Gluonic observables and the scalar spectrum of twelve-flavour QCD

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on behalf of the **LatKMI** collaboration

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Lattice 2013, Mainz

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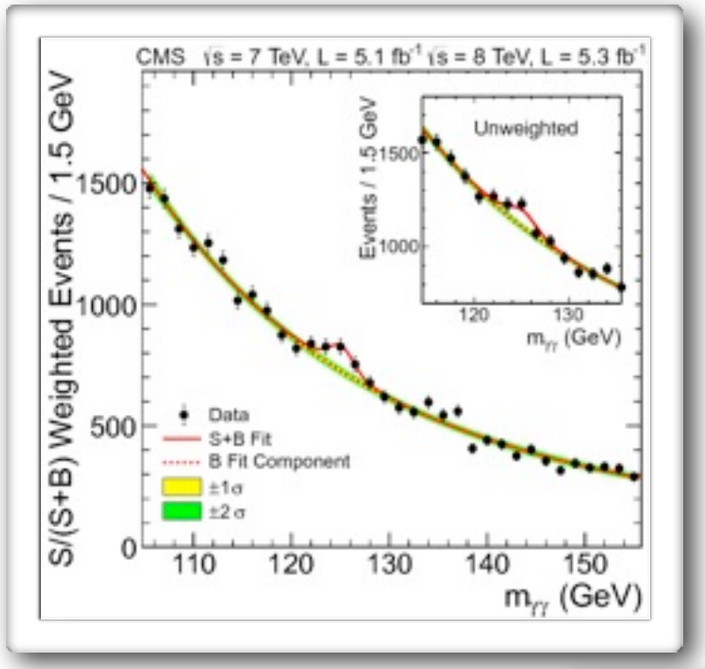
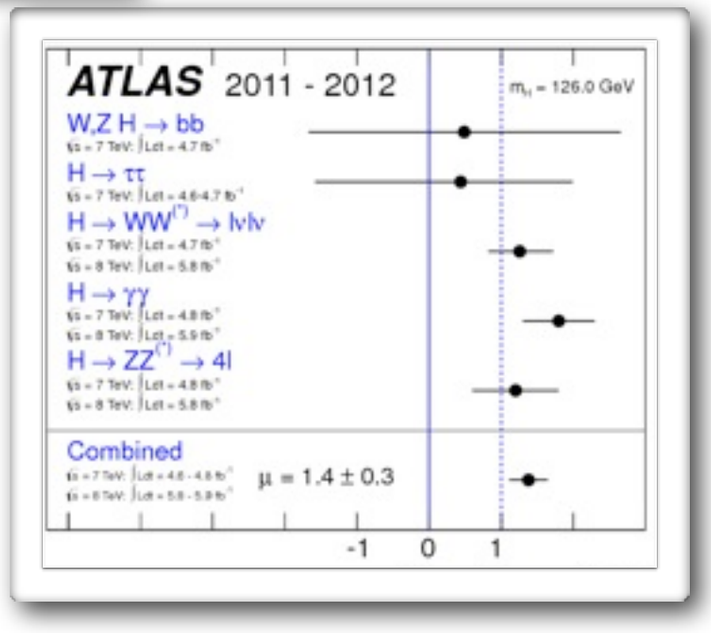
KEK

Motivations

- a Higgs boson has been found at LHC
- a “composite” Higgs-like scalar can also reproduce the LHC data
- this is realised in Walking Technicolor models
- interest in finding the lower edge of the Conformal Window
- a light scalar appears as pseudo-NG boson



CMS, PLB716 (2012) 30-61



ATLAS, PLB716 (2012) 1-29

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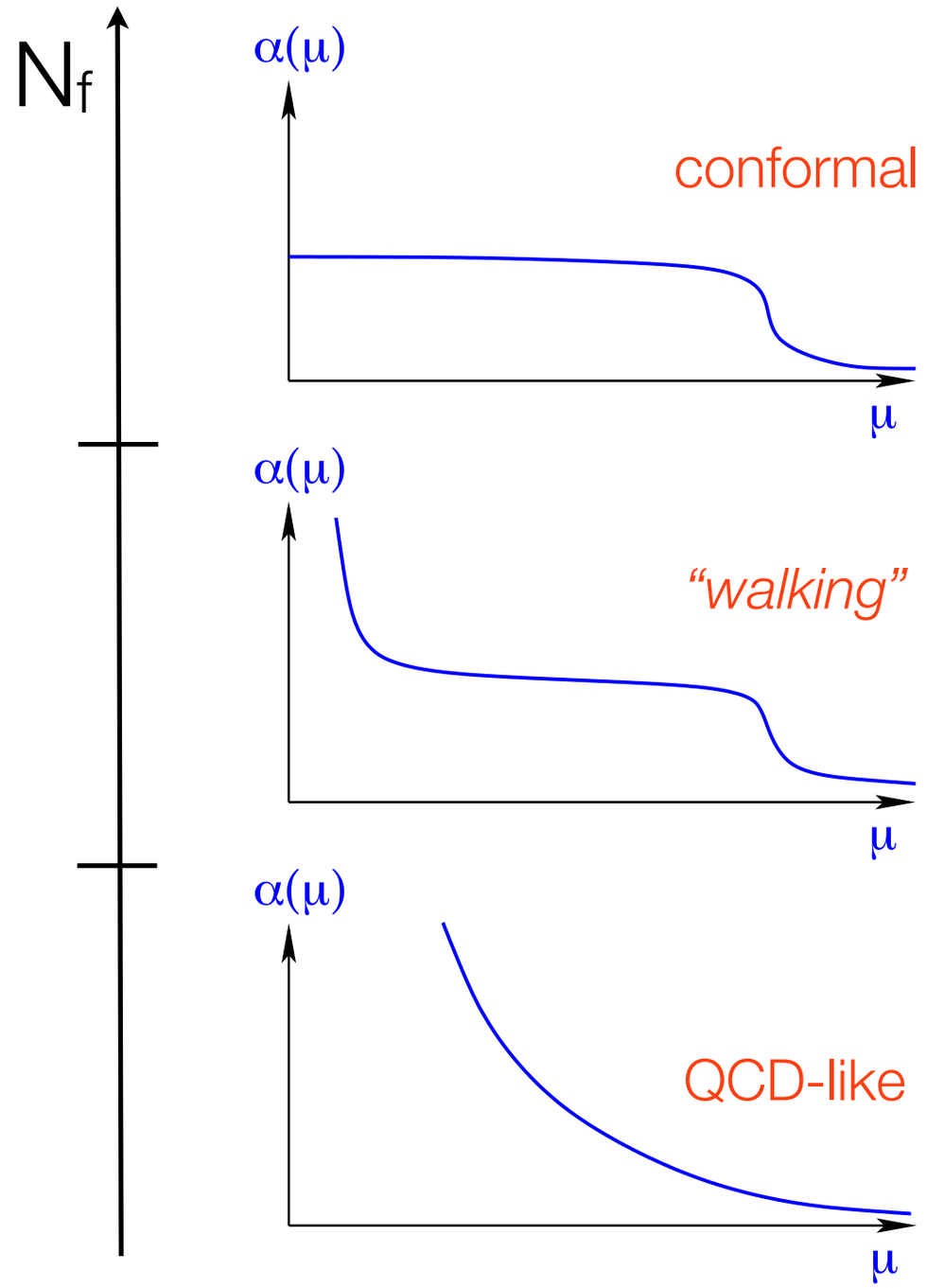


Lattice simulations of twelve-flavour QCD

- SU(3) gauge field + 12 dynamical fermions
  - gauge action → tree level Symanzik
  - fermion action → HISQ (Highly Improved Staggered Quarks)
  - modified HMC based on MILC code (v7)
- fixed gauge coupling → **$\beta = 4.0$**
- four volumes → **$V=18^3, 24^3, 30^3, 36^3$**
- six bare fermion masses → **$am_f = 0.05 - 0.16$**
- long trajectories → **>15000 MD units**

L	T	am_f	confs.
18	24	0.06	3090
18	24	0.08	5250
18	24	0.10	5130
18	24	0.12	5200
18	24	0.16	4200
24	32	0.05	9440
24	32	0.06	14000
24	32	0.08	15340
24	32	0.10	9480
30	40	0.05	7500
30	40	0.06	14000
30	40	0.08	11450
30	40	0.10	5000
36	48	0.05	5100
36	48	0.06	6000

Glueball spectroscopy

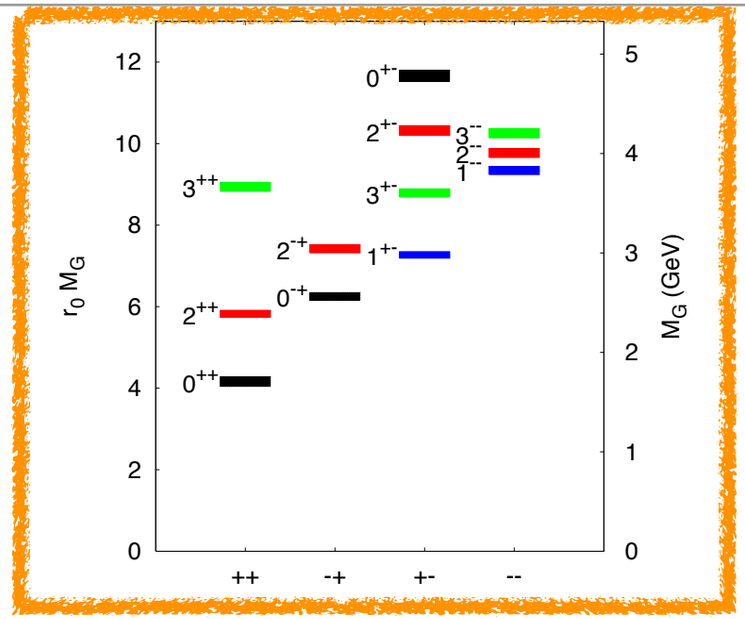
- the glueball spectrum is well studied in SU(N) Yang-Mills theories

- the lightest glueballs are starting to be investigated in full LQCD

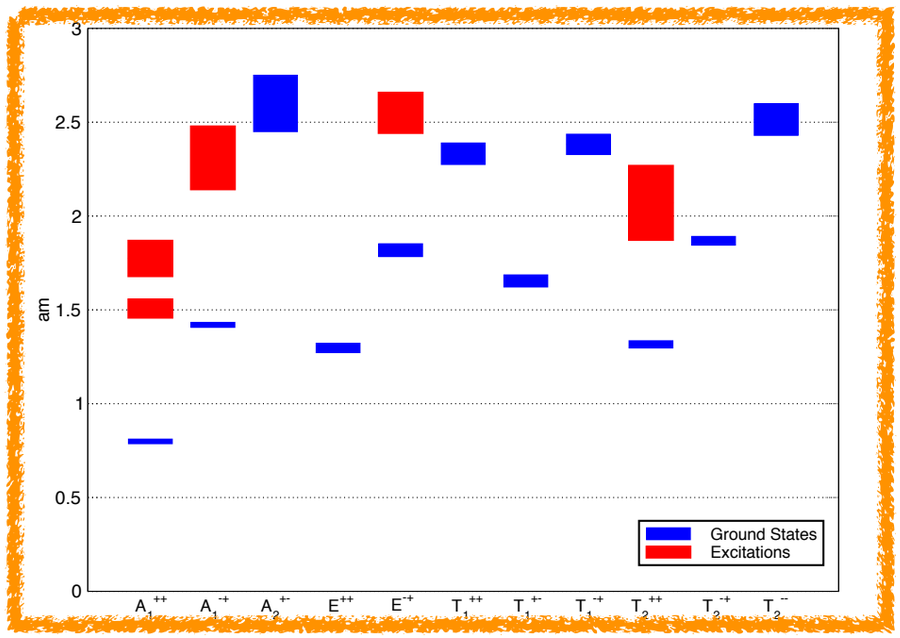
- this talk focus on many-flavour QCD

  - typically heavy mass, determined by Λ_{QCD}

  - typically noisy and require large statistic



Chen et al., PRD73(2006)



Lucini, Rago, ER, JHEP08(2010)

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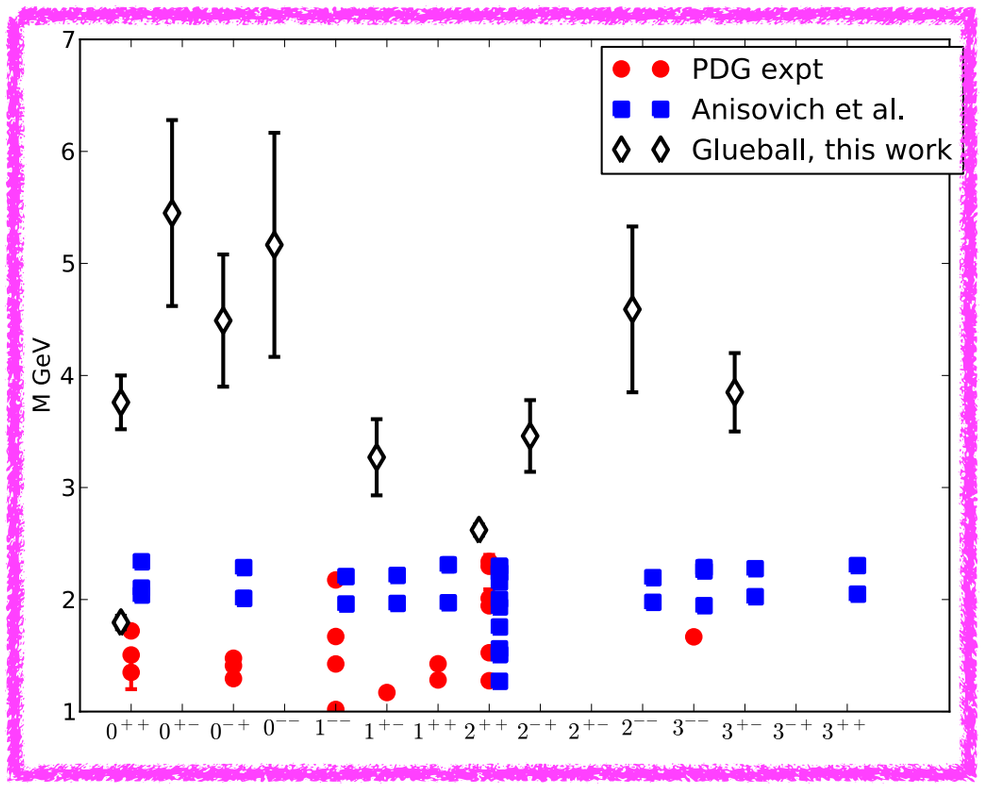

Gregory, Irving, Lucini, McNeile, Rago, Richards, ER, JHEP10(2012)

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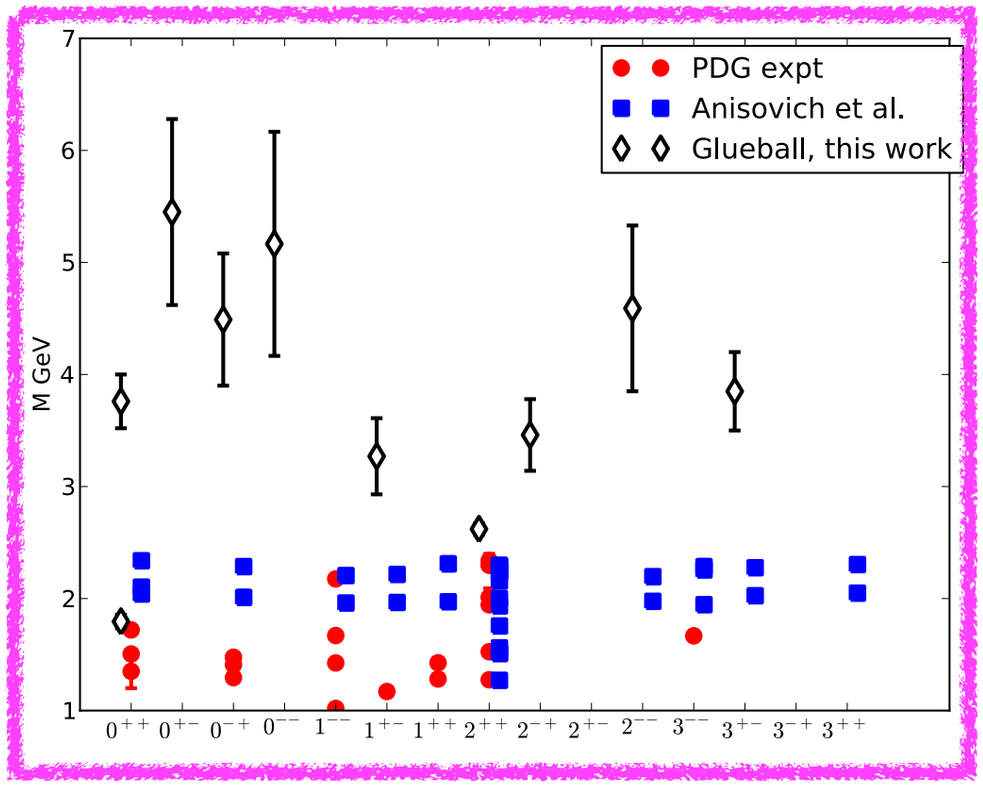


Gregory, Irving, Lucini, McNeile, Rago, Richards, ER, JHEP10(2012)

Glueball spectroscopy: operators

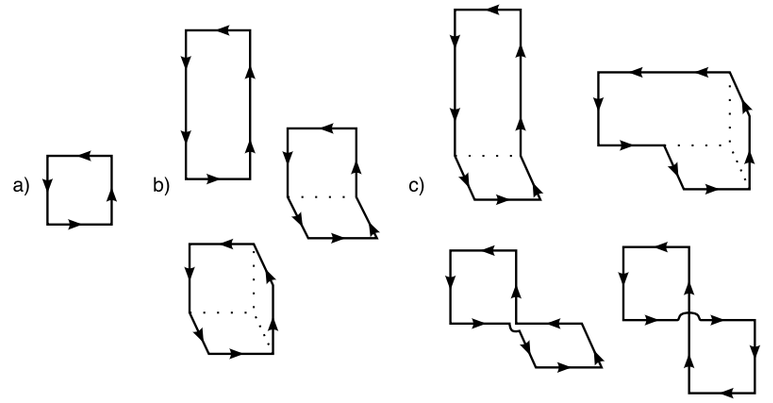
- eigenstates of the Hamiltonian are classified according to the irreducible representations of the cubic group

$$\{A_1(1),\, A_2(1),\, E(2),\, T_1(3),\, T_2(3)\}$$

J	A_1	A_2	E	T_1	T_2
0	1	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	1	0	1	1
4	1	0	1	1	1

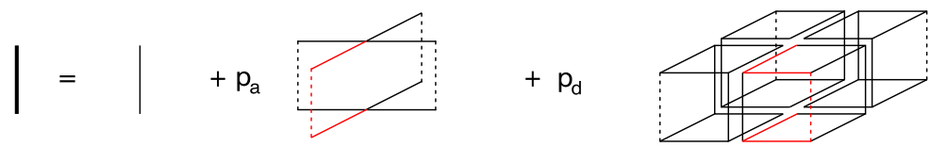
- suitable gauge-invariant operators must be constructed that respect the symmetries

$$\mathcal{O}_G(t) = \frac{1}{L^3}\sum_{x\in L^3} \mathrm{Tr}\left(\prod_{l\in\mathcal{W}(x)} U_l\right) \qquad \mathcal{O}_G^{(R)}(t) = \sum_{\alpha=1}^{24} a_\alpha^{(R)} \mathcal{R}_\alpha\left[\mathcal{O}_G(t)\right]$$



- vacuum contributions must be subtracted in the scalar case

$$\mathcal{O}^{(A_1)}(t) - \langle 0|\mathcal{O}^{(A_1)}|0\rangle$$



- improved operators are obtained by blocking and smearing algorithms

Lucini, Rago, ER
JHEP08(2010)

Glueball spectroscopy: variational analysis

- basis of operators ➔

$$\{\mathcal{O}_1(t), \ldots, \mathcal{O}_n(t)\}$$

- matrix of correlators ➔

$$\mathcal{C}_{ij}(t) = \sum_{\tau} \langle 0 | \mathcal{O}_i^{\dagger}(\tau + t) \mathcal{O}_j(\tau) | 0 \rangle$$

- generalised eigenvalue problem ➔

$$\mathcal{C}_{ij}(t) v_j^{\alpha} = \lambda^{\alpha} v_i^{\alpha}$$

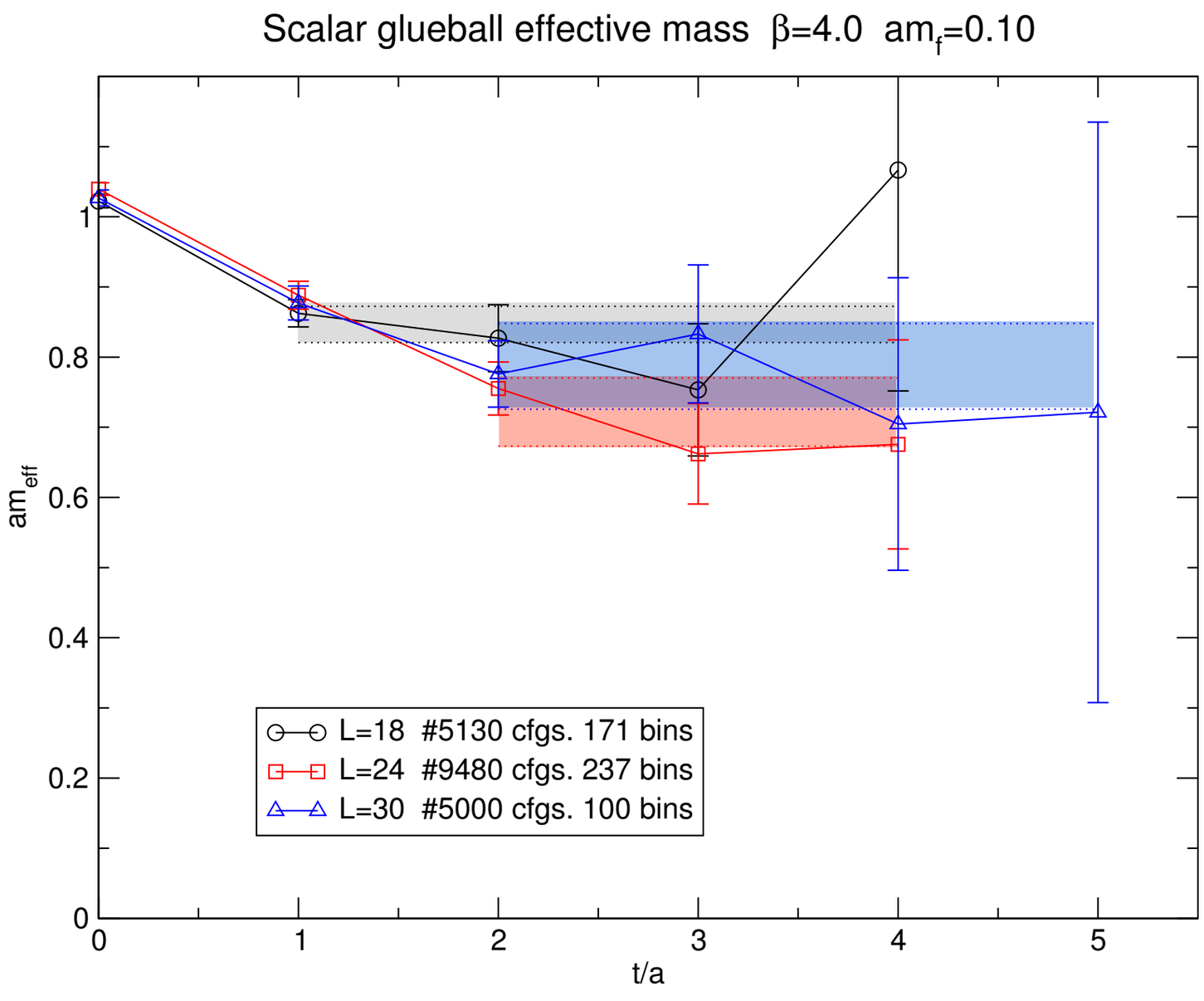
$$\Phi_{\alpha}(t) = \sum_{i=1}^{n} v_i^{\alpha} \mathcal{O}_i(t)$$

- ground state correlator fit (α=0) ➔

$$\langle \Phi_{\alpha}^{\dagger}(t) \Phi_{\alpha}(0) \rangle = |c_{\alpha}|^2 \left(e^{-m_{\alpha} t} + e^{-m_{\alpha}(T-t)} \right)$$

- the effective mass plateaux is used to determine the fitting window on the correlator

Example: scalar glueball effective mass

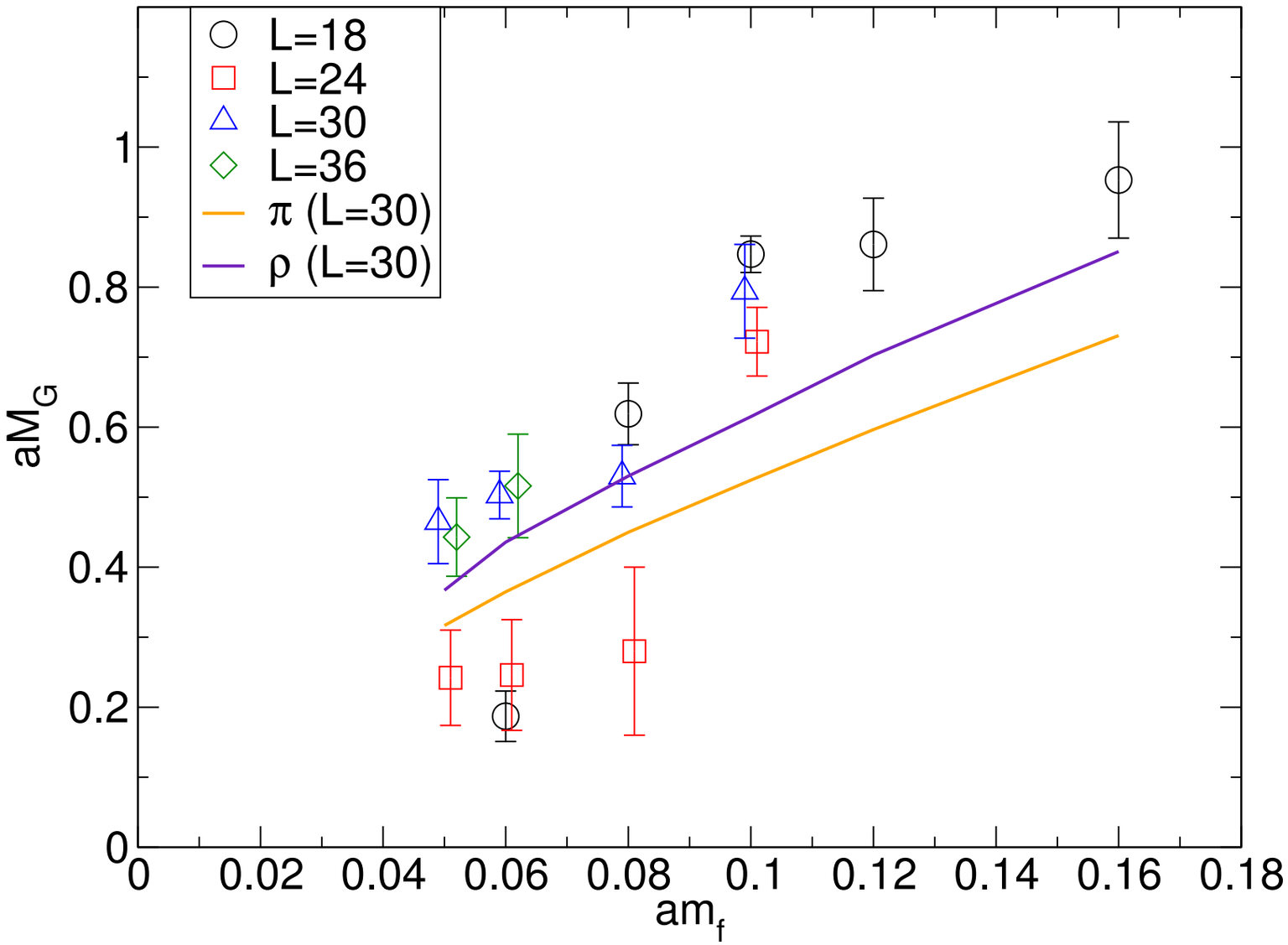


The fitted mass is shown on top of the corresponding effective mass enabling us to establish the goodness of the estimate

Finite-size effects appear to be negligible at larger fermion masses

Results: scalar glueball spectrum

- The scalar glueball mass decreases with the bare fermion mass
- it is expected to become independent of m_f in the quenched limit
- at light quark mass and on small volumes it is consistent with the mass of a fermionic bound state **[talk by T.Yamazaki 3F]**
- finite volume effects and mixing with a light scalar fermionic state need to be better investigated



Pseudoscalar and vector data are taken from LatKMI, PRD86(2012)054506

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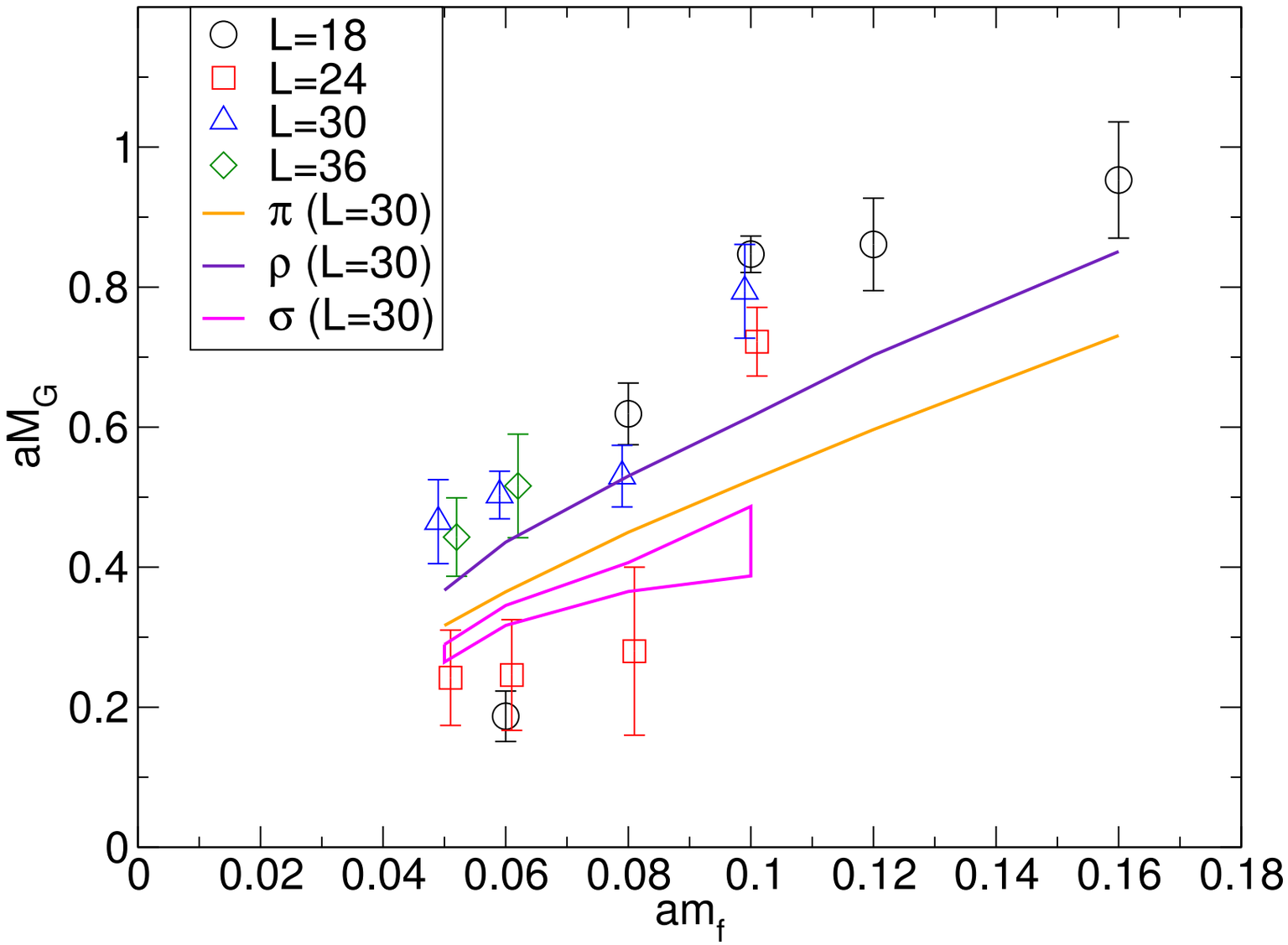
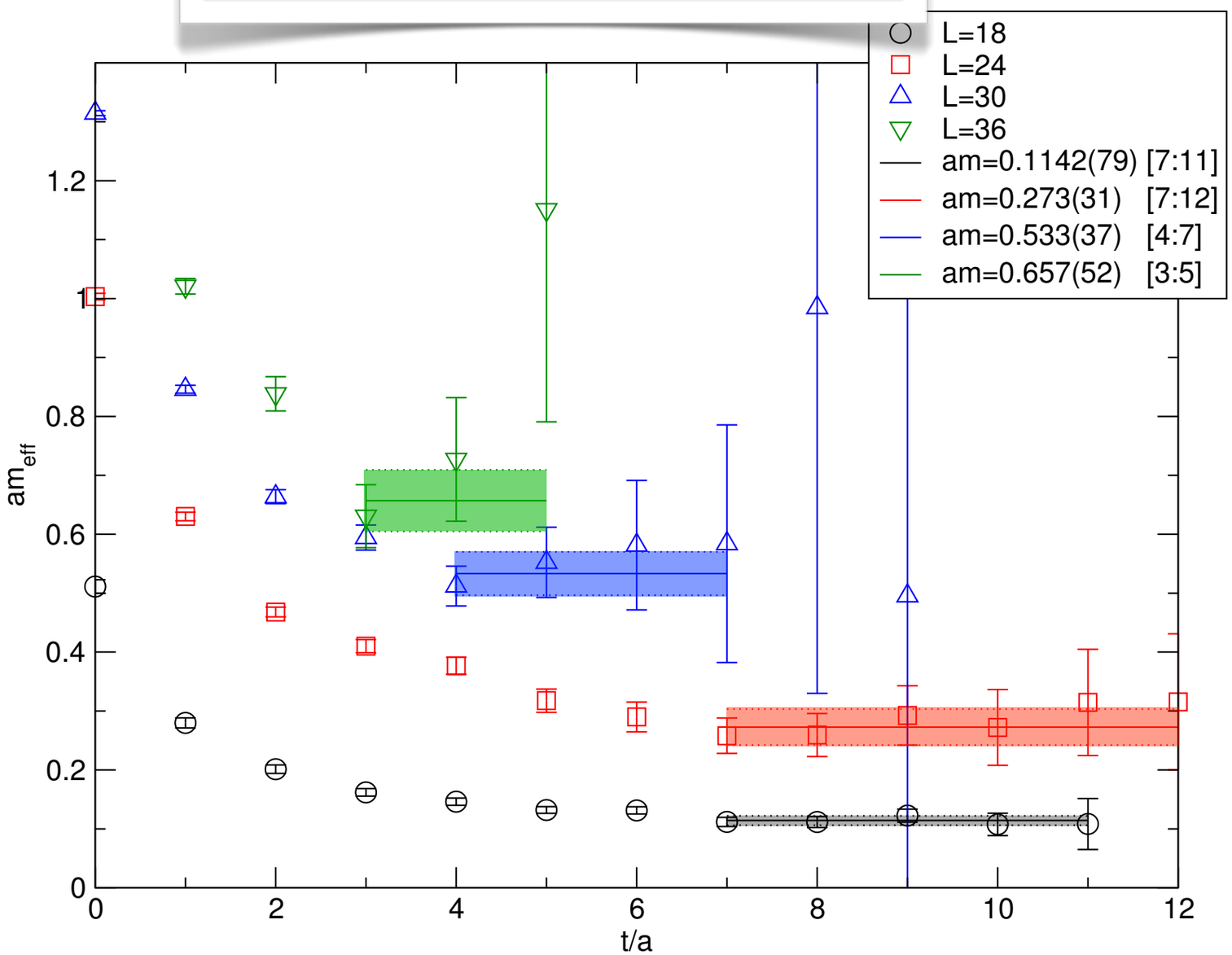


Pseudoscalar and vector data are taken from LatKMI, PRD86(2012)054506
Scalar data are taken from LatKMI, arxiv:1305.6006

Example: spatial torelon effective mass (am_f=0.06)

$$am_{\rm tor}(L) = a^2\sigma L - \frac{\pi}{3L} - \frac{\pi^2}{18L^3}\frac{1}{a^2\sigma}$$



- The torelon ground state mass is obtained from a variational analysis of smeared spatial Polyakov loop correlators
- The string tension is extracted from the L dependence of the torelon ground state mass
- A stable effective mass plateaux is found for each volume and the fitted mass is shown on top of the effective mass data
- The formula applied for the largest volumes implies a common string tension value

Example: spatial torelon effective mass (L=24)

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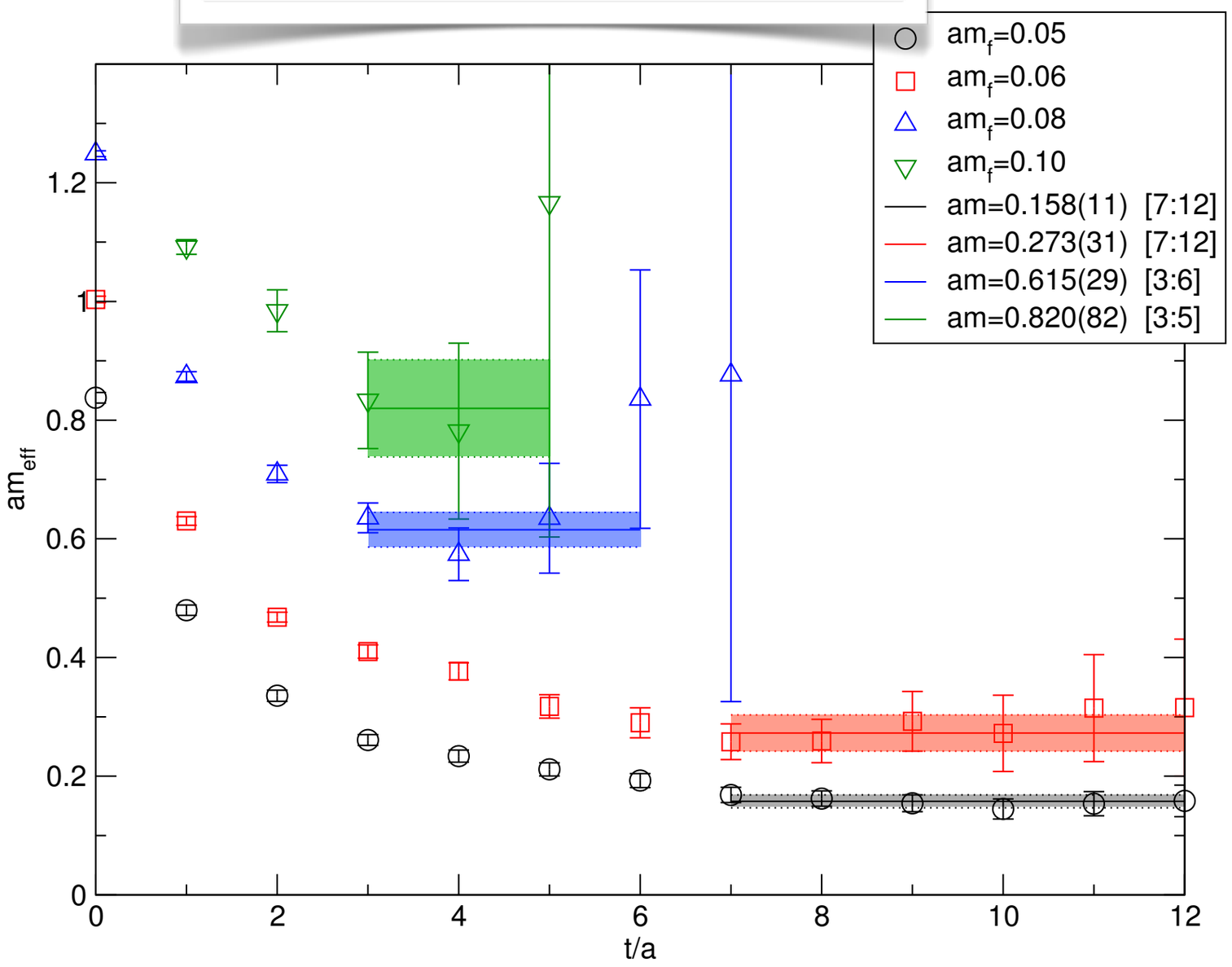


- Torelon effective masses on a fixed volume show the bare fermion mass dependence

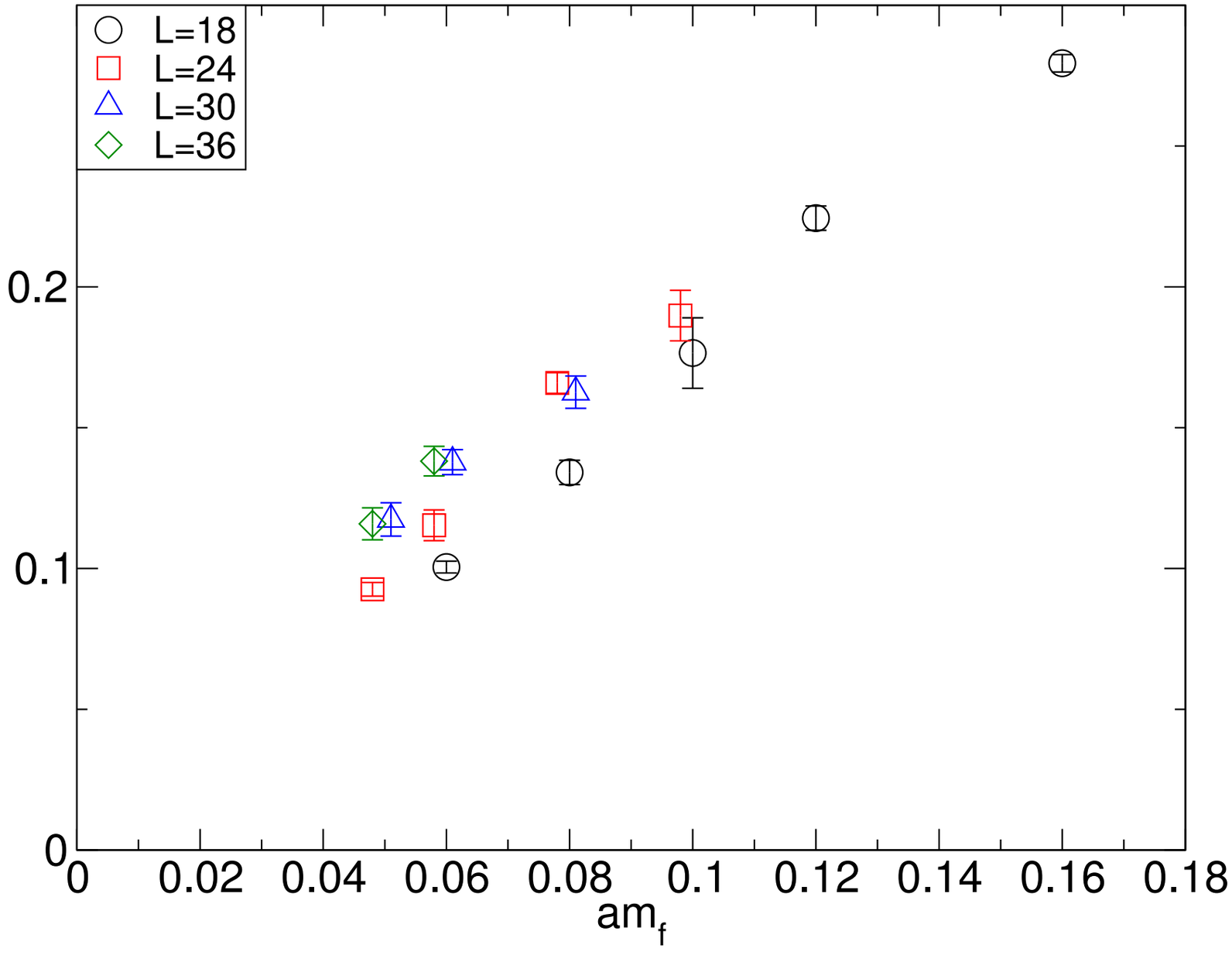
- When the bare quark mass increases we obtain a larger torelon mass

Results: string tension

- The string tension is obtained for the full range of simulated masses and volumes with 1% to 4% statistical errors

- The finite-size effects seem under control as date on two volumes with compatible string tension exist for all light fermion masses

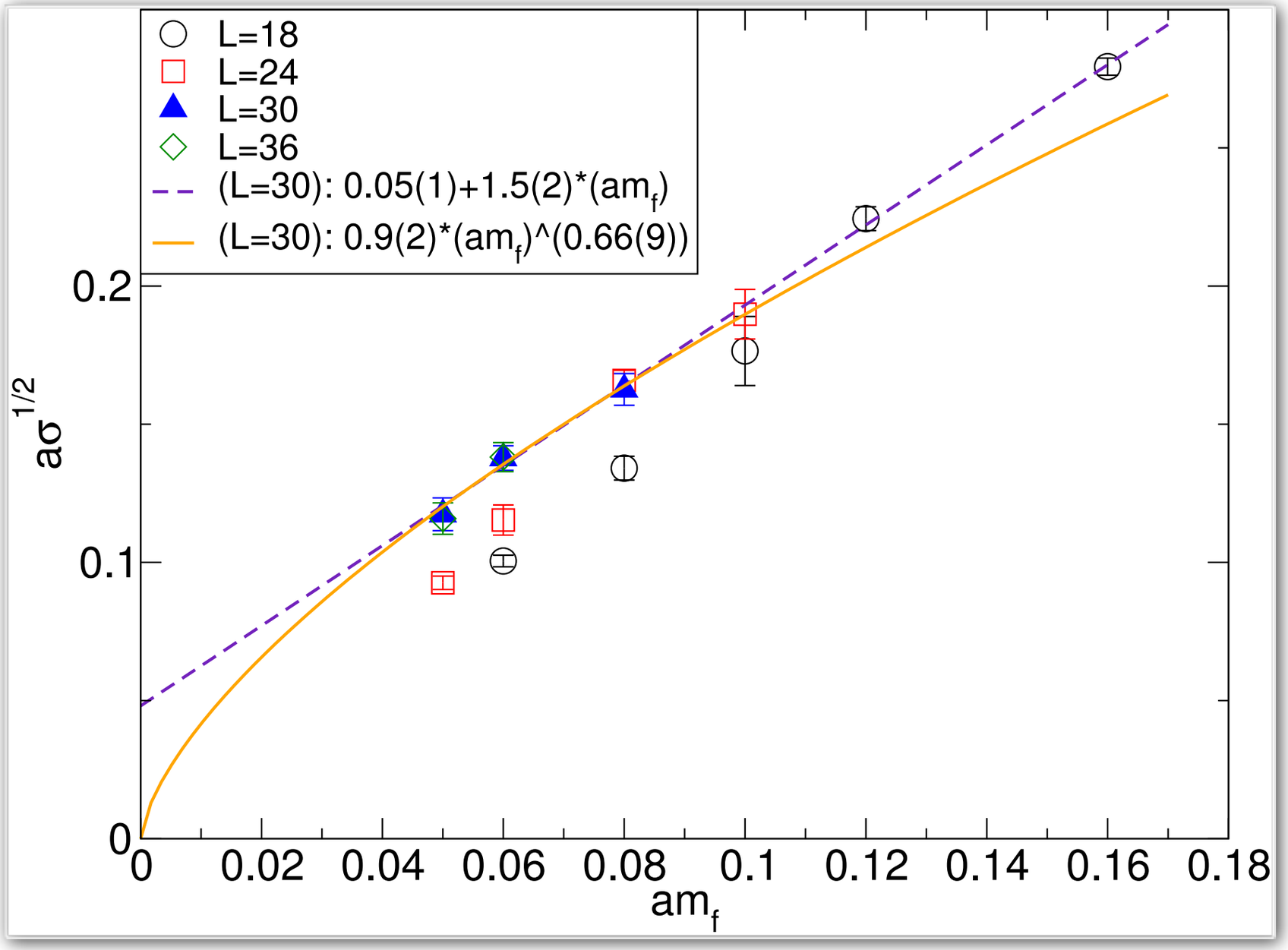
- There is a clear dependence of the string tension on the bare fermion mass



Scaling of the string tension

$$a\sqrt{\sigma} = c_0 + c_1\ am_f$$

$$a\sqrt{\sigma} = c\ am_f^{\frac{1}{1+\gamma}}$$



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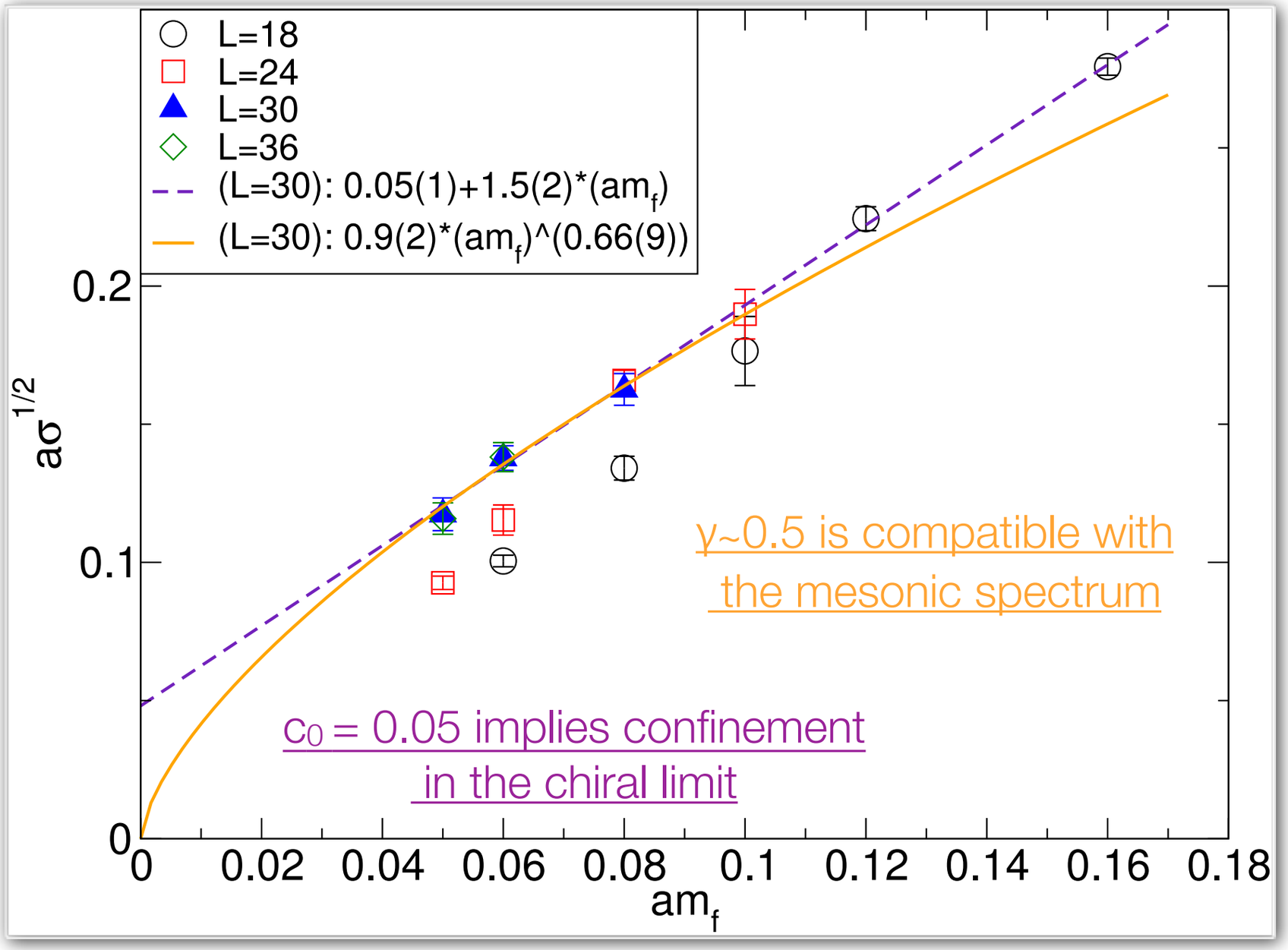

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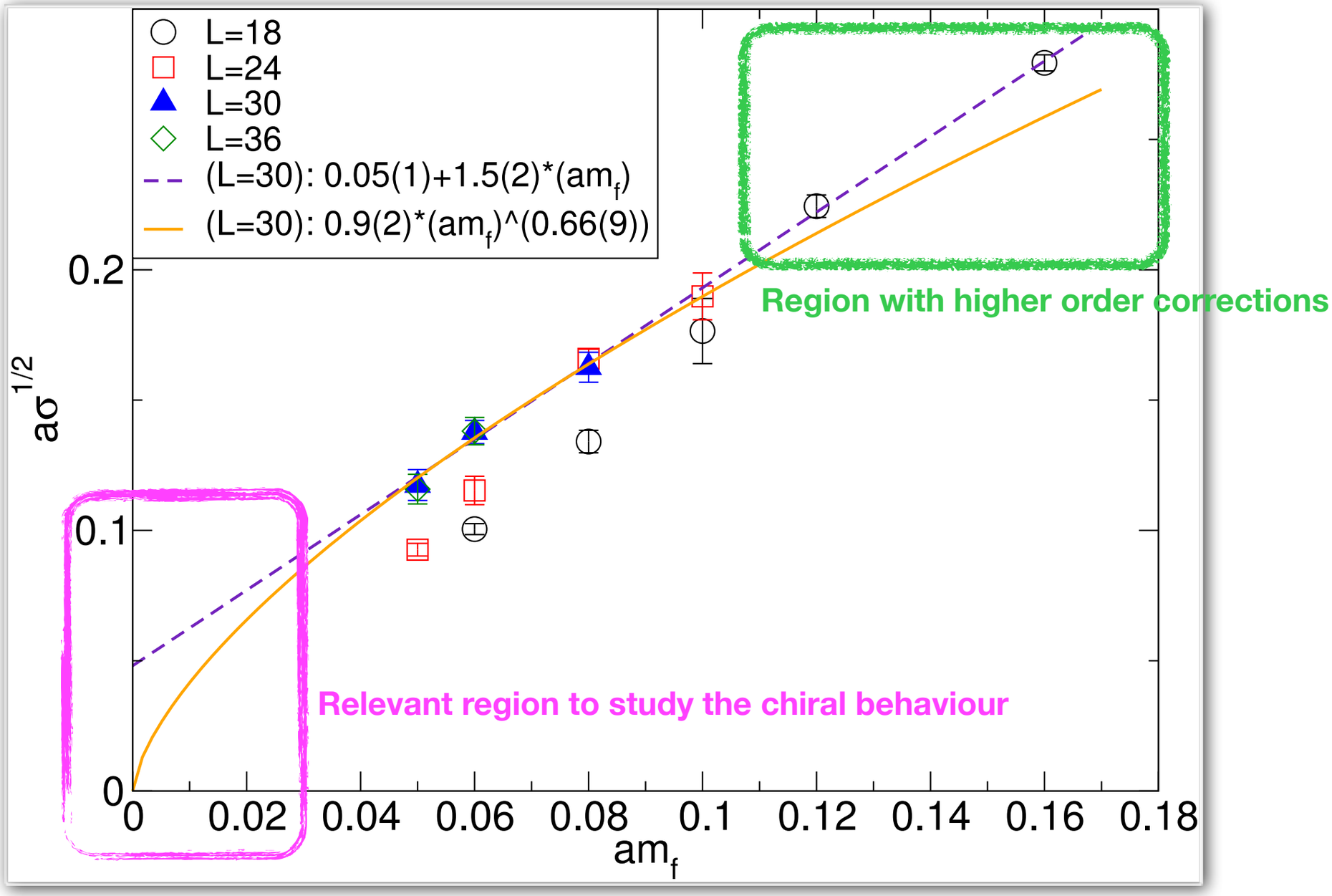
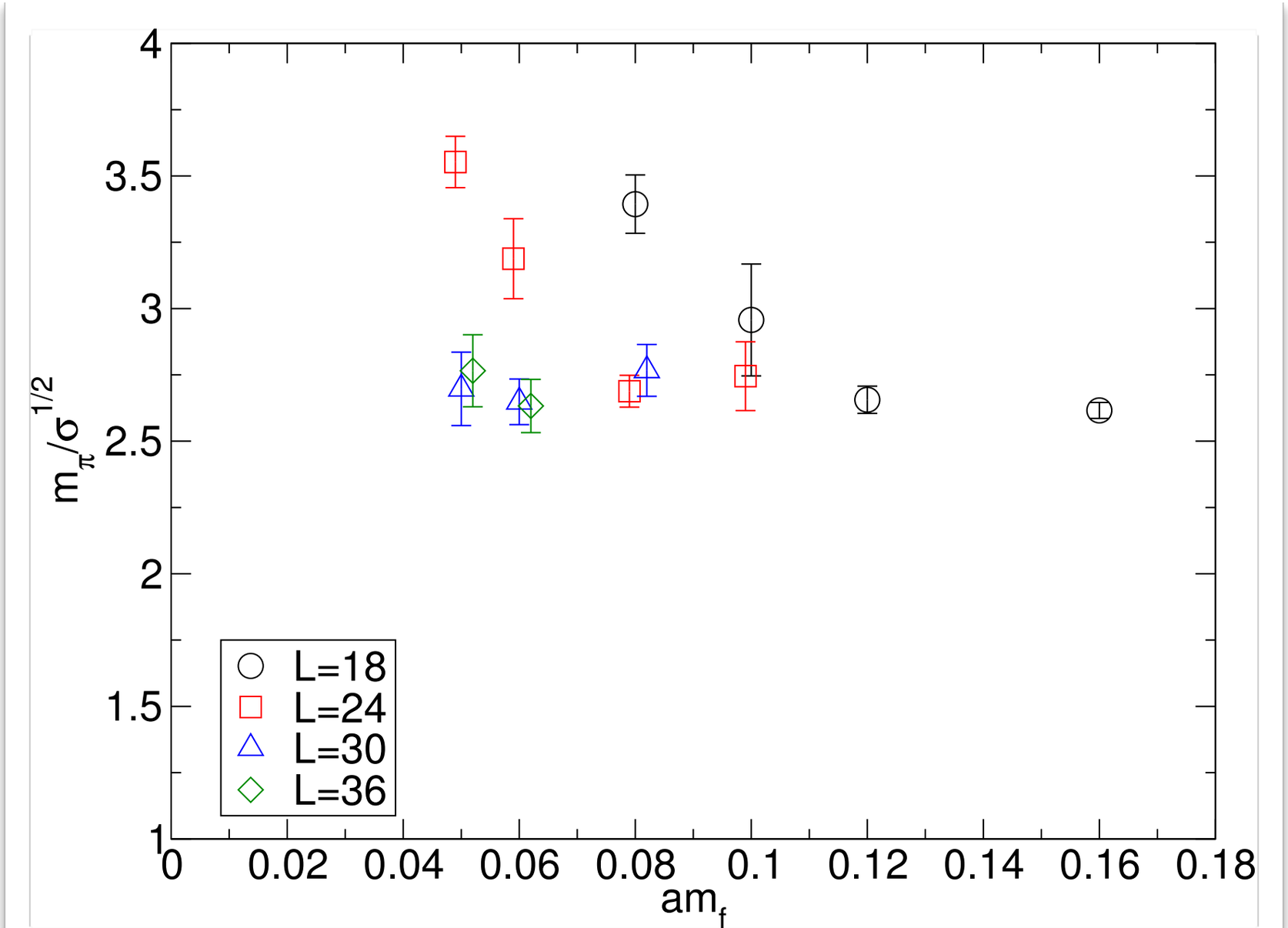


Ratios of gluonic and mesonic scales

Fermionic pseudoscalar state and string tension



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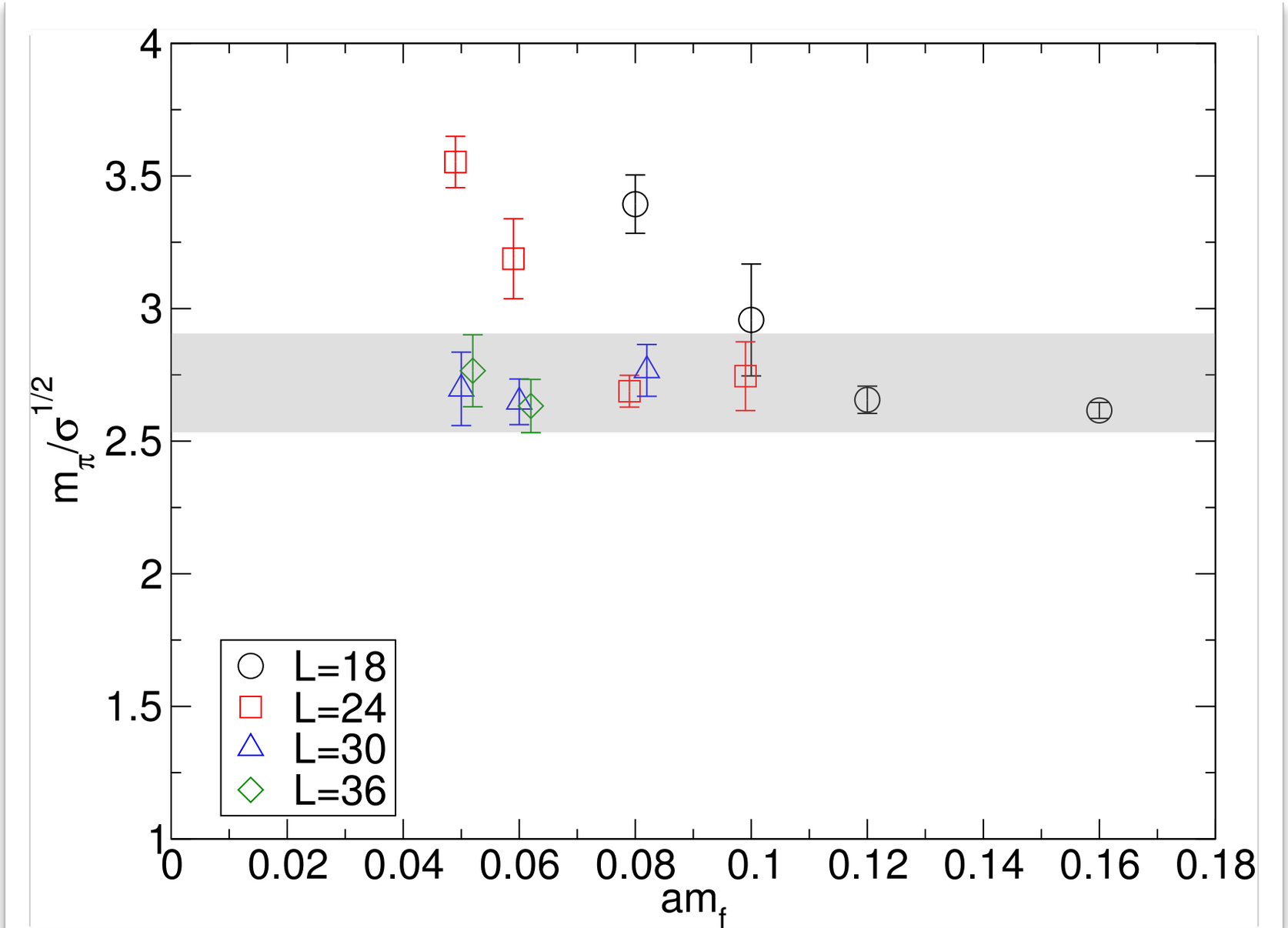
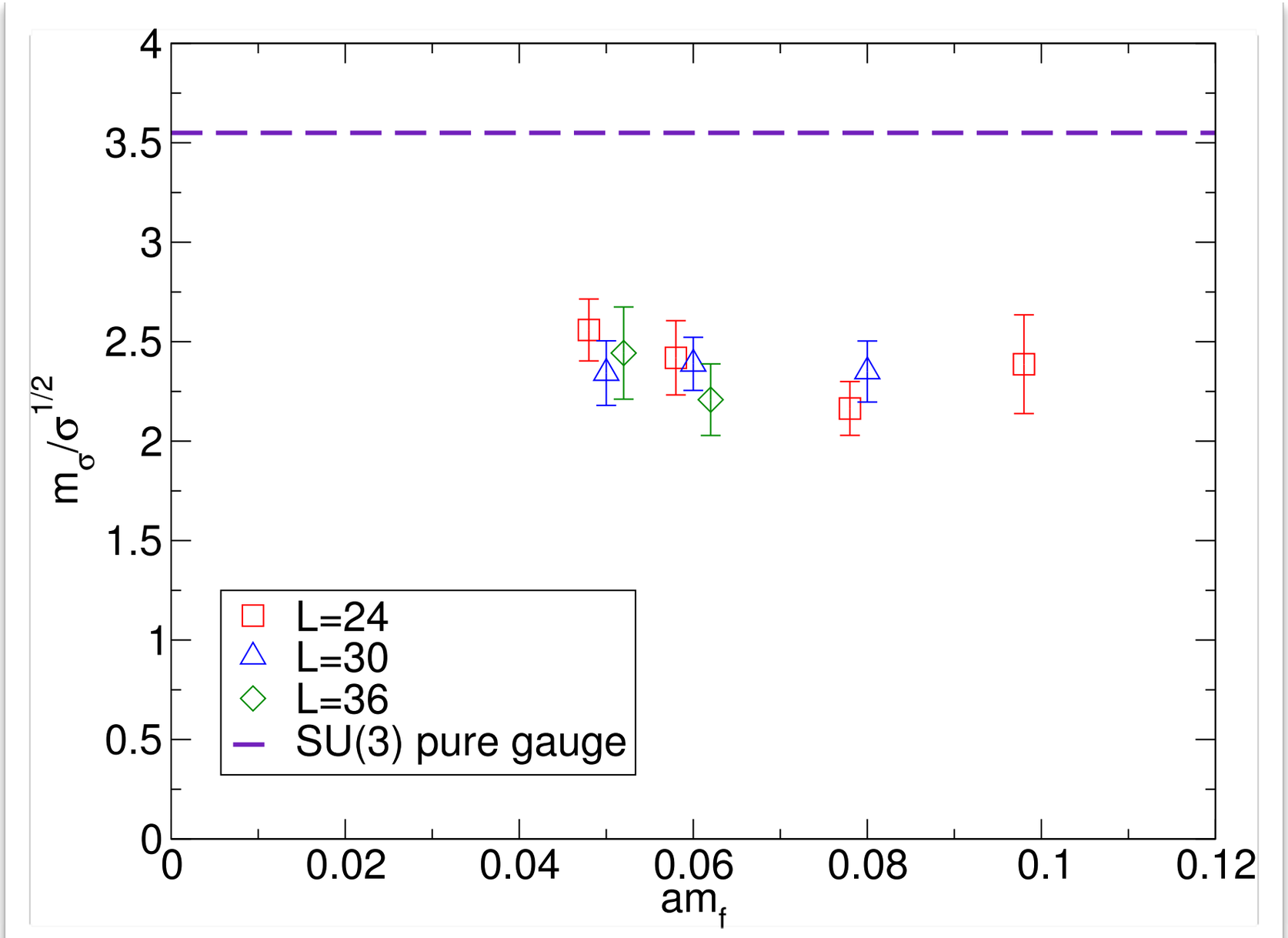


Ratios of gluonic and mesonic scales

Flavour-singlet fermionic scalar state and string tension



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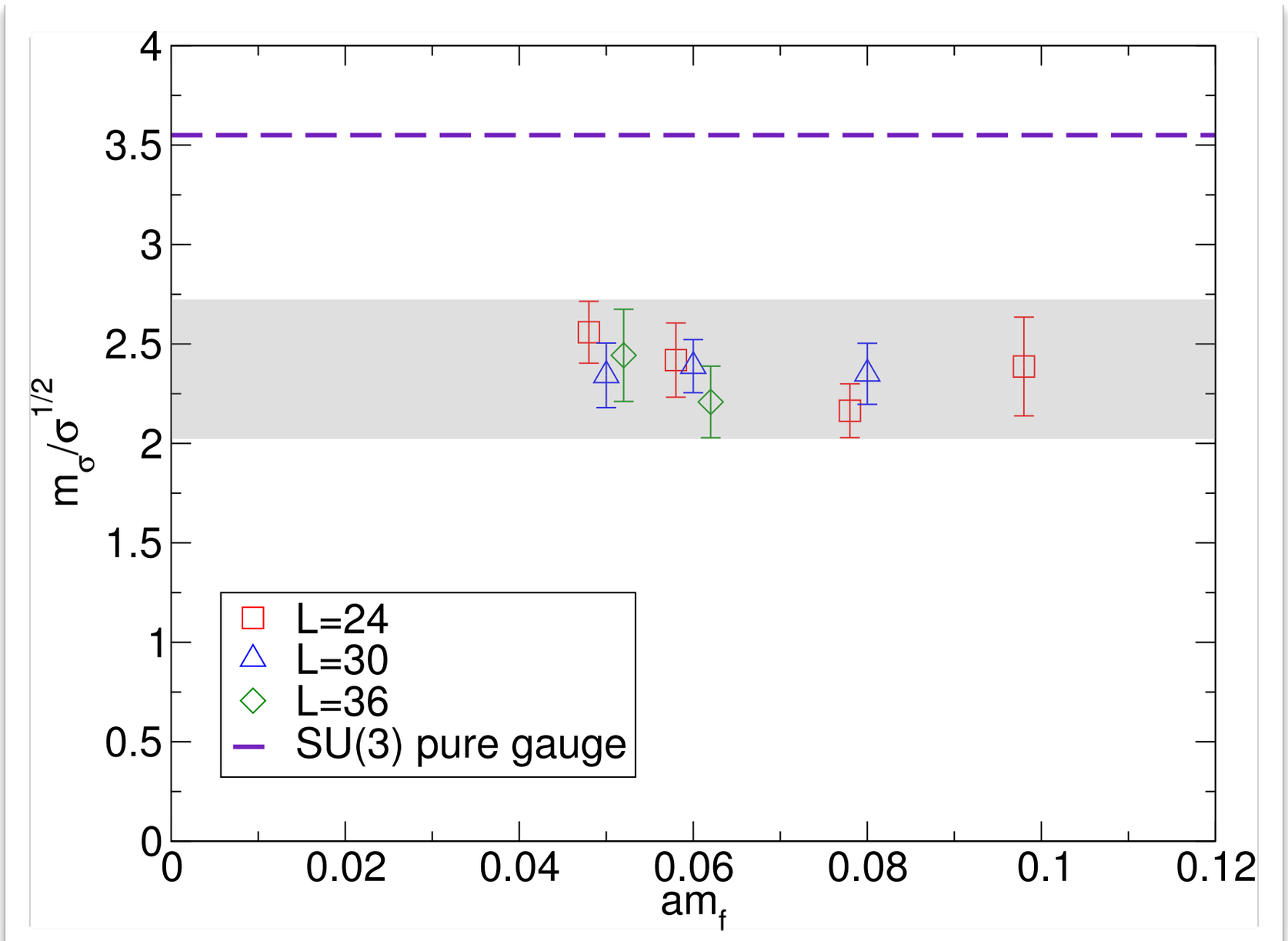


Conclusions

- we study the scalar spectrum in many-flavour QCD, focused on N_f=12
- scalar glueball and scalar meson interpolating operators are used
- both glueball operators and mesonic operators couple to this state
    - important to understand the mixing of such contributions
- we measure the string tension in N_f=12 QCD and its fermion mass dependence
    - it is difficult to extrapolate the results to the chiral limit
- ratios of fermionic pseudoscalar and scalar state with the string tension are compatible with a constant value, down to the chiral limit

Outlook

- larger volumes at light quark masses are necessary to carefully estimate finite size errors

- possible discretisation effects need to be studied by simulating at different lattice spacing

- lighter quark masses are needed to better identify scaling properties of the string tension

- a variational analysis including both gluonic and mesonic operators in the scalar channel is needed: it will give informations about the mixing between the two kind of operators

- a similar study is in progress on $N_f=8$ QCD configurations
  [talks by K.Nagai and H.Ohki 1F]

Thank you

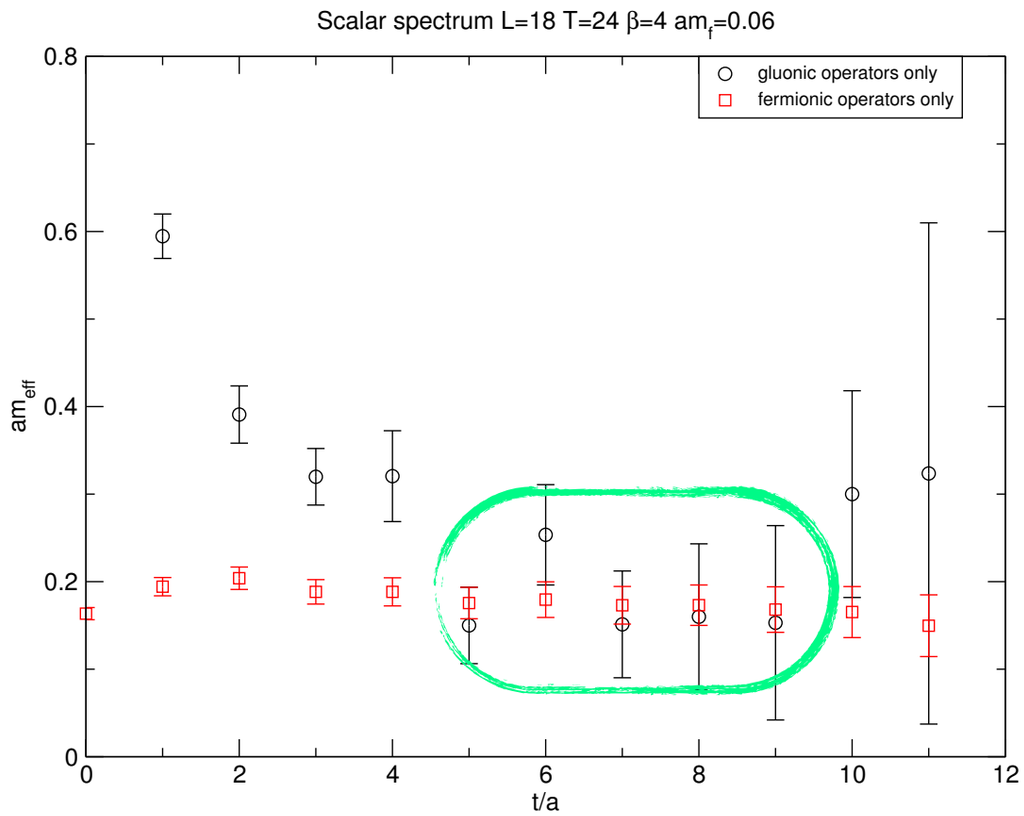
Appendix

additional slides with results

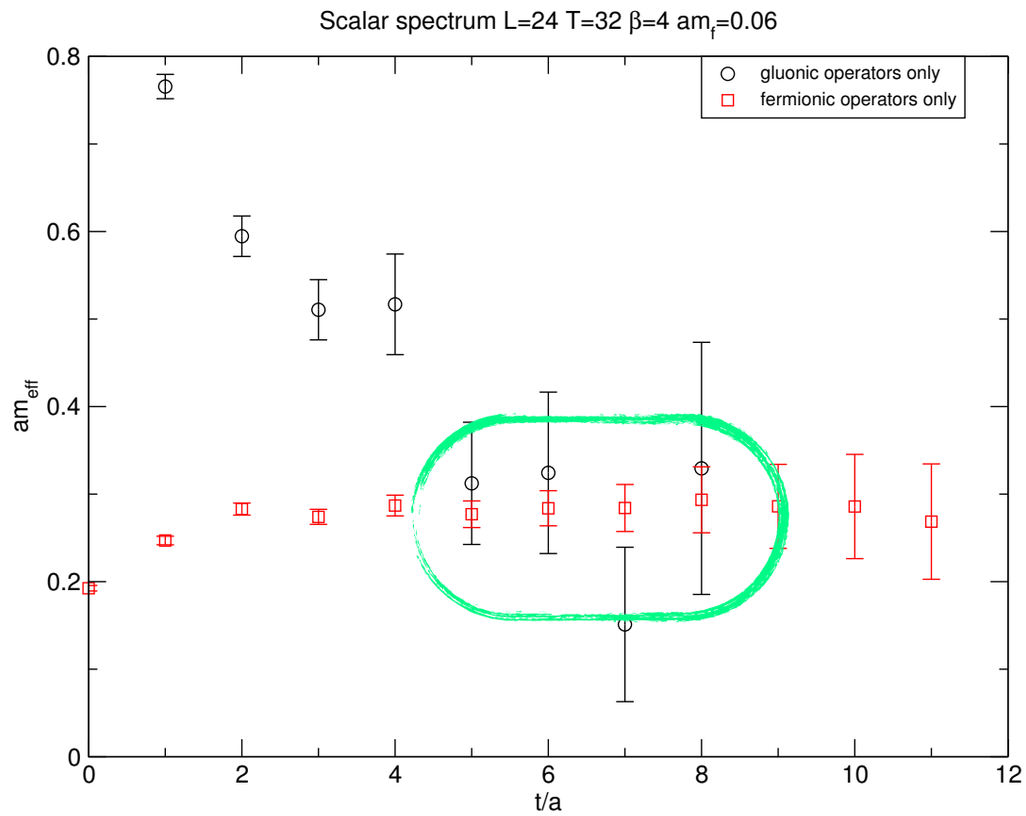
Comparison with gluonic observables

Preliminary

L = 18 and $am_f = 0.06$



L = 24 and $am_f = 0.06$



Comparison with gluonic observables

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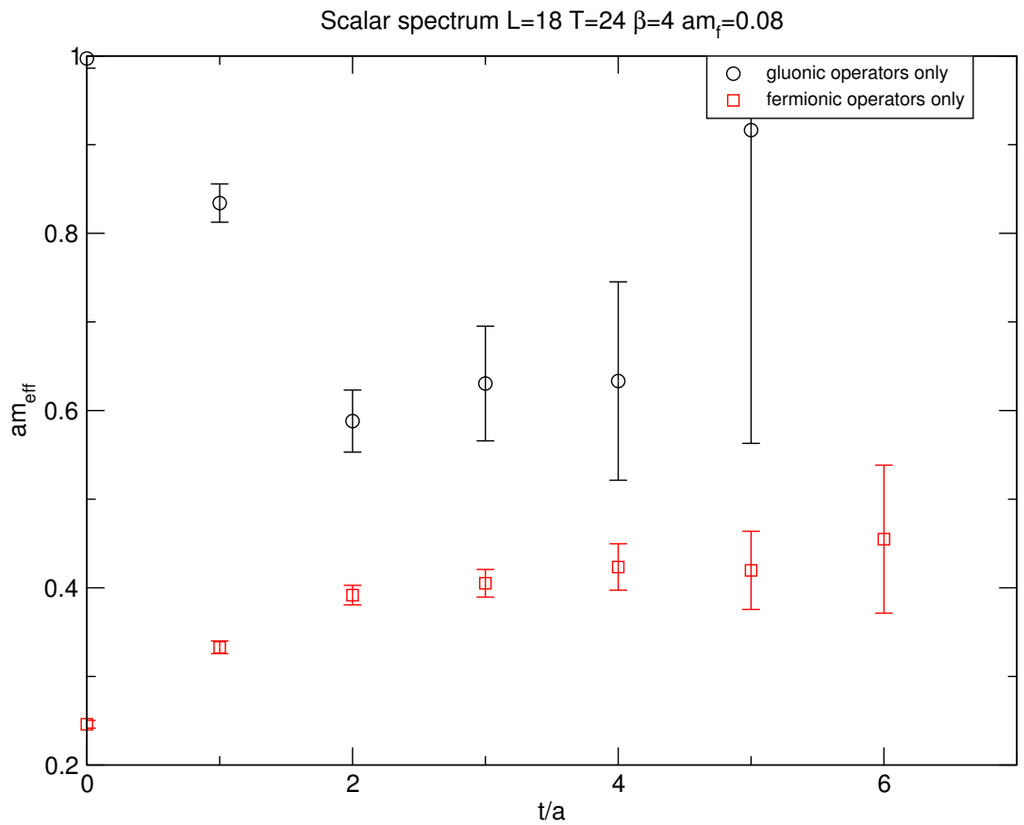

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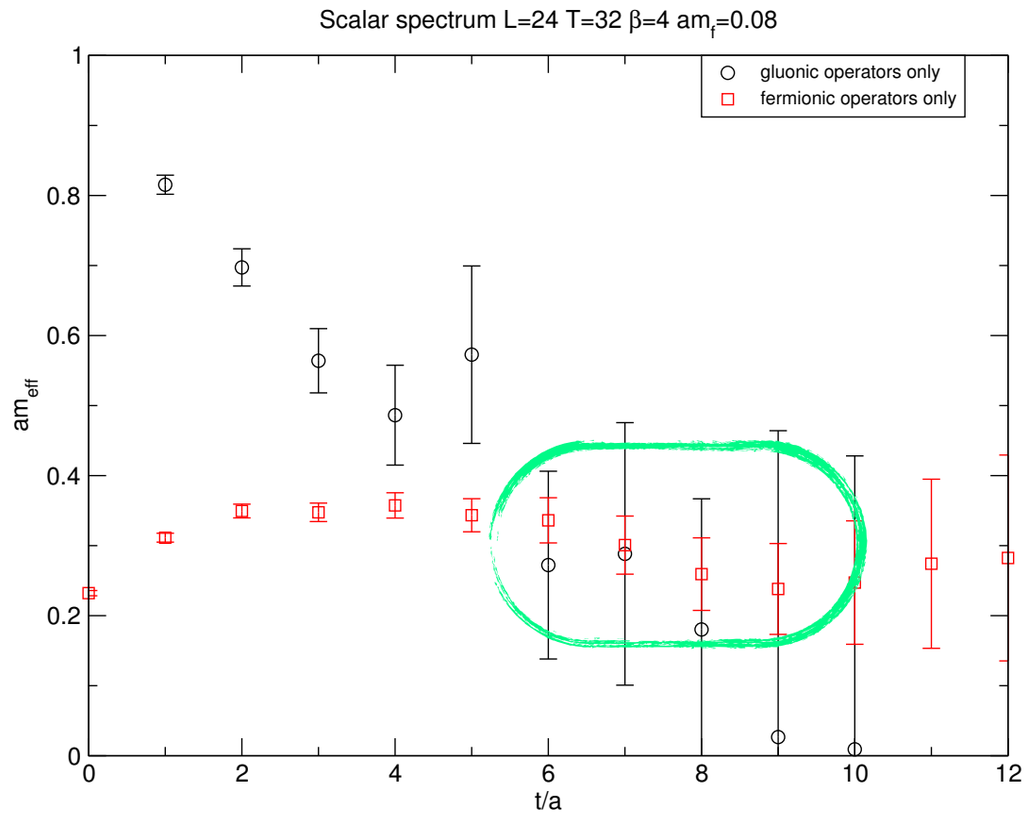

Comparison with gluonic observables

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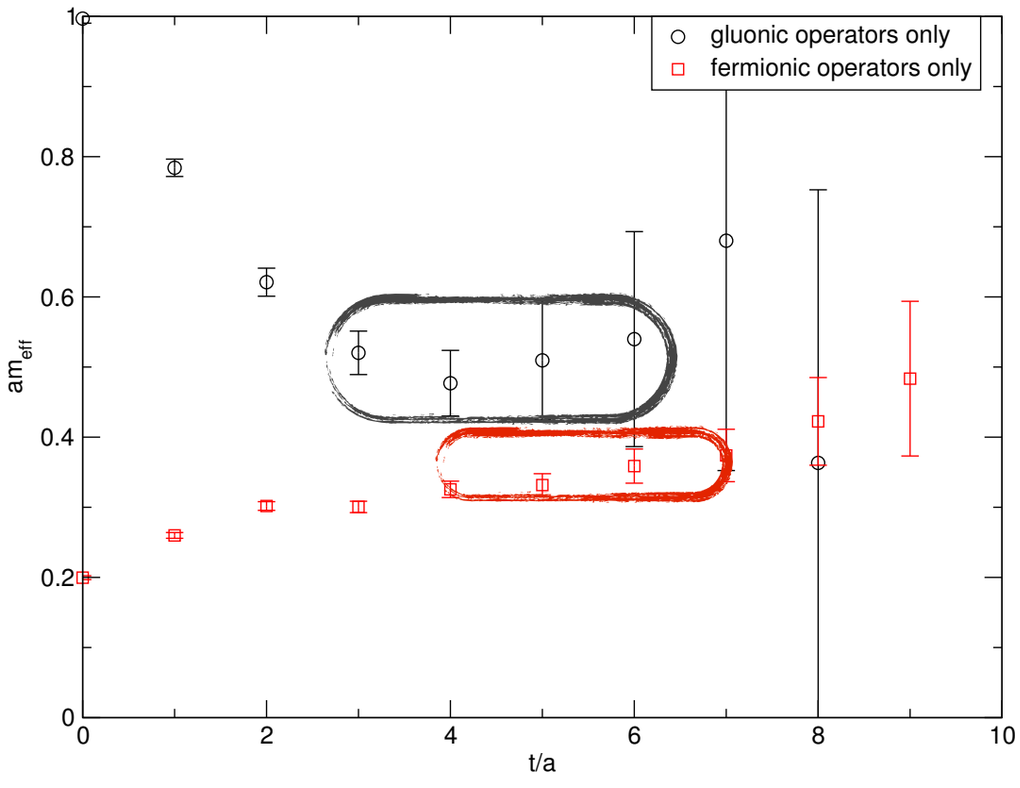
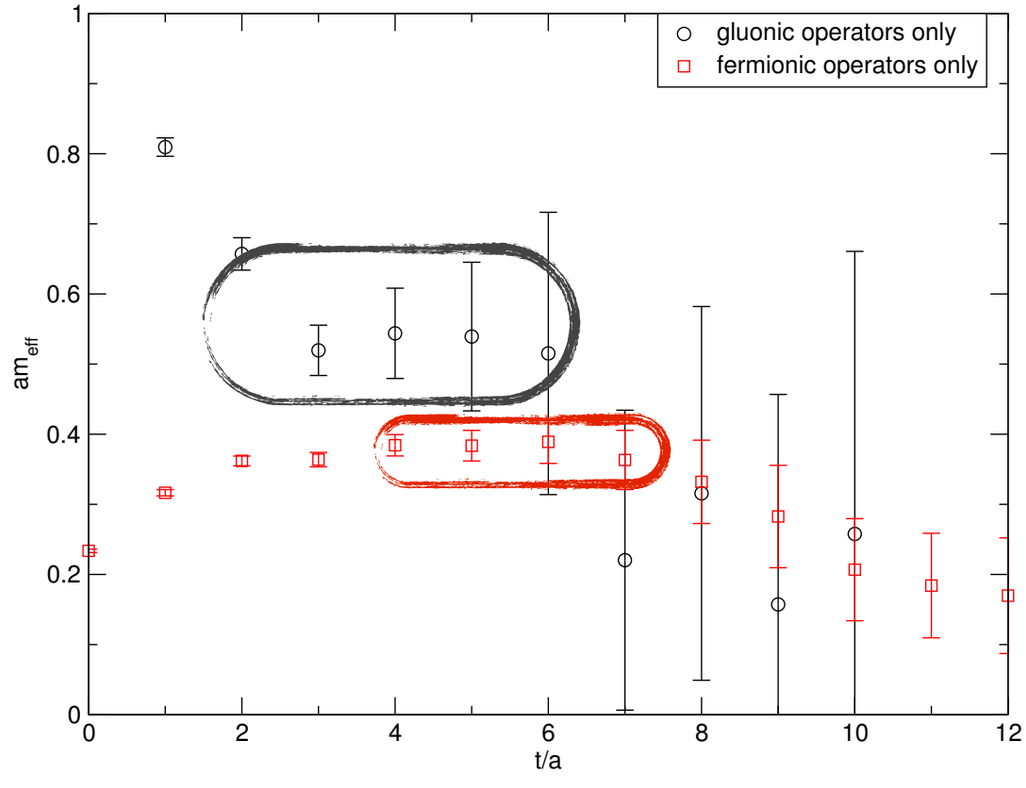
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