Finite size effects in lattice RI-MOM

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RI-MOM (or its RI'-MOM variant) is one of the most polular renormalization schemes for Lattice QCD; being regulator independent, it can be effectively adopted in a lattice regularization. RI-MOM is defined in infinite volume. This is in principle a fundamental problem for the lattice, since any simulation is performed in a finite volume. From a practical point of view, one most often verifies a posteriori (by performing computations on different physical volumes) the expectation that renormalization constants, determined in the RI-MOM scheme at large momenta, should not be affected by significant finite size effects.

In the context of Numerical Stochastic Perturbation Theory, we have in recent years devised a novel method to explicitly look and correct for finite size effects (in a convenient window). This is an account of what we learnt, trying to put it in a perspective. We review the method, discussing how it can be applied in a non-perturbative formulation as well.

Agenda

- RI'-MOM: definitions and standard best practice
- NSPT for RI'-MOM renormalization constants and FINITE SIZE EFFECTS
  A little history of what we understood
- Taming FINITE SIZE EFFECTS in NSPT
  An account of our NSPT best practice
- Can we go the SAME WAY in the NON-PERTURBATIVE approach?
  A proposal for a “better best practice”

RI’-MOM: definitions and standard best practice

The celebrated RI'-MOM

$$\int \mathrm{d}x \langle p|\psi(x)\Gamma\psi(x)|p\rangle = G_\Gamma(pa)$$

$$G_\Gamma(pa) \rightarrow \Gamma_\Gamma(pa) = S^{-1}(pa) G_\Gamma(pa) S^{-1}(pa)$$

$$O_\Gamma(pa) = \mathrm{Tr}\left(\hat{P}_{O_\Gamma}\Gamma_\Gamma(pa)\right)$$

$$Z_{O_\Gamma}(\mu a, g(a)) Z_q^{-1}(\mu a, g(a)) O_\Gamma(pa)\Big|_{p^2=\mu^2} = 1$$

$$Z_q(\mu a, g(a)) = -\mathrm{i}\,\frac{1}{12}\,\frac{\mathrm{Tr}(\not{p} S^{-1}(pa))}{p^2}\Bigg|_{p^2=\mu^2}$$

A few remarks:

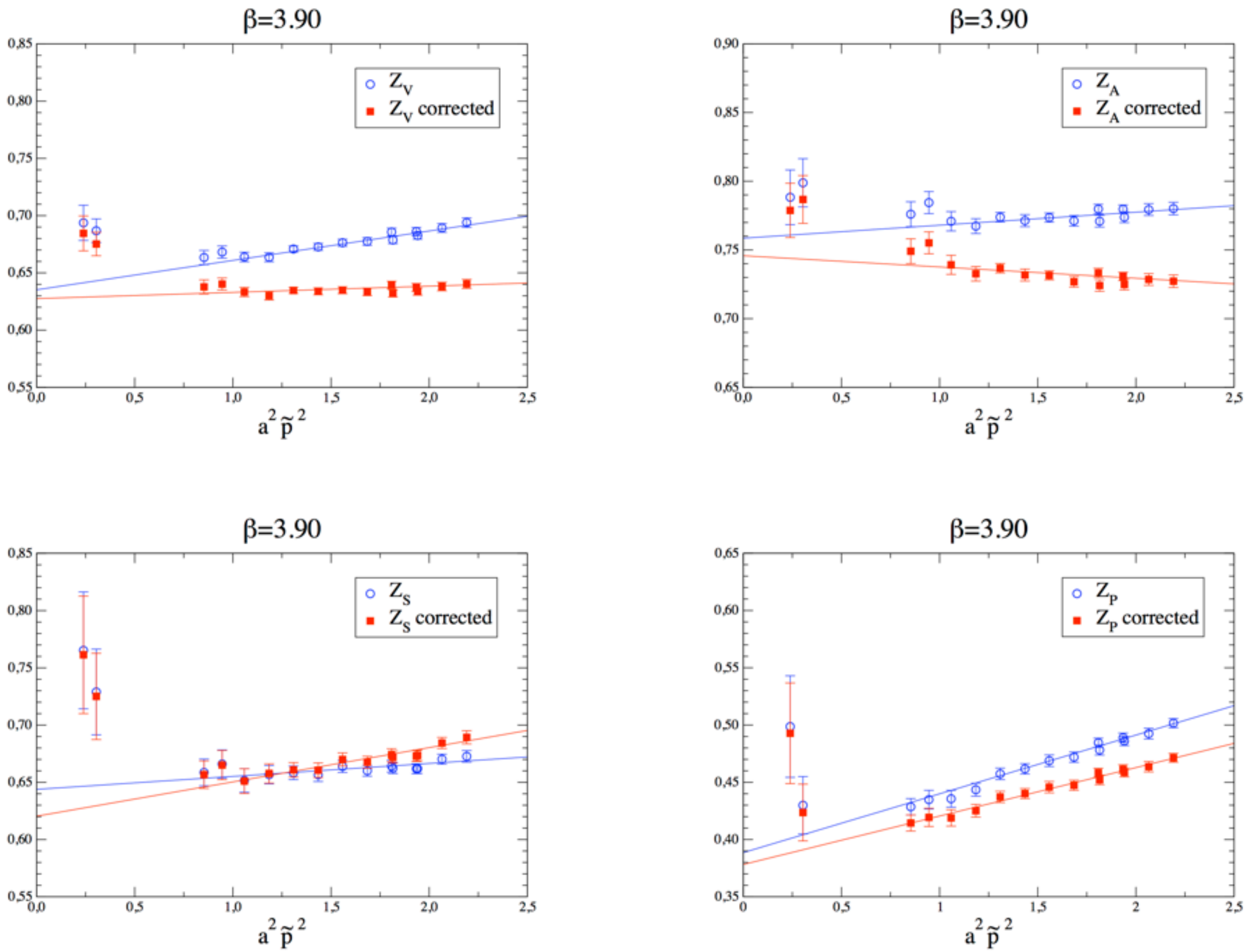
- defined in the chiral limit (needs extrapolation)

- natural in momentum space

  (still sources for inverting propagators are not always taken diagonal in momentum)

- defined in INFINITE VOLUME

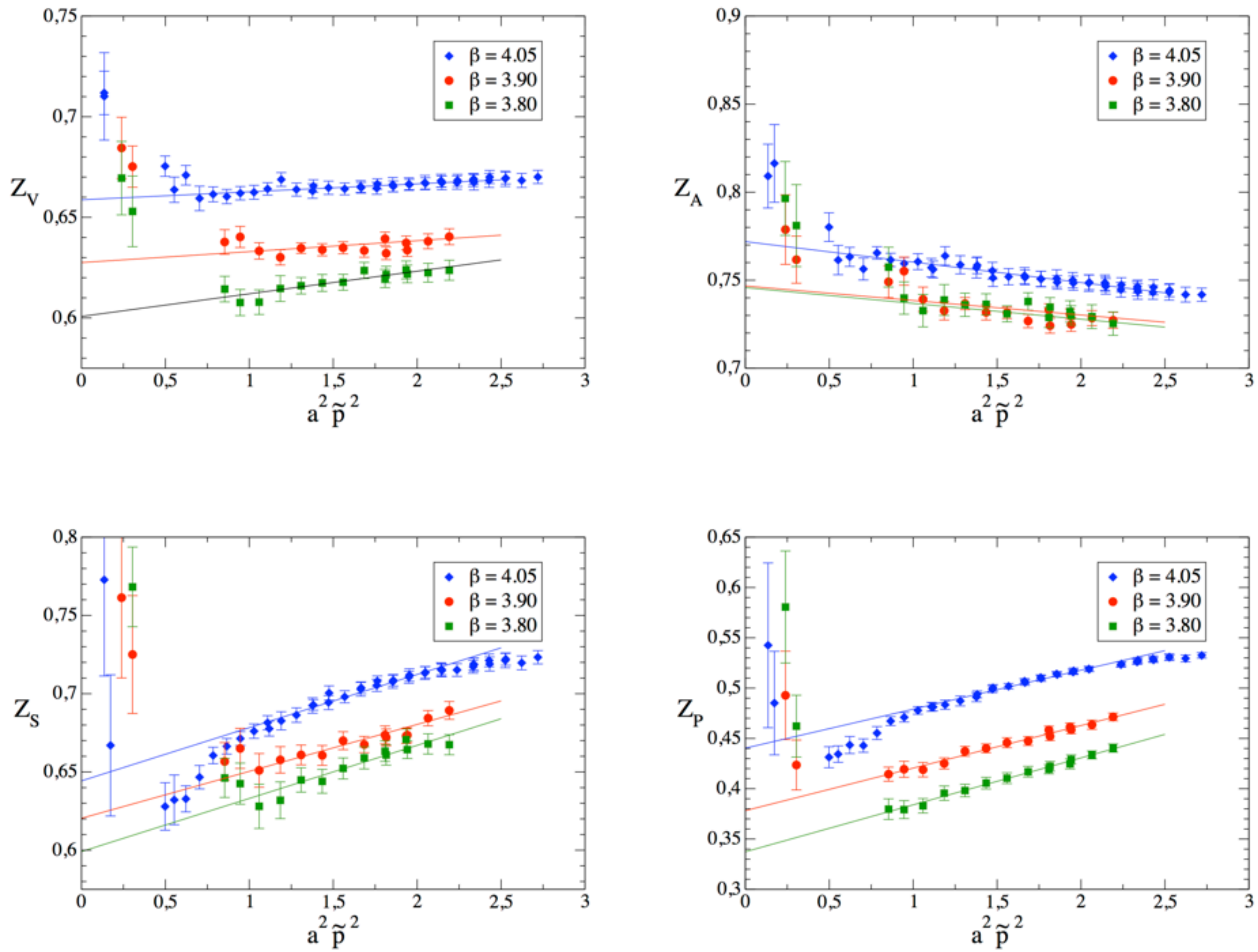
The standard best practice (from ETMC JHEP1008(2010)068)



In order to control finite lattice spacing, one-loop irrelevant contributions are (if possible, i.e. if known) subtracted

The standard best practice (2)

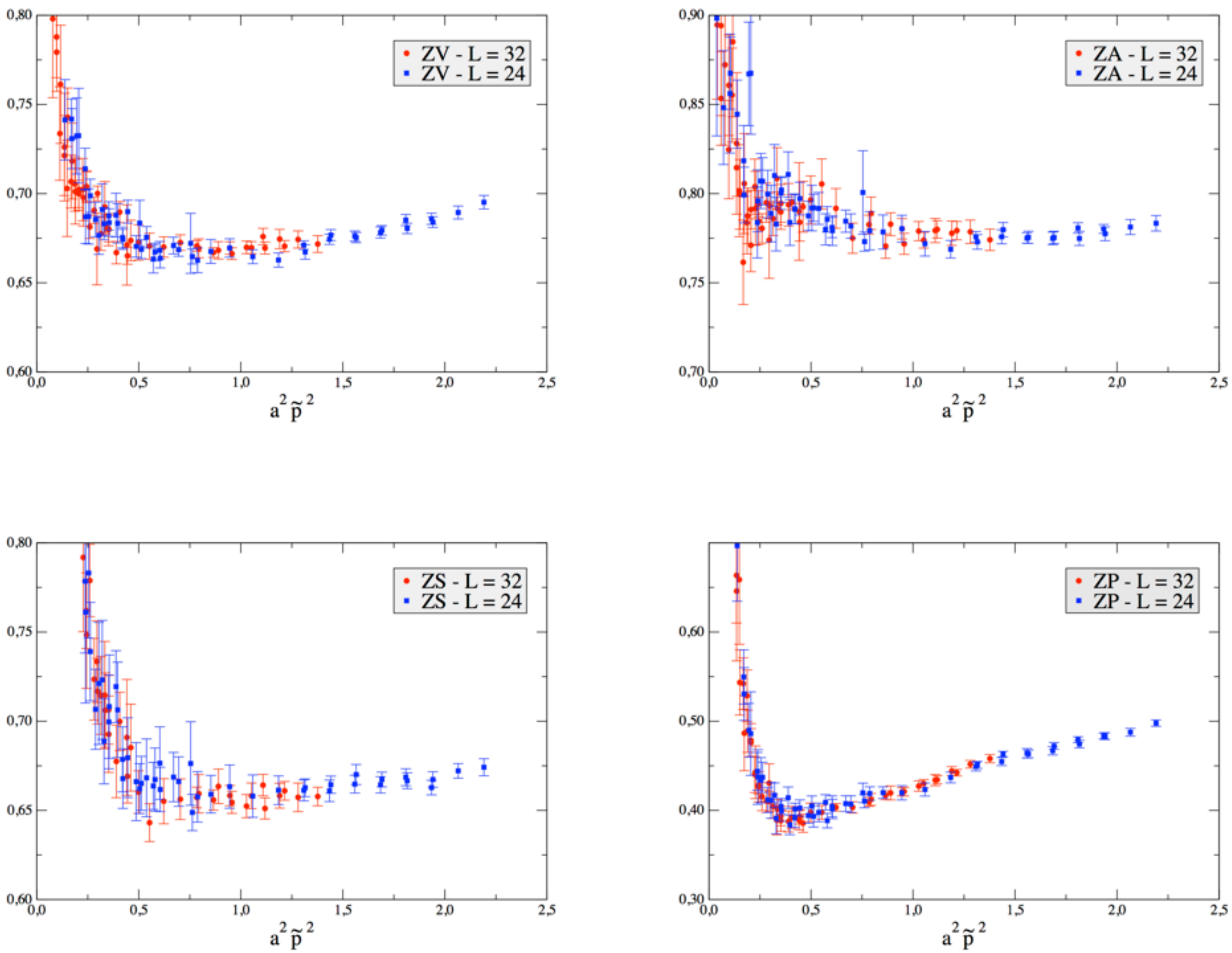
An extra extrapolation can be needed, i.e. an effective higher-order fit ...



... and here FINITE SIZE EFFECTS could pop in, since one is pinching the IR ...

The standard best practice (3)

A posteriori one looks for confirmation of not-significant FINITE SIZE EFFECTS



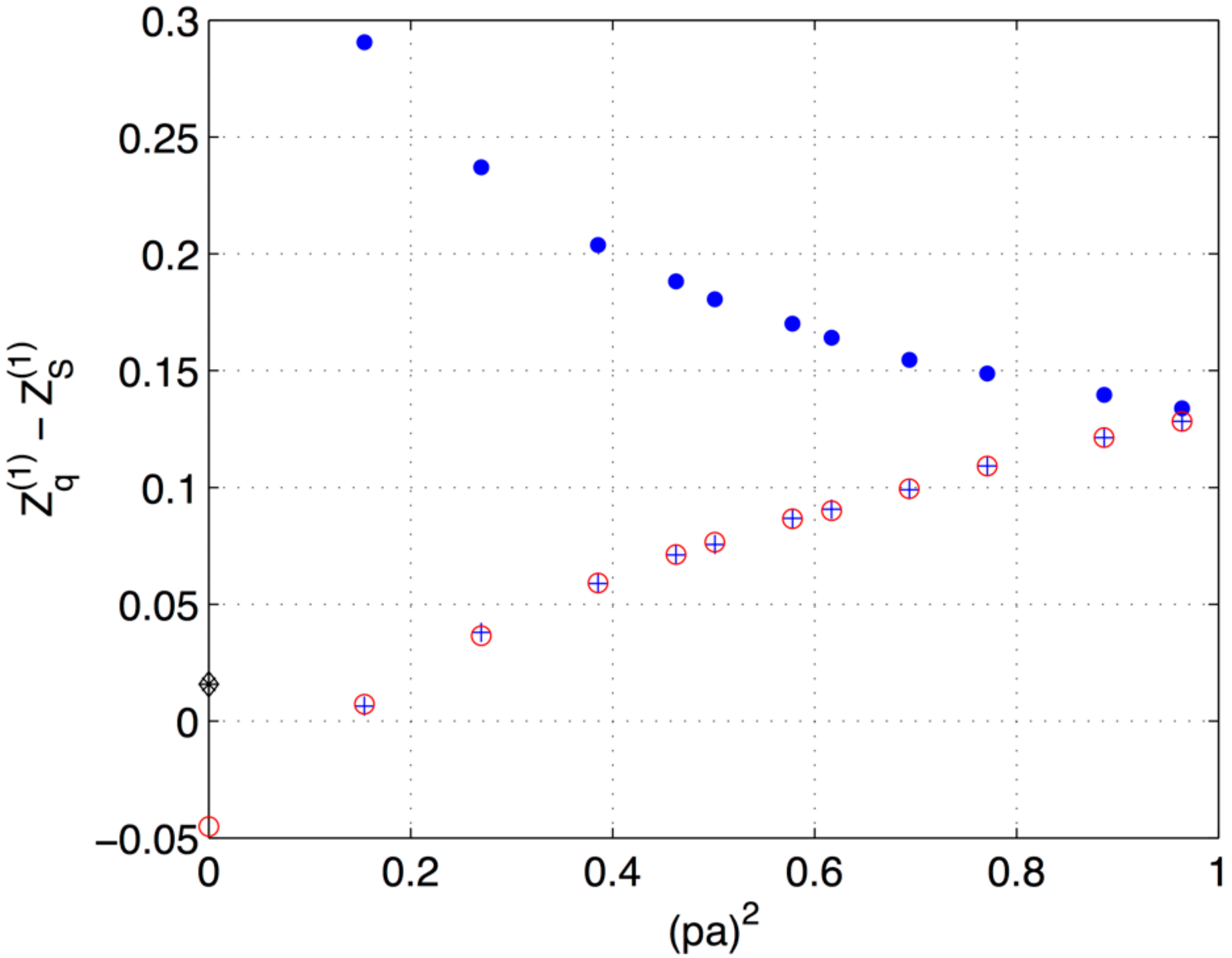
... our claim is that there could be more information buried into than looked at ...

NSPT for RI'-MOM renormalization constants and FINITE SIZE EFFECTS

How FSE popped into NSPT computations (Parma group EPJC51(2007)645)

In order to fit the relevant part in $Z_{O_\Gamma}(\mu a, g(a)) Z_q^{-1}(\mu a, g(a)) O_\Gamma(pa)\big|_{p^2=\mu^2} = 1$

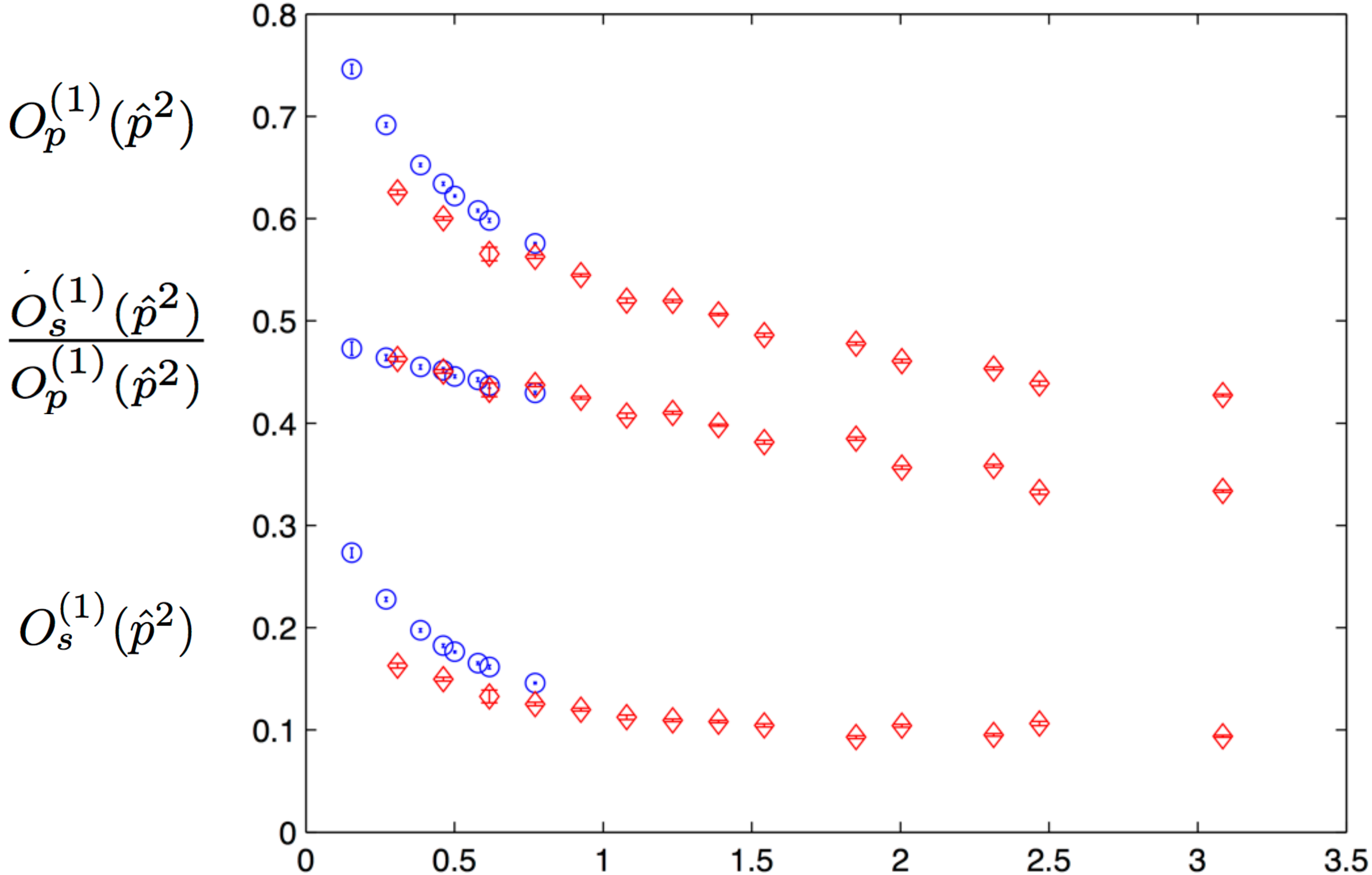
we need to subtract the logs (known) $z_q^{(1)} - z_s^{(1)} = O_s^{(1)}(\hat{p}^2) - \gamma_s^{(1)} \log(\hat{p}^2)$



... but we were failing!

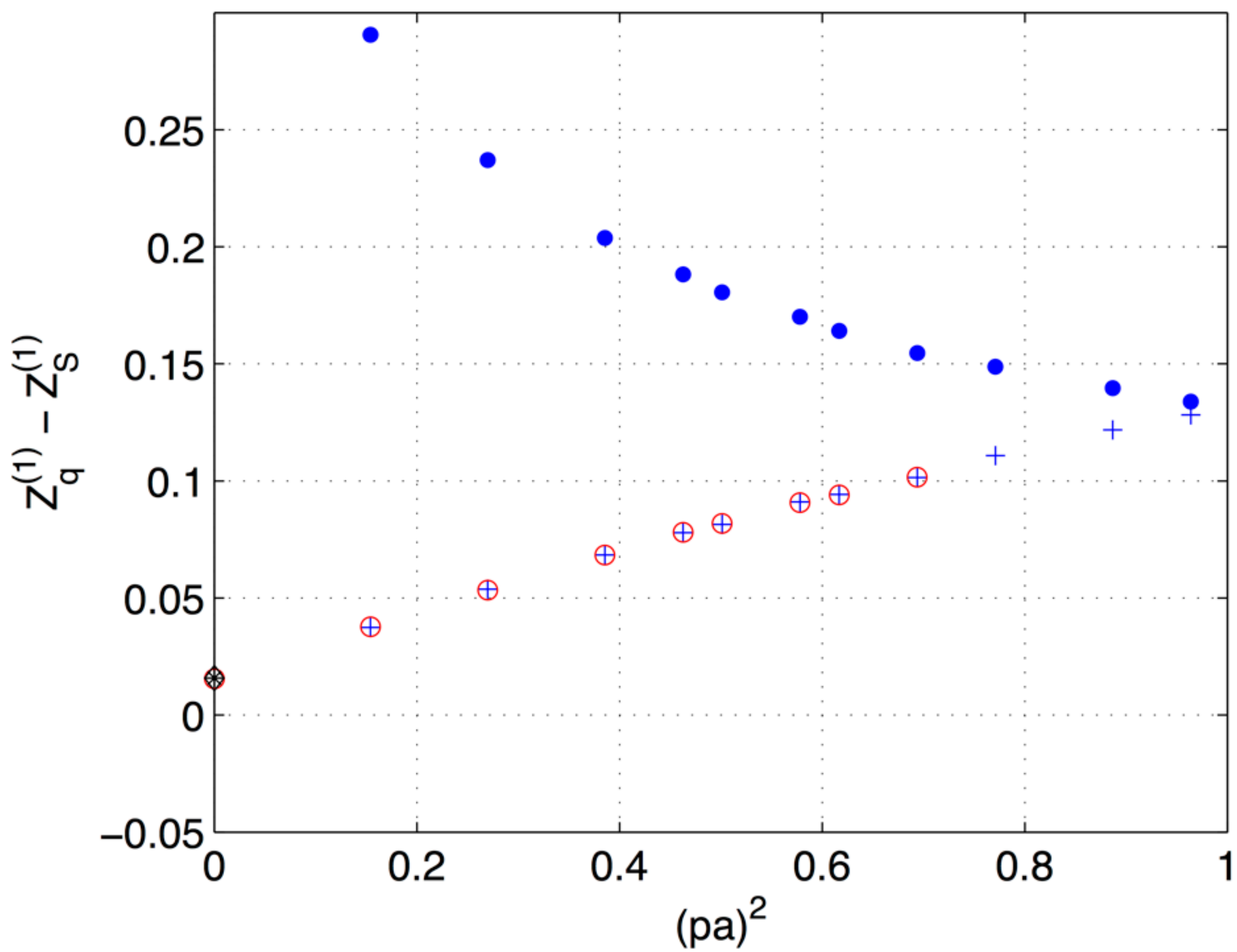
How FSE popped into NSPT computations (2)

By comparing (unsubtracted) results at different volumes, it was clear that it was due to FINITE SIZE EFFECTS ...



We had a first solution at that time (Parma group EPJC51(2007)645)

By computing what is left of a logarithm in a finite volume, we were at that time able to recover the correct 1 loop result ...



... but that was de facto only workin at 1 loop ...

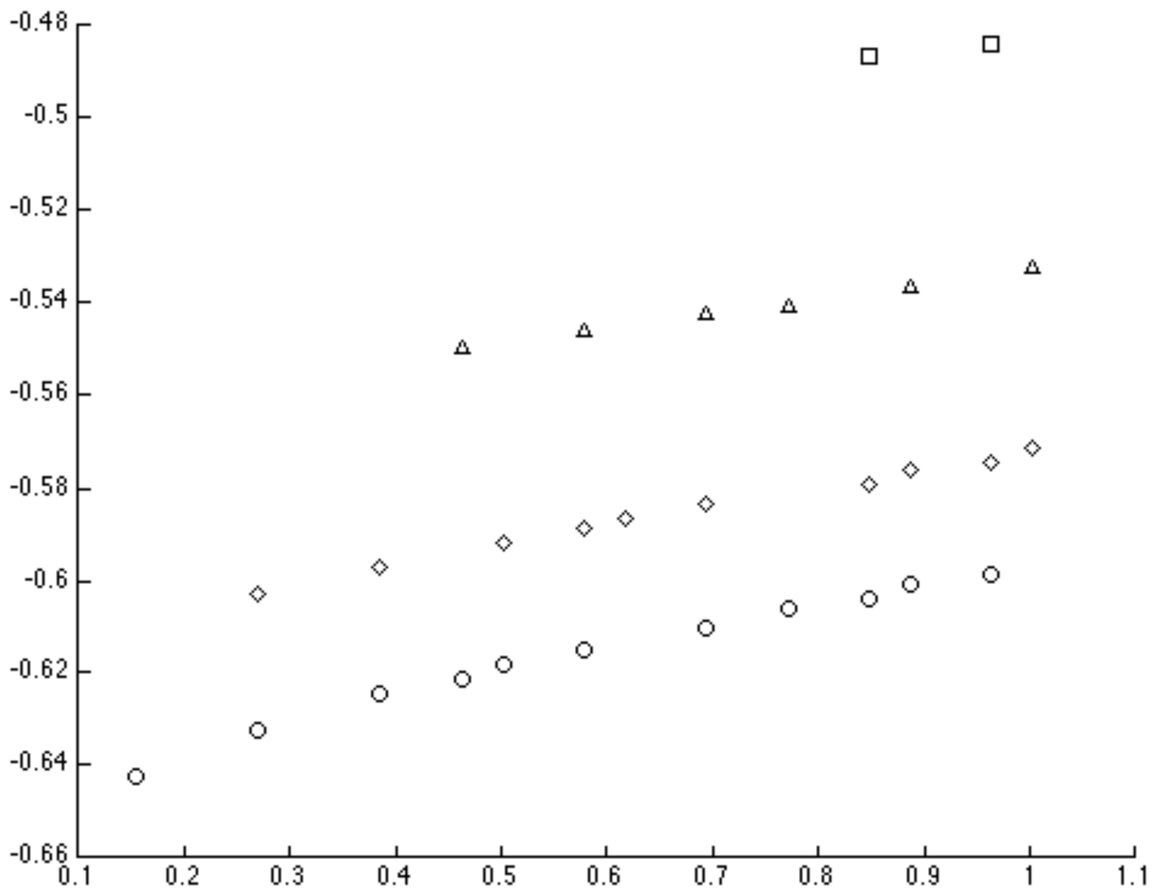
Taming FINITE SIZE EFFECTS in NSPT

There is a big information built-in the hypercubic group!

Let's consider the QUARK SELF ENERGY $a\Gamma_2(\hat{p},\hat{m}_{cr},\beta^{-1}) = aS(\hat{p},\hat{m}_{cr},\beta^{-1})^{-1}$

$$= i\hat{\not{p}} + \hat{m}_W(\hat{p}) - \hat{\Sigma}(\hat{p},\hat{m}_{cr},\beta^{-1})$$

$$\hat{\Sigma}(\hat{p},\hat{m}_{cr},\beta^{-1}) = \hat{\Sigma}_c(\hat{p},\hat{m}_{cr},\beta^{-1}) + \hat{\Sigma}_V(\hat{p},\hat{m}_{cr},\beta^{-1}) + \hat{\Sigma}_o(\hat{p},\hat{m}_{cr},\beta^{-1})$$



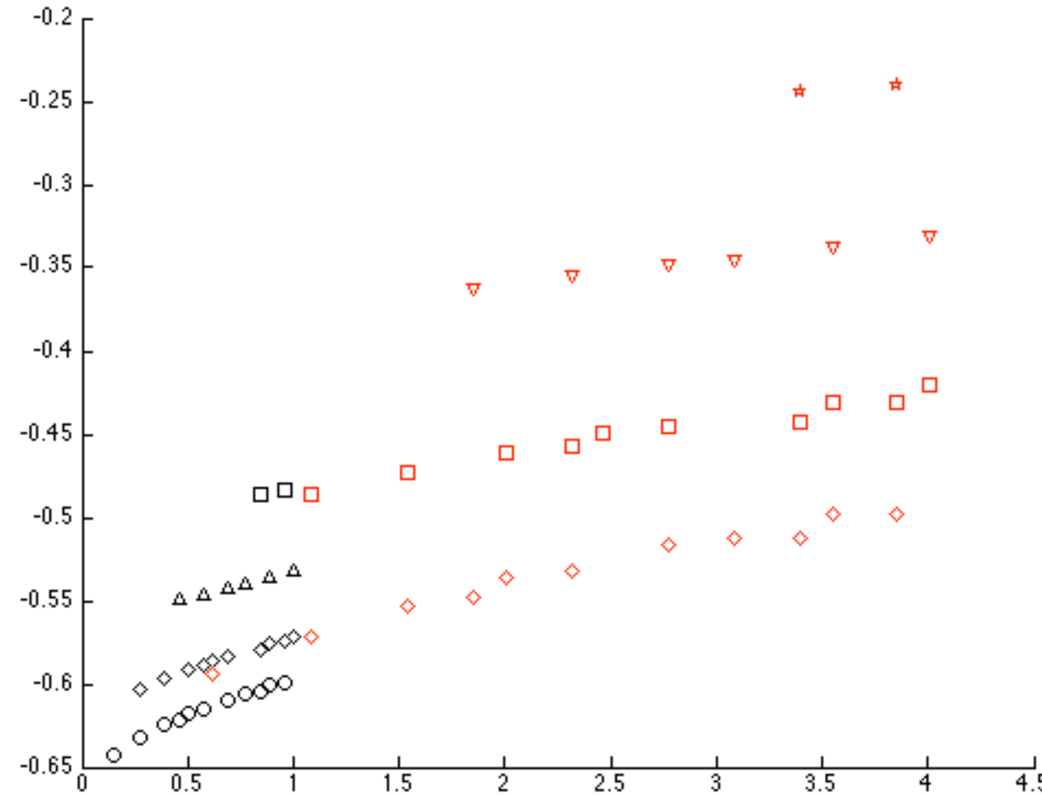
There is a clear sign of symmetry $$\hat{\Sigma}_V = i\sum_{\mu}\gamma_\mu \hat{p}_\mu \left(\hat{\Sigma}_V^{(0)} + \hat{p}_\mu^2\hat{\Sigma}_V^{(1)} + \hat{p}_\mu^4\hat{\Sigma}_V^{(2)} + \ldots\right)$$

Notice that $$\hat{\Sigma}_V^{(n)} = \alpha_1^{(n)} 1 + \alpha_2^{(n)}\sum_{\nu}\hat{p}_\nu^2 + \alpha_3^{(n)}\sum_{\nu}\hat{p}_\nu^4/p^2 + \mathcal{O}(a^4)$$

... and there was an important ingredient missing ...

Let's have a first look at 2 volumes

(32^4 and 16^4)



The real break-through was realizing that there was something missing

(first put at work in Di Renzo, Ilgenfritz, Perlt, Schiller, Torrero NPB831(2010)262)

Expect rather

$$\hat{\Sigma}_V^{(n)}(\hat{p}, pL) = \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \left(\hat{\Sigma}_V^{(n)}(\hat{p}, pL) - \hat{\Sigma}_V^{(n)}(\hat{p}, \infty)\right)$$
$$= \hat{\Sigma}_V^{(n)}(\hat{p}, \infty) + \Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL)$$

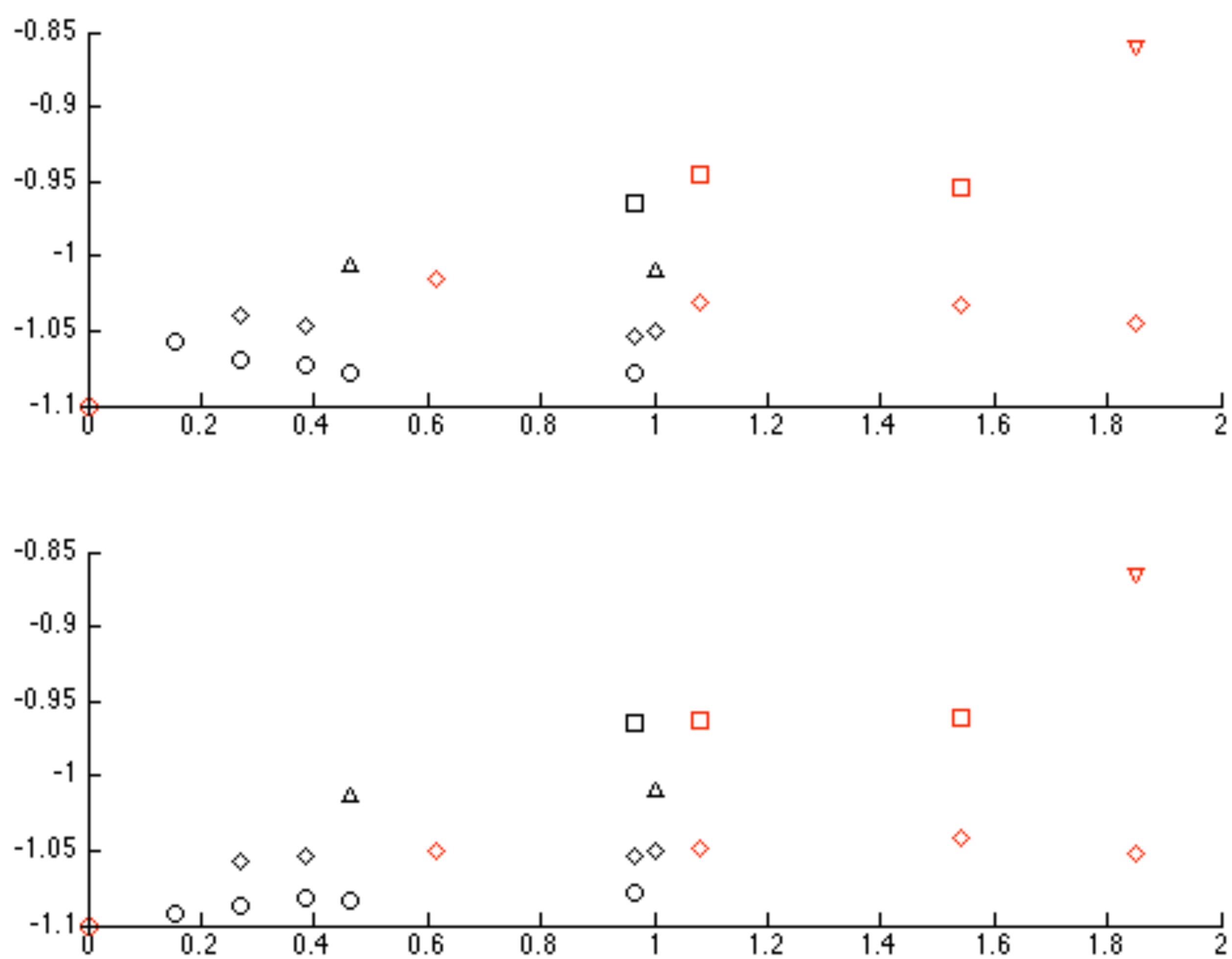
and a better fit is

$$\hat{\Sigma}_V^{(n)}(\hat{p}, pL) = \alpha_1^{(n)} 1 + \alpha_2^{(n)} \sum_\nu \hat{p}_\nu^2 + \alpha_3^{(n)} \sum_\nu \hat{p}_\nu^4 / p^2 +$$
$$+ \Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL) + \ldots$$

What we really learnt of FSE from NSPT

Let's solve for a single renorm. constant $\left. Z_{O_\Gamma}(\mu a, g(a)) Z_q^{-1}(\mu a, g(a)) O_\Gamma(pa) \right|_{p^2=\mu^2} = 1$

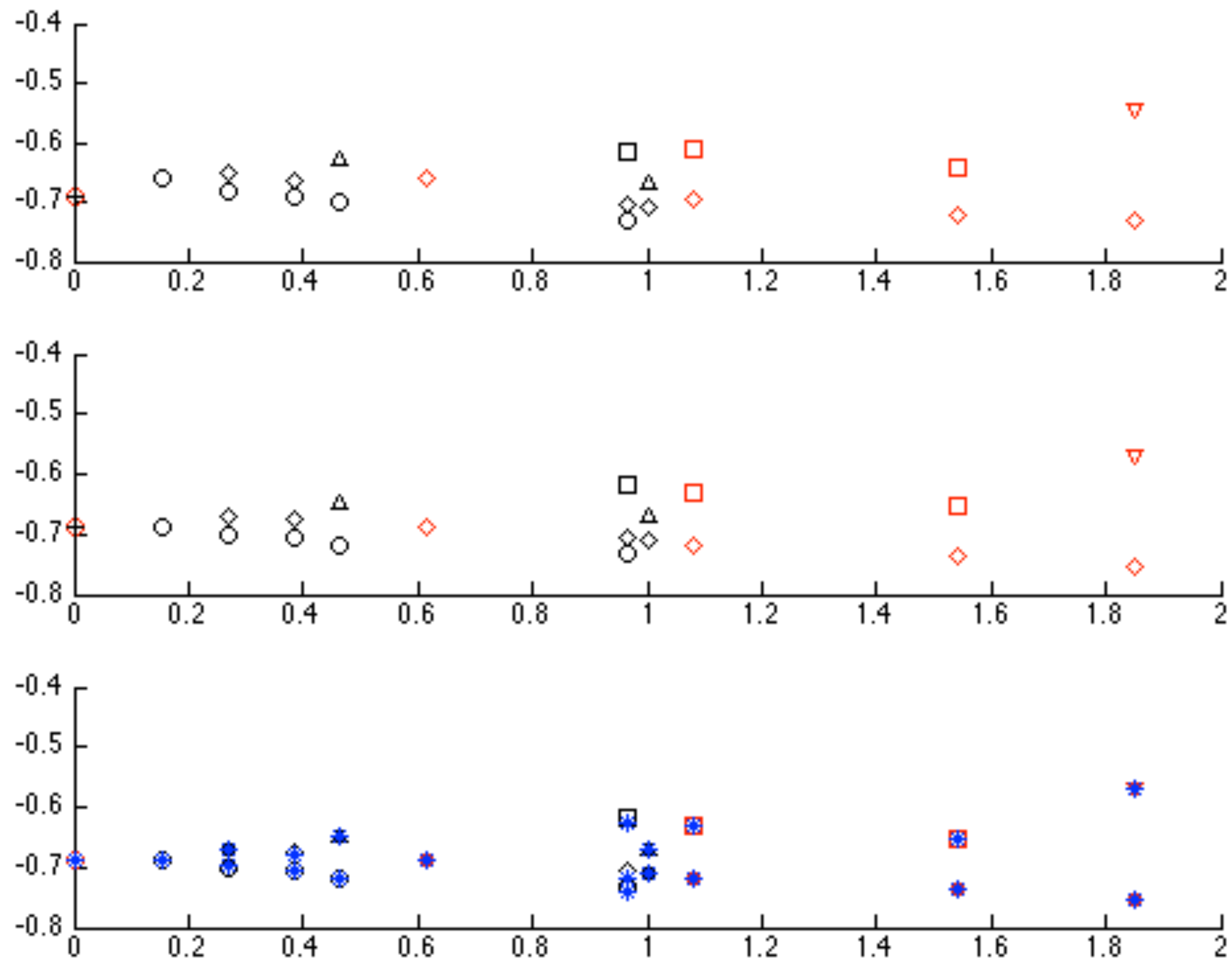
And let's plot 32^4 and 16^4 data toghether with and without the new fitting form (Z_P)
(Brambilla, Di Renzo, Hasegawa to be published - see PoS LATTICE2012 242)



If $\quad \Delta\hat{\Sigma}_V^{(n)}(\hat{p}, pL) \sim \Delta\hat{\Sigma}_V^{(n)}(pL) \quad$ then $\quad p_\mu L = \frac{2\pi n_\mu}{L} L = 2\pi n_\mu$

and we only have to care for one number for a given $\{n_1, n_2, n_3, n_4\}$

But if it works in NSPT (Brambilla, Di Renzo, Hasegawa to be published)...



... why don't we try to go the same way in a NP approach?

We need to:

- "really" work in momentum space
- always check the hyperbolic effects
- put different volumes toghether

We have no conclusions ...

... but we want to advertize work going on

- in assessing finite-a and finite-V effects in NSPT computations

- in implementing the same method of looking for finite size effects also in a NON PERTURBATIVE framework

... well aware that PT and NP will be very different, but confident that there is more information in RI'-MOM computations that we usually look into.

We can think of a "better best practice"

A possible approach can entail:

- a high order NSPT computation of renormalization constants

- a check (and possibly) correction for finite size effects for both NSPT and NP

- a better subtraction of irrelevant effects

Let me finally advertize the poster by Jakob Simeth