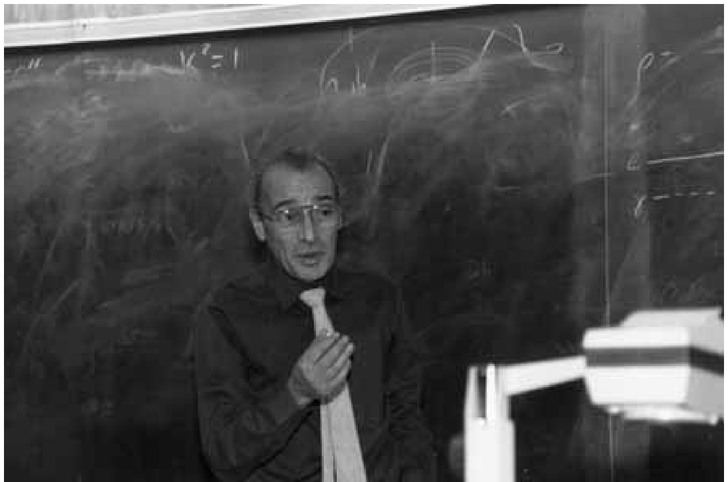
EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Faculty of Science
Theoretical Physics



Scanned at the American Institute of Physics

INSTABILITY OF NON-ABELIAN GAUGE THEORIES AND IMPOSSIBILITY OF CHOICE OF COULOMB GAUGE*

V. N. Gribov

ABSTRACT

In this lecture it is demonstrated that by virtue of the impossibility of introducing Coulomb gauge for large fields and of the growth of the invariant charge at large distances, non-Abelian gauge theories may not be formulated as a theory of interacting massless particles. This assertion appears as a strong argument in favor of the idea that the spectrum of states in non-Abelian theories is substantially different from the spectrum of states in perturbation theory.

Confinement in Coulomb gauge

What does the lattice teach us?

Lattice 2013 - Mainz, July 29th 2013, Giuseppe Burgio

Together with:

- M. Quandt
- H. Reinhardt
- M. Schröck
- H. Vogt

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Faddeev-Popov insufficient beyond perturbation theory

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to either:

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to either:
  - $-\vec{D} \cdot \vec{\nabla} > 0$: Gribov Region Ω, local maxima $F(A)$ easy

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to either:
  - $-\vec{D} \cdot \vec{\nabla} > 0$: Gribov Region Ω, local maxima $F(A)$ — easy
  - absolute maxima $F(A)$: Fundamental Modular Region Λ — hard

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Faddeev-Popov insufficient beyond perturbation theory
- restrict gauge functional $F(A)$ to either:
  - $-\vec{D} \cdot \vec{\nabla} > 0$: Gribov Region Ω, local maxima $F(A)$ easy
  - absolute maxima $F(A)$: Fundamental Modular Region Λ hard
- $(-\vec{D} \cdot \vec{\nabla})^{-1}$ singular at $\partial\Omega$ $(\partial\Lambda)$!

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Singularity restricts functional integral to Ω (Λ)

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Singularity restricts functional integral to Ω (Λ)
- Radius of Ω (Λ) introduces an IR scale (Gribov mass M_G)

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Singularity restricts functional integral to Ω (Λ)
- Radius of Ω (Λ) introduces an IR scale (Gribov mass M_G)
- $M_G \simeq O(\Lambda_{QCD}, \sigma_W) \Rightarrow$ confinement?

GZ-Confinement [Gribov NPB 1978; Zwanziger NPB 1997]

- Singularity restricts functional integral to Ω (Λ)
- Radius of Ω (Λ) introduces an IR scale (Gribov mass M_G)
- $M_G \simeq O(\Lambda_{QCD}, \sigma_W) \Rightarrow$ confinement?
- Natural test: A_μ, ψ dispersion relations; static potential

All gauges are equal...

All gauges are equal...

...but some are more equal than others!

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...
- In CG (lattice) Gribov problem milder. $\Omega \sim \Lambda$?

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...
- In CG (lattice) Gribov problem milder. $\Omega \sim \Lambda$?
- In CG physical content clearer and/or easier to extract:

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...
- In CG (lattice) Gribov problem milder. $\Omega \sim \Lambda$?
- In CG physical content clearer and/or easier to extract:
  - Static Coulomb potential V_C

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...
- In CG (lattice) Gribov problem milder. $\Omega \sim \Lambda$?
- In CG physical content clearer and/or easier to extract:
  - Static Coulomb potential V_C
  - (quasi-)particles dispersion relations ω_A, ω_ψ

All gauges are equal...

...but some are more equal than others!

- FP-ghost depends on the gauge. If/how GZ works also...
- In CG (lattice) Gribov problem milder. $\Omega \sim \Lambda$?
- In CG physical content clearer and/or easier to extract:
  - Static Coulomb potential V_C
  - (quasi-)particles dispersion relations ω_A, ω_ψ
  - fermion mass $M(\vec{p})$

GZ scenario in CG

- Ghost form factor $d(\vec{p})$ IR divergent $\Rightarrow$

  ω_A grows at large distances - no free gluons! [Gribov NPB 1978]

GZ scenario in CG

- Ghost form factor $d(\vec{p})$ IR divergent $\Rightarrow$

  ω_A grows at large distances - no free gluons! [Gribov NPB 1978]

- $V_C \geq V_{phys} \quad \Rightarrow \quad V_C(r) \simeq \sigma_C r$

  $V_C(\vec{p}) = 8\pi\sigma_C|\vec{p}|^{-4}, \quad \sigma_C \geq \sigma_W$

  necessary condition for confinement! [Zwanziger PRL 2003]

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

GZ scenario in CG

- Ghost form factor $d(\vec{p})$ IR divergent $\Rightarrow$

  ω_A grows at large distances - no free gluons! [Gribov NPB 1978]

- $V_C \geq V_{phys} \quad \Rightarrow \quad V_C(r) \simeq \sigma_C r$

  $V_C(\vec{p}) = 8\pi\sigma_C|\vec{p}|^{-4}, \quad \sigma_C \geq \sigma_W$

  necessary condition for confinement! [Zwanziger PRL 2003]

- Does χ-symmetry breaking show in $M(\vec{p})$?

GZ scenario in CG

- Ghost form factor $d(\vec{p})$ IR divergent $\Rightarrow$

  ω_A grows at large distances - no free gluons! [Gribov NPB 1978]

- $V_C \geq V_{phys} \quad \Rightarrow \quad V_C(r) \simeq \sigma_C r$

  $V_C(\vec{p}) = 8\pi\sigma_C|\vec{p}|^{-4}, \quad \sigma_C \geq \sigma_W$

  necessary condition for confinement! [Zwanziger PRL 2003]

- Does χ-symmetry breaking show in $M(\vec{p})$?

- What about ω_ψ?

Results from Hamiltonian approach

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}
  - $\kappa_{gl} + 2\kappa_{gh} = 1$
    $\gamma_{gl} + 2\gamma_{gh} = 1$

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}
  - $\kappa_{gl} + 2\kappa_{gh} = 1$
    $\gamma_{gl} + 2\gamma_{gh} = 1$
  - $\kappa_{gl} = -1$, $\kappa_{gh} = 1$, $\gamma_{gl} = 0$, $\gamma_{gh} = 1/2$

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}
  - $\kappa_{gl} + 2\kappa_{gh} = 1$
    $\gamma_{gl} + 2\gamma_{gh} = 1$
  - $\kappa_{gl} = -1$, $\kappa_{gh} = 1$, $\gamma_{gl} = 0$, $\gamma_{gh} = 1/2$
- $d(\vec{p}) \propto \epsilon(\vec{p})^{-1}$ vacuum dielectric function $\Rightarrow$ Dual superconductor! [Reinhardt PRL 2008]

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}
  - $\kappa_{gl} + 2\kappa_{gh} = 1$
    $\gamma_{gl} + 2\gamma_{gh} = 1$
  - $\kappa_{gl} = -1$, $\kappa_{gh} = 1$, $\gamma_{gl} = 0$, $\gamma_{gh} = 1/2$
- $d(\vec{p}) \propto \epsilon(\vec{p})^{-1}$ vacuum dielectric function $\Rightarrow$
  Dual superconductor! [Reinhardt PRL 2008]
- Extension to fermions: running mass $M(\vec{p})$

Results from Hamiltonian approach

- Asymptotics for $\omega_A(\vec{p})$, $d(\vec{p})$
  - IR power laws κ_{gl}, κ_{gh}
  - UV anomalous dimensions γ_{gl}, γ_{gh}
  - $\kappa_{gl} + 2\kappa_{gh} = 1$
    $\gamma_{gl} + 2\gamma_{gh} = 1$
  - $\kappa_{gl} = -1$, $\kappa_{gh} = 1$, $\gamma_{gl} = 0$, $\gamma_{gh} = 1/2$
- $d(\vec{p}) \propto \epsilon(\vec{p})^{-1}$ vacuum dielectric function $\Rightarrow$ Dual superconductor! [Reinhardt PRL 2008]
- Extension to fermions: running mass $M(\vec{p})$
- Everything static "by construction"!

Goals of CG lattice investigation

From d, V_C, ω_A, $M(\vec{p})$ (and possibly ω_ψ) test:

Goals of CG lattice investigation

From d, V_C, ω_A, $M(\vec{p})$ (and possibly ω_ψ) test:

- GZ confinement scenario

Goals of CG lattice investigation

From d, V_C, ω_A, $M(\vec{p})$ (and possibly ω_ψ) test:

- GZ confinement scenario
- Results from variational approach

Goals of CG lattice investigation

From d, V_C, ω_A, $M(\vec{p})$ (and possibly ω_ψ) test:

- GZ confinement scenario
- Results from variational approach
- Extend GZ to the quark sector

Goals of CG lattice investigation

From d, V_C, ω_A, $M(\vec{p})$ (and possibly ω_ψ) test:

- GZ confinement scenario
- Results from variational approach
- Extend GZ to the quark sector
- "Built-in" differences to continuum. Issues to be addressed!

Lattice calculations

Correlators in CG

Lattice calculations

Correlators in CG

1. Static "by construction"

- $G(\vec{p}) = |\vec{p}|^{-2} d(\vec{p}) = \delta^{ab} \langle \bar{c}^a(\vec{p}) c^b(-\vec{p}) \rangle = \langle (-\vec{D} \cdot \vec{\nabla})^{-1} \rangle$
- $V_C(\vec{p}) = g^2 \, \delta^{ab} \langle (-\vec{D} \cdot \vec{\nabla})^{-1} \vec{\nabla}^2 (-\vec{D} \cdot \vec{\nabla})^{-1} \rangle$

Lattice calculations

Correlators in CG

1. Static "by construction"
   - $G(\vec{p}) = |\vec{p}|^{-2} d(\vec{p}) = \delta^{ab}\langle \bar{c}^a(\vec{p}) c^b(-\vec{p})\rangle = \langle(-\vec{D}\cdot\vec{\nabla})^{-1}\rangle$
   - $V_C(\vec{p}) = g^2\,\delta^{ab}\langle(-\vec{D}\cdot\vec{\nabla})^{-1}\vec{\nabla}^2(-\vec{D}\cdot\vec{\nabla})^{-1}\rangle$

   (Almost) directly comparable to continuum

Lattice calculations

Correlators in CG

1. Static "by construction"

   - $G(\vec{p}) = |\vec{p}|^{-2} d(\vec{p}) = \delta^{ab}\langle \bar{c}^a(\vec{p}) c^b(-\vec{p})\rangle = \langle(-\vec{D}\cdot\vec{\nabla})^{-1}\rangle$
   - $V_C(\vec{p}) = g^2\,\delta^{ab}\langle(-\vec{D}\cdot\vec{\nabla})^{-1}\vec{\nabla}^2(-\vec{D}\cdot\vec{\nabla})^{-1}\rangle$

   (Almost) directly comparable to continuum

2. Time (i.e. energy!) dependent

   - $D(\vec{p}, p_0) = \delta^{ab}\delta_{ij}\langle A_i^a(\vec{p}, p_0) A_j^b(-\vec{p}, -p_0)\rangle$
   - $S(\vec{p}, p_0) = \delta^{AB}\langle\bar{\psi}^A(\vec{p}, p_0)\psi^B(-\vec{p}, -p_0)\rangle$

Lattice calculations

Correlators in CG

1. Static "by construction"
   - $G(\vec{p}) = |\vec{p}|^{-2} d(\vec{p}) = \delta^{ab} \langle \bar{c}^a(\vec{p}) c^b(-\vec{p}) \rangle = \langle (-\vec{D} \cdot \vec{\nabla})^{-1} \rangle$
   - $V_C(\vec{p}) = g^2 \, \delta^{ab} \langle (-\vec{D} \cdot \vec{\nabla})^{-1} \vec{\nabla}^2 (-\vec{D} \cdot \vec{\nabla})^{-1} \rangle$

   (Almost) directly comparable to continuum

2. Time (i.e. energy!) dependent
   - $D(\vec{p}, p_0) = \delta^{ab} \delta_{ij} \langle A_i^a(\vec{p}, p_0) A_j^b(-\vec{p}, -p_0) \rangle$
   - $S(\vec{p}, p_0) = \delta^{AB} \langle \bar{\psi}^A(\vec{p}, p_0) \psi^B(-\vec{p}, -p_0) \rangle$

   Equal-time component $\Leftrightarrow$ integrate over p_0.
   Sizable cut-off effects: scale with a_t^{-1}...

Minimizing lattice artifacts I

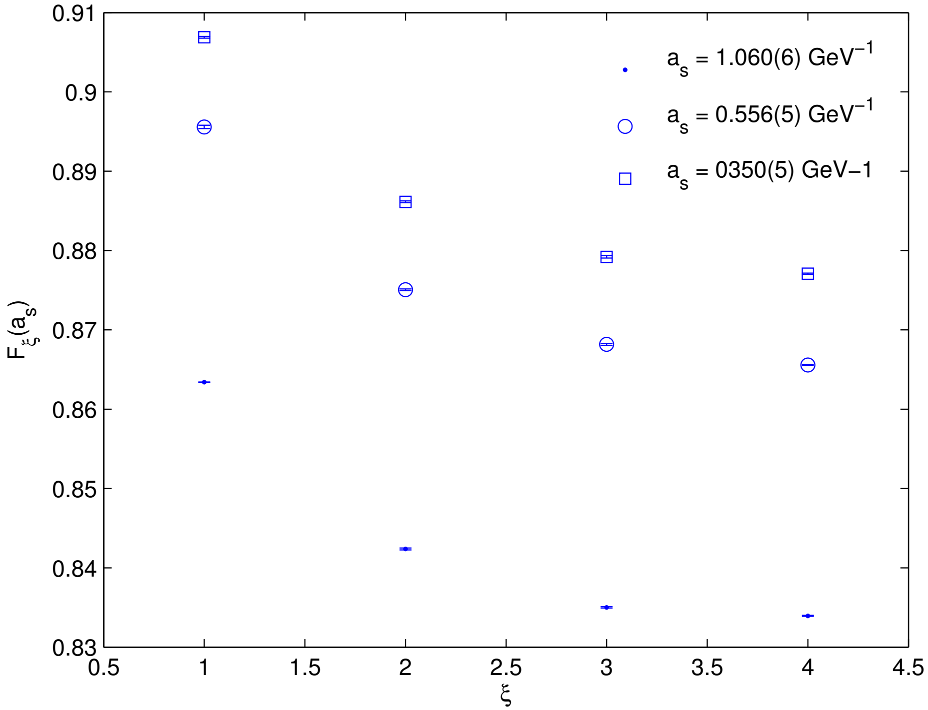
Anisotropic action. Closer to the Hamiltonian limit $a_t \to 0$!

$$S = \beta \sum_x \left\{ \gamma \sum_{j>i=1}^{d} \left(1 - \frac{1}{N_c} \Re \left[\mathrm{Tr} \left(P_{ij}(x) \right) \right] \right) + \frac{1}{\gamma} \sum_{i=1}^{d} \left(1 - \frac{1}{N_c} \Re \left[\mathrm{Tr} \left(P_{i,d+1}(x) \right) \right] \right) \right\}$$

γ bare anisotropy. Must fix $\xi = \frac{a_s}{a_t}$ non-perturbatively!

$T = 0$ simulations on a $L^3 \times (\xi L)$ lattice.
Need $\xi \gg 1$ for good scaling!

Strong effect, e.g. on $F(A)$ for fixed a_s



Large corrections, scale with a_t^2, a_t^4 (glueball spectrum!)

Minimizing lattice artifacts II

Scaling violations in static gluon $\sum_{p_0} D(\vec{p}, p_0)$

Minimizing lattice artifacts II

Scaling violations in static gluon $\sum_{p_0} D(\vec{p}, p_0)$

- model explicit p_0 dependence to do $\int dp_0$ "analytically"

Minimizing lattice artifacts II

Scaling violations in static gluon $\sum_{p_0} D(\vec{p}, p_0)$

- model explicit p_0 dependence to do $\int dp_0$ "analytically"
- Static $D(\vec{p})$ renormalizable, "agrees" with Gribov's formula

$$\omega_A(|\vec{p}|) \propto \sqrt{|\vec{p}|^2 + \frac{M_G^4}{|\vec{p}|^2}}$$

$M_G = 0.856(8) \mathrm{G}eV$. See H. Vogt's talk...

Minimizing lattice artifacts III

- Lesson from Landau gauge:

  full QCD exhibits strong scaling violations

Minimizing lattice artifacts III

- Lesson from Landau gauge:

  full QCD exhibits strong scaling violations

  Must use improved actions!

Minimizing lattice artifacts III

- Lesson from Landau gauge:

  full QCD exhibits strong scaling violations

  Must use improved actions!

  - Generate them yourself: expensive...

Minimizing lattice artifacts III

- Lesson from Landau gauge:

  full QCD exhibits strong scaling violations

  Must use improved actions!

  - Generate them yourself: expensive...

- Download configurations from ILDG, e.g. Asqtad (MILC) ☺

Minimizing lattice artifacts III

- Lesson from Landau gauge:

  full QCD exhibits strong scaling violations

  Must use improved actions!

  - Generate them yourself: expensive...

- Download configurations from ILDG, e.g. Asqtad (MILC) ☺

  - Parameters rarely test the “deep” IR ☹

Ghost from factor *d* (see also H. Vogt's talk)

Ghost from factor *d* (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$

Ghost from factor d (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$
- IR behavior: $\kappa_{gh} \gtrsim 0.5$. Agrees with GZ scenario!

Ghost from factor *d* (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$
- IR behavior: $\kappa_{gh} \gtrsim 0.5$. Agrees with GZ scenario!
    - IR sum rule violated?

Ghost from factor d (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$
- IR behavior: $\kappa_{gh} \gtrsim 0.5$. Agrees with GZ scenario!
  - IR sum rule violated?
  - Exceptional configurations with high contributions to d

    FP inversion numerically difficult; need better algorithms

Ghost from factor d (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$
- IR behavior: $\kappa_{gh} \gtrsim 0.5$. Agrees with GZ scenario!
  - IR sum rule violated?
  - Exceptional configurations with high contributions to d

    FP inversion numerically difficult; need better algorithms

Best estimate for κ_{gh} might just be lower bound

Ghost from factor *d* (see also H. Vogt's talk)

- UV behaviour: well fitted by $\gamma_{gh} = \frac{1}{2}$
- IR behavior: $\kappa_{gh} \gtrsim 0.5$. Agrees with GZ scenario!
  - IR sum rule violated?
  - Exceptional configurations with high contributions to *d*

  FP inversion numerically difficult; need better algorithms

Best estimate for κ_{gh} might just be lower bound

and/or static vertex might not be trivial...

Extracting Coulomb string tension

- Asymptotic + leading corrections:

$$V_C(r) = \sigma_C r + \mu - \frac{\lambda}{r} + \mathcal{O}(\frac{1}{r^2}) \, ,$$

Extracting Coulomb string tension

- Asymptotic $+$ leading corrections:

$$V_C(r) = \sigma_C r + \mu - \frac{\lambda}{r} + \mathcal{O}(\frac{1}{r^2}),$$

- Introduce an intermediate IR cutoff...

$$|\vec{p}|^4 \, V_C(\vec{p}) = 8\pi\sigma_C + 4\pi\lambda|\vec{p}|^2 + \mathcal{O}(|\vec{p}|^3)$$

μ term vanishes with IR cutoff...

Extracting Coulomb string tension

- Asymptotic $+$ leading corrections:

$$V_C(r) = \sigma_C r + \mu - \frac{\lambda}{r} + \mathcal{O}(\frac{1}{r^2})\,,$$

- Introduce an intermediate IR cutoff...

$$|\vec{p}|^4\, V_C(\vec{p}) = 8\pi\sigma_C + 4\pi\lambda|\vec{p}|^2 + \mathcal{O}(|\vec{p}|^3)$$

  μ term vanishes with IR cutoff...

- From λ term: no IR plateau!

Extracting Coulomb string tension

- Asymptotic $+$ leading corrections:

$$V_C(r) = \sigma_C r + \mu - \frac{\lambda}{r} + \mathcal{O}(\frac{1}{r^2}),$$

- Introduce an intermediate IR cutoff...

$$|\vec{p}|^4\, V_C(\vec{p}) = 8\pi\sigma_C + 4\pi\lambda|\vec{p}|^2 + \mathcal{O}(|\vec{p}|^3)$$

  μ term vanishes with IR cutoff...

- From λ term: no IR plateau!

- Optimist: fit using such information $\sigma_C = 2.2(2)\,\sigma$

Extracting Coulomb string tension

- Asymptotic $+$ leading corrections:

$$V_C(r) = \sigma_C r + \mu - \frac{\lambda}{r} + \mathcal{O}(\frac{1}{r^2}) \,,$$

- Introduce an intermediate IR cutoff...

$$|\vec{p}|^4 \, V_C(\vec{p}) = 8\pi\sigma_C + 4\pi\lambda|\vec{p}|^2 + \mathcal{O}(|\vec{p}|^3)$$

  μ term vanishes with IR cutoff...

- From λ term: no IR plateau!

- Optimist: fit using such information $\sigma_C = 2.2(2)\,\sigma$
  Hard to get clean fit: see H. Vogt's talk.

Quark

- From the Dirac operator in CG we get

  $S^{-1}(\vec{p}, p_0) = i\vec{\not{p}} A_s(\vec{p}) + i\not{p}_0 A_t(\vec{p}) + B_m(\vec{p})$

Quark

- From the Dirac operator in CG we get

$$S^{-1}(\vec{p}, p_0) = i\vec{\not p} A_s(\vec{p}) + i\not p_0 A_t(\vec{p}) + B_m(\vec{p})$$

- If renormalizable:

$$S^{-1}(\vec{p}, p_0) = Z^{-1}(\vec{p}) \left[i\vec{\not p} + i\not p_0 \alpha(\vec{p}) + M(\vec{p}) \right]$$

Quark

- From the Dirac operator in CG we get

  $$S^{-1}(\vec{p}, p_0) = i\vec{\not p} A_s(\vec{p}) + i\not p_0 A_t(\vec{p}) + B_m(\vec{p})$$

- If renormalizable:

  $$S^{-1}(\vec{p}, p_0) = Z^{-1}(\vec{p}) \left[i\vec{\not p} + i\not p_0 \alpha(\vec{p}) + M(\vec{p}) \right]$$

  - $\alpha(\vec{p})$ and $M(\vec{p})$ must be cut-off independent

Quark

- From the Dirac operator in CG we get

  $S^{-1}(\vec{p}, p_0) = i\not{\vec{p}} A_s(\vec{p}) + i\not{p}_0 A_t(\vec{p}) + B_m(\vec{p})$

- If renormalizable:

  $S^{-1}(\vec{p}, p_0) = Z^{-1}(\vec{p}) \left[i\not{\vec{p}} + i\not{p}_0 \alpha(\vec{p}) + M(\vec{p}) \right]$

  - $\alpha(\vec{p})$ and $M(\vec{p})$ must be cut-off independent
  - $Z(\vec{p})$ must be renormalizable

Quark

- From the Dirac operator in CG we get

  $S^{-1}(\vec{p}, p_0) = i\vec{\not{p}} A_s(\vec{p}) + i\not{p}_0 A_t(\vec{p}) + B_m(\vec{p})$

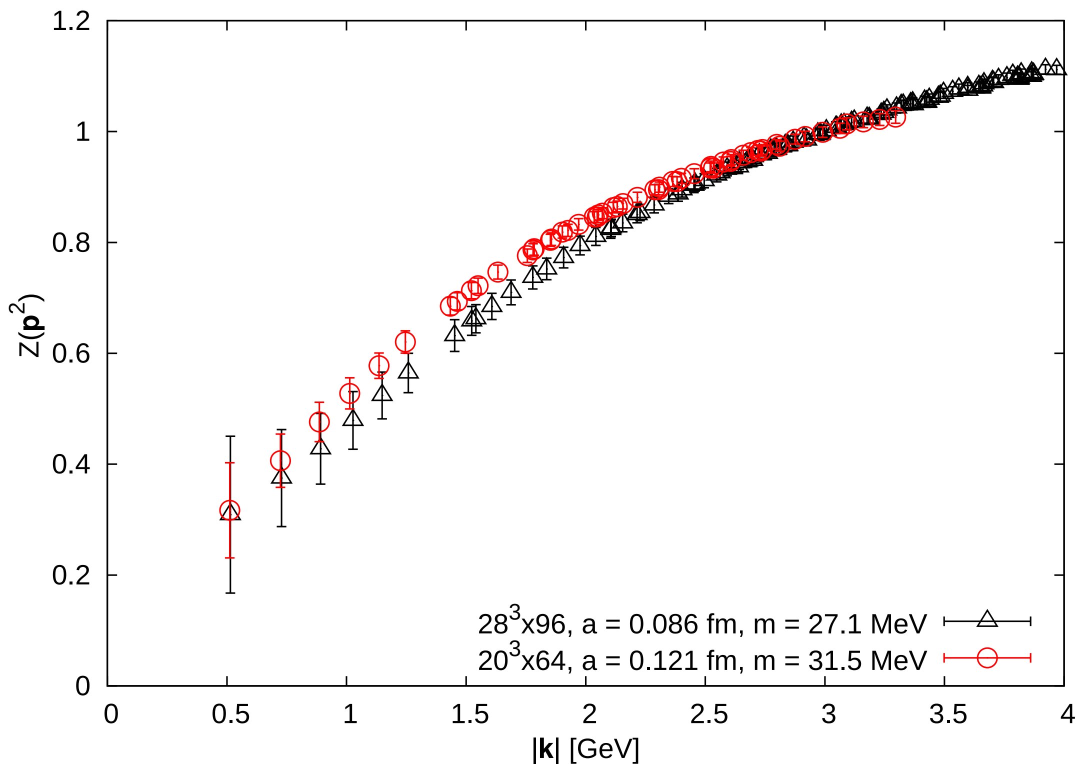
- If renormalizable:

  $S^{-1}(\vec{p}, p_0) = Z^{-1}(\vec{p}) \left[i\vec{\not{p}} + i\not{p}_0 \alpha(\vec{p}) + M(\vec{p}) \right]$

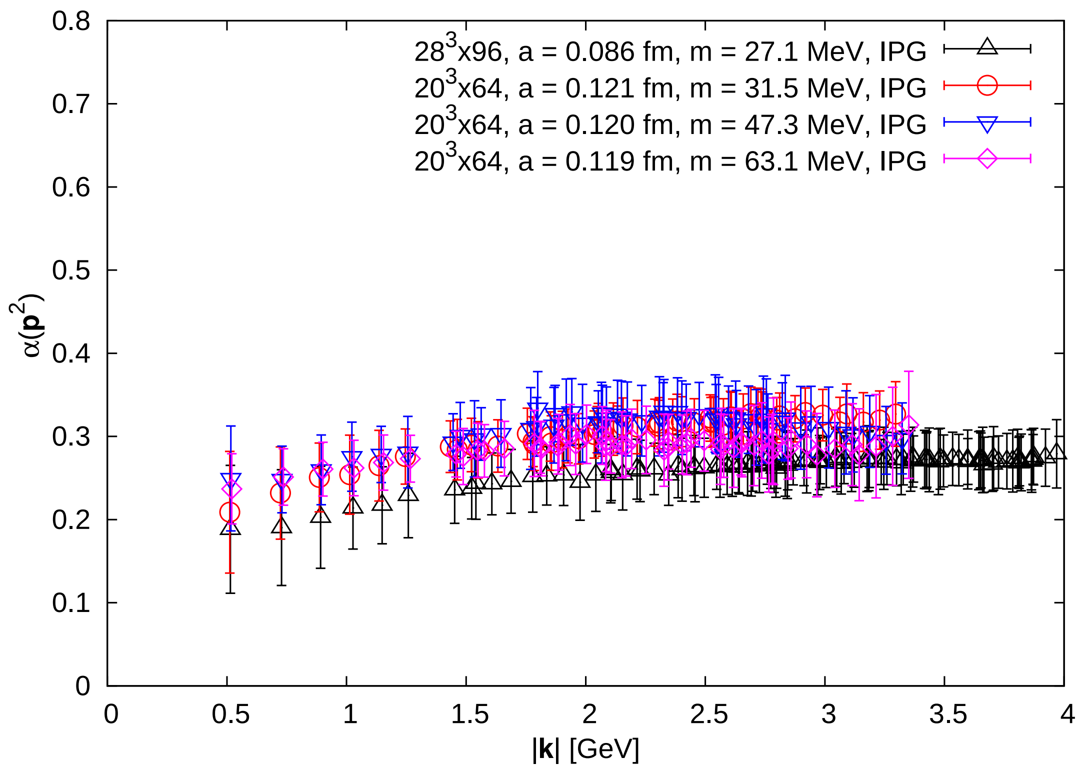
  - $\alpha(\vec{p})$ and $M(\vec{p})$ must be cut-off independent
  - $Z(\vec{p})$ must be renormalizable

- Define $S(\vec{p})$, ω_ψ from $S^{-1}(\vec{p}, p_0)$...

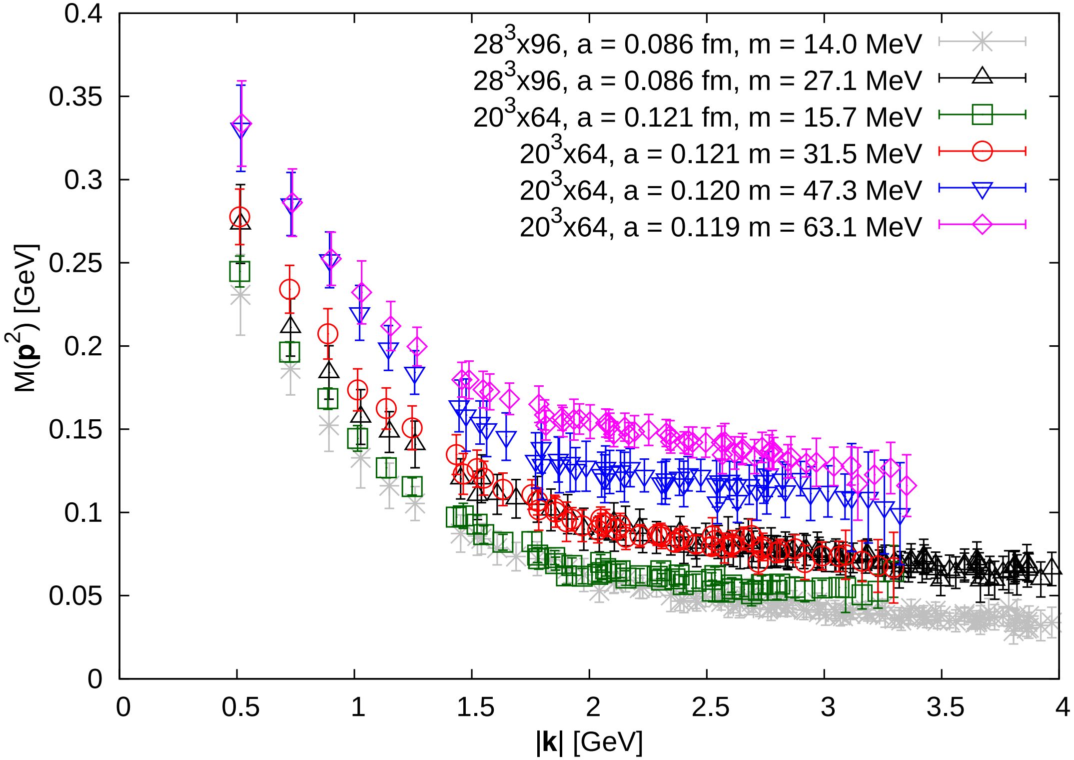
$Z(\vec{p})$ renormalizable



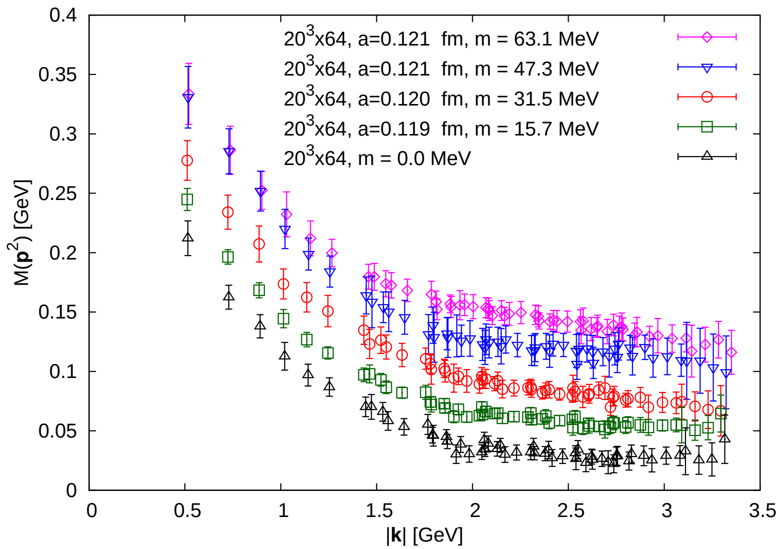
$\alpha(\vec{p})$ scale invariant



$M(\vec{p})$ scale invariant (for fixed quark mass!)



Chiral limit for $M(\vec{p})$



$$M(|\vec{k}|, m_b) = \frac{m_\chi(m_b)}{1 + b\,\frac{|\vec{k}|^2}{\Lambda^2}\,\log\left(e + \frac{|\vec{k}|^2}{\Lambda^2}\right)^{-\gamma}} + \frac{m_r(m_b)}{\log\left(e + \frac{|\vec{k}|^2}{\Lambda^2}\right)^{\gamma}}$$

$b = 2.9(1)$, $\gamma = 0.84(2)$, $\Lambda = 1.22(6)$ GeV, $m_\chi(0) = 0.31(1)$ GeV, χ^2/d.o.f.$= 1.06$

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ I

Analogy with free fermion: Hamiltonian

$S^H(\vec{p}) = \int dp_0\, S(\vec{p}, p_0) \propto H$

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ I

Analogy with free fermion: Hamiltonian

$S^H(\vec{p}) = \int dp_0 \, S(\vec{p}, p_0) \propto H$

- $$S^H(\vec{p}) = \frac{Z(\vec{p})}{\alpha(\vec{p})} \frac{\sqrt{\vec{p}^2 + M^2(|\vec{p}|)}}{i\vec{\not{p}} + M(\vec{p})} = \frac{Z(\vec{p})}{\alpha(\vec{p})} \frac{-i\vec{\not{p}} + M(\vec{p})}{\sqrt{\vec{p}^2 + M^2(|\vec{p}|)}}$$

  No divergences ☺

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ I

Analogy with free fermion: Hamiltonian

$S^H(\vec{p}) = \int dp_0\, S(\vec{p}, p_0) \propto H$

- $$S^H(\vec{p}) = \frac{Z(\vec{p})}{\alpha(\vec{p})} \frac{\sqrt{\vec{p}^2 + M^2(|\vec{p}|)}}{i\vec{\not p} + M(\vec{p})} = \frac{Z(\vec{p})}{\alpha(\vec{p})} \frac{-i\vec{\not p} + M(\vec{p})}{\sqrt{\vec{p}^2 + M^2(|\vec{p}|)}}$$

  No divergences ☺

- Coefficient^{-1} of $-i\vec{\not p} + M(\vec{p})$ eigenvalue of H: quark effective energy!

$$\omega_\psi^H(|\vec{p}|) = \frac{\alpha(|\vec{p}|)}{Z(|\vec{p}|)} \sqrt{\vec{p}^2 + M^2(|\vec{p}|)}$$

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ II

Analogy with free fermion: Euclidean

Consistency between $\int dp_0\, S(\vec{p}, p_0)$ and $\int dp_0\, S^{-1}(\vec{p}, p_0)$

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ II

Analogy with free fermion: Euclidean

Consistency between $\int dp_0\, S(\vec{p}, p_0)$ and $\int dp_0\, S^{-1}(\vec{p}, p_0)$

- $S^E(\vec{p}) = \Lambda \dfrac{Z(\vec{p})}{i\vec{p}\!\!\!/ + M(\vec{p})}$

  $\Lambda \propto a_t^{-1} \to \infty$ ☹

Define $S(\vec{p})$, $\omega_\psi(|\vec{p}|)$ II

Analogy with free fermion: Euclidean

Consistency between $\int dp_0\, S(\vec{p}, p_0)$ and $\int dp_0\, S^{-1}(\vec{p}, p_0)$

- $S^E(\vec{p}) = \Lambda \dfrac{Z(\vec{p})}{i\vec{\not{p}} + M(\vec{p})}$

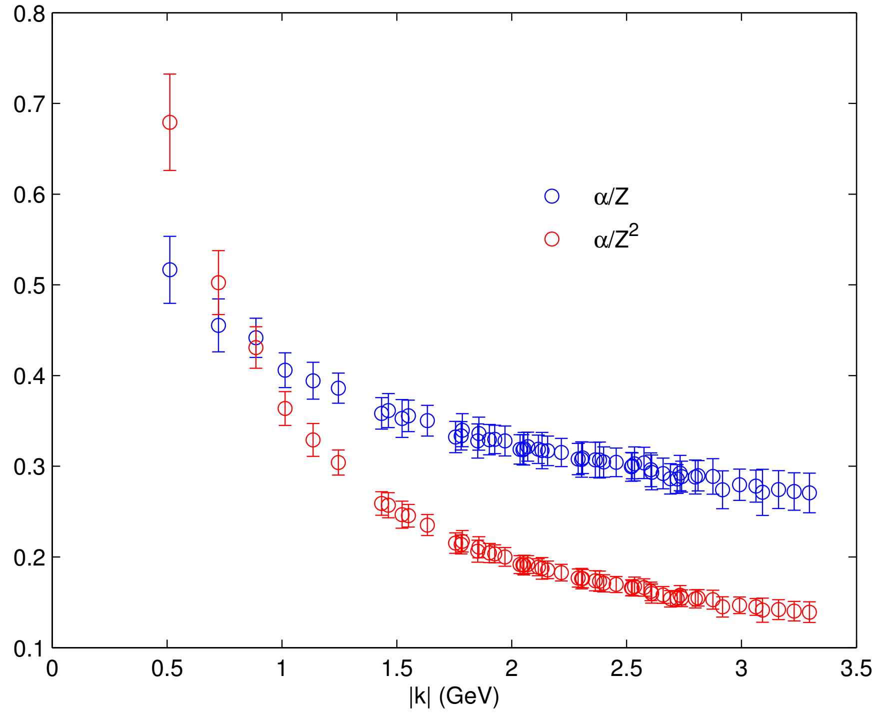
  $\Lambda \propto a_t^{-1} \to \infty$ ☹

- quark effective energy

$$\omega_\psi^E(|\vec{p}|) = \int dp_0\, S^2(\vec{p}, p_0) = \frac{\alpha(|\vec{p}|)}{Z^2(|\vec{p}|)} \sqrt{\vec{p}^2 + M^2(|\vec{p}|)}$$

$$\omega_{\psi}^{H,E}(|\vec{p}|) = \frac{\alpha(|\vec{p}|)}{Z^{(2)}(|\vec{p}|)}\sqrt{\vec{p}^2 + M^2(|\vec{p}|)}$$

$M(|\vec{p}|) \rightarrow m_\chi$. Only $\frac{\alpha}{Z^{(2)}}$ relevant for IR...



Both IR enhanced! What happens at lower momenta?

Summary

- Static propagators in CG renormalizable

Summary

- Static propagators in CG renormalizable
- $d(\vec{p}) \gtrsim |\vec{p}|^{-1/2}$ IR divergent; GZ works!

Summary

- Static propagators in CG renormalizable
- $d(\vec{p}) \gtrsim |\vec{p}|^{-1/2}$ IR divergent; GZ works!
- $\sigma_C > \sigma$. Trust extrapolation to 2.2(2) σ?

Summary

- Static propagators in CG renormalizable
- $d(\vec{p}) \gtrsim |\vec{p}|^{-1/2}$ IR divergent; GZ works!
- $\sigma_C > \sigma$. Trust extrapolation to 2.2(2) σ?
- ω_A, ω_ψ IR divergent, as expected from confinement

Summary

- Static propagators in CG renormalizable
- $d(\vec{p}) \gtrsim |\vec{p}|^{-1/2}$ IR divergent; GZ works!
- $\sigma_C > \sigma$. Trust extrapolation to 2.2(2) σ?
- ω_A, ω_ψ IR divergent, as expected from confinement
- $M(\vec{p})$ well describes χ-symmetry breaking

Outlook

Outlook

- See H. Vogt's talk!

Thanks!