Computation of the strong coupling in $N_f = 4$ QCD

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GRADUIERTEN KOLLEG Masse-Spektrum-Symmetrie

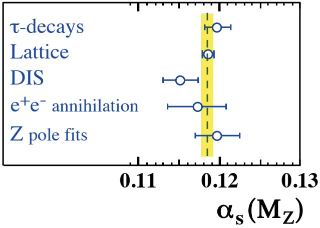
ALPHA Collaboration

Computational Theoretical Particle Physics KA AC B SFB TR9

Motivation

- Error coming from the theory is dominating
- Lattice QCD: *ab initio* determination of QCD parameters
- Full control over systematics
  - non-perturbative renormalization
  - chiral extrapolation
  - continuum extrapolation

[PDG '12]



N_f	$\Lambda_{\overline{\mathrm{MS}}}$	Theory
0	238(19)	LGT, ALPHA Collaboration [Della Morte'05]
2	310(20)	LGT, ALPHA Collaboration [Fritzsch'12]
5	160(11)	NNLO PT, fits to PDFs [Alekhin'12]
5	198(16)	NNLO PT, fits to PDFs [Martin'09]
5	275(57)	4-loop PT at M_Z

Strategy for the computation of $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$ in physical units

$$\boxed{\frac{\Lambda^{(5)}_{\overline{MS}}}{f_{\mathrm{K}}} = \frac{1}{f_{\mathrm{K}} L_{\mathrm{max}}} \times \frac{L_{\mathrm{max}}}{L_k} \Lambda^{(4)}_{SF} L_k \times \frac{\Lambda^{(4)}_{\overline{MS}}}{\Lambda^{(4)}_{SF}} \times \frac{\Lambda^{(5)}_{\overline{\mathrm{MS}}}}{\Lambda^{(4)}_{\overline{MS}}}}$$

- Scale setting: chiral and continuum extrapolation needed
- Intermediate scheme: connecting low and high energies
  1-loop PT $\Lambda^{(4)}_{\overline{\mathrm{MS}}}/\Lambda^{(4)}_{SF} = 2.9065$ [Sint'96, Sommer'11]
- Decoupling across the b-threshold [Chetyrkin'00]

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Problem of the lattice formulation

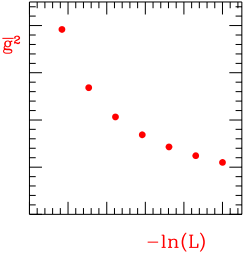
$$\underset{\text{box size}}{\underset{\uparrow}{L}} \gg \underset{\text{conf. scale, } m_\pi}{\underset{\uparrow}{\frac{1}{0.2\mathrm{GeV}}}} \gg \frac{1}{\mu} \sim \frac{1}{10\mathrm{GeV}} \gg \underset{\text{spacing}}{\underset{\uparrow}{a}}$$

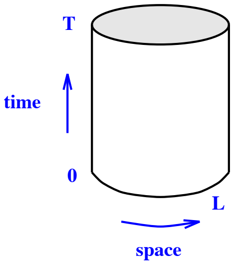
$$\Downarrow$$

$$L/a \gg 50$$

- ALPHA Collaboration Solution: $\boxed{L = 1/\mu}$ finite size scaling **[Lüscher, Weisz, Wolff 1991]**
- Integrated *beta* function - step scaling function
  $\sigma(s, u) = \bar{g}^2(sL)$, with $u = \bar{g}^2(L)$ [s=2]
- determines the non-perturbative running:

$$\begin{aligned} u_0 &= \bar{g}^2(L_{\max}) \\ &\downarrow \\ \sigma(2, u_{k+1}) &= u_k \\ &\downarrow \\ u_k &= \bar{g}^2(2^{-k} L_{\max}) \end{aligned}$$





[Lüscher, Narayanan, Weisz, Wolff 1992]

$$
\begin{aligned}
e^{-\Gamma} &= \int \mathcal{D}U\mathcal{D}\overline{\psi}\psi e^{\{-S[U,\overline{\psi},\psi]\}} \\
U(x,k)|_{x_0=0} &= \exp\{aC_k(\eta)\}, \\
U(x,k)|_{x_0=T} &= \exp\{aC'_k(\eta)\}
\end{aligned}
$$

$$\Gamma' = \left.\frac{\partial \Gamma}{\partial \eta}\right|_{\eta=0} = \frac{k}{\bar{g}^2(L)} \propto \langle\, F^8_{0k}|_{\mathrm{boundary}} \rangle$$

Advantages of SF approach:

- Infrared safe: one can simulate massless quarks ($\lambda^2_{min}(\gamma_5 D) \approx 1/L^2$)
- improvement coefficients can be computed non-perturbatively
- $\frac{\partial \Gamma}{\partial \eta}$ easily computed in Monte Carlo simulations $\langle \mathrm{d}S/\mathrm{d}\eta \rangle$

The lattice step scaling function $\boxed{\Sigma(s, \bar{g}^2(L), a/L)}$

- On the lattice:

  $\boxed{\text{additional dependence on the resolution } a/L}$

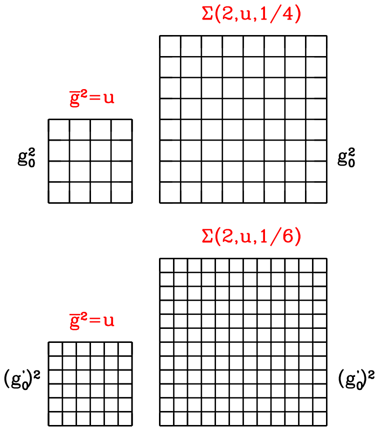
$\rightarrow$ g_0 fixed, L/a doubled:

$$\bar{g}^2(L) = u, \qquad \bar{g}^2(sL) = u' ,$$
$$\Sigma(s, u, a/L) \quad = \quad u'$$

$\rightarrow$ continuum limit:

$$\Sigma(s, u, a/L) = \sigma(s, u) + \mathrm{O}(a/L)$$

- In our work: $s = 2$
- $m = 0$ (PCAC mass defined in $(L/a)^4$ lattice)



Lattice setup

- $N_f = 4$ O(a) improved *massless Wilson fermions*

New code for SF simulations

- $N_f = 0, 2, 4$ τ-language *APE* $\rightarrow$ flexible *PC code*
- *SF-MP-HMC*: Hasenbusch preconditioning, $N_{\mathrm{PF}} = 2$
- $L \equiv T$ $\rightarrow$ CG solver, good scalability for SF lat. sizes

Set of $N_f = 4$ SF ensembles

- High statistics: O(160000) MDU trajectories
  - $L/a = 6 \rightarrow 2L/a = 12$ (9 couplings)
  - $L/a = 8 \rightarrow 2L/a = 16$ (10 couplings)
  - $L/a = 12 \rightarrow 2L/a = 24$ (1 coupling - *check of the cutoff effects*)
- Goal: to reduce the errors from [Tekin et. al 2010] by a factor of 2

Lattice setup

- $N_f = 4$ O(a) improved *massless Wilson fermions*
  - *Achieved by tuning* $\kappa_c(\beta, a/L)$, *s.t.* $m_{PCAC}(L) = 0$
  - *Tuning criteria:* $|$*syst. err. from* $m_{PCAC}| <$ *aimed precision of* $\sigma(\overline{g}^2(L))$

Lattice setup

- $N_f = 4$ O(a) improved *massless Wilson fermions*

O(a) improvement

- NP determined $c_{SW}(g_0)$ for $N_f = 4$ theory [Tekin, Sommer, Wolff, 2009]
- Boundary O(a)-terms, e.g. $c_t(g_0)a^4 \sum_x F_{0k}F_{0k}$ at $x_0 = 0$ and $x_0 = T$
  2-loop computation by [Bode, Weisz, Wolff 1999]
- Remaining cutoff effects: [De Divitiis et al. 1993, ALPHA Collaboration 1993-1999]

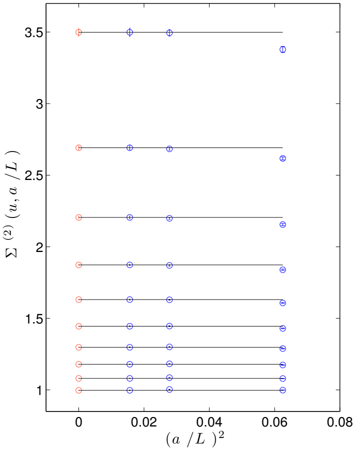
$$\delta(u, a/L) = \frac{\Sigma(u,a/L) - \sigma(u)}{\sigma(u)} = \delta_1(a/L)u + \delta_2(a/L)u^2 + \dots$$

$$\boxed{\Sigma^{(2)}(u, a/L) = \frac{\Sigma(u,a/L)}{1+\delta_1(a/L)u+\delta_2(a/L)u^2}}$$

- Improved lattice SSF: $\Sigma^{(2)}(u, a/L) = \sigma(u) + O(a^2)$

Lattice SSF for $N_f = 4$

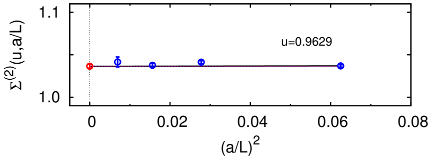
- $L/a = 4$ O(50000) MDU [Tekin '10]
- $L/a = 6, 8$ O(200000) MDU [this work + Tekin '10, combined]



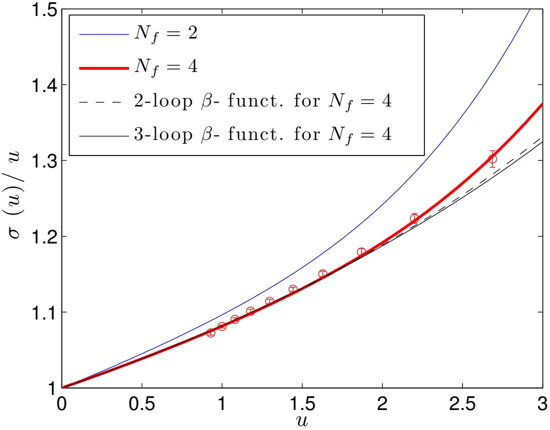
$$\Sigma^{(2)}(u, a/L) = \sigma(u), \; L/a = 8$$

Check of the cutoff effects:

- $L/a = 4, 6, 8, 12$,
  $u = \bar{g}^2(L) = 0.9629$.
- **constant fit of $L/a = 8$ data**

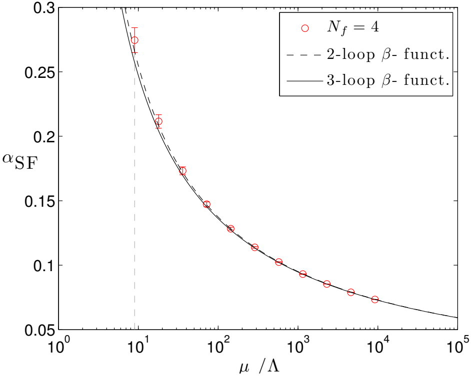


Continuum step scaling function in the SF scheme $\boxed{N_f = 4}$



[ALPHA Collaboration [Tekin '10 + this work; combined result]]

Non-perturbative running of α in the SF scheme $\boxed{N_f = 4}$



energies μ

$0.7\,\mathrm{GeV}$

.

.

.

$700\,\mathrm{GeV}$

$\overline{g}^2(L_{\max}) = 3.45$

$\boxed{\ln(\Lambda L_{\max}) = -2.195(80)}$ $\Lambda \equiv \Lambda_{SF}$

[old. res. $\ln(\Lambda L_{\max}) = -2.294(153)$ [Tekin '10] corrected errorbars]

Still missing!

$$\frac{\Lambda^{(5)}_{\overline{MS}}}{f_{\mathrm{K}}} = \boxed{\frac{1}{f_{\mathrm{K}} L_{\max}}} \times \frac{L_{\max}}{L_k} \Lambda^{(4)}_{SF} L_k \times \frac{\Lambda^{(4)}_{\overline{MS}}}{\Lambda^{(4)}_{SF}} \times \frac{\Lambda^{(5)}_{\overline{\mathrm{MS}}}}{\Lambda^{(4)}_{\overline{MS}}}$$

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Summary and outlook

Summary

- Precise determination Λ from non-perturbative methods needed to resolve discrepancies of PT estimates
- Finite-size scaling + Schrödinger functional scheme
- SF-MP-HMC code: good scalability, CG solver, flexible for different PC clusters
- 2^{nd} computation of the $N_{\mathrm{f}} = 4$ SF running coupling
- Lambda parameter in the units of L_{max}
- Ultimate goal: $N_{\mathrm{f}} = 4$ Lambda parameter in physical units

Outlook

- Gradient flow coupling **[See talks: P.Fritzsch (Mon, 14.20h, 1C) & A. Ramos (Tue, 12h 6E)]**
- Implementation of advanced solvers
- Matching to the physical units
  precise setting of the scale for $N_{\mathrm{f}} = 2 + 1 + 1$ is needed

Thank you !

- $S_W[U] \;\; = \beta \sum_P w(P)(1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr} U_P), \qquad \beta = 6/g_0^2$

$$w(P) = \begin{cases} 1 & U_P \text{ is the plaquette in the bulk,} \\ c_t(g_0) & U_P \text{ is the time-like touching } x_0 = 0 \text{ or } x_0 = T \text{ boundary,} \\ \frac{1}{2} c_s(g_0) & U_P \text{ is the space-like plaquette at } x_0 = 0, T. \end{cases}$$

- Fermionic fields also: periodic in space

$$\begin{aligned} \psi(x + L\hat{k}/a) &= e^{i\theta_k}\psi(x), \\ \overline{\psi}(x + L\hat{k}/a) &= e^{-i\theta_k}\overline{\psi}(x). \end{aligned}$$

fixed on the time boundaries

$$P_+\psi(x)|_{x_0=0} = \rho(x), \qquad P_-\psi(x)|_{x_0=T} = \rho'(x),$$
$$\overline{\psi}(x)P_-|_{x_0=0} = \overline{\rho}(x), \qquad \overline{\psi}(x)P_+|_{x_0=T} = \overline{\rho}'(x),$$

- $S_F = S_f^W + S_{SW} + S_b$

$$\begin{aligned} S_{\mathrm{b}}[U,\psi,\overline{\psi}] = \frac{a^4}{2}\sum_x \quad & \{(\tilde{c}_s(g_0) - 1)[(\overline{\rho}(x)\gamma_k(\nabla_{\mathbf{k}}^* + \nabla_{\mathbf{k}})\rho(x)) \\ - & (\overline{\rho}'(x)\gamma_k(\nabla_{\mathbf{k}}^* + \nabla_{\mathbf{k}})\rho'(x))] \\ + & (\tilde{c}_t(g_0) - 1)[(\overline{\psi}(x)(P_-\nabla_0 + P_+\overleftarrow{\nabla}_0^*)\psi(x))_{x_0=a} \\ - & (\overline{\psi}(x)(P_-\nabla_0 + P_+\overleftarrow{\nabla}_0^*)\psi(x))_{x_0=T-a}]\} \end{aligned}$$

- Additioanl improvement coefficients: on-shell improvement of the axial current c_A

$N_f = 4$ running coupling - Backup slides

u	$\sigma(u)$	$\Delta(\sigma(u))$
0.93	0.998567	0.00164622
1	1.07943	0.00148896
1.08128	1.17405	0.00187277
1.1787	1.28861	0.00237175
1.29717	1.42987	0.00280331
1.44354	1.60788	0.00342647
1.62854	1.83945	0.00493976
1.87	2.1551	0.00761488
2.20033	2.61708	0.0105432
2.68704	3.37799	0.0213151

n	u_n	$\ln(\Lambda L_{max})$	$\ln(\Lambda L_{max})^{(2-loop)}$
0	3.450	-2.028	-2.117
1	2.658(17)	-2.076(21)	-2.146
2	2.177(17)	-2.109(31)	-2.166
3	1.850(15)	-2.132(38)	-2.182
4	1.612(13)	-2.149(45)	-2.193
5	1.431(12)	-2.163(52)	-2.201
6	1.287(11)	-2.173(59)	-2.208
7	1.171(10)	-2.182(66)	-2.214
8	1.0737(93)	-2.189(73)	-2.218
9	0.9921(86)	-2.195(80)	-2.222
10	0.9222(80)	-2.200(87)	-2.225

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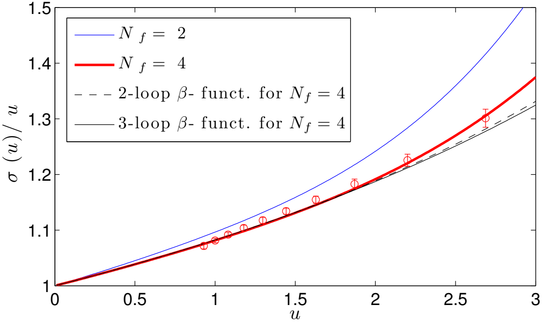

[ALPHA Collaboration 2010, F.Tekin, *Corrected errorbars*]