

# Eight light flavors on large lattice volumes — — — a USQCD BSM project — — —

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Lattice 2013, Mainz, 29 July



[bsm.physics.yale.edu](http://bsm.physics.yale.edu)

# USBSM participants

Members of USQCD using leadership computing resources  
to study strongly-coupled physics beyond the standard model

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[www.usqcd.org](http://www.usqcd.org)

# USBSM projects

(cf. USQCD white paper)

- **SU(3) with  $N_F = 8$  fundamental**
- Pseudo-dilaton in SU(3) with  $N_F = 2$  sextet
- Scalar pseudo-Goldstones in SU(2) with  $N_F = 2$  fundamental
- Lattice supersymmetry ( $\mathcal{N} = 1$  SYM;  $\mathcal{N} = 4$  SYM;  $\mathcal{N} = 1$  SQCD)

## Argonne Leadership Computing Facility



## SU(3) with $N_F = 8$ fundamental

2008–2010: Deuzeman, Lombardo & Pallante; Jin & Mawhinney;  
Fodor, Holland, Kuti, Nogradi & Schroeder; Hasenfratz  
Boulder, [arXiv:1301.1355](https://arxiv.org/abs/1301.1355) – large mass anomalous dimension  
 $\gamma_m \sim 1$  across wide range of energy scales  
LatKMI, [arXiv:1302.6859](https://arxiv.org/abs/1302.6859) – chirally broken with  $\gamma_m \sim 1$

**Goal:** Large-volume  $p$ -regime lattice ensembles for community use  
Pursue every possible analysis!

This talk (after overview of lattice generation):

- Initial results for the hadron spectrum
- Chiral condensate, GMOR relation, Dirac eigenvalues
- Finite-size scaling
- **Time permitting:** Valence domain wall measurements
- **Backup:** Thermalization, autocorrelations, topological suscept.

# Eight-flavor lattice generation strategy

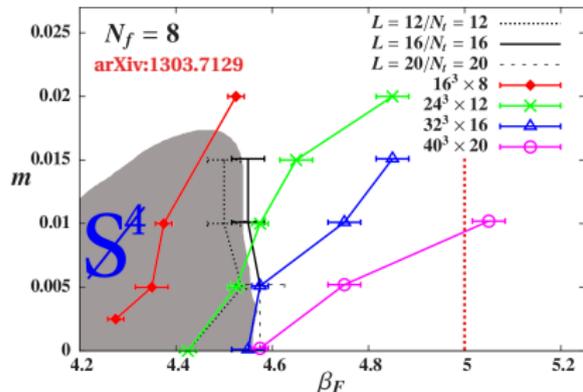
**Goal:** Large-volume  $p$ -regime lattice ensembles for community use

- Fix gauge coupling at relatively strong value
  - Compensate for effects of many light fermions
  - Be cautious of strong-coupling lattice artifacts

nHYP-smearred staggered lattice action

(new since May)

- Fundamental-plaquette  $\beta_F = 5.0$ , adjoint-plaquette  $\beta_A = -1.25$
- Implemented in QHMC/FUEL (“Framework for Unified Evolution of Lattices”)



Lattice phase diagram  
already explored independently

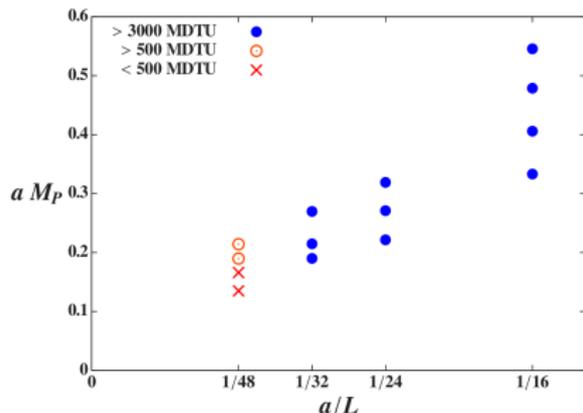
$N_T \leq 20$  thermal transitions  
hit bulk transition around  $\beta_F \approx 4.6$

Chiral limit requires large volumes

# Eight-flavor lattice generation strategy

**Goal:** Large-volume  $p$ -regime lattice ensembles for community use

- Push towards chiral limit on largest possible volumes
  - $\chi$ PT radius of convergence shrinks with  $N_F$  (arXiv:1002.3777)
  - Monitor finite-volume effects from overlapping ranges of masses



Ensembles up to  $32^3 \times 64$  complete,  
 $48^3 \times 96$  in production

Fermion mass  $0.008 \leq m \leq 0.05$ ,  
0.004 and 0.006 in production

Pseudoscalar mass  $0.19 \leq M_P \leq 0.55$ , with  $M_P \approx 0.135$  in production  
( $5.3 \leq M_P L \leq 10.3$ )

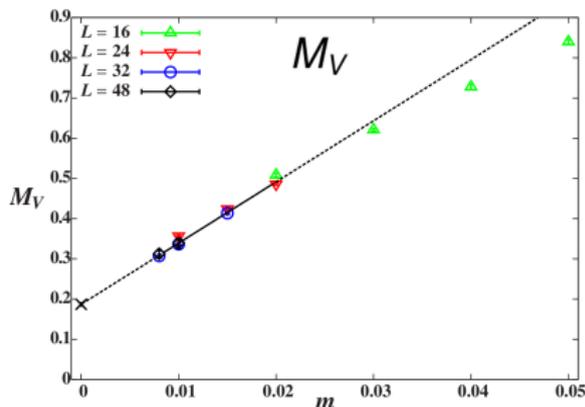
# Initial results for the hadron spectrum

Linear fits mainly to guide the eye

( $M_P L \geq 6$ )

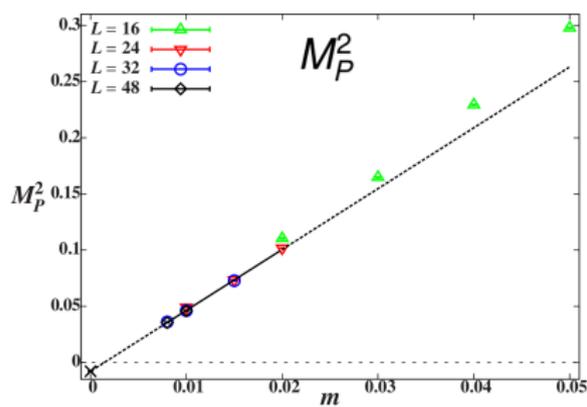
Always fit only  $L \geq 24$  and  $m \leq 0.02$ ,

omitting  $L = 24$  with  $m = 0.01$



Intercept: 0.187(4)

$\chi^2/\text{dof}$ : 18/5

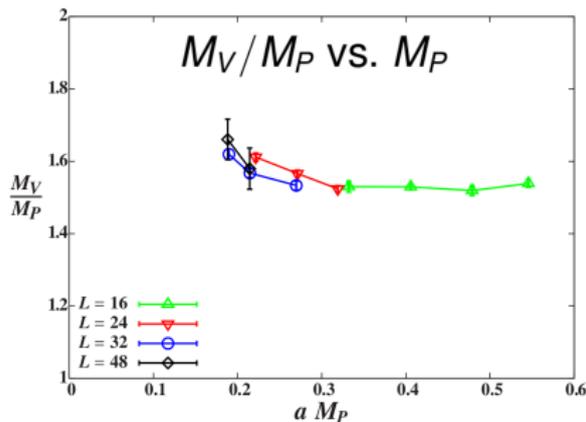
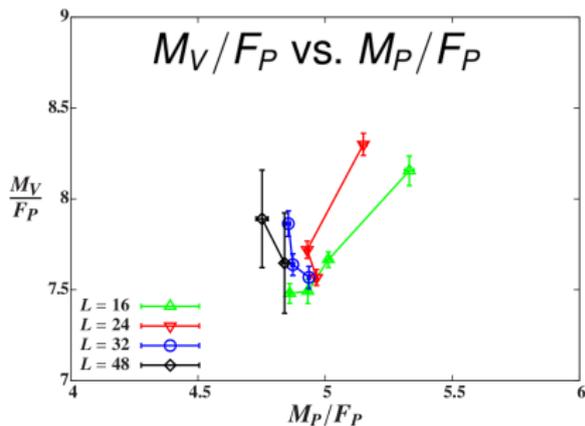


Intercept:  $-0.00793(11)$

$\chi^2/\text{dof}$ : 272/5

- Clear deviations from linearity, especially as  $m$  increases
- Seem unlikely to be due to finite-volume effects or chiral logs...

# More fun with the hadron spectrum



**Left:** Finite-volume effects increase  $M$ , decrease  $F_P$

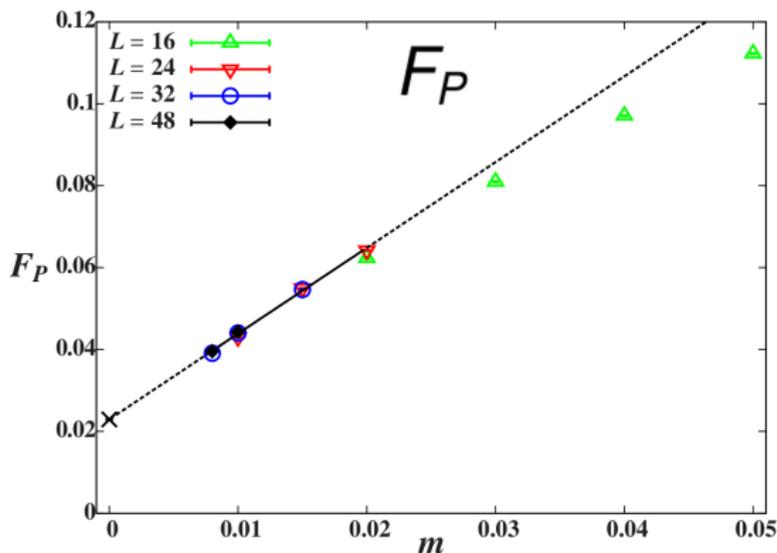
Clearly visible in lightest points for  $L = 16$  and  $24$  (both  $M_P L = 5.3$ ), all other points essentially on top of each other ( $\sim 4\%$  variation)

**Right:** In chirally broken systems,  $M_V/M_P \rightarrow \infty$  as  $M_P \rightarrow 0$

Ratio may be starting to turn up, but not significantly ( $\sim 5\%$  variation)

→ Need  $m \leq 0.006$  ensembles to probe spontaneous  $\chi$ SB

## Initial results for pseudoscalar decay constant



$$1 \leq F_P L \leq 2.1$$

Fitted points:  $F_P L \geq 1.25$

Intercept: 0.0229(7)

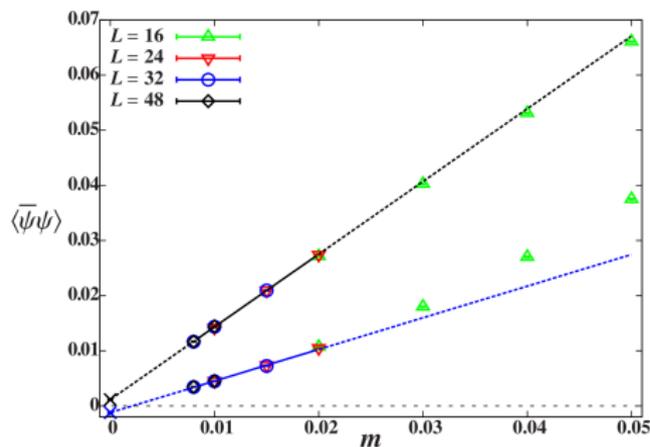
$\chi^2/\text{dof}$ : 391/5

As for  $M_P^2$ , large  $\chi^2$  from  $m \leq 0.02$  linear fit,  
with additional deviation as  $m$  increases

Motivates investigation of chiral condensate...

## Initial results for the chiral condensate

- In chiral limit, order parameter of spontaneous  $\chi$ SB
- Direct measurements sensitive to valence mass in term  $\propto m_v/a^2$
- Leading-order  $\chi$ PT (GMOR relation):  $\langle \bar{\psi}\psi \rangle = M_P^2 F_P^2 / 2m$



$$\langle \bar{\psi}\psi \rangle \text{ and } M_P^2 F_P^2 / 2m$$

Intercepts: 0.001146(18)  
and -0.001223(20)

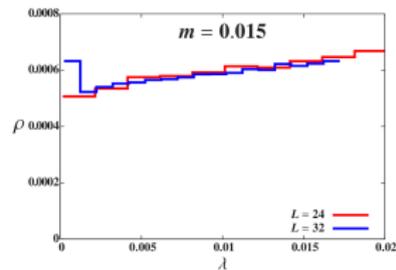
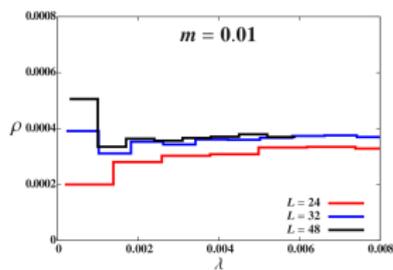
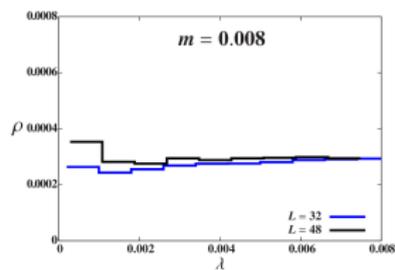
$\chi^2/\text{dof}$ : 51/5 and 285/5

- Direct measurements require order-of-magnitude extrapolation
- GMOR predictions less sensitive to  $m_v$ , fit has larger  $\chi^2$
- Third option: Dirac eigenvalue spectrum  $\rho(\lambda \rightarrow 0)$

# Chiral condensate from Dirac eigenmode number

Address valence mass effects in  $\langle \bar{\psi}\psi \rangle$   
by analyzing the eigenvalues of the massless Dirac operator

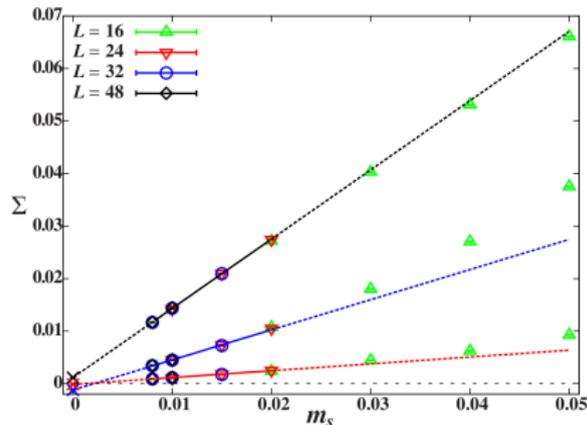
Compare  $\rho(\lambda)$  on different volumes with fixed sea mass:



Good agreement up to expected finite-volume effects,  
and topological zero-mode effects in first bin

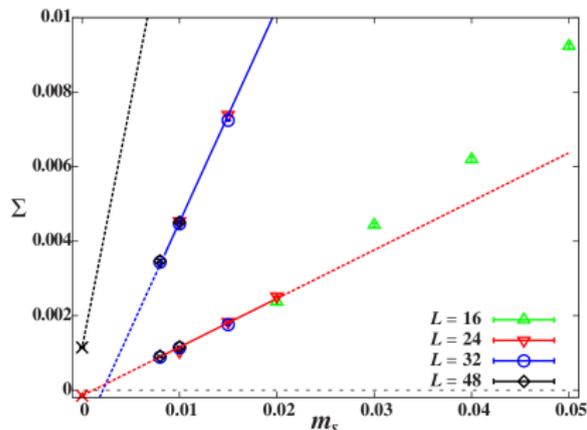
Extract  $\Sigma_{m_s} \equiv \pi \rho(\lambda \rightarrow 0)$  from derivative of mode number  $\nu \sim \int \rho d\lambda$

# Chiral condensate from all three approaches



Recall: 0.001146(18)  
and -0.001223(20)

$\chi^2/\text{dof}$ : 51/5 and 285/5



**Zoom in on  $\Sigma$**

Intercept: -0.000151(16)

$\chi^2/\text{dof}$ : 32/5

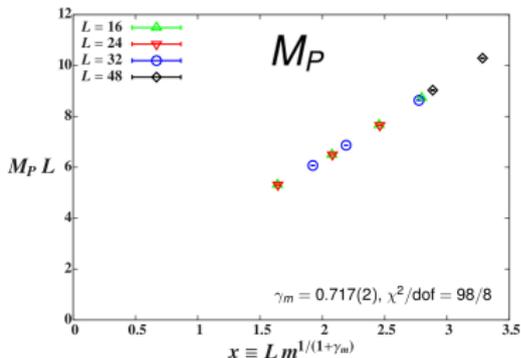
- $\Sigma$  from eigenvalues seems to be the best controlled
- $\lim_{m_s \rightarrow 0} \langle \bar{\psi}\psi \rangle$  is small or vanishing — motivates finite-size scaling...

(Future fun with the eigenmode number:

extract running mass anomalous dimension – Anqi Cheng, 11:40 Weds.)

## Initial results for finite-size scaling

- IR conformality  $\implies ML = f(x)$  with scaling variable  $x \equiv Lm^{1/(1+\gamma_m)}$
- Search for anomalous dimension  $\gamma_m$  that optimizes curve collapse
- I use method of Houdayer and Hartmann, [cond-mat/0402036](https://arxiv.org/abs/cond-mat/0402036)



Relatively small number of points for FSS

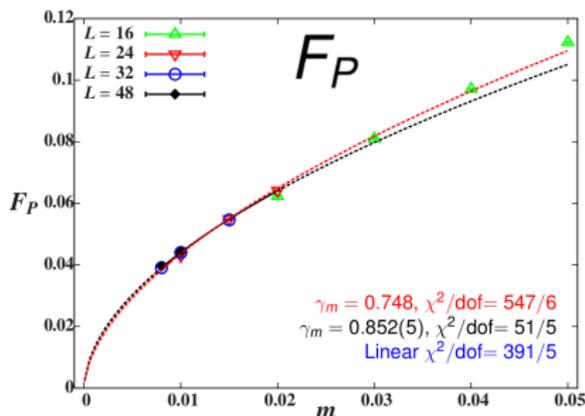
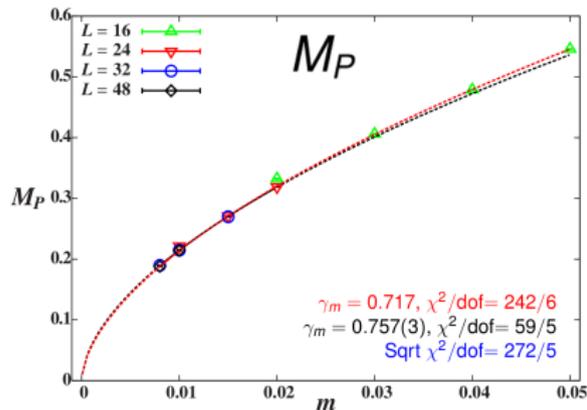
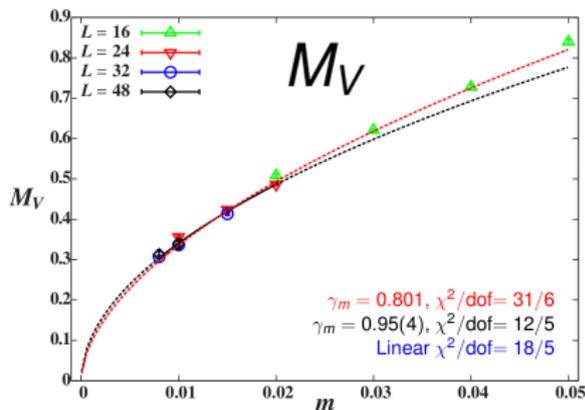
Obs.	$\gamma_m$	$\chi^2/\text{dof}$
$M_P$	0.717(2)	98/8
$M_V$	0.801(14)	40/8
$F_P$	0.748(2)	1444/8

- Roughly consistent  $\gamma_m \sim 0.75$ , though widely-varying quality
- Try using these  $\gamma_m$  in power-law fits to hadron spectrum...

(Future fun with finite-size scaling:

account for nearly-marginal gauge coupling – Anna Hasenfratz, 14:20 Tues.)

# Revisit hadron spectrum: Power-law fits



Same seven points in fits

Red fits fix  $\gamma_m$  from FSS

Black fits let  $\gamma_m$  float

( $\gamma_m$  always increases)

$\chi^2/\text{dof}$  smaller than linear fits,

$\gamma_m \gtrsim 0.75$  preferred

# Motivation for valence domain wall analyses

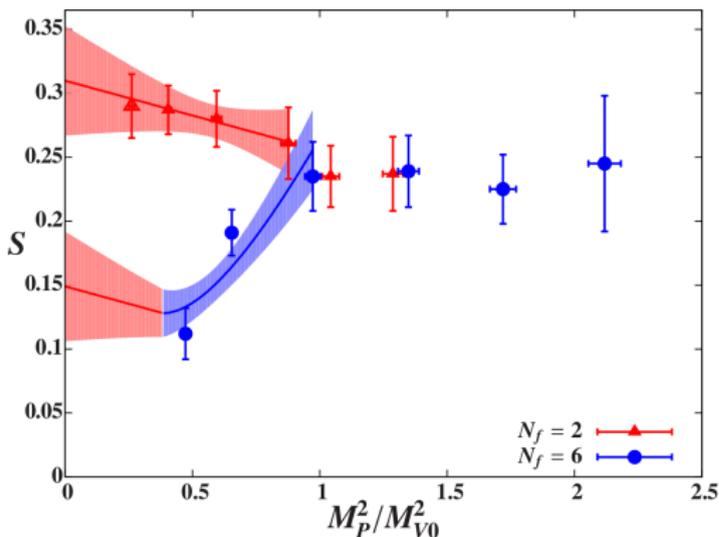
Experiment requires small electroweak  $S$  parameter,

$$S = 0.03(10) \text{ with } M_H = 125 \text{ GeV}$$

Lattice Strong Dynamics Collaboration found reduction in  $S$  for  $N_F = 6$

(PRL **106**:231601, 2011)

Important observable to explore for  $N_F = 8$



LSD analysis:  $32^3 \times 64$   
domain wall fermions  
( $M_H^{(ref)} \sim 1 \text{ TeV}$ )

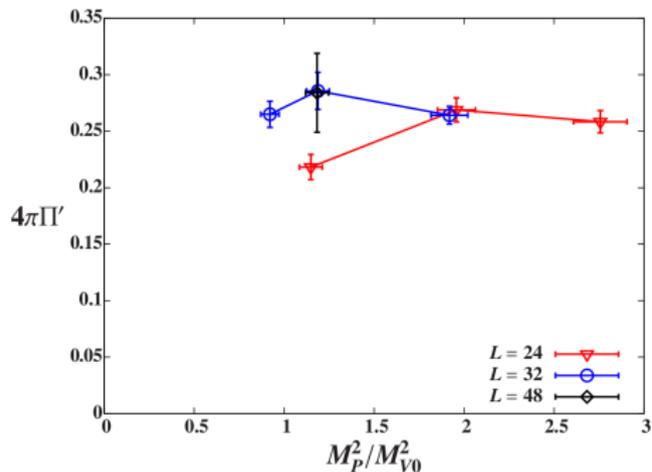
Want to make sure  
decrease is not  
finite-volume effect

# Initial results for $V-A$ vacuum polarization

$S$  parameter depends on  $Q^2 \rightarrow 0$  slope of transverse  $\Pi_{V-A}$

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu b}(0) \rangle - \langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu b}(0) \rangle \right]$$



We've already seen  
clear finite-volume effects  
in lightest  $24^3 \times 48$  point

$N_F = 6$  reduction began  
only for  $M_P^2 \lesssim M_{V0}^2$

Looking forward to smaller masses on  $48^3 \times 96$  !

# Recapitulation: SU(3) with $N_F = 8$ fundamental

This USBSM project is off to a good start!

**Goal:** Large-volume  $p$ -regime lattice ensembles for community use

- Ensembles up to  $48^3 \times 96$  becoming available
- Initial analyses suggest small or vanishing chiral condensate
- Finite-size scaling prefers large  $\gamma_m \gtrsim 0.7$
- Prospects for  $S$  parameter from valence domain wall

Looking forward to more fun in the future

- Running mass anomalous dimension from eigenmode number
- Unitary staggered analysis of vacuum polarization
- Other USBSM projects: light scalars; lattice supersymmetry



# Thank you!

# Thank you!

## Contributors to this talk

George Fleming, Anna Hasenfratz, Meifeng Lin, Ethan Neil, James Osborn

## The rest of the USBSM community

Tom Appelquist, Rich Brower, Mike Buchoff, Simon Catterall, Michael Cheng, Joel Giedt, Kieran Holland, Joe Kiskis, Julius Kuti, Heechang Na, Gregory Petropoulos, Claudio Rebbi, Chris Schroeder, Don Sinclair, Gennady Voronov, Pavlos Vranas, Oliver Witzel



## Backup: Status of ensemble generation

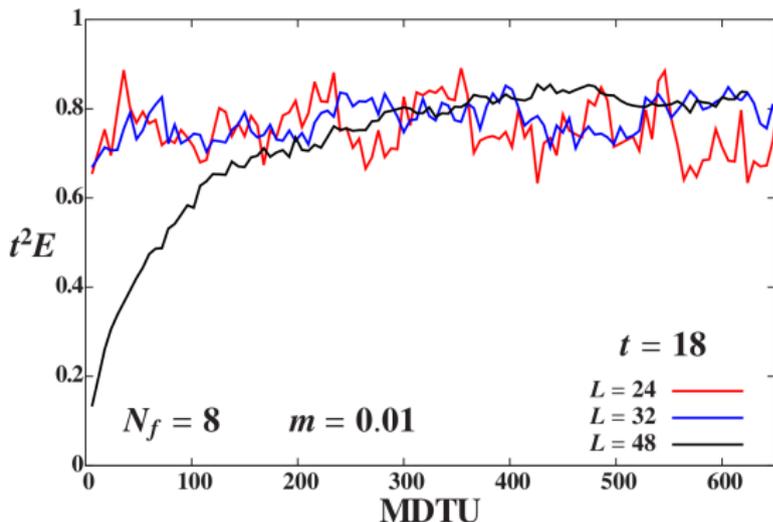
Many-flavor lattice systems may have long autocorrelations

Expect observables related to topology to be sensitive (arXiv:1204.6000)

Wilson flow to monitor thermalization, autocorrelations

Integrate infinitesimal stout smearing steps out to flow time  $t$

with  $\sqrt{8t}$  comparable to  $L/2$



$$E(t) = -\frac{1}{2} \text{ReTr} F_{\mu\nu} F^{\mu\nu}(t)$$

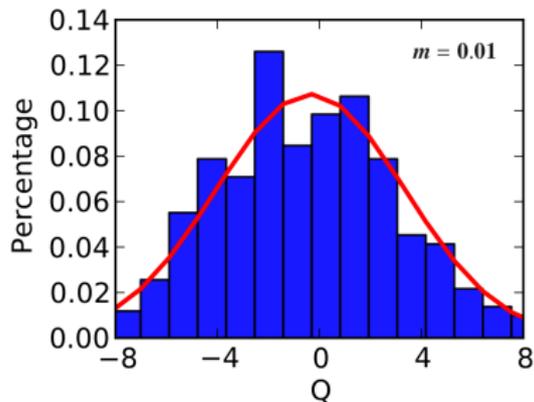
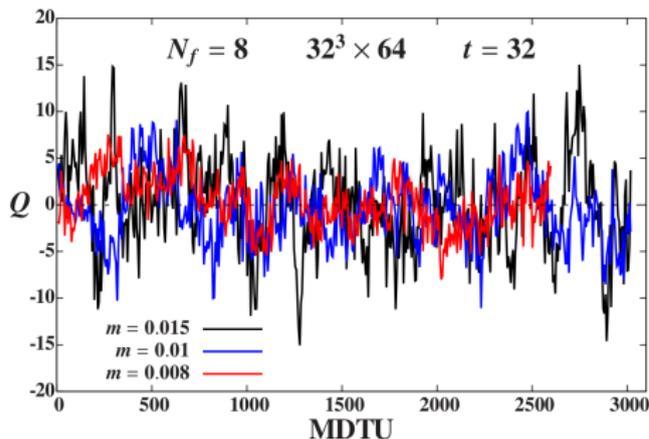
$$\sqrt{8t}/24 = 0.5$$

$$\sqrt{8t}/32 = 0.375$$

$$\sqrt{8t}/48 = 0.25$$

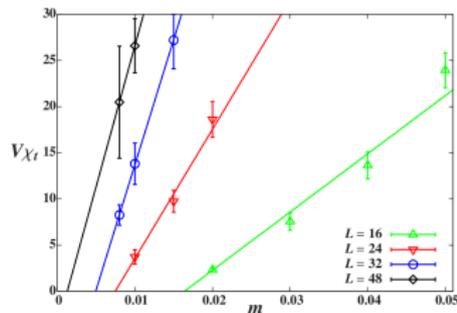
# Backup: Topo. charge and $\chi_t$ from the Wilson flow

$$32\pi^2 Q = \text{ReTr} [\epsilon_{\mu\nu\sigma\tau} F_{\mu\nu} F^{\sigma\tau}] \text{ after flowing to } \sqrt{8t} = L/2$$



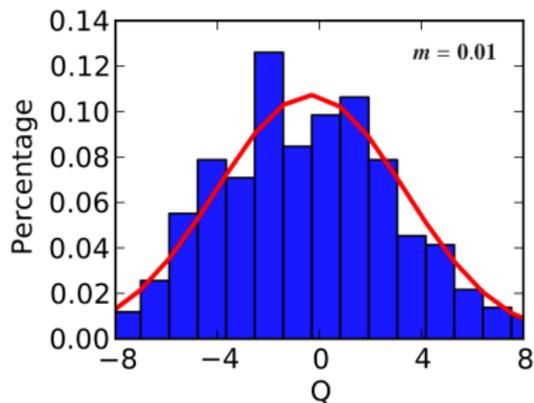
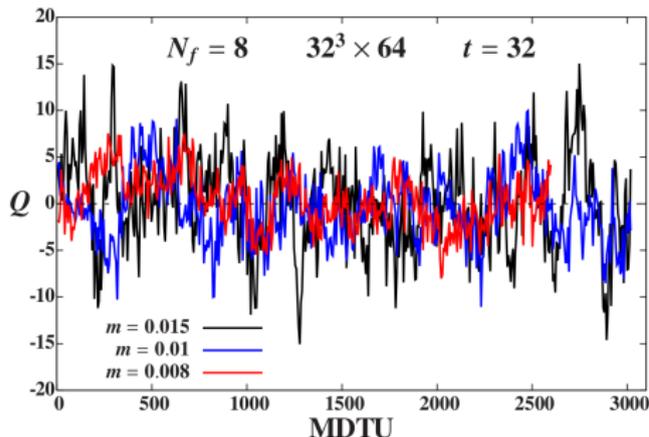
$m$ -dependence of topological suscept.

Linear  $m \rightarrow 0$  extrapolations go negative



# Backup: Topo. charge and $\chi_t$ from the Wilson flow

$$32\pi^2 Q = \text{ReTr}[\epsilon_{\mu\nu\sigma\tau} F_{\mu\nu} F^{\sigma\tau}] \text{ after flowing to } \sqrt{8t} = L/2$$

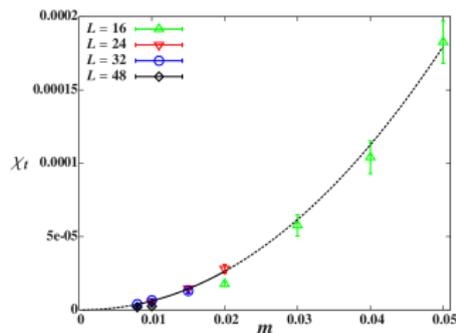


## $m$ -dependence of topological suscept.

For IR-conformal system,  $\chi_t \propto m^4/(1+\gamma_m)$

Neglecting poorly-determined  $L = 48$ ,

$$\gamma_m = 0.91(16) \text{ with } \chi^2/\text{dof} = 1.5/3$$

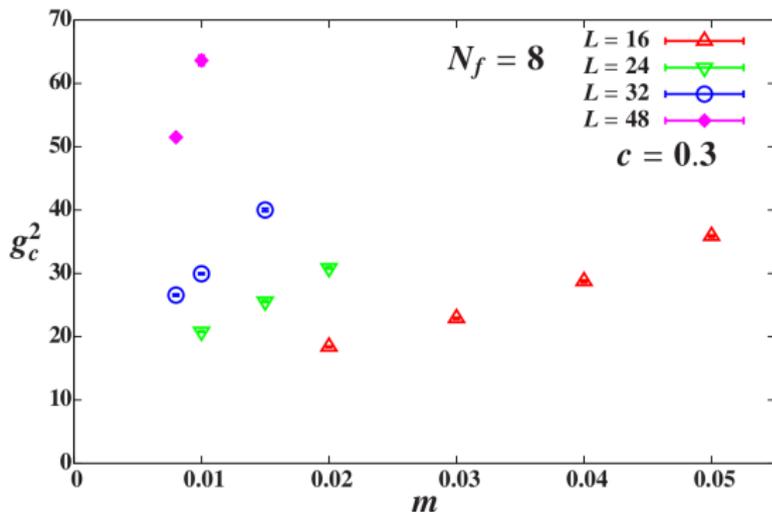


# Backup: Wilson flow running coupling (arXiv:1208.1051)

Z. Fodor, K. Holland, J. Kuti, D. Negradi and C. H. Wong

define SU( $N$ ) running coupling from Wilson flow  $\langle t^2 E(t) \rangle$ :

$$g_c^2(L) = \frac{128\pi^2 \langle t^2 E(t) \rangle}{3(N^2 - 1)(1 + \delta_c)}$$



$$c \equiv \sqrt{8t}/L = 0.3$$

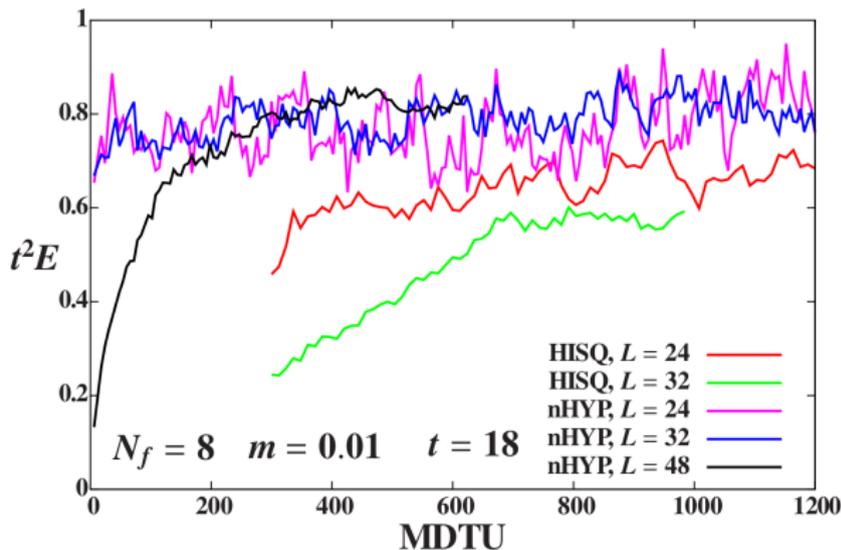
$$1 + \delta_c \approx 0.97$$

$$g_c^2 \sim \mathcal{O}(10)$$

Sensitive to non-zero fermion mass as well as lattice volume

## Backup: Trouble with HISQ at strong coupling

Our new nHYP action is **orders of magnitude** faster than HISQ  
at comparably strong couplings



As above,  
Wilson flow  $t^2 E$   
as measure of  
thermalization

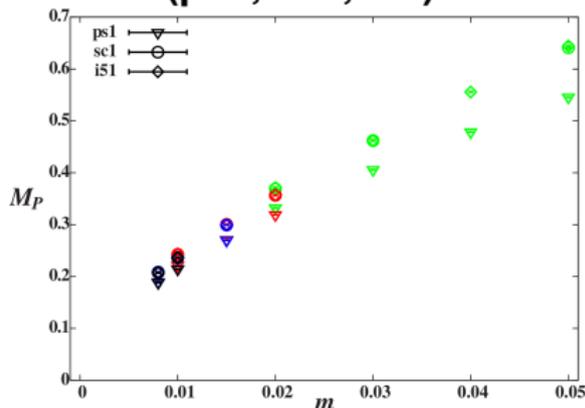
HISQ strong-coupling numerical instabilities  $\rightarrow$  short trajectories  
 $\rightarrow$  long thermalization/autocorrelation times

# Backup: nHYP taste splitting

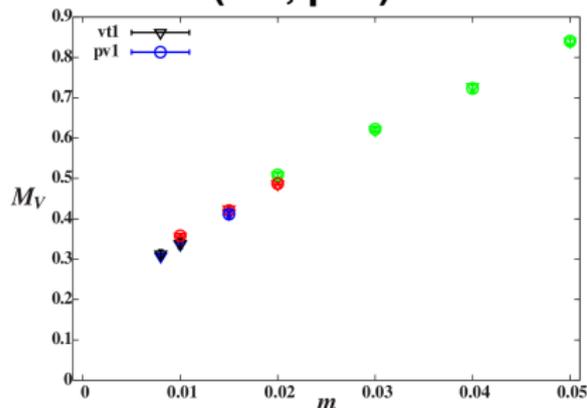
The nHYP-smeared action

exhibits excellent control over staggered taste splitting

**Pseudoscalar states**  
(ps1, sc1, i51)



**Vector states**  
(vt1, pv1)



These are very preliminary results;

pv1 masses are not yet determined for some  $L = 32$  and 48 runs

## Backup: Valence domain wall procedure (LHPC, arXiv:0705.4295)

- HYP smear to reduce  $m_{res}$  and get renormalization factors  $Z \sim 1$
- Tune domain wall height  $M_5$  and length  $L_5$  of fifth direction so that residual chiral symmetry breaking  $m_{res} \ll m$
- Tune bare valence mass  $m_f$  so that  $M_P$  matches unitary value

$$M_5 = 1.8 \text{ and } L_5 = 16$$

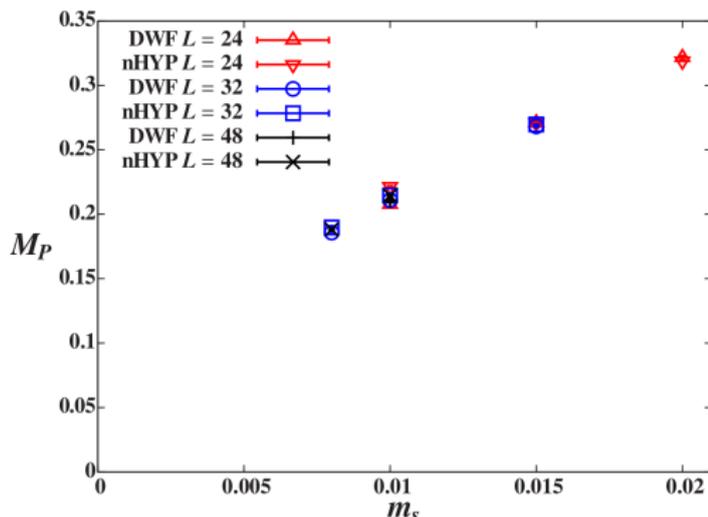
$$\rightarrow Z_V \approx Z_A \approx 1.08$$

$$\rightarrow m_{res} \approx 0.001 \lesssim m_f/13$$

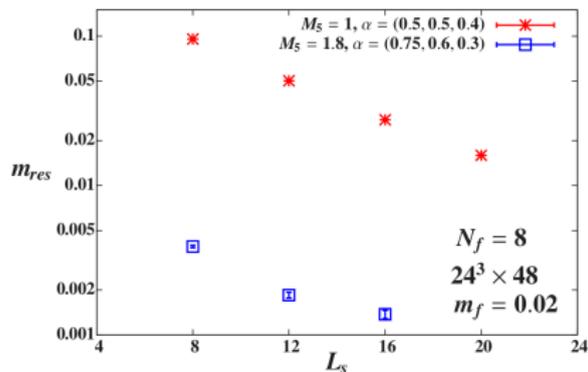
Need  $m_f > m_s$  to match  $M_P$ :

$$1.7 \lesssim m/m_s \lesssim 2.05$$

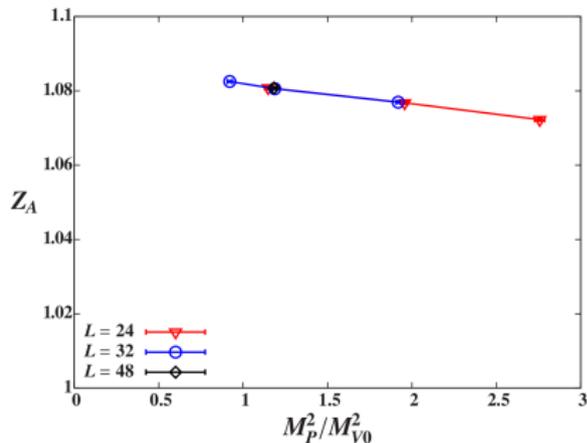
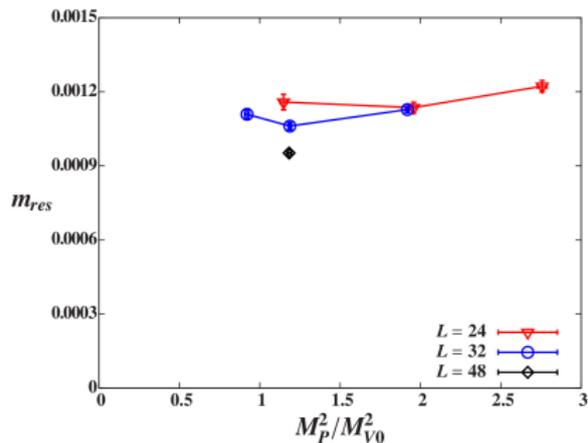
where  $m \equiv m_f + m_{res}$



# Backup: Valence domain wall $m_{res}$ and $Z_A$



- Want  $m_{res} \ll m$
- Want  $Z_A \sim 1$



Constrain the physics of electroweak symmetry breaking  
from its effects on vacuum polarizations  $\Pi(Q)$  of EW gauge bosons



(independent of flavor physics/ETC)

$$S = 4\pi N_D \lim_{Q^2 \rightarrow 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H)$$

$$\textcircled{1} \quad \Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \langle \mathcal{V}^{\mu a}(x) \mathcal{V}^{\nu b}(0) \rangle - \langle \mathcal{A}^{\mu a}(x) \mathcal{A}^{\nu b}(0) \rangle \right]$$

$\textcircled{2}$   $N_D \geq 1$  is the number of doublets with chiral electroweak couplings

$\textcircled{3}$   $\Delta S_{SM}(M_H)$  subtracted so that  $S = 0$  in the standard model

Removes three eaten modes, depends on Higgs mass