Vector correlator and scale determination in lattice QCD

Anthony Francis, Georg von Hippel, Fred Jegerlehner, Harvey Meyer

SRFN

Lattice 2013, Mainz 29 July-3 August 2013

Outline

- scale setting from the vector correlator
- comparison of the vector correlator with a phenomenological study.
- finite-volume effects on the correlator.

Scale setting in Lattice QCD

Relative scale setting: ability to compare dimensionful quantities computed at different lattice spacings. Quite a few choices available:

- r_0, r_1: static potential [Sommer hep-lat/9310022]; [MILC, hep-lat/0002028].
- $\nu_{\rm R}/V$: renormalized eigenvalue density [Giusti, Lüscher 0812.3638]
- t_0 and w_0: Wilson flow [Lüscher 1006.4518; BMW 1203.4469]

Absolute scale setting: the quantity must be known experimentally.

- M_Ω: spectrum
- F_π or F_K: decay constants.

Here: explore a proposal to set the absolute scale using the vector correlator [Bernecker, HM 1107.4388].

Point of view: for now, phenomenology is more accurate than lattice QCD in calculating $\Pi(Q^2)$ and $a_\mu^{\rm HLO}$.

Basic relations

Let $j_\mu^{\rm em}(x)$ be the electromagnetic current of hadrons.

$$G^{\rm em}(t) \equiv \int \mathrm{d}\boldsymbol{x}\, \langle j_z^{\rm em}(t,\boldsymbol{x}) j_z^{\rm em\,\dagger}(0)\rangle$$

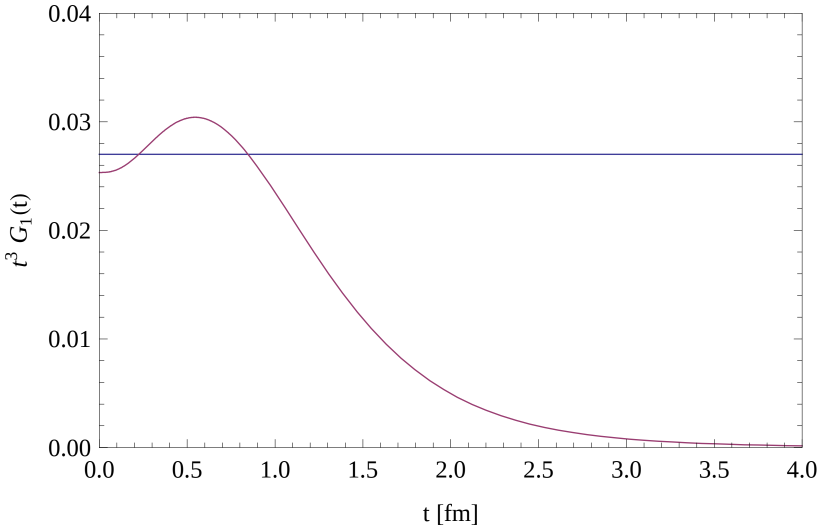
Spectral representation:

$$\begin{aligned} G(t) &= \int_0^\infty \mathrm{d}\omega\, \omega^2 \rho(\omega^2) e^{-\omega|t|}, \\ \rho(s) &= \frac{R(s)}{12\pi^2}, \qquad R(s) \equiv \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{4\pi\alpha(s)^2/(3s)}. \end{aligned}$$

- if exclusive channels are measured experimentally, can do flavor separation in a model independent way
- introduce $R_1(s)$ defined as $R(s)$, but final state required to be isovector
- By G-parity, this implies an even number of pions
- above 2GeV, perturbation theory considered to be reliable.

On the lattice, we need only compute Wick-connected diagram for the isovector correlator $G(t)$.

Form of $\tau^3 G(\tau)$ expected from phenomenology [1107.4388]

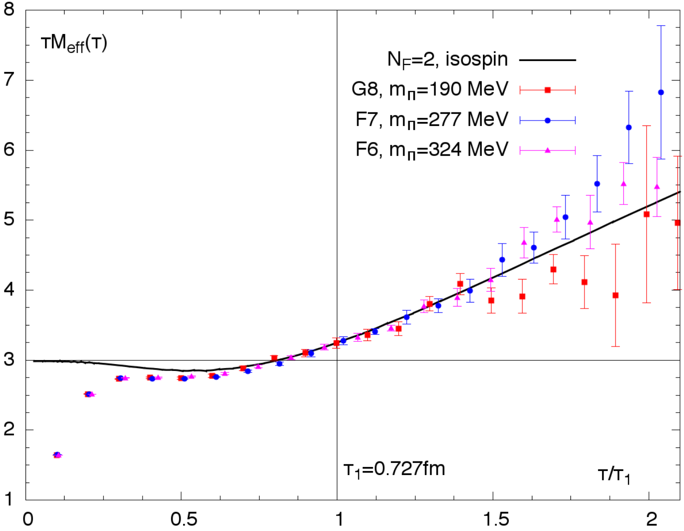


CLS Ensembles

β	label	lat. dimensions	$m_\pi[\mathrm{MeV}]$
5.5	N_5	$48^3 \times 96$	440
	N_6	$48^3 \times 96$	340
	O_7	$64^3 \times 128$	270
5.3	F_6	$48^3 \times 96$	310
	F_7	$48^3 \times 96$	270
	G_8	$64^3 \times 128$	190
5.2	A_4	$32^3 \times 64$	380
	A_5	$32^3 \times 64$	330

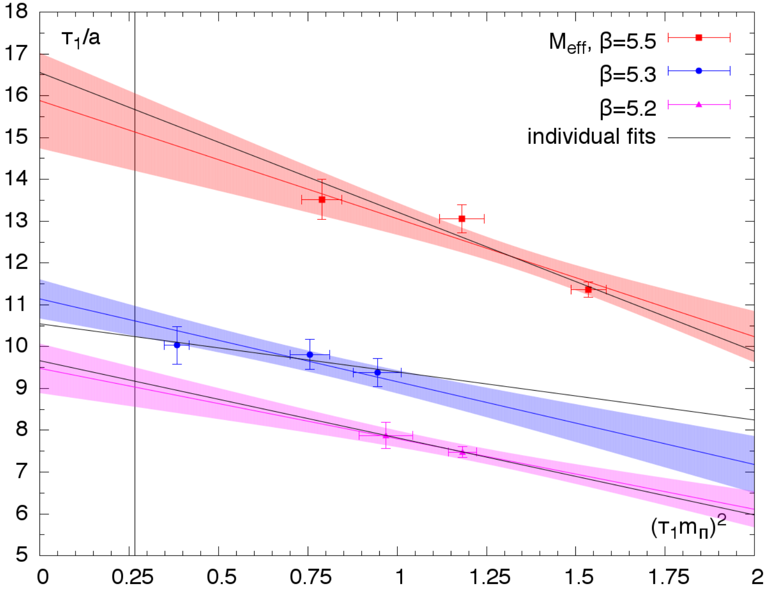
$m_\pi L > 4$ in all ensembles.

Determination of τ_1: $f(\tau_1) \equiv 3.25$, $\quad f(t) = \frac{t}{a} \log \frac{G(t)}{G(t+a)}$

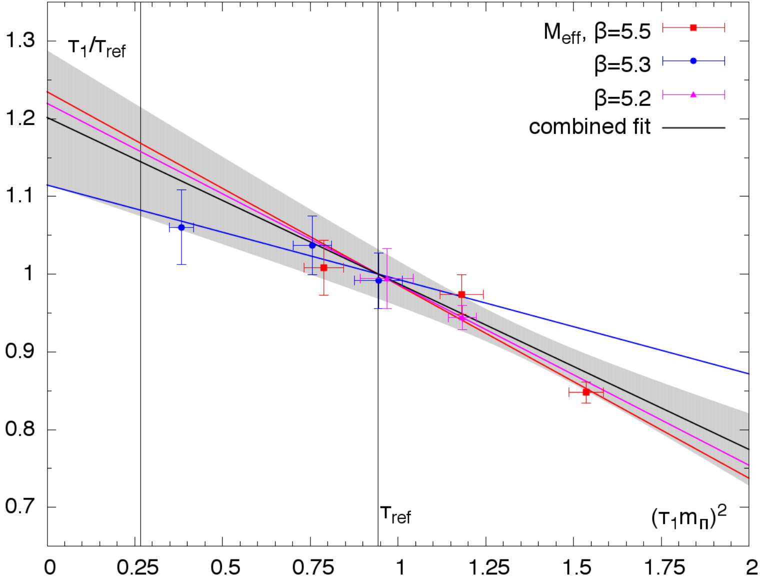


Error propagation: $\frac{\delta\tau_1}{\tau_1} = \left(\frac{\tau_1}{f}\frac{df}{d\tau_1}\right)^{-1} \frac{\delta m_{\rm eff}(\tau_1)}{m_{\rm eff}(\tau_1)} \overset{f(\tau_1)=3.25}{\approx} 1.5\, \frac{\delta m_{\rm eff}(\tau_1)}{m_{\rm eff}(\tau_1)}$

Chiral extrapolation of τ_1



Cutoff effects on the quark mass dependence of τ_1

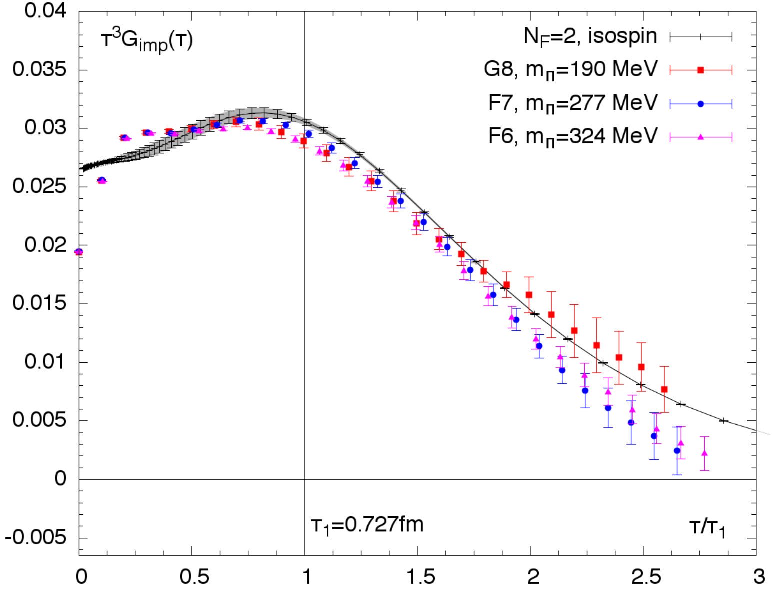


Scale determination: comparison of methods

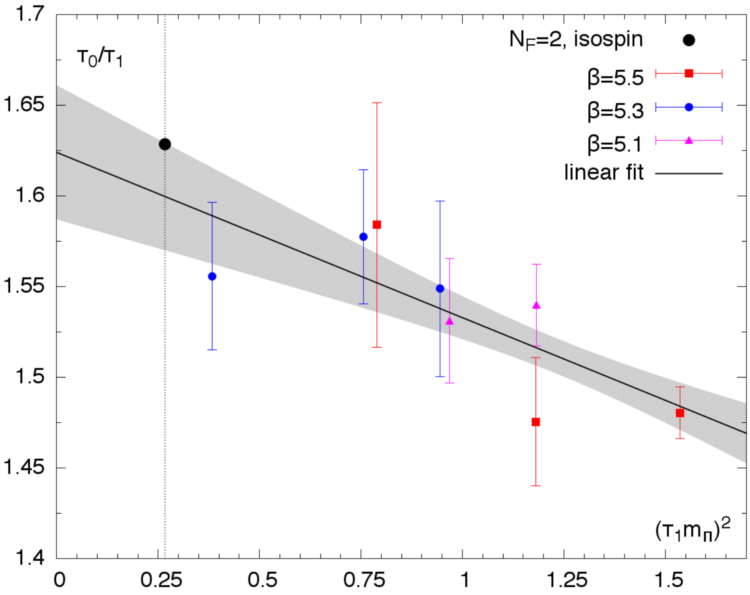
β	a/fm from τ_1	a/fm from m_Ω (*)	a/fm from F_K (**)
5.5	0.048(3)	0.050(2)(2)	0.0486(4)(5)
5.3	0.0685(23)	0.063(2)(2)	0.0658(7)(7)
5.2	0.081(4)	0.079(3)(2)	0.0755(9)(7)

(*) G. von Hippel et al. 1110.6365
(**) ALPHA collaboration, 1205.5380.

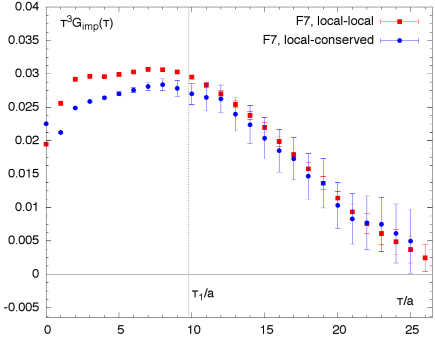
Comparison of $\tau^3 G(\tau)$ with phenomenology



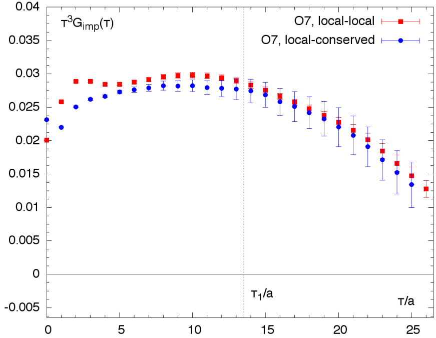
Quark mass dependence of τ_0/τ_1: $\tau_0^3 G(\tau_0) \equiv 0.021$



Dependence of $\tau^3 G(\tau)$ on the discretization of the vector current



$\beta = 5.3$, $a = 0.068$fm

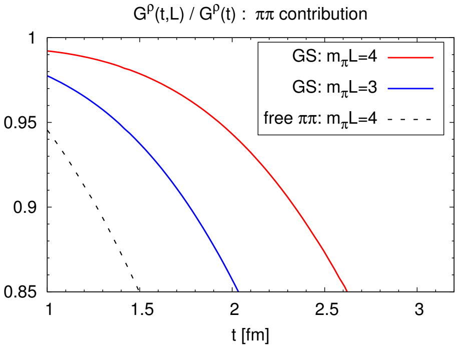


$\beta = 5.5$, $a = 0.048$fm

- $m_\pi \approx 270$MeV
- relatively large O(a) effect for $t < \tau_1$?
- O(a) improvement:
  $(V_{\rm I})^a_\mu = (1 + b_V a m_{\rm q}) Z_V(g_0) \left(V^a_\mu + a c_V \partial_\nu \bar{\psi} i \sigma_{\mu\nu} \frac{\tau^a}{2} \psi \right)$

Finite-size effects [Francis et al. 1306.2532]

- because $T = 2L$, dominant effect associated with finite L
- low-lying spectrum and matrix elements predicted by Lüscher finite-volume formalism
  [Lüscher '91, HM 1105.1892]



- non-interacting pions:

$$\begin{aligned}
G(x_0, L) &= \frac{1}{L^3} \sum_{\boldsymbol{k}} k_z^2 \, \frac{e^{-2E_k|x_0|}}{E_k^2} \\
&= \frac{m_\pi^3}{6\pi^2} \sum_{\boldsymbol{n}} \int_0^\infty dx \, \frac{x^4}{x^2+1} \, \frac{\sin(m_\pi L|\boldsymbol{n}|x)}{m_\pi L|\boldsymbol{n}|x} \, e^{-2m_\pi|x_0|\sqrt{x^2+1}} \\
&\simeq G(x_0) + \sqrt{\frac{m_\pi}{\pi^3|x_0|^5}} \, \frac{e^{-2m_\pi|x_0|}}{48} \sum_{\boldsymbol{n}\neq 0} \left(3 - \frac{m_\pi L^2 \boldsymbol{n}^2}{2|x_0|}\right) \exp\left(-\frac{m_\pi L^2 \boldsymbol{n}^2}{4|x_0|}\right).
\end{aligned}$$

Outlook

- the scale setting method via the vector current works in practice: quark mass dependence mild, finite-size effects expected to be under control for $t < 1\mathrm{fm}$.
- apply O(a) improvement systematically, increase statistics, stochastic methods; disconnected diagrams $\Rightarrow$ no flavor separation required.
- apply method in 2+1 flavor theory.