On two- and three-point functions of Landau gauge Yang-Mills theory

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MARIE CURIE ACTIONS

Non-perturbative approaches to QCD

	lattice	functional continuum methods
volume	calc. for finite volume	finite vol. possible
scale separations	☹	☺
errors	finite size & lattice spacing, statistics	truncations
propagators	☺	☺
vertices	☹	☺

$\Rightarrow$ Methods can ideally complement each other with their specific strengths.

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⇒ Methods can ideally complement each other with their specific strengths.

temperature	☺	☺
chemical potential	☺	sign problem
analytic structure	☺	not directly

Landau Gauge Yang-Mills theory

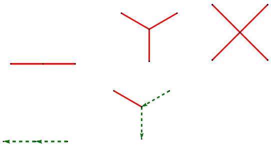
Gluonic sector of quantum chromodynamics: Yang-Mills theory

$$\mathcal{L} = \frac{1}{2}F^2 + \mathcal{L}_{gf} + \mathcal{L}_{gh}$$
$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + i\,g\,[\mathbf{A}_\mu, \mathbf{A}_\nu]$$

Propagators and vertices are gauge dependent
$\rightarrow$ choose any gauge, ideally one that is convenient.

Landau gauge

- simplest one for functional equations
- $\partial_\mu \mathbf{A}_\mu = 0$: $\quad \mathcal{L}_{gf} = \frac{1}{2\xi}(\partial_\mu \mathbf{A}_\mu)^2, \quad \xi \rightarrow 0$
- requires ghost fields: $\quad \mathcal{L}_{gh} = \bar{\mathbf{c}}\,(-\Box + g\,\mathbf{A}\,\times)\,\mathbf{c}$



Truncated propagator Dyson-Schwinger equations

Standard truncation:

- No four-point interactions
- models for ghost-gluon and three-gluon vertices

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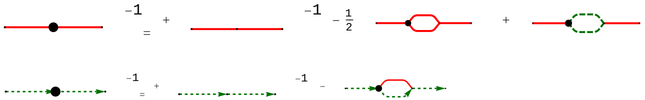

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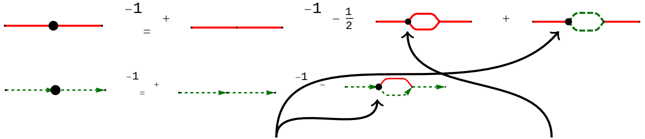

Standard: bare ghost-gluon vertex and three-gluon vertex model

$$D^{ab}_{gl,\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{\mathbf{Z(p^2)}}{p^2}\delta^{ab}$$

Influence of three-point functions?

$$D^{ab}_{gh}(p) = -\frac{\mathbf{G(p^2)}}{p^2}\delta^{ab}$$

Testing and improving truncations

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Test reliability of truncations by

- calculate influence of neglected quantities = enlarge truncation. difficult ↯
- compare results with other methods. lattice ☺

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		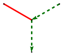	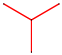	year	
✓	0	0	model	1979	qual. wrong
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		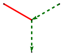	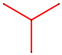	year	
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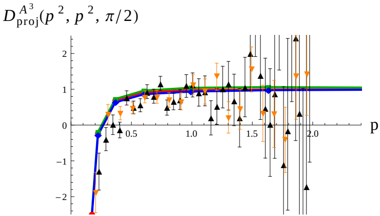
Three-gluon vertex: Infrared

Three-gluon vertex might have a zero crossing.

$d = 2, 3$: seen on lattice [Cucchieri, Maas, Mendes, PRD77 (2008); Maas, PRD75 (2007)]

$d = 2$: seen with DSEs [MQH, Maas, von Smekal, JHEP11 (2012)]

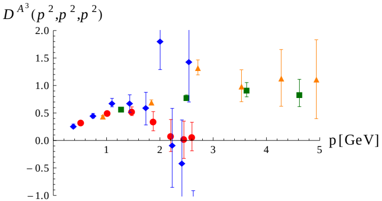
$d = 2$:



[Maas, PRD75; MQH, Maas, von Smekal, JHEP11 (2012)]

Three-gluon vertex: Infrared

$d = 4$:



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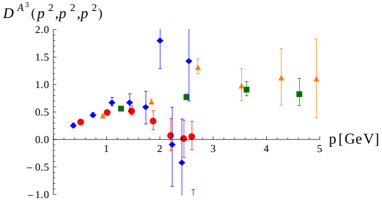


[Cucchieri, Maas, Mendes, PRD77 (2008)]

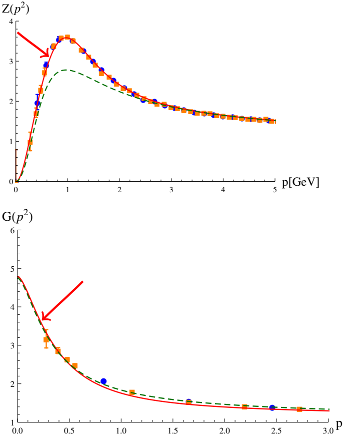
$$D^{A^3,IR}(x,y,z) = \mathbf{h_{IR}} G(x+y+z)^3 (f^{3g}(x) f^{3g}(y) f^{3g}(z))^4 \qquad \text{with} \qquad f^{3g}(x) := \frac{\Lambda_{3g}^2}{\Lambda_{3g}^2 + x}$$

Zero crossing confirmed with leading order DSE calculation

[MQH, von Smekal, JHEP04 (2013)].

Dynamic ghost-gluon vertex: Propagator results

Dynamic ghost-gluon vertex, opt. eff.
three-gluon vertex [MQH, von Smekal, JHEP04 (2013)]



lattice data: [Sternbeck (2006)]

Good quantitative agreement for ghost *and* gluon dressings.

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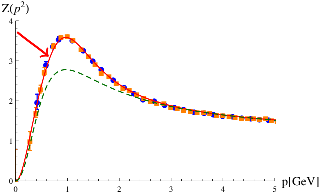
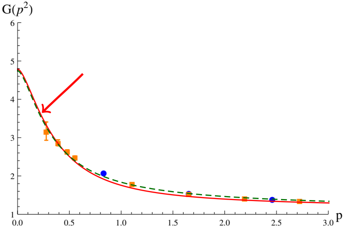
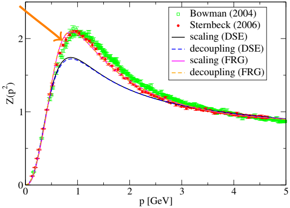


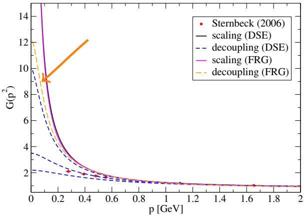


lattice data: [Sternbeck (2006)]

FRG results

[Fischer, Maas, Pawlowski, AP324 (2009)]



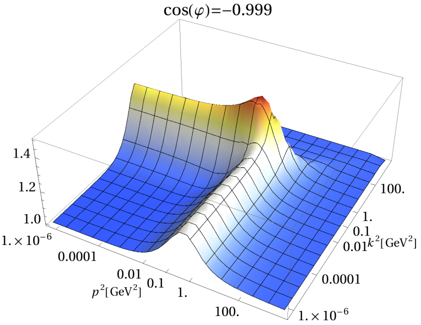


Good quantitative agreement for ghost *and* gluon dressings.

Ghost-gluon vertex: Selected configurations (decoupling)

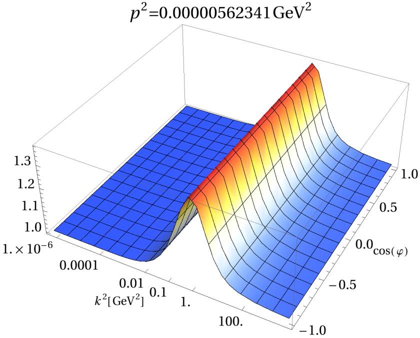
$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\left(p_\mu A(k;p,q) + k_\mu B(k;p,q)\right)$$

Fixed angle:



[MQH, von Smekal, JHEP04 (2013)]

Fixed anti-ghost momentum:



Ghost-gluon vertex: Comparison with lattice data

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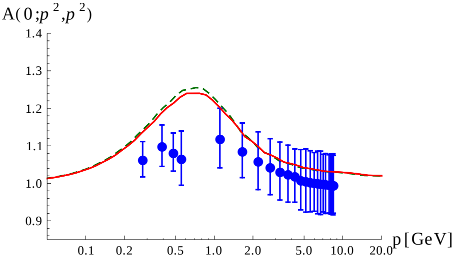
Orthogonal configuration $k^2 = 0$, $q^2 = p^2$:



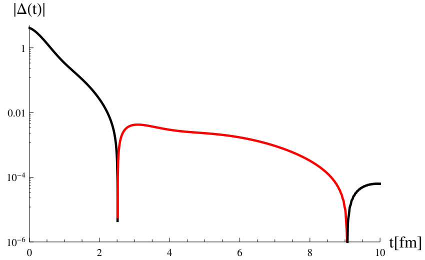
DSE calculation: [MQH, von Smekal, JHEP04 (2013)]
lattice data: [Sternbeck, hep-lat/0609016]

- constant in the IR
- relatively insensitive to changes of the three-gluon vertex (red/green lines: different three-gluon vertex models)

Schwinger function

Schwinger function $\Delta(t)$:

$$\Delta(t) = \frac{1}{\pi}\int dq\,\cos(q\,t)\frac{Z(q^2)}{q^2}$$



[MQH, von Smekal, PoS CONFX 062 (2013)]

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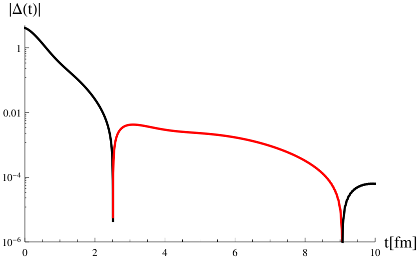

[MQH, von Smekal, PoS CONFX 062 (2013)]

$$\Delta(t) = \int_0^\infty d\nu\, \rho(\nu^2)e^{-\nu t} = \mathcal{L}(\rho)$$

ρ: spectral density, must be positive for physical particles

Positivity violation of propagators $\rightarrow$ confinement.

Towards the phase diagram of QCD with DSEs

Lattice results helpful in several aspects:

- for comparison.

Towards the phase diagram of QCD with DSEs

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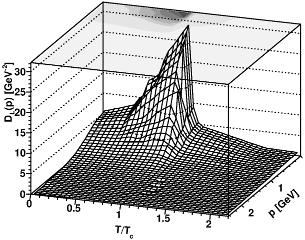
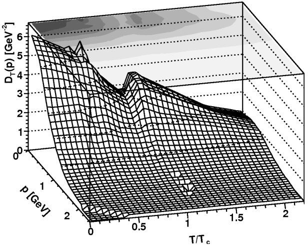
- for comparison.
- as input.

Ghost propagator

First steps towards full system: Take some lattice input.

Gluon propagator: lattice based fits [Fischer, Maas, Müller, EPJC68 (2010)]





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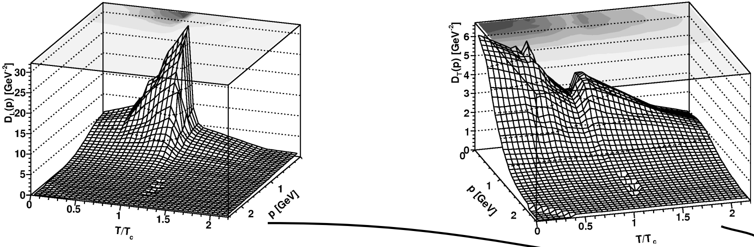

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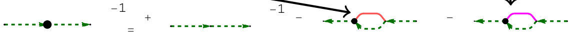
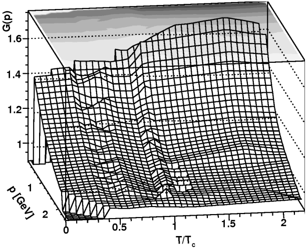


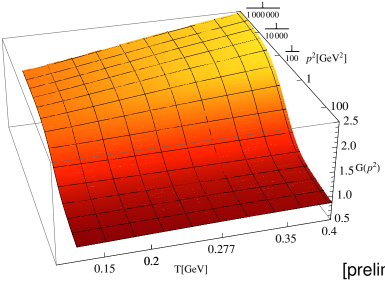
For zeroth Matsubara no contribution from (dominant) zeroth Matsubara summand.
⇒ Ghost does not react to phase transition.

Ghost propagator

Ghost propagator at various T:



[Fischer, Maas, Müller, EPJC68 (2010)]



[preliminary]

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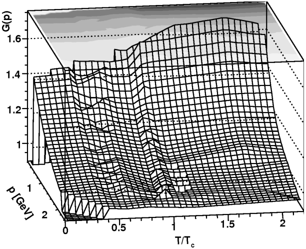

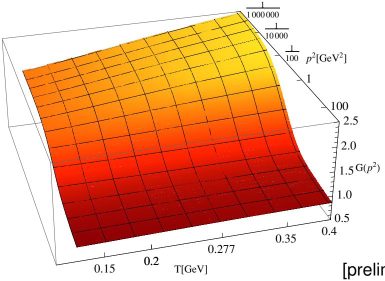

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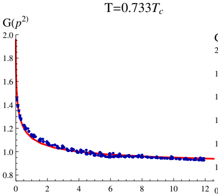

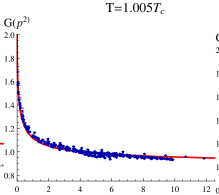

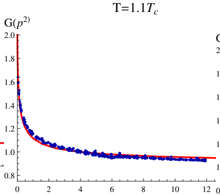

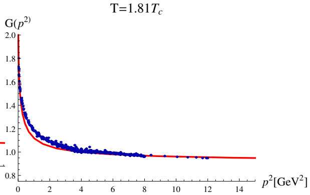

Lattice: [Maas, Pawlowski, Spielmann, von Smekal, PRD85 (2012)]

Ghost-gluon vertex

Simple approximation:

Fully iterated ghost propagator
Gluon propagator from the lattice
[Fischer, Maas, Müller, EPJC68 (2010)]

$\Bigg\}$ Ghost-gluon vertex semi-perturbatively at symmetric point ($p^2 = q^2 = k^2$)

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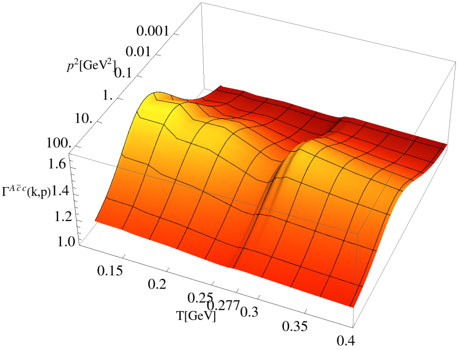

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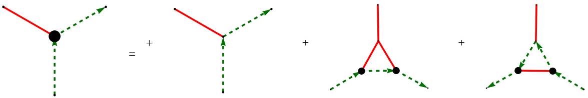
Thank you for your attention.

Ghost-gluon vertex

IR and UV consistent truncation:



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gluon and ghost propagators + ghost-gluon vertex

Only unfixed quantity in present truncation: three-gluon vertex.

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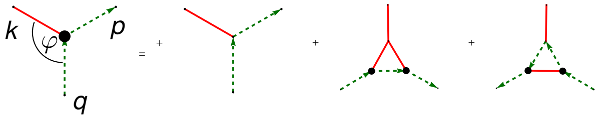

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$$\Gamma_\mu^{A\bar{c}c,abc}(k;p,q) := i\,g\,f^{abc}\left(p_\mu \mathbf{A(k;p,q)} + k_\mu B(k;p,q)\right)$$

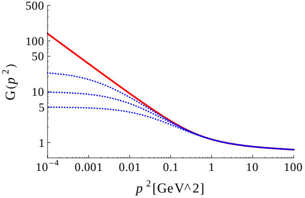
Note:
$B(k;p,q)$ is irrelevant in Landau gauge (but it is not the pure longitudinal part). Taylor argument applies only to longitudinal part (it's an STI).

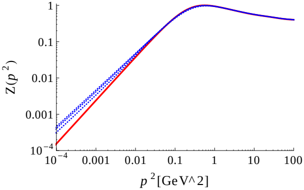
Solutions of functional equations: Decoupling and scaling

- Two types of solutions with functional methods that differ only in deep IR [Boucaud et al., JHEP 0806, 012; Fischer, Maas, Pawlowski, AP 324 (2009)]:
  scaling [von Smekal, Alkofer, Hauck PRL97],
  decoupling [Aguilar, Binosi, Papavassiliou PRD78; Fischer, Maas, Pawlowski, AP 324 (2009)]
- Lattice calculations find only decoupling type solution for $d = 3, 4$ and scaling for $d = 2$
- Decoupling emerges also from Refined Gribov-Zwanziger framework [Dudal, Sorella, Vandersickel, Verschelde, PRD77]

Decoupling and scaling solutions

DSEs: Vary ghost boundary condition [Fischer, Maas, Pawlowski, AP 324 (2009)]

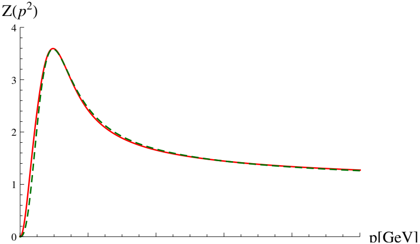


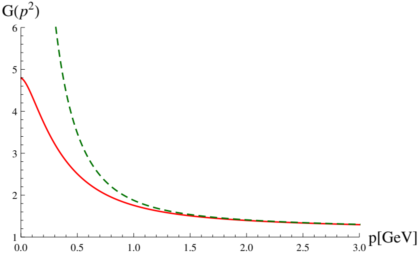


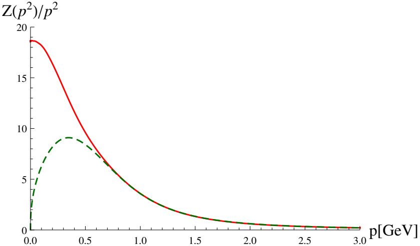
- Dependence of propagators on Gribov copies,
  e.g., [Bogolubsky, Burgio, Müller-Preussker, Mitrjushkin, PRD 74 (2006); Maas, PR 524 (2013)]
- Ideas:
  - [Sternbeck, Müller-Preussker, 1211.3057]: choose Gribov copies by lowest eigenvalue of the Faddeev-Popov operator $\rightarrow$ modification of both dressings
  - [Maas, PLB689 (2010)]: choose Gribov copies by value of ghost propagator

$d = 2$: Analytic and numerical arguments from DSEs for scaling only [Cucchieri, Dudal, Vandersickel, PRD85 (2012); MQH, Maas, von Smekal, JHEP11 (2012)] as well as from analysis of Gribov region [Zwanziger, PRD87].

Scaling solution: Propagators



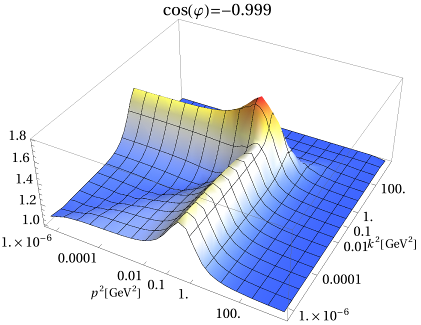




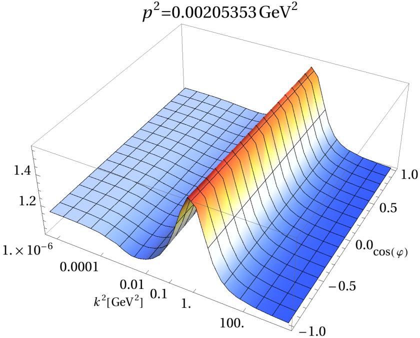
- Scaling solution
- Decoupling solution
- Differences only at low momenta.

Scaling solution: Ghost-gluon vertex

Fixed angle:



Fixed momentum:



- Dressing not 1 in the IR $\leftarrow$ Contributions from loop corrections (for decoupling they are suppressed)
- Scaling/decoupling also seen in ghost-gluon vertex