fermions $P(s_p) \sim (a/r_0(m))^{10}$

How well is the topological charge Q sampled?



Q: top. charge measured at t_0

$$P_\nu = \frac{e^{\frac{-\nu^2}{2Q^2}}}{\sqrt{2\pi Q^2}} \left(1 + O(V^{-1})\right)$$

[Giusti, Lüscher, Weisz, Wittig,'03]

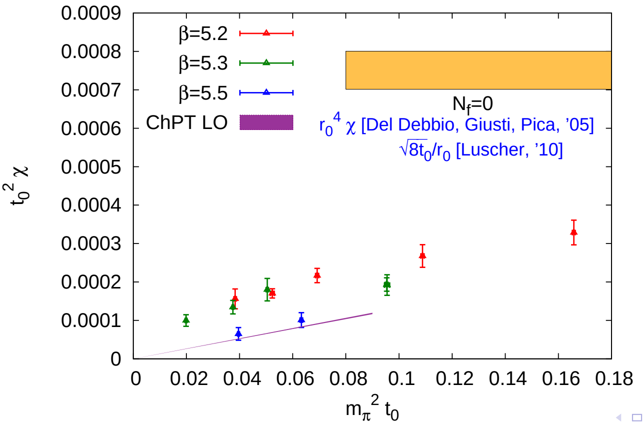
1. $m_\pi = 280$ MeV, $L = 3.6$ fm
2. $m_\pi = 190$ MeV, $L = 4.2$ fm
3. $m_\pi = 340$ MeV, $L = 2.3$ fm

Reasonable sampling of charge distribution

Susceptibility

We computed $t_0^2\chi = t_0^2\frac{\langle Q^2\rangle}{V}$ and we compare it with

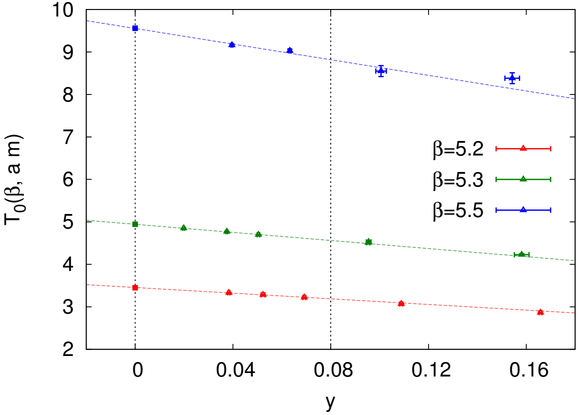
$$\text{ChPT LO}: \chi = \frac{m}{2}\Sigma\left(1 + O(m)\right) = \frac{1}{8}f_\pi^2 m_\pi^2\left(1 + O(m_\pi^2)\right)$$



Input:
$f_\pi = 130.4$ MeV
$t_0 = 0.02396(37)$ fm^2

Suppression by fermion determinant is clearly seen

t_0: chiral extrapolation



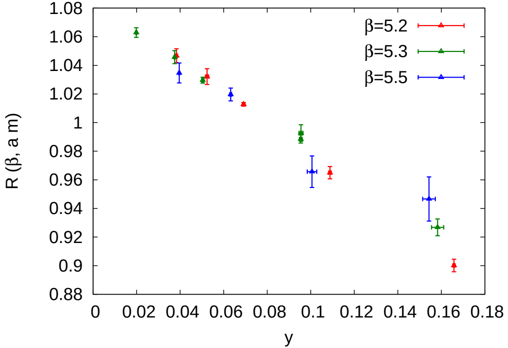
Definitions:

- $am = am_{PCAC}$
- $T_0(\beta, am) = \frac{t_0}{a^2}$
- $M_\pi(\beta, am) = am_\pi$
- $y \equiv T_0 M_\pi^2$

- $T_0(\beta, am_{12}) = A(\beta)(1 + By) \rightarrow \left(\frac{t_0}{a^2}\right)^{chiral} \equiv T_0(\beta, 0) = A(\beta)$

Remaining $O(a)$ effects?

- Does B depend on a: $B(a) = B + O(a)$?



$$R(\beta, am) = \frac{T_0(\beta, am)}{T_0(\beta, am)|_{y=0.08}}$$

We see neither a nor a^2 effects with few per-mille precision

Symanzik effective theory $\rightarrow \tilde{g}_0 = g_0(1 + b_g am_q)$

- $b_g = (0.012 \times N_f)g_0^2 + O(g_0^4)$ in PT
- we used $b_g = 0$ because of the smallness of 1-loop effects and it is confirmed to be ok non-perturbatively

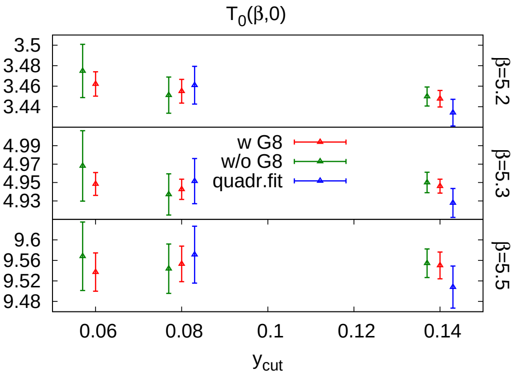
t_0: systematics

We checked the extrapolations are stable by:

- cutting the pion mass $m_\pi <$cut

cut/MeV	y_{cut}
520	0.14
400	0.08
330	0.06

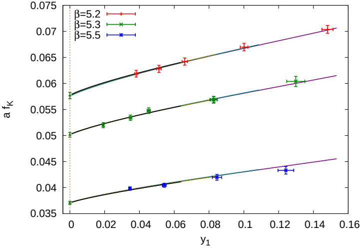
- cutting the pion mass $m_\pi > 190$ MeV
- adding higher terms (e.g. $O(y^2)$)



t_0: continuum limit

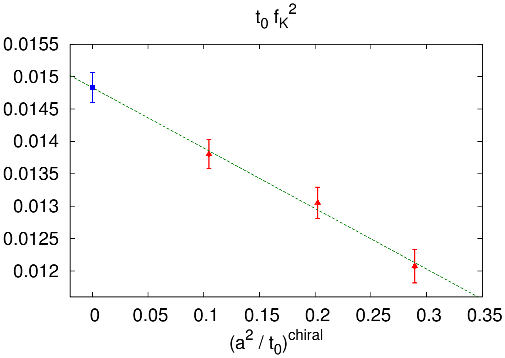
Input:

- af_K at phys. quark masses,



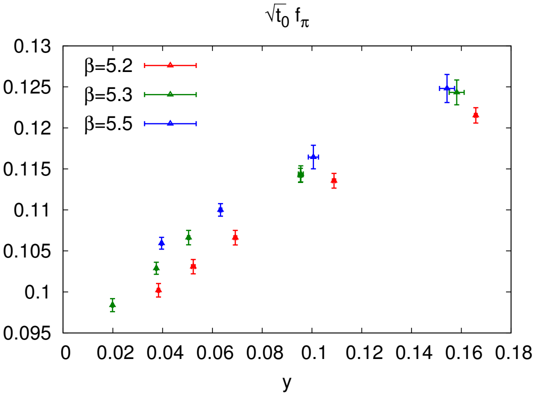
see S. Lottini's talk

- $f_{K,phys} = 155\text{MeV}$



$$t_0 = \frac{\lim_{a\to 0}(t_0 f_K^2)}{f_{K,phys}^2} = 0.02396(37)\,\text{fm}^2$$

t_0: possible uses



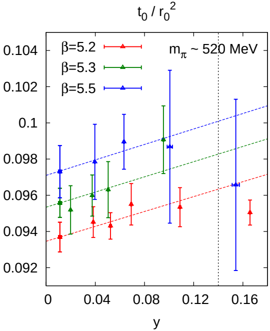
- small discretisation effects
- more precise extrapolations w.r.t. $r_0 f_\pi$
- see S. Lottini's talk

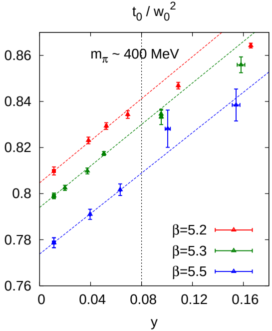
Dynamical fermion effects in gluonic observables

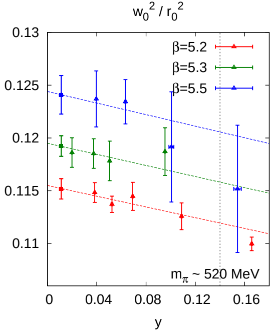
- 4% discretis. effects
- weak quark mass dependence

- 5% discretis. effects
- stronger quark mass dependence

- 9% discretis. effects
- weak quark mass dependence

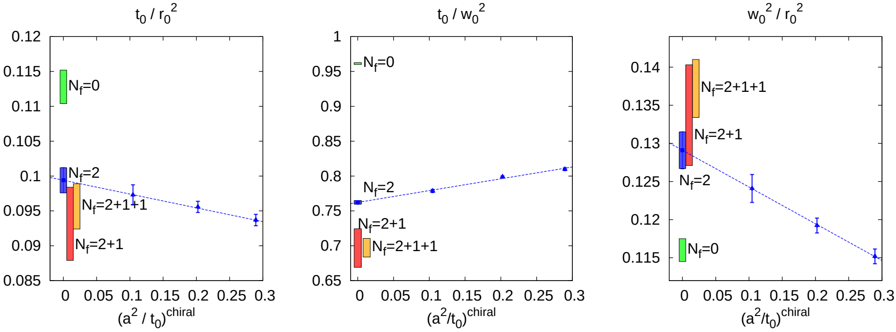






Definition w_0: $t\frac{d}{dt}t^2\langle E(t)\rangle\Big|_{t=w_0^2} = 0.3$ [BMW, '12]

N_f dependence



- $N_f = 0$ Lüscher, '10
- $N_f = 2+1$

Quantity [fm]	ref.
$r_0 = 0.480(10)(4)$	RBC, '12
$\sqrt{t_0} = 0.1465(21)(13)$	BMW, '12
$w_0 = 0.1755(18)(4)$	BMW, '12

- $N_f = 2+1+1$

Quantity	ref.
$r_0/r_1 = 1.508$	HotQCD, '11
$\sqrt{t_0}/w_0 = 0.835(8)$	HPQCD, '13
$r_1/w_0 = 1.790(25)$	HPQCD, '13

Conclusions

Simulations with $19 < stat/\tau_{exp} < 165$:

- autocorrelations effects just under control
- open bc help even at largest lattice spacing
- weak quark mass dependence of t_0, and r_0

$$B_{t_0} = -0.96(5)\,, \quad B_{r_0} = -0.7(2)$$

- weaker quark mass dependence of t_0/r_0^2

$$B_{t_0/r_0^2} = 0.22(16)$$

- dynamical fermion suppression of topology clearly seen
- ... but still a long way to quantitative understanding of topology