Walking signals in Nf=8 QCD on the lattice

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Lattice 2013, 29 July 2013 @ Mainz

LatKMI collaboration

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E. Rinaldi, K. Yamawaki, T. Yamazaki



A. Shibata

In LatKMI Collaboration;

♠ Lesson from SD about the mass correction in the (finite-size) hyperscaling relation.

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Plan of the Talk:

1. Introduction
2. Simulation setup in Nf=8 QCD
3. Result of Nf=8 case (walking?)
4. Summary

★ Phenomenological (Light scalar (0^{++}), S-param.)

$\Rightarrow$ Ohki, Rinaldi, Yamazaki and Aoki

1. Introduction

- LQCD with many fermions
- ⇒ Candidate of the (walking) technicolor

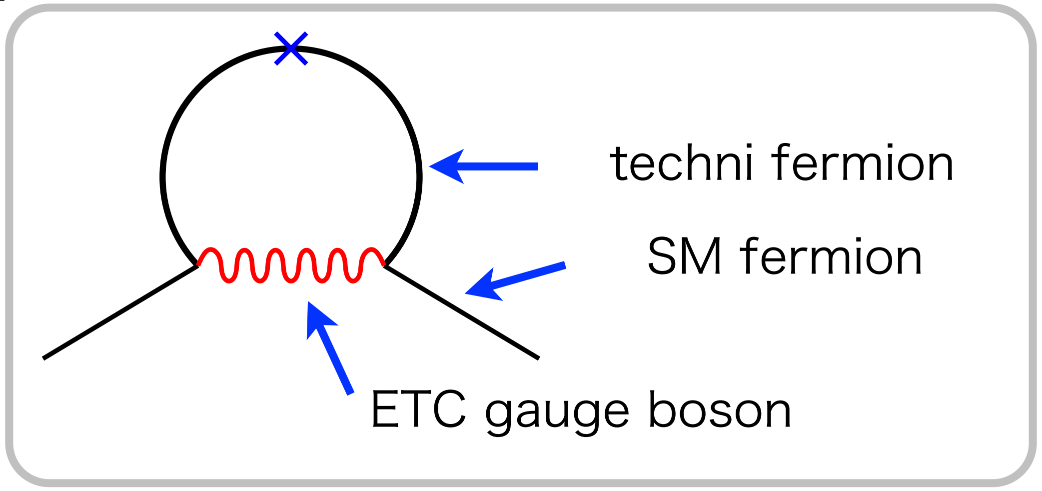
Requirements for the successful WTC theory

- spontaneous chiral symmetry breaking
- running coupling "walks" = slowly changing with μ: → nearly conformal
- large mass anomalous dimension: $\gamma_m \sim 1$
- light scalar 0^{++} (m_H = 126 GeV @ LHC !)
  - with input $F_\pi = 246 / \sqrt{N}$ GeV (N: # weak doublet in techni-sector)
    - to reproduce $W^\pm$ mass
  - typical QCD like theory: $M_{Had} >> F_\pi$ (ex.: QCD: $m_\rho/f_\pi \sim 8$)
    - Naive TC: M_{Had} > 1,000 GeV
    - 0^{++} is a special case: pseudo Nambu-Goldstone boson of scale inv.

    ➡ is it really so ?

Extended Technicolor (ETC)

- fermion masses → extended technicolor (ETC)
- New strong interaction of $SU(N_{ETC})$: $N_{ETC}>N_{TC}$, $\boxed{T_{ETC}=(\ T,\ f\ ):}$ $T\in TC$, $f\in SM$
- SSB: $SU(N_{ETC}) \to SU(N_{TC})$ x SM @ Λ_{ETC} ($\gg\Lambda_{TC}$)
- $$\frac{1}{\Lambda_{ETC}^2}\overline{T}T\overline{f}f \to m_f = \frac{\langle\overline{T}T\rangle_{ETC}}{\Lambda_{ETC}^2}$$
- $$\frac{1}{\Lambda_{ETC}^2}\overline{f}f\overline{f}f$$ FCNC



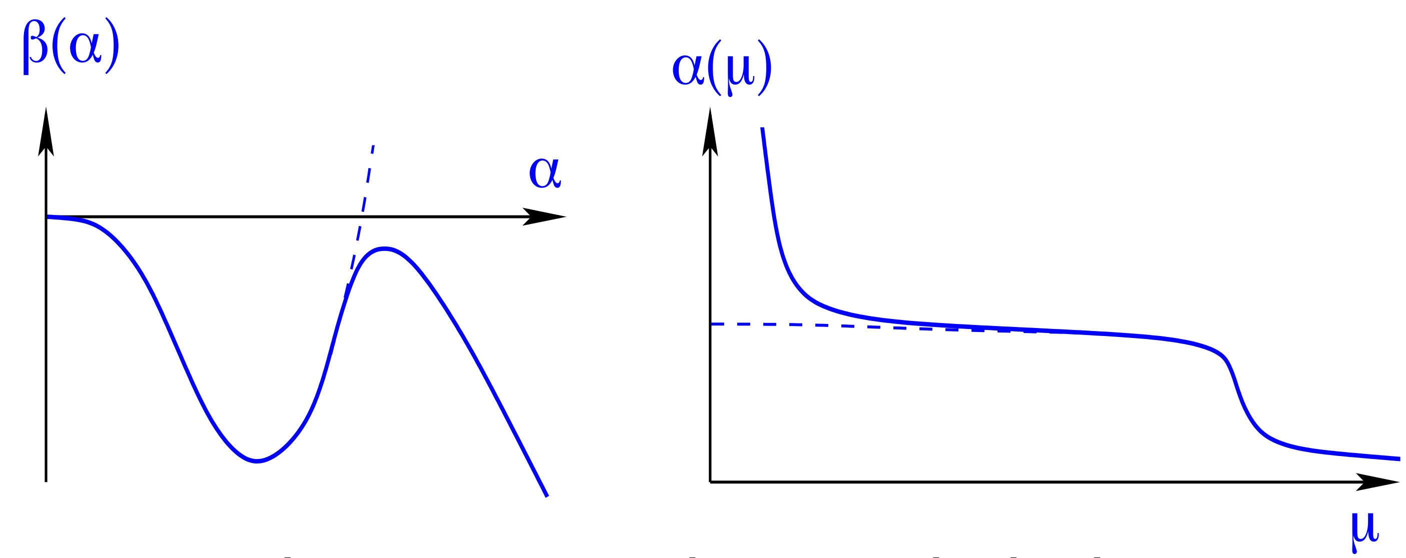
- FCNC should be small ⇔ top or bottom quark mass should be produced

➡ $\boxed{\text{walking TC}}$

Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Holdom, Yamawaki-Bando-Matsumoto]

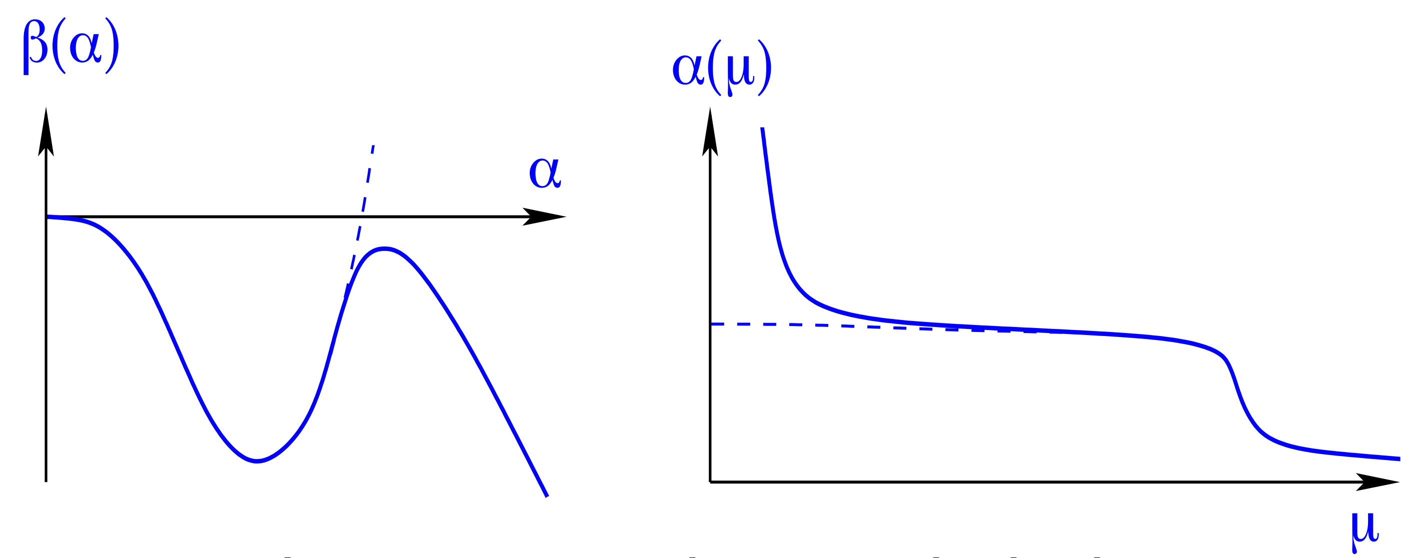


- renormalized gauge coupling
  - to run very slowly (walking)
  - logarithmically divergent at low energies → to produce techni-pions
- mass anomalous dimension
  - large: $\gamma_m \sim 1$

Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Holdom, Yamawaki-Bando-Matsumoto]

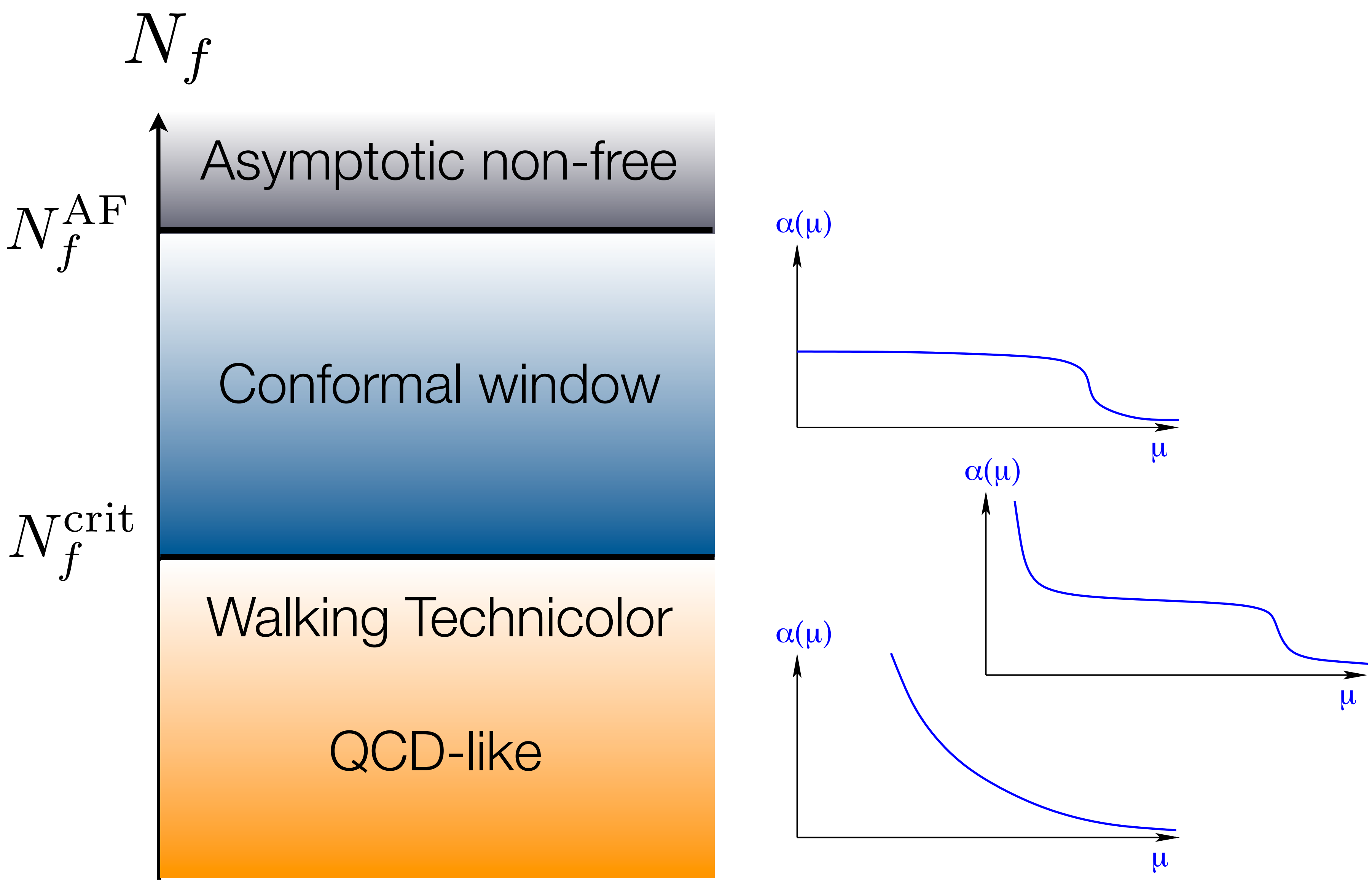


  - renormalized gauge coupling
    - to run very slowly (walking)
    - logarithmically divergent at low energies → to produce techni-pions
  - mass anomalous dimension
    - large: $\gamma_m \sim 1$

Is it possible to construct such a theory ?

conformal window and walking coupling

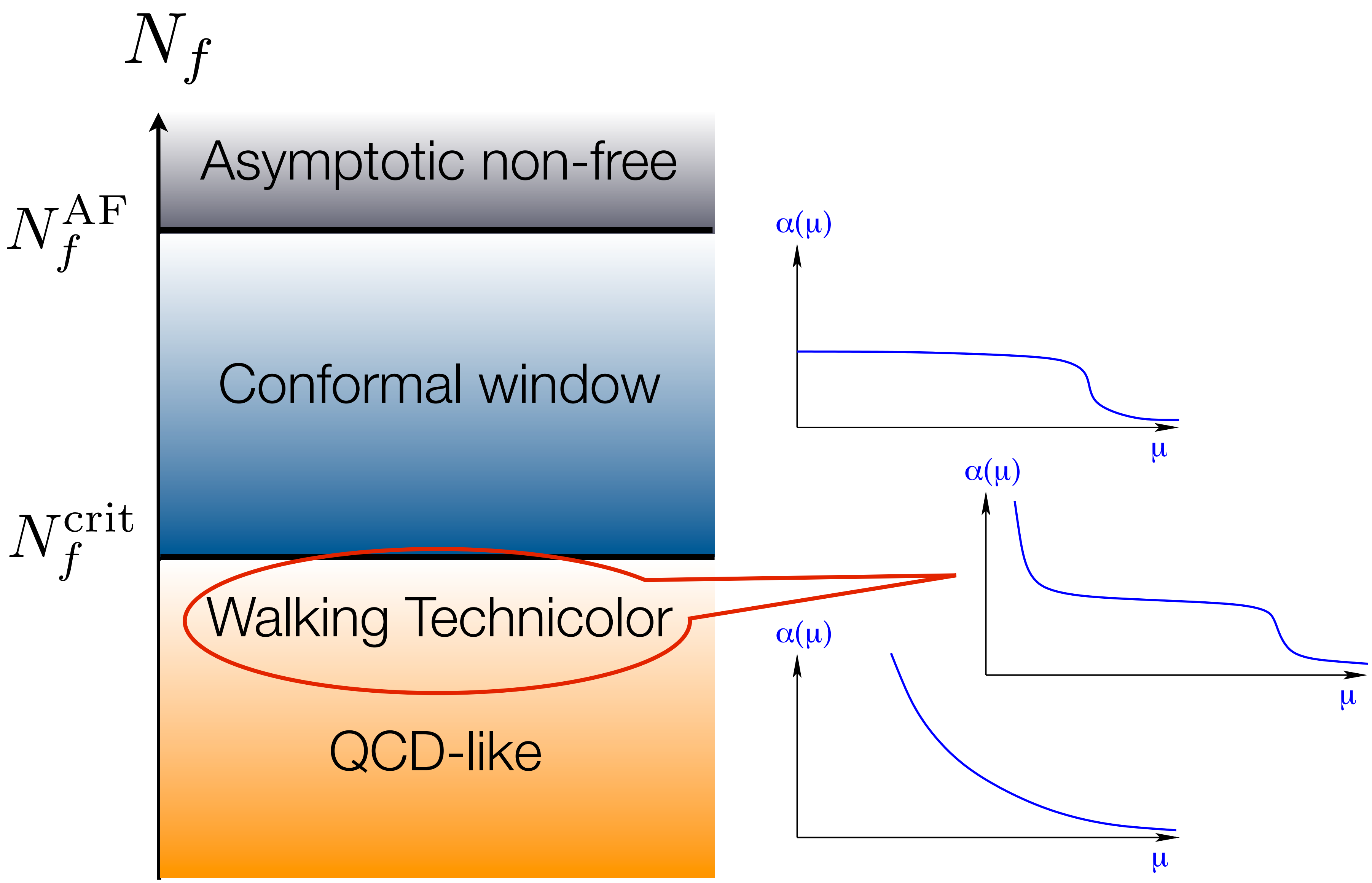
- non-Abelian gauge theory with N_f massless fermions -



- Walking Techinicolor could be realized just below the conformal window
- crucial information: N_f^{crit} & mass anomalous dimension around N_f^{crit}

conformal window and walking coupling

- non-Abelian gauge theory with N_f massless fermions -



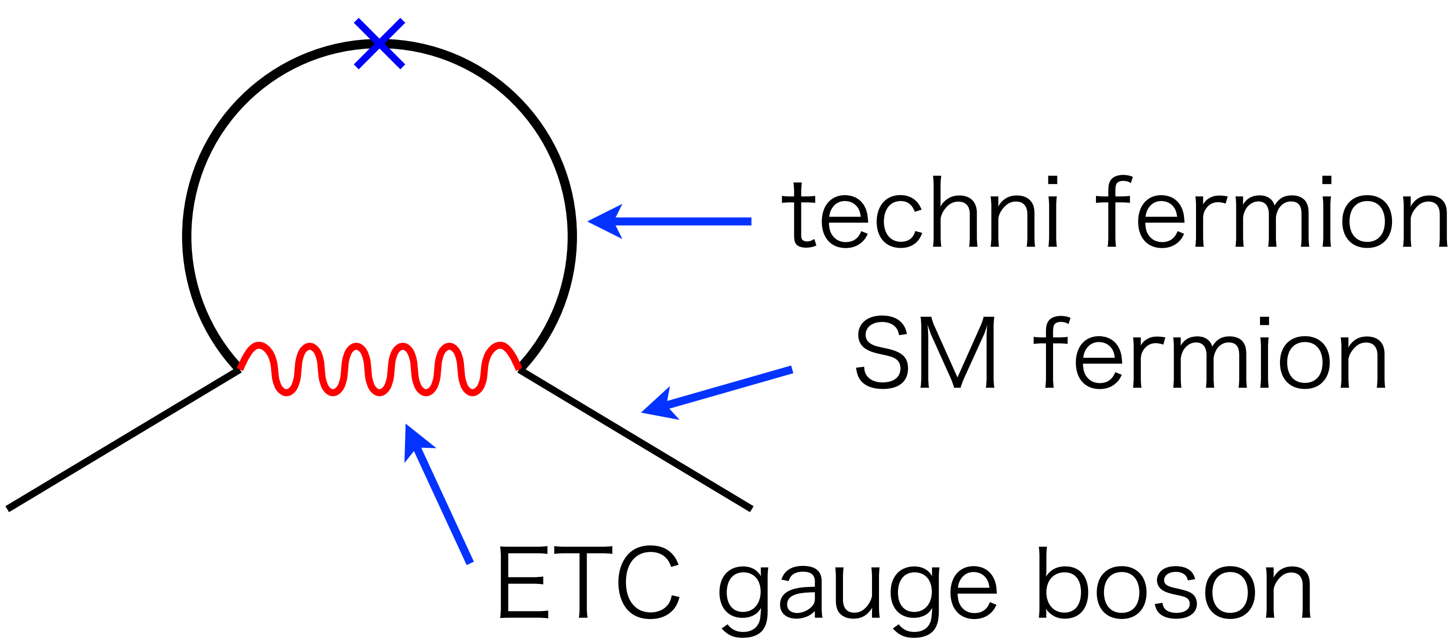
- Walking Techinicolor could be realized just below the conformal window
- crucial information: N_f^{crit} & mass anomalous dimension around N_f^{crit}

One-family Extended TechniColor model

$$
\begin{pmatrix} u \\ d \\ u \\ d \\ u \\ d \\ e \\ \nu_e \end{pmatrix}
\begin{pmatrix} c \\ s \\ c \\ s \\ c \\ s \\ \mu \\ \nu_\mu \end{pmatrix}
\begin{pmatrix} t \\ b \\ t \\ b \\ t \\ b \\ \tau \\ \nu_\tau \end{pmatrix}
\begin{pmatrix} U_1 \\ D_1 \\ U_1 \\ D_1 \\ U_1 \\ D_1 \\ E_1 \\ N_1 \end{pmatrix}
\cdots
\begin{pmatrix} U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ U_{N_{\rm TC}} \\ D_{N_{\rm TC}} \\ E_{N_{\rm TC}} \\ N_{N_{\rm TC}} \end{pmatrix}
$$

$$SU(3+N_{\rm TC}) \xrightarrow{\Lambda_1} SU(2+N_{\rm TC}) \xrightarrow{\Lambda_2} SU(1+N_{\rm TC}) \xrightarrow{\Lambda_3} SU(N_{\rm TC})$$

8-flavor $\mathrm{SU}(N_{\rm TC})$ technicolor



We consider $N_{\rm TC} = 3$

Many flavor QCD

$\Rightarrow$ Candidate of walking/conformal

Our investigation in Nf=12

$\rightarrow$ consistent with the conformal with γ=0.4--0.5 .

not favor as WTC (model building)

Thus, we investigate Nf=8 QCD.

strong coupling dynamics and non-perturbative

Lattice simulation of Nf=8 QCD

Investigation of Nf=8 (direct and indirect):

LHC (J. Kuti et.al.) → SχSB

A. Hasenfratz et.al. → If Nf=8 would be walking, ...
In comparison with Nf=4 and 12.

M. Lombardo et.al. → broken, but near the conformal edge

Y. Iwasaki et.al. → conformal
(Yukawa-type correlator in Nf=7 and 16)

LatKMI → In this talk (walking?)

2. Simulation setup in Nf=8 case

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Simulation for Nf=8

lattice action

(8 flavor Hybrid Monte-Carlo simulation)

- Tree-level Symanzik gauge action
- Highly Improved Staggered Quarks = **HISQ**

(without tadpole improvement and mass correction in Naik term)

★ parameter set

- $\beta\,(\equiv 6/g^2) =$ **3.8**, (3.7, 3.9 4.0) , T/L=4/3 fixed.

V	12^3 x 16	18^3 x 24	24^3 x 32	30^3 x 40	36^3 x 48
mf	0.04~0.16	0.04~0.1	0.02~0.1	0.02~0.07	0.015~0.03

★ Measurements (P+AP method ⇒ double size in T-dir.)

- M_π, F_π, M_ρ, chiral condensate
- $M_\pi L$ > 4--5

HISQ with Nf=8:

effective mass for the lowest mf(=0.015)
on the largest size(L=36)
→ good plateau

$M_\pi{}^{PS}=M_\pi{}^{SC}$, $M_\rho{}^{PV}=M_\rho{}^{VT}$
→ good flavor symmetry

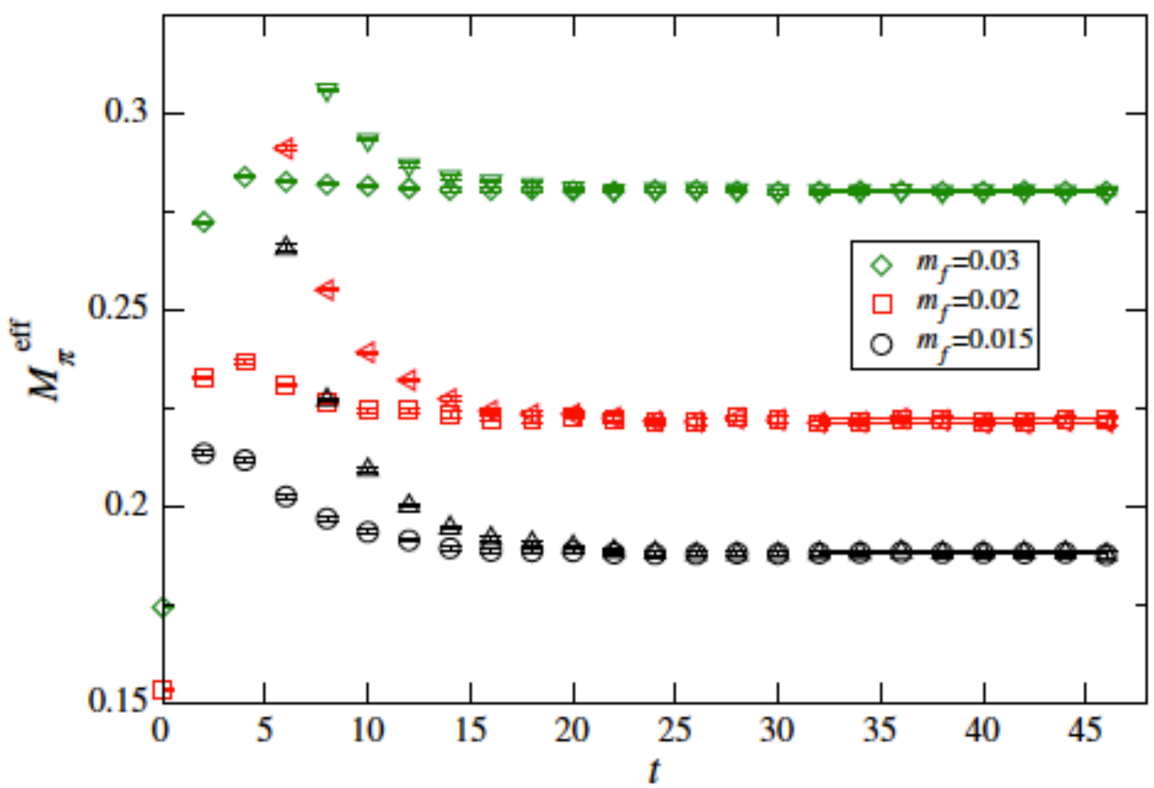


FIG. 2 (color online). Effective masses of PS meson, $M_\pi^{\rm eff}$, at $L = 36$. Triangles and other symbols denote results from point sink correlators with random wall source and corner wall source, respectively. Fit results with error band obtained from random wall source correlator are also plotted by solid lines.

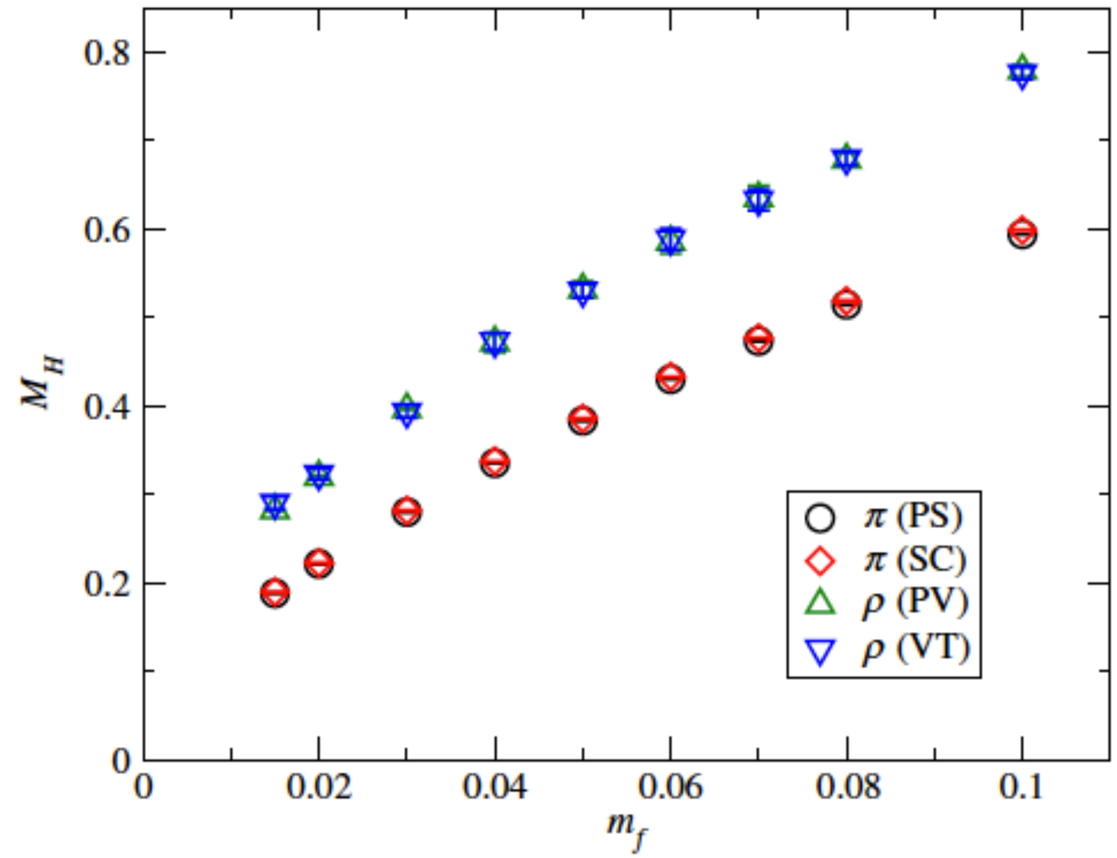


FIG. 4 (color online). Comparisons of M_π and $M_{\rm SC}$, and of $M_{\rho({\rm PV})}$ and $M_{\rho({\rm VT})}$ as a function of m_f with largest volume data at each m_f.

3. Result of Nf=8 case

F_π/M_π for Nf=8, 12 and 4 (flat or divergent in χ-limit?)

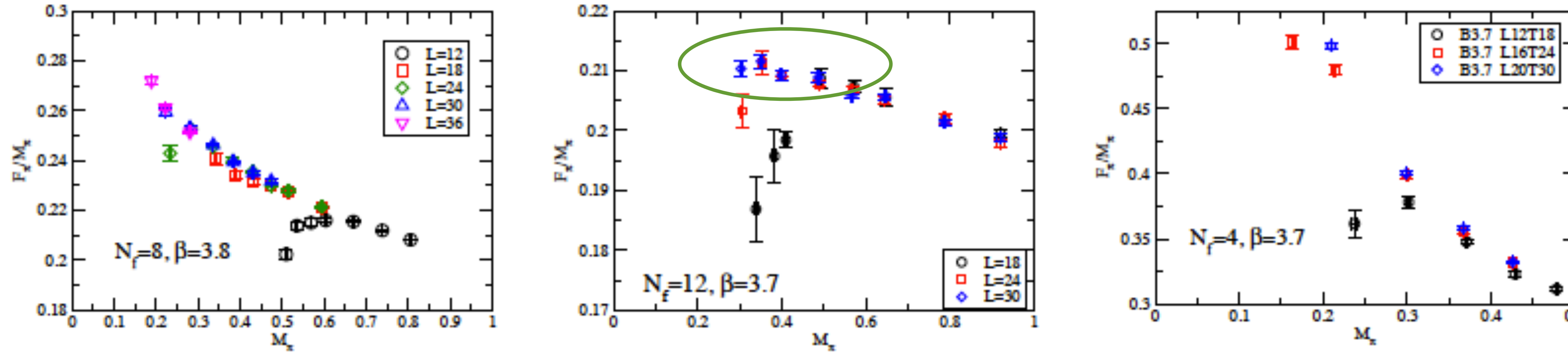


FIG. 5. F_π/M_π as a function of M_π for $N_f = 8$ (left), $N_f = 12$ at $\beta = 3.7$ (center), and $N_f = 4$ at $\beta = 3.7$ (right).

M_ρ/M_π for Nf=8, 12 and 4 (flat or divergent in χ-limit?)

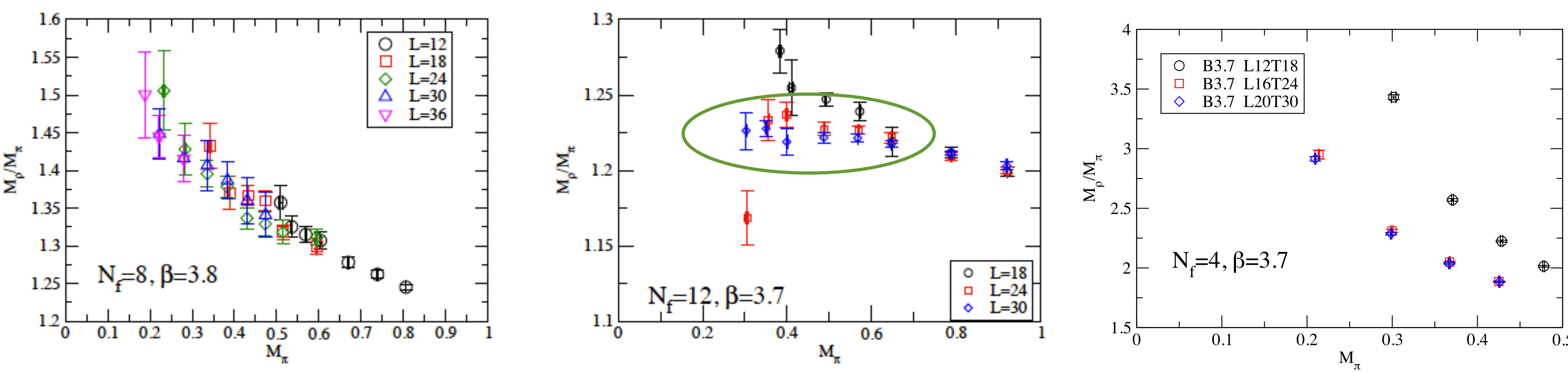


FIG. 6. M_ρ/M_π as a function of M_π for $N_f = 8$ (left) and $N_f = 12$ at $\beta = 3.7$ (right).

Nf=8 ⇒ spontaneous chiral symmetry breaking?

(SχSB)

Chiral Perturbation Theory test (ChPT)

In SχSB; $M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \cdots, \quad F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \cdots,$

⇒ Polynomial fit

We regard the data on the largest volume at each mf
as the ones on the infinite volume. (Backup figs.)

We don't discuss the chiral log behavior in this talk.

However, we discussed the chiral log in the published paper.

ChPT analysis (quadratic fit) of F_π

Chiral limit value, F, is zero or non-zero?

Natural chiral expansion parameter

$$\boxed{\chi = N_f \left(\frac{M_\pi}{4\pi F} \right)^2}$$

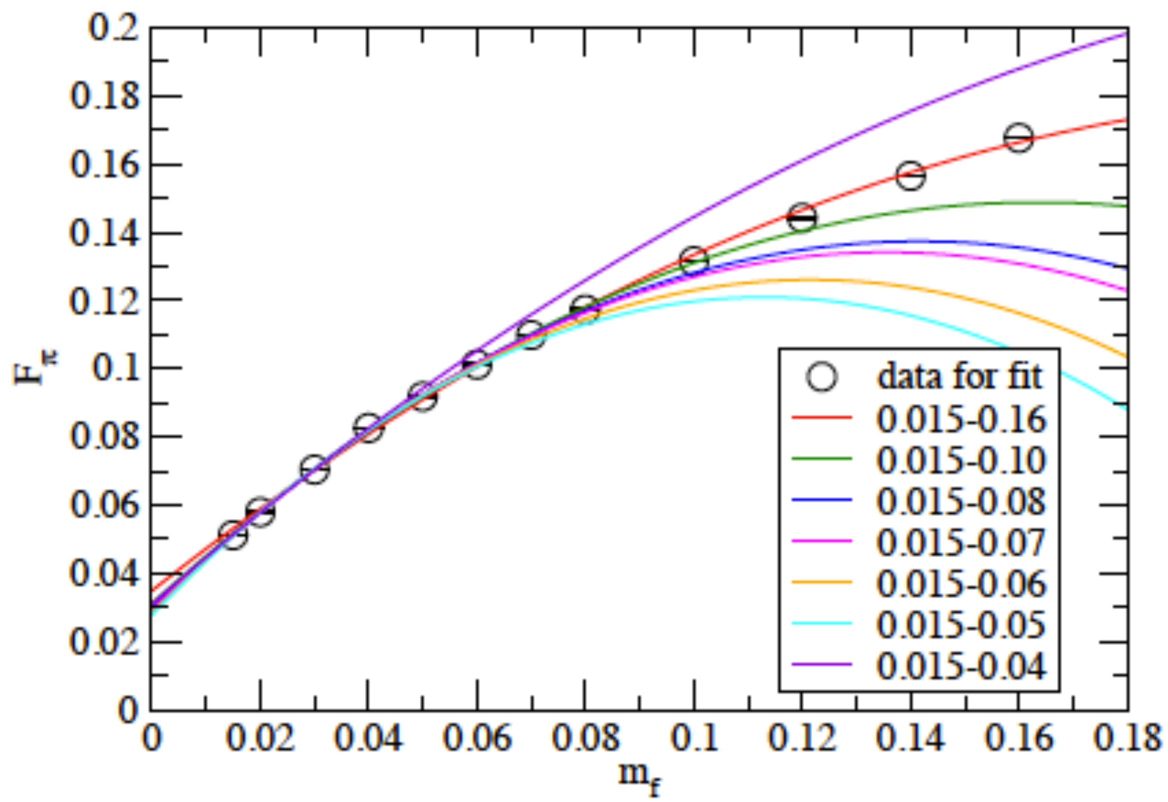


TABLE I. Results of chial fit of F_π with $F_\pi = F + C_1 m_f + C_2 m_f^2$ for various fit ranges.

fit range (m_f)	F	$\mathcal{X}(m_f^{\min} = 0.015)$	$\mathcal{X}(m_f = m_{\max})$	χ^2/dof	dof
0.015–0.04	0.0310(13)	3.74	11.80	0.46	1
0.015–0.05	0.0278(8)	4.64	19.28	5.56	2
0.015–0.06	0.0284(6)	4.44	23.2	4.09	3
0.015–0.07	0.0293(5)	4.18	26.5	4.46	4
0.015–0.08	0.0296(4)	4.10	30.6	4.06	5
0.015–0.10	0.0311(3)	3.70	37.0	7.85	6
0.015–0.16	0.0349((2)	2.94	54.0	34.2	9

The above analysis suggests that our result in $N_f = 8$ is consistent with SχSB phase with $F = 0.0310(13)$.

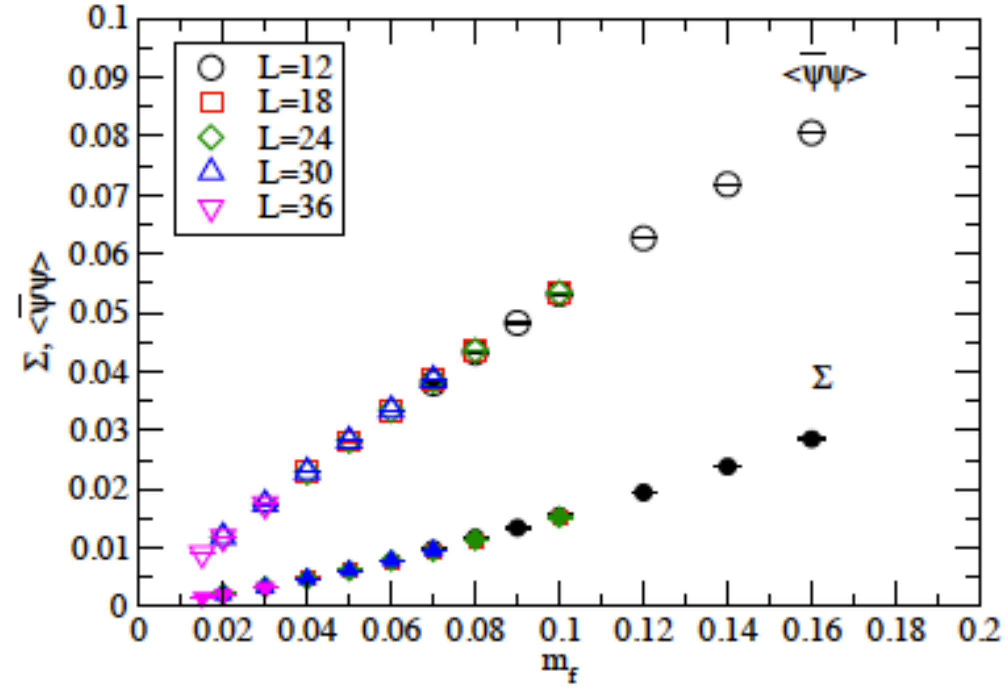
The chiral log is not seen in our simulation region of mf.

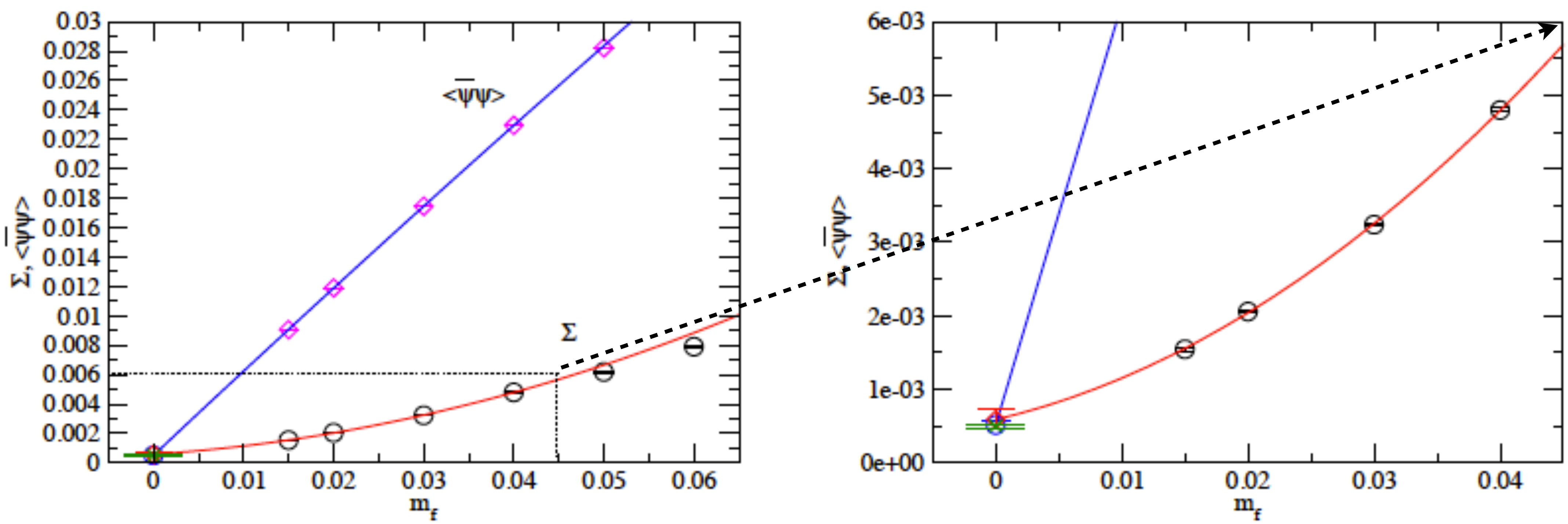
Chiral condensate (direct and indirect calc.)

direct: = Tr[D_{HISQ}^{-1}(x,x)]/4

indirect: $\Sigma = F_\pi^2 M_\pi^2/(4mf)$

based on the GMOR relation





In chiral limit (quadratic fit in 0.015≦mf≦0.04)

$$\langle\bar{\psi}\psi\rangle\big|_{m_f\to 0} = 0.00052(5), \quad \Sigma\big|_{m_f\to 0} = 0.00059(13).$$

$$F^2 \cdot \left(\frac{M_\pi^2}{4m_f}\right)\bigg|_{m_f\to 0} = 0.00050(3)$$

Summary-1, ChPT analysis

- The quadratic fit was done in $0.015 \leqq mf \leqq 0.04$.
- $F_{\pi}>0$, $M_{\rho}>0$, Condensate>0, $M_{\pi}=0$ in the χ-limit.
- Nf=8 is consistent with SχSB in the small mf region.
- The expansion parameter $\chi=O(1)$ in the smallest mf. (self-consistent)
- The chiral log is not seen.
- ⇒ simple SχSB phase?

Scenario of Walking Dynamics (2-loop/ladder);

Case-1: probe mf << m_D → SχSB-like

Case-2: probe mf >> m_D → conformal-like

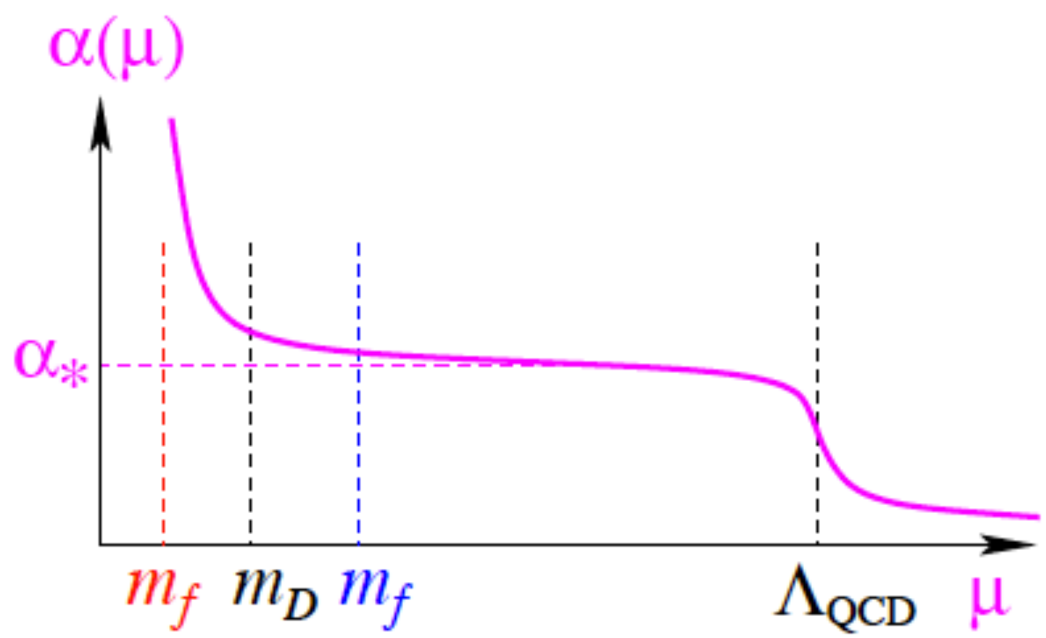
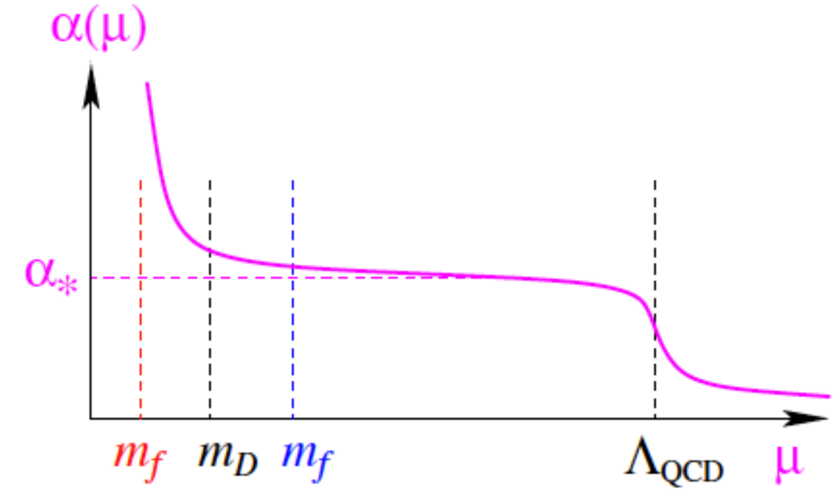


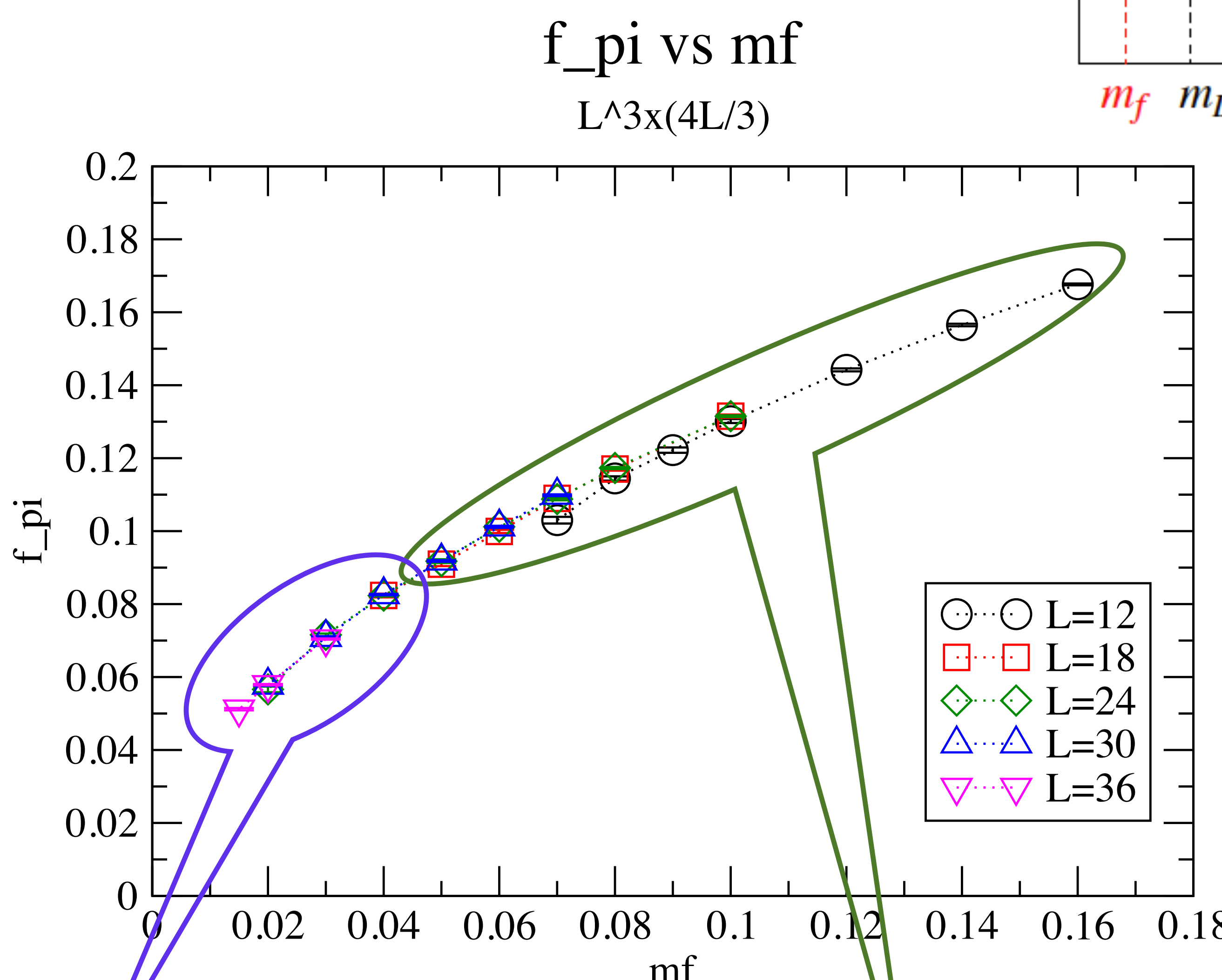
FIG. 1. Schematic two-loop/ladder picture of the gauge coupling of the massless large N_f QCD as a walking gauge theory in the SχSB phase near the conformal window. m_D is the dynamical mass of the fermion generated by the SχSB. The effects of the bare mass of the fermion m_f would be qualitatively different depending on the cases: Case 1: $m_f \ll m_D$ (red dotted line) well described by ChPT, and Case 2: $m_f \gg m_D$ (blue dotted line) well described by the hyper scaling.

How to translate into

⇒ $\boxed{\text{Spectrum?}}$ SχSB and conformal ?

F_π vs mf :





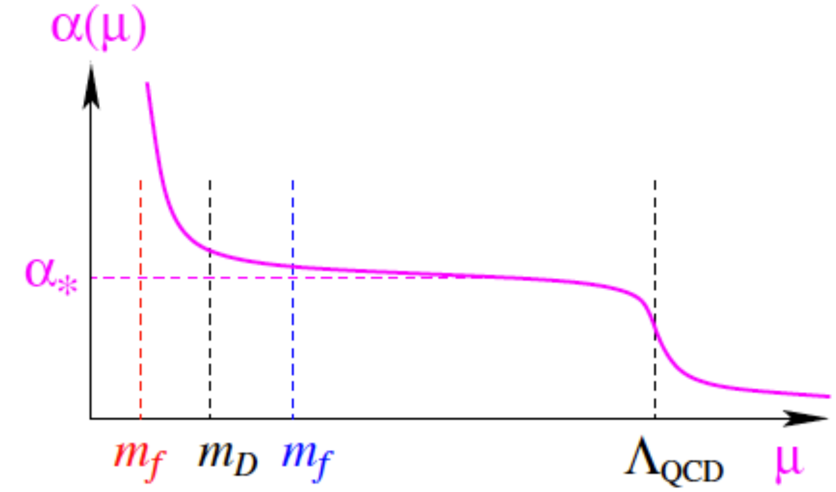
Polynomial-like behavior

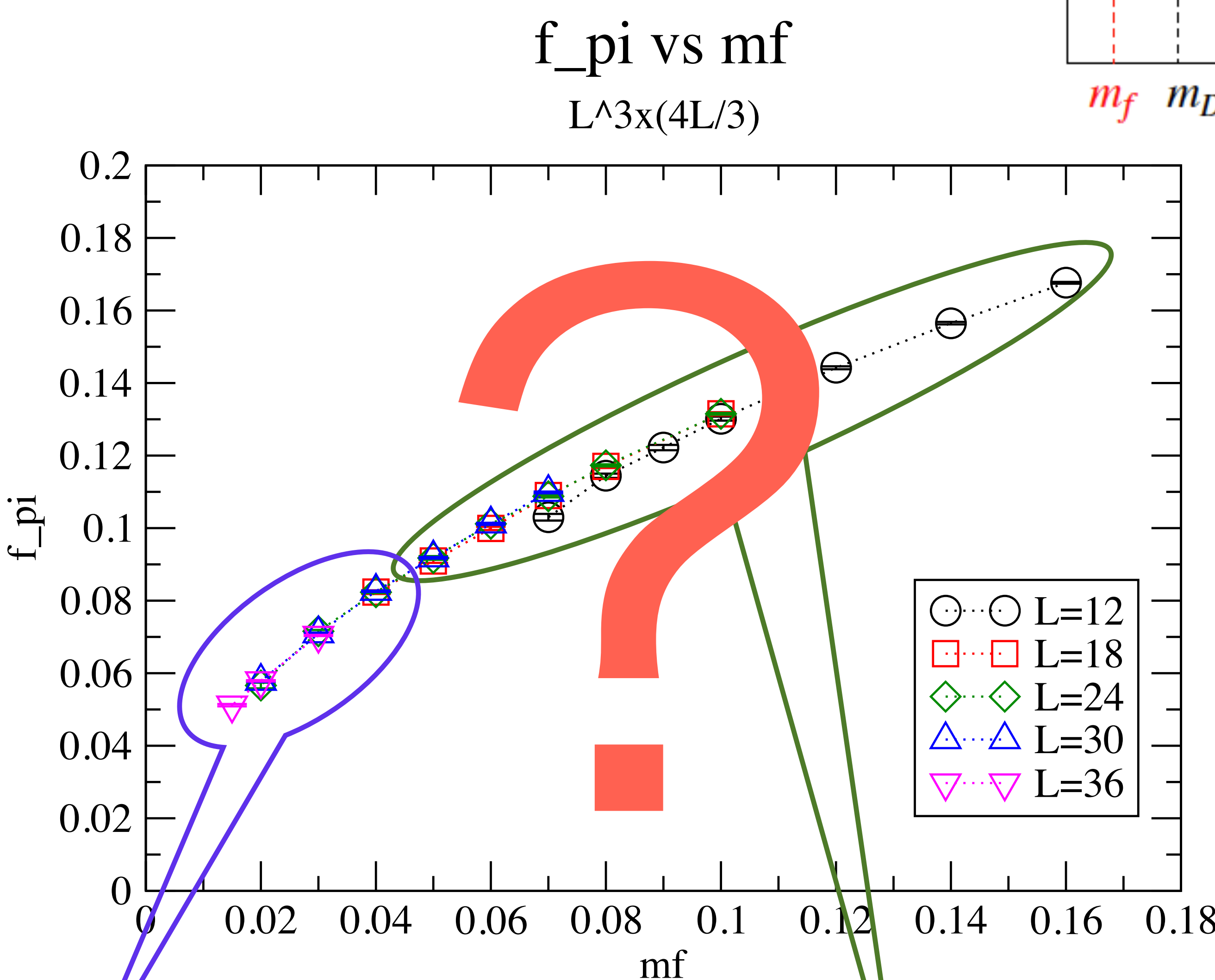
(ChPT-like)

Power-like behavior ?

(Remnant (Impact) of conformality ?)

F_π vs mf :





Polynomial-like behavior

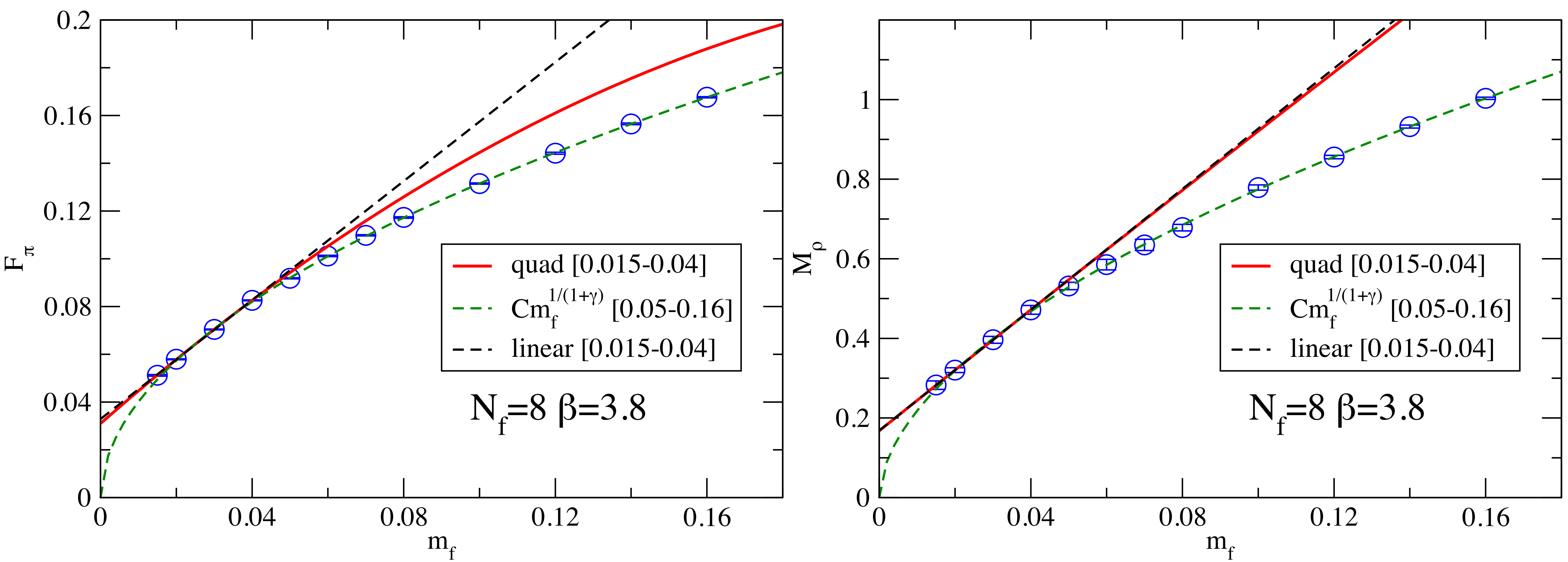
(ChPT-like)

Power-like behavior ?

(Remnant (Impact) of conformality ?)

Fπ (left panel) and Mρ (right panel):

quadratic and linear fit in 0.015 ≦ mf ≦ 0.04
power fit in 0.05 ≦ mf ≦ 0.16 (green dotted line)



To confirm the conformal(-like) behavior,

⇒ Finite-size Hyperscaling analysis

Finite size Hyperscaling analysis

(conformal test)

Finite size Hyperscaling analysis

(conformal test)

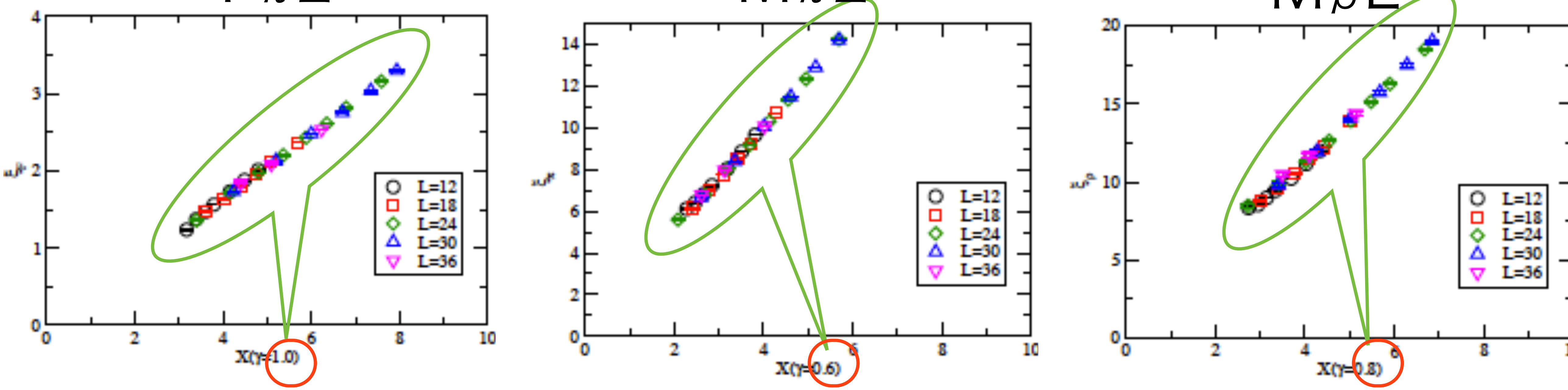
Finite size Hyperscaling behavior with universal γ

$$\boxed{LM_H = \mathcal{F}_H(x), LF_H = \mathcal{G}_F(x)}$$

$$x \equiv L\, m^{1/1+\gamma}$$

(DeGrand, Del Debbio et al.)

$F_\pi L$ $M_\pi L$ $M_\rho L$

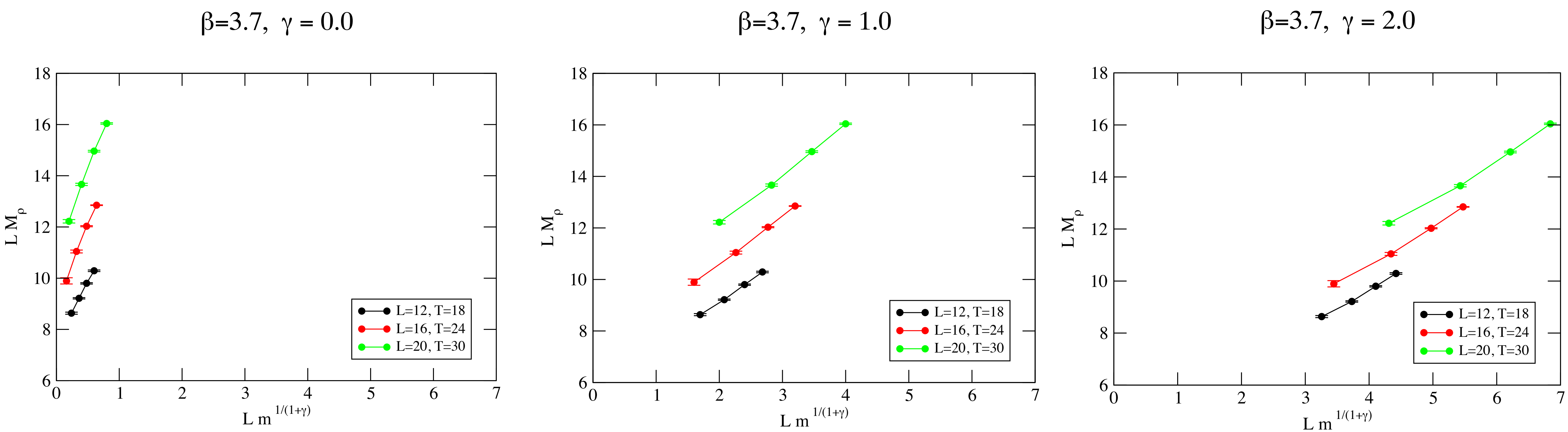


$\gamma(M_\pi) \neq \gamma(F_\pi)$, $\gamma(F_\pi)$~1.0

not conformal;

Nf=8 may be in SχSB phase.

Comparison with Nf=4
: hyperscaling of $M_\rho L$ (in SχSB)



M_ρ : no scaling for $0 < \gamma < 2$

⇒ In this sense,

it seems that Nf=8 is not SχSB of ordinary QCD.

(data scaling, but not universal γ)

Simultaneous fit of hyperscaling with mass corrections

$\xi_H = C_0^H + C_1^H X + C_2^H L m_f^\alpha$. (same method with Nf=12)

Hyperscaling? in the middle region of mf (mf≧0.05 and ξ_π (=MπL) ≧8)

The mass corrections might be needed, as done in Nf=12, from the lesson in SD analysis.

TABLE XI. Simultaneous FSHS fit with a correction term, $\xi = C_0^H + C_1^H X + C_2^H L m_f^\alpha$ using several choices of α. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$.

$\alpha = 0.889(55)$	C_0^H	C_1^H	C_2^H
ξ_π	$-0.005(25)$	1.338(96)	1.494(37)
ξ_F	$-0.0275(98)$	0.4435(36)	—
ξ_ρ	0.53(16)	2.476(39)	—
	$\gamma = 0.9130(76)$, $\chi^2/\text{dof} = 1.73$, dof $= 33$		

$\alpha = 1$ fixed	C_0^H	C_1^H	C_2^H
ξ_π	$-0.014(24)$	1.61(10)	1.31(15)
ξ_F	$-0.012(10)$	0.484(30)	$-0.068(44)$
ξ_ρ	0.01(19)	2.60(17)	0.25(24)
	$\gamma = 0.874(25)$, $\chi^2/\text{dof} = 0.75$, dof $= 32$		

$\alpha = \frac{3-2\gamma}{1+\gamma}$ fixed	C_0^H	C_1^H	C_2^H
ξ_π	0.020(24)	1.52(39)	1.17(35)
ξ_F	$-0.011(10)$	0.572(34)	$-0.158(52)$
ξ_ρ	0.03(19)	2.91(30)	$-0.15(36)$
	$\gamma = 0.775(56)$, $\chi^2/\text{dof} = 0.93$, dof $= 32$		

⇒ good χ^2/dof, but unclear which α is better.

Summary-2, Finite-size Hyperscaling analysis

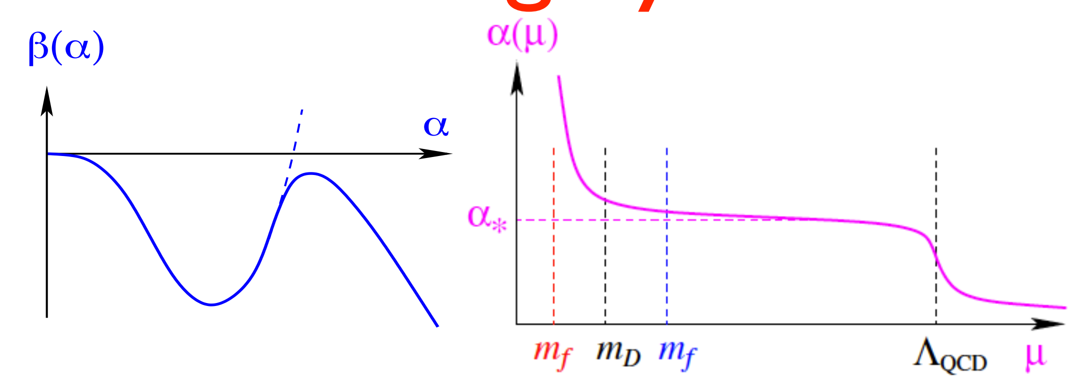
- In the region of mf≧0.05, hyperscaling is seen. (Remnant ("Impact") of conformality?)
- γ for each observable in the finite-size hyperscaling are not universal.
- Mass correction for mf≧0.05 (heavier region)? ⇐ Lesson from the SD analysis with the (large) mass.
- Simultaneous fit of hyperscaling with mass correction terms gives the common γ=0.78--0.93 ～1.
- It suggests that in Nf=8 there may exists the "remnant" of the conformality in the middle range of mf.

Summary

◆ SU(3) gauge theories with 4, 12 and 8 HISQ quarks.

◆ Nf=8; consistent with SχSB in the small mass region of our simulation and the remnant of the conformality in the middle region of mf with γ~1. (In contrast to Nf=4 and 12 cases.)

Nf=8 → Candidate of Walking dynamics



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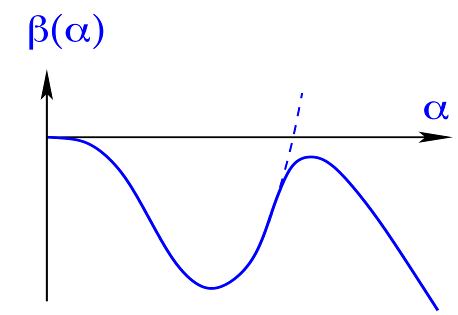
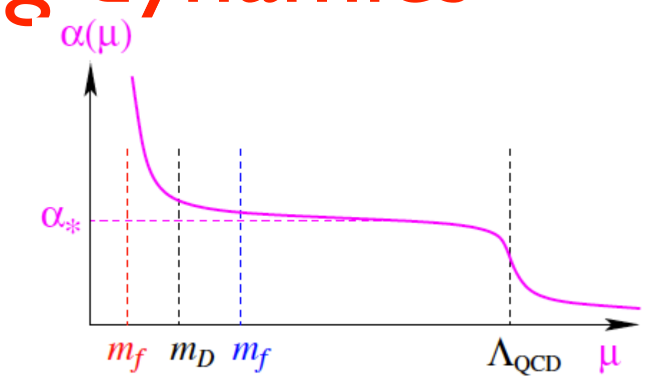




Future plan

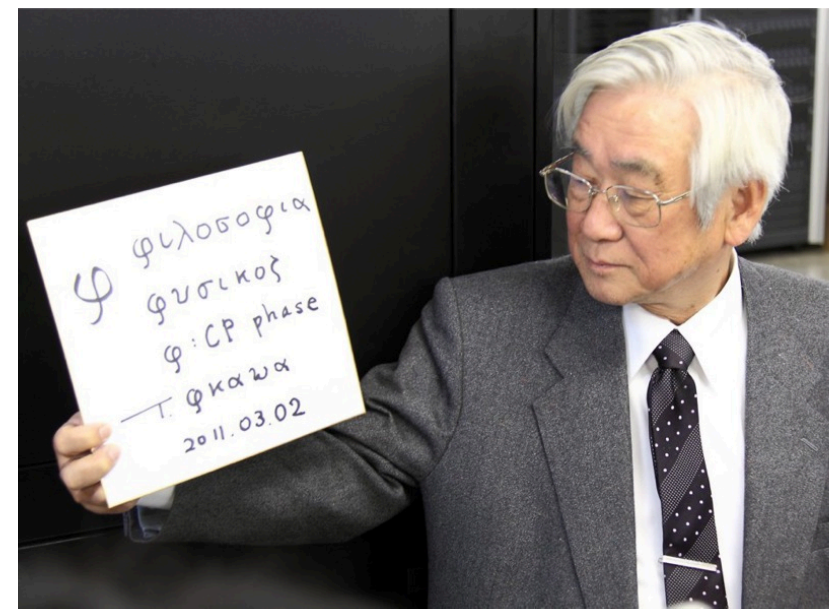
- Simulation on larger volumes at lighter masses (mf→0)
- Finite Size Scaling (due to the difficulty to take V=∞)
- Lattice spacing dependence (Enhancement) ←many β
- Spectroscopy ($M_{glueball}$, $M_{\text{"dilaton"}}$, M_{baryon}, M_{meson}, $F_{\rho/\sigma}$, S-param. etc.)
- $M_{scalar(singlet)}$, $M_{glueball}$, M_H ⇒ 126GeV? (O(100GeV))

- → LatKMI (Ohki, Rinaldi, Yamazaki, Aoki), Go!

Backup

KMI computer, φ

- non GPU nodes
  - 148 nodes
  - 2x Xenon 3.3 GHz
  - 24 TFlops (peak)
- GPU nodes
  - 23 nodes
  - 3x Tesla M2050
  - 39 TFlops (peak)



Spectroscopy (raw data) at β=3.8

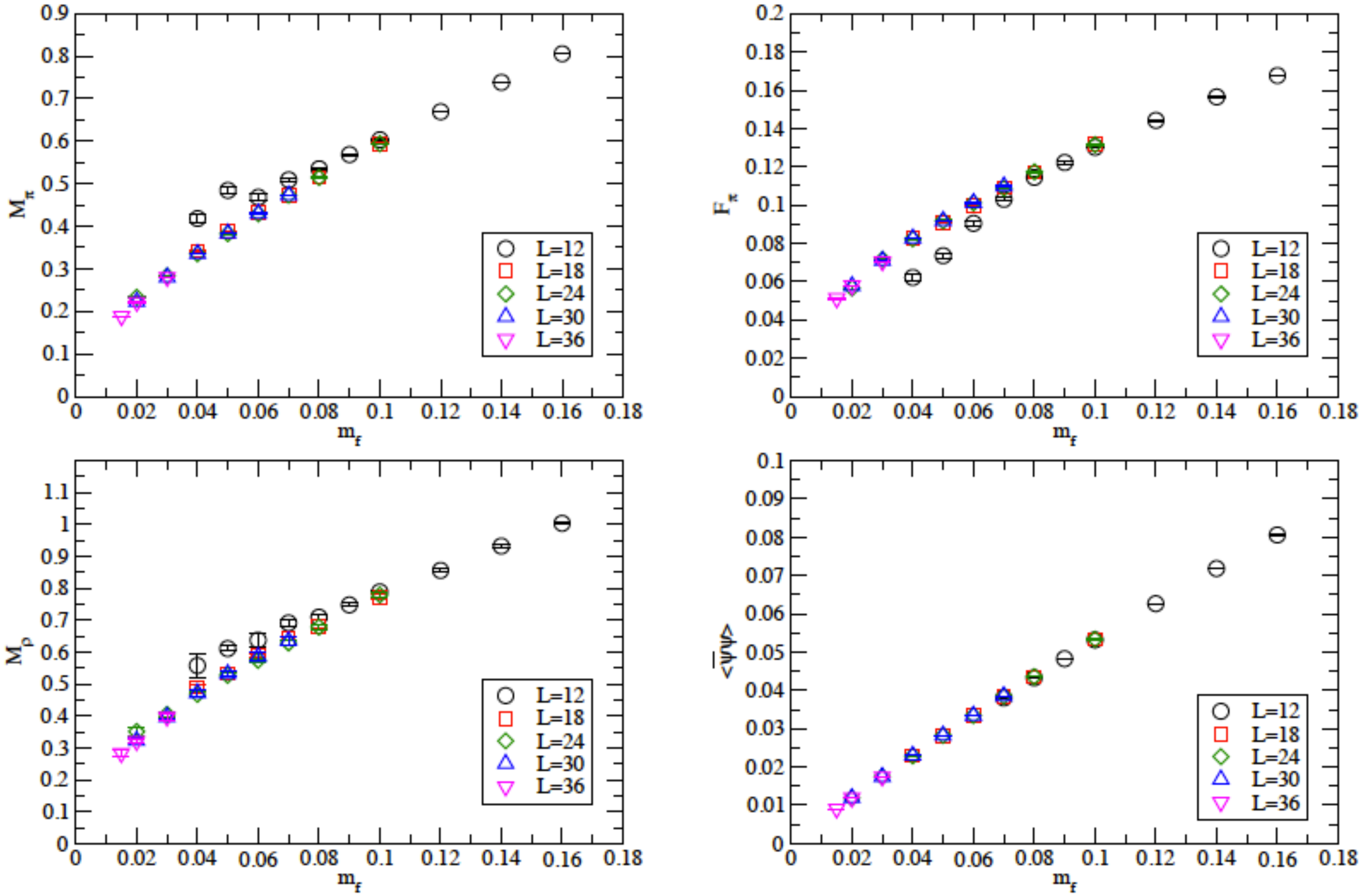


FIG. 3. Raw data of observables as a function of m_f for M_π (top left), F_π (top right), M_ρ (bottom left) and $\langle\bar{\psi}\psi\rangle$ (bottom right).

Size dependence of Mπ and Fπ at β=3.8

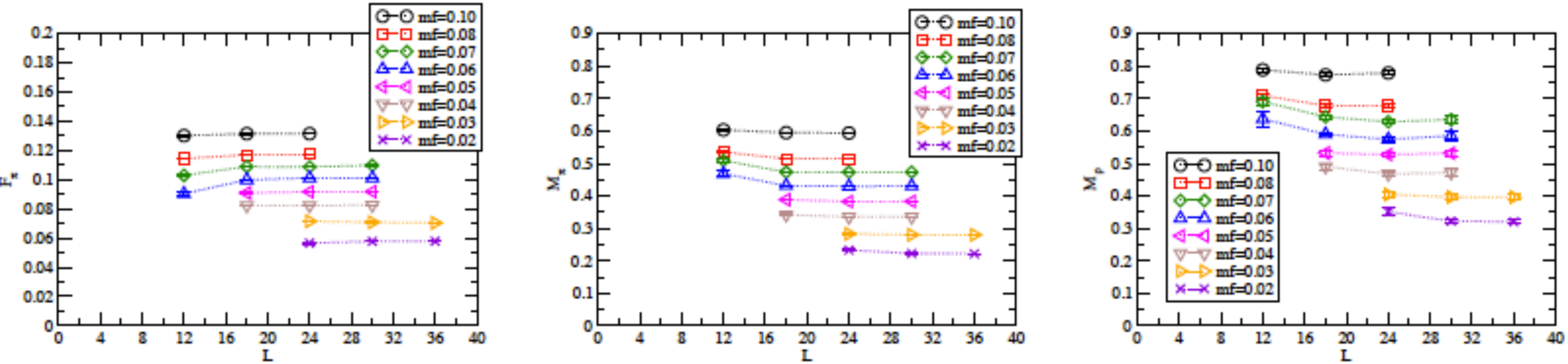


FIG. 7. F_π (left), M_π (center) and M_ρ (right) as functions of L.

On the larger volume, there is not (or very tiny) size dependence.

We use the data on the largest volume at each mf.

M_ρ vs mf

M_π^2 vs mf

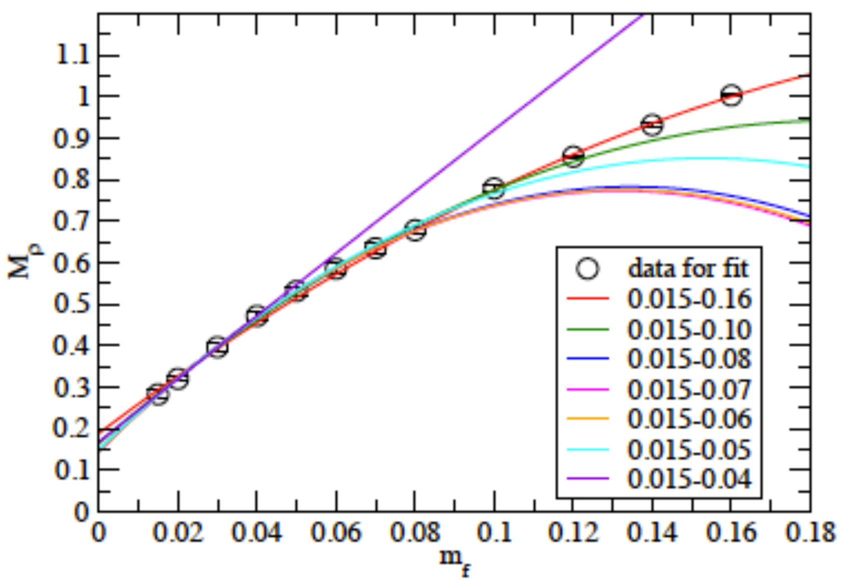


FIG. 9. Results of quadratic fit of M_ρ for various fit ranges.

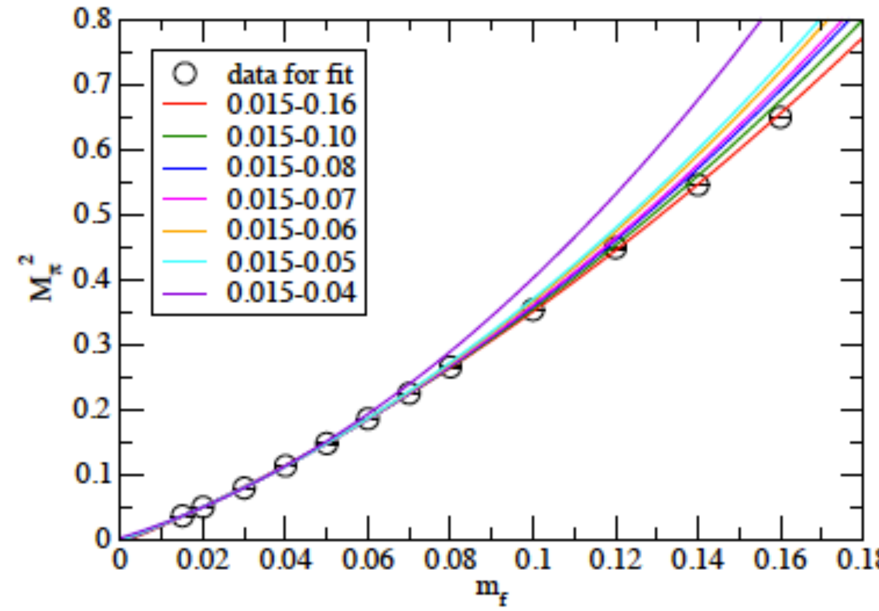


FIG. 11. Quadratic fit of M_π^2 for various fit ranges.

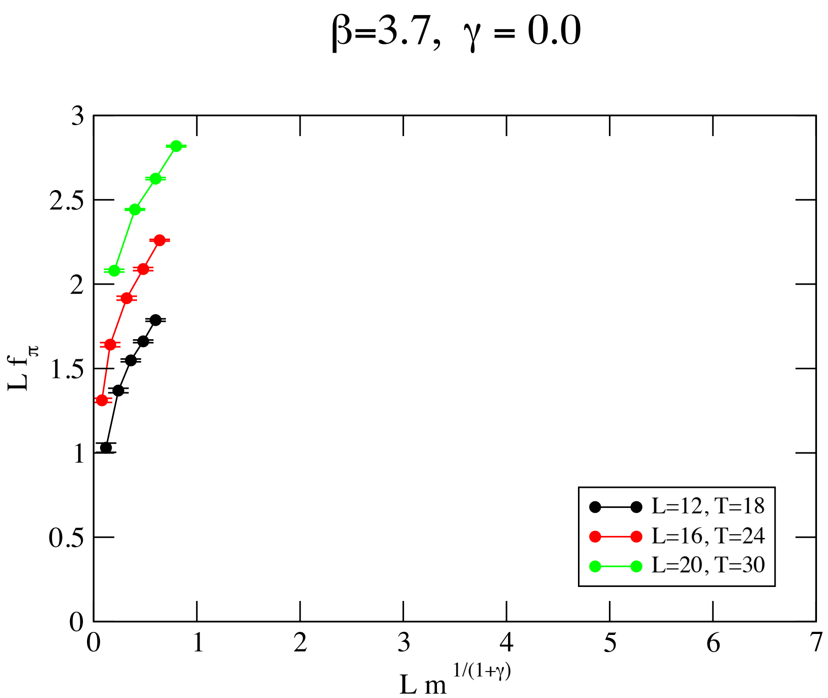
TABLE II. Chiral fit of M_ρ with $M_\rho = C_0^\rho + C_1^\rho m_f + C_2^\rho m_f^2$ for various fit ran

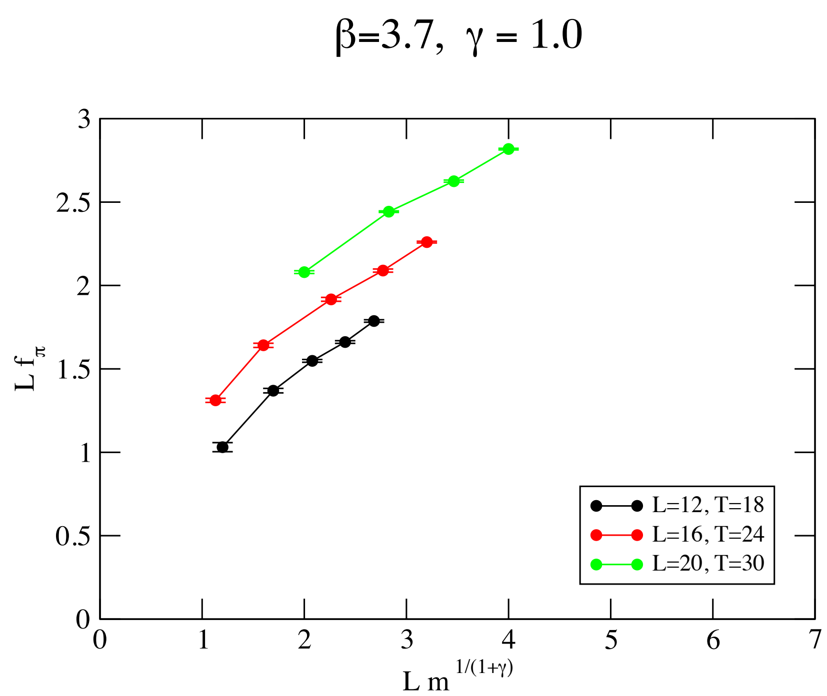
fit range (m_f)	C_0^ρ	χ^2/dof	dof
0.015–0.04	0.168(32)	0.0017	1
0.015–0.05	0.149(33)	0.098	2
0.015–0.06	0.145(25)	0.084	3
0.015–0.07	0.144(20)	0.063	4
0.015–0.08	0.146(16)	0.052	5
0.015–0.10	0.164(12)	0.57	6
0.015–0.16	0.189(7)	1.48	9

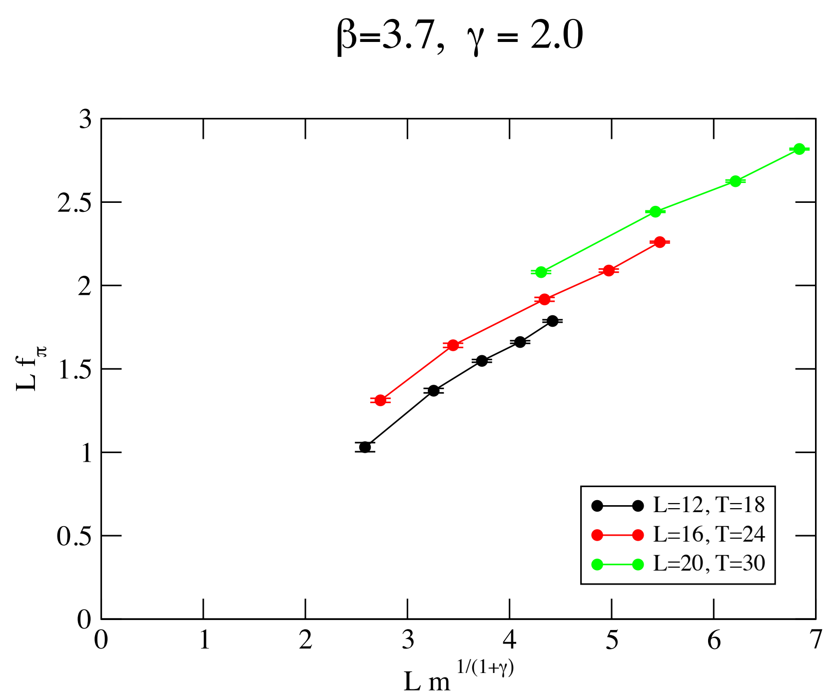
TABLE III. Chiral fit results for M_π^2 with $M_\pi^2 = C_0^\pi + C_1^\pi m_f + C_2^\pi m_f^2$ for various fit ranges.

fit range (m_f)	C_0^π	χ^2/dof	dof
0.015–0.04	0.0016(13)	1.21	1
0.015–0.05	−0.0017(9)	5.90	2
0.015–0.06	−0.0022(6)	4.18	3
0.015–0.07	−0.0032(5)	5.00	4
0.015–0.08	−0.0037(5)	5.44	5
0.015–0.10	−0.0049(4)	7.28	6
0.015–0.16	−0.0071(3)	14.8	9

Comparison with $N_f = 4$
: hyperscaling of F_π (in SχSB)







F_π : no scaling for $0 < \gamma < 2$

⇒ In this sense,

it seems that Nf=8 is not SχSB of ordinary QCD.

Detour: (sideways)

In the previous, the quadratic fit in the wide range of mf gives worse χ^2/dof.

⇒ Why? What happens?

Power fit trial:(conformal-like)

TABLE V. Power fit results of F_π for various fit ranges, using $F_\pi = C_1 m_f^{1/(1+\gamma)}$. The left table shows the results for the ranges with minimum mass set to the lightest, $m_f = 0.015$, while the right one does those with maximum mass being the heaviest $m_f = 0.16$.

fit range (m_f)	C_1	γ	χ^2/dof
0.015–0.04	0.415(7)	0.988(19)	14.8
0.015–0.05	0.414(5)	0.991(15)	9.84
0.015–0.06	0.418(4)	0.979(12)	7.88
0.015–0.07	0.424(3)	0.963(9)	7.35
0.015–0.08	0.425(3)	0.961(8)	6.15
0.015–0.10	0.426(2)	0.958(7)	5.31
0.015–0.16	0.428(1)	0.952(4)	3.98

Case including the small mf region

good χ^2/dof

fit range (m_f)	C_1	γ	χ^2/dof
0.02–0.16	0.429(1)	0.947(4)	2.22
0.03–0.16	0.431(1)	0.942(5)	1.94
0.04–0.16	0.429(2)	0.950(10)	1.23
0.05–0.16	0.431(2)	0.941(7)	0.66
0.06–0.16	0.429(2)	0.948(9)	0.44
0.07–0.16	0.429(3)	0.950(10)	0.52
0.08–0.16	0.431(3)	0.939(14)	0.20
0.10–0.16	0.432(4)	0.934(19)	0.23

Case excluding the small mf region

Power fit in the middle region of mf (0.05 ≦ mf) shows the good χ^2/dof.

Linear fit in the hyperscaling relation

Note: here, Figs. are plotted as a function of mf, not x

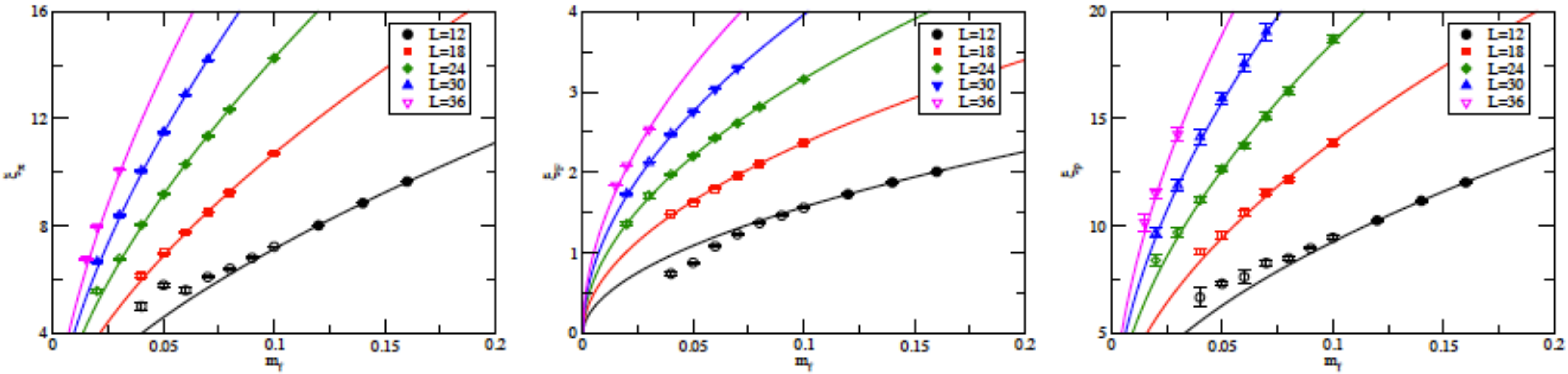


FIG. 18. Linear fits for the FSHS of M_π (left), F_π (center), and M_ρ (right). The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$.

TABLE VI. The γ fitted by the linear ansatz. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$.

	γ	C_0^H	C_1^H	χ^2/dof
ξ_π	0.5668(26)	0.049(22)	2.57766(99)	2.52
ξ_F	0.9279(79)	−0.17(10)	0.4372(38)	0.73
ξ_ρ	0.798(20)	0.04(19)	2.779(69)	0.66

γ is not universal. $\gamma(M_\pi) \neq \gamma(F_\pi)$, $\gamma(F_\pi) \sim 1.0$