Lattice study on chiral dynamics of two-color six-flavors QCD

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collaborated with

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LATTICE 2013 in Mainz, July 29, 2013

SU(2)$_C$ and EW Breaking

- The motivation of this study is to search the gauge system which breaks EW symmetry, so-called technicolor.
- We focus here on a series of $\mathbf{SU(2)_C}$ gauge theories.

- $\mathbf{SU(2)_C}$ gauge theory is one of $\mathbf{SP(\boldsymbol{N})}$ gauge theories, not $\mathbf{SU(\boldsymbol{N})}$.
- Fundamental representation is pseudo-real.
- Chiral symmetry is enhanced to $\mathbf{SU(2\boldsymbol{N_f})}$.
- Plausible breaking pattern is $\mathbf{SU(2\boldsymbol{N_f}) \rightarrow SP(2\boldsymbol{N_f})}$.
- The chiral dynamics of two-color QCD is different from those of three-color QCD.

- From the point of view of the application to the dynamical realization of the EW breaking, the effective Higgs sector of $\mathbf{SU(2)_C}$ is different from those of $\mathbf{SU(\boldsymbol{N_C})_C} \quad (\boldsymbol{N_C} \geq \mathbf{3})$.
- This fact also motivates us to perform lattice simulation to grasp the properties of its quantum-mechanical dynamics such as spectra of bound states.

Why $N_f = 6$?

☆ What N_f^{crt} for $\mathrm{SU}(2)_C$?

- To break EW symmetry, it needs the chiral symmetry breaking.
- N_f^{crt} is the largest N_f where the system breaks the chiral symmetry.

➢ Perturbative approaches suggest $N_f^{\mathrm{crt}} = 6 \sim 9$

➢ Some study by other groups

- Phase structure of Wilson fermions

  Y. Iwasaki, et.al., Phys. Rev. D69(2004)014507 [arXiv:hep-lat/0309159]

  ⇒ $N_f = 3$ is conformal in the IR limit.

- Running coupling constant for $N_f = 6$ defined under SF boundary condition

  F. Bursa, et.al., Phys. lett. B696(2011)374 arXiv:1007.3067[hep-lat]

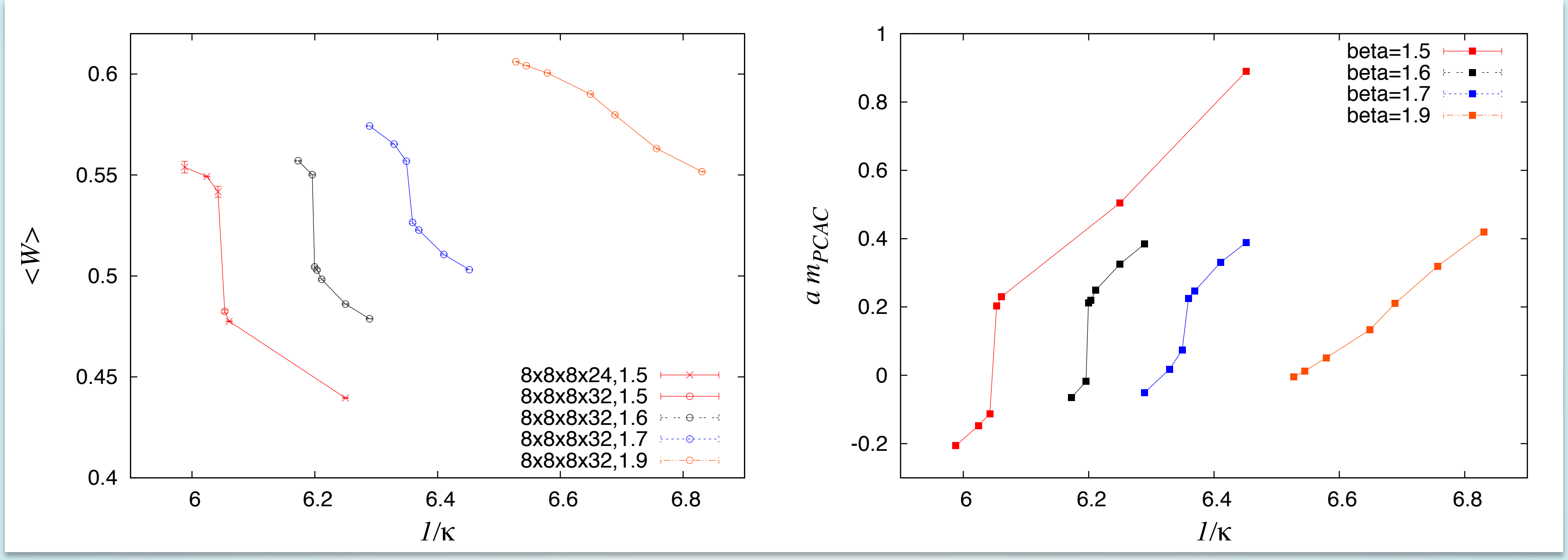
  ⇒ $N_f = 6$ is conformal in IR limit.

➢ We report our preliminary results about chiral dynamics of gauge system with two-color and six-flavors by showing the quark mass dependence of some quantities.

Phase Structure

☆ What β should we choose?

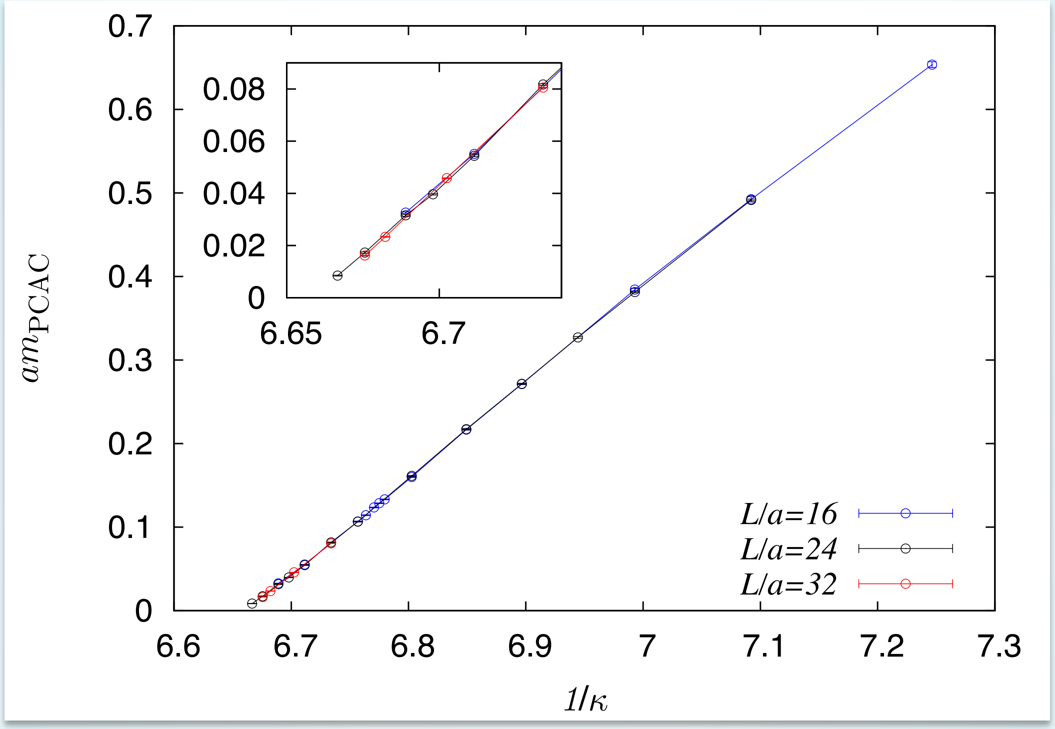
- We studied phase structure of 6 Wilson fermions by surveying κ-dependence of averaged plaquettes $\langle W \rangle$ and PCAC mass m_{PCAC} for some β ; 1.5 to 1.9



- We chose $\beta = 2.0$ where the system seems not to occur the bulk phase transition.

PCAC Mass for $\beta = 2.0$

This is PCAC mass for $\beta = 2.0$ as a function of $1/\kappa$

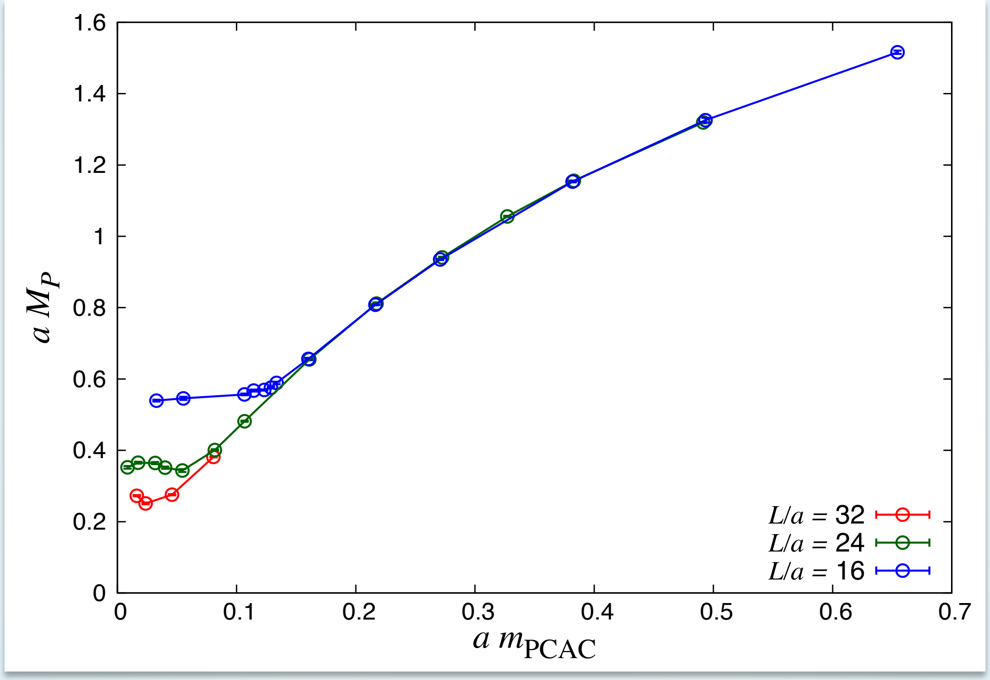


- There is no visible volume dependence.
- It depends linearly (and monotonically) on $1/\kappa$.

➢ We can convert the dependence of various observable on κ to that on PCAC mass, which is often referred to as the quark mass.

Mass of P-channel

We first survey the quark mass dependence of the mass of the lightest pseudoscalar meson, M_P.

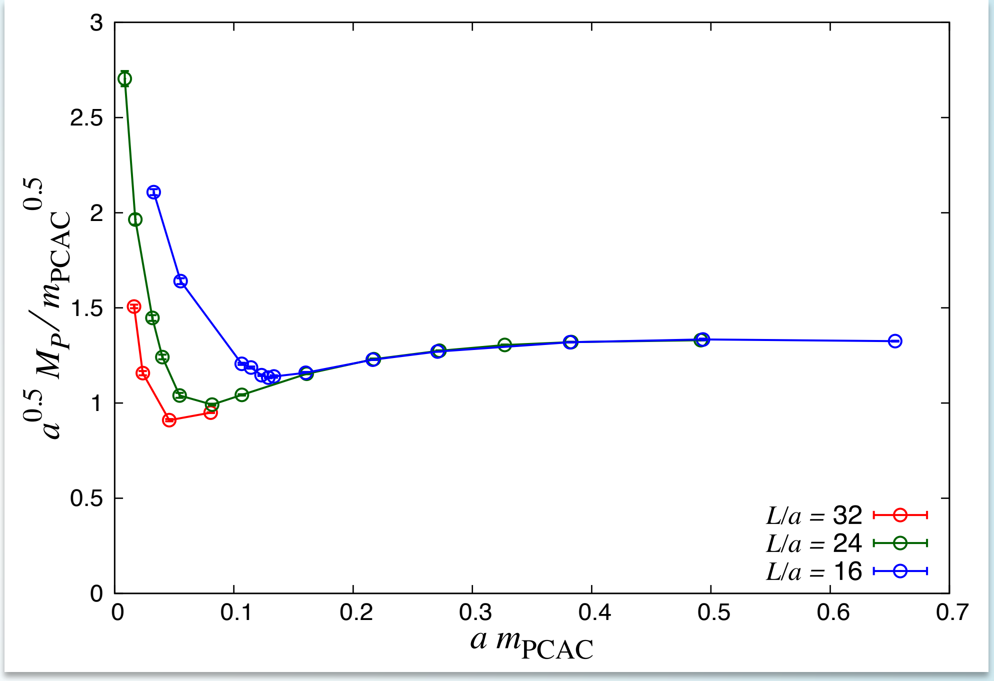


☆ The mass is bounded from below and cannot approach to 0 for $m_{\mathrm{PCAC}} \to 0$ because of the finite size effect (FSE)

Mass of P-channel

The quark mass dependence of $a^{0.5} M_P / \sqrt{m_{\rm PCAC}}$

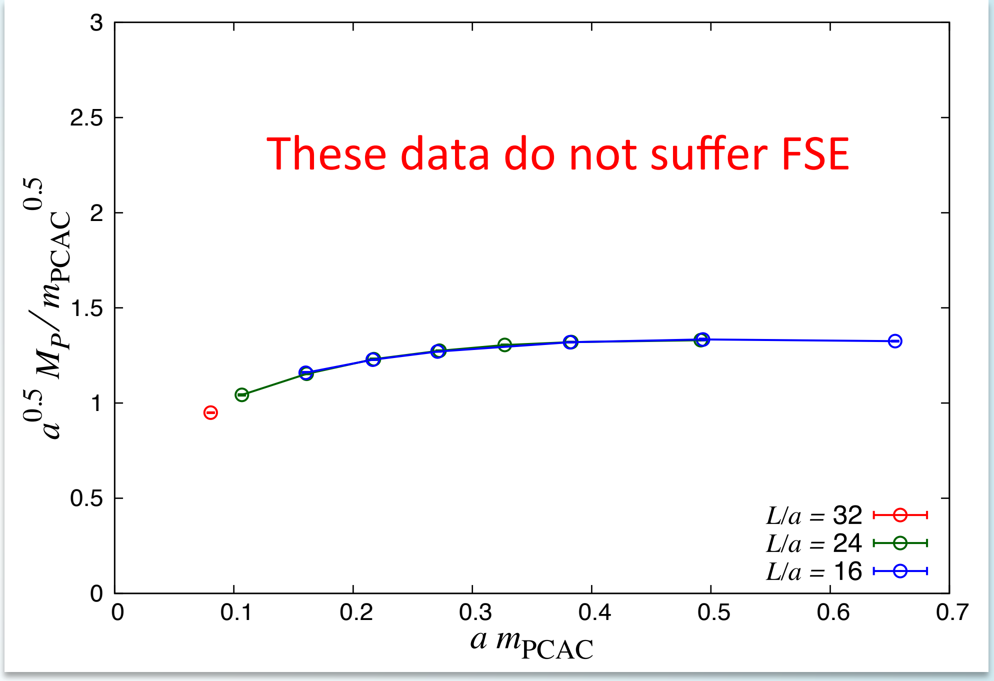
- It starts to increase at some $m_{\rm PCAC}$ depending on L/a, and diverges in the chiral limit due to FSE on M_P.
  We ignore these data.



Mass of P-channel

The quark mass dependence of $a^{0.5}M_P/\sqrt{m_{\mathrm{PCAC}}}$

- It starts to increase at some m_{PCAC} depending on L/a, and diverges in the chiral limit due to FSE on M_P. We ignore these data.
- It implies $M_P \propto \sqrt{m_{\mathrm{PCAC}}}$ in the region $am_{\mathrm{PCAC}} \gtrsim 0.35$.

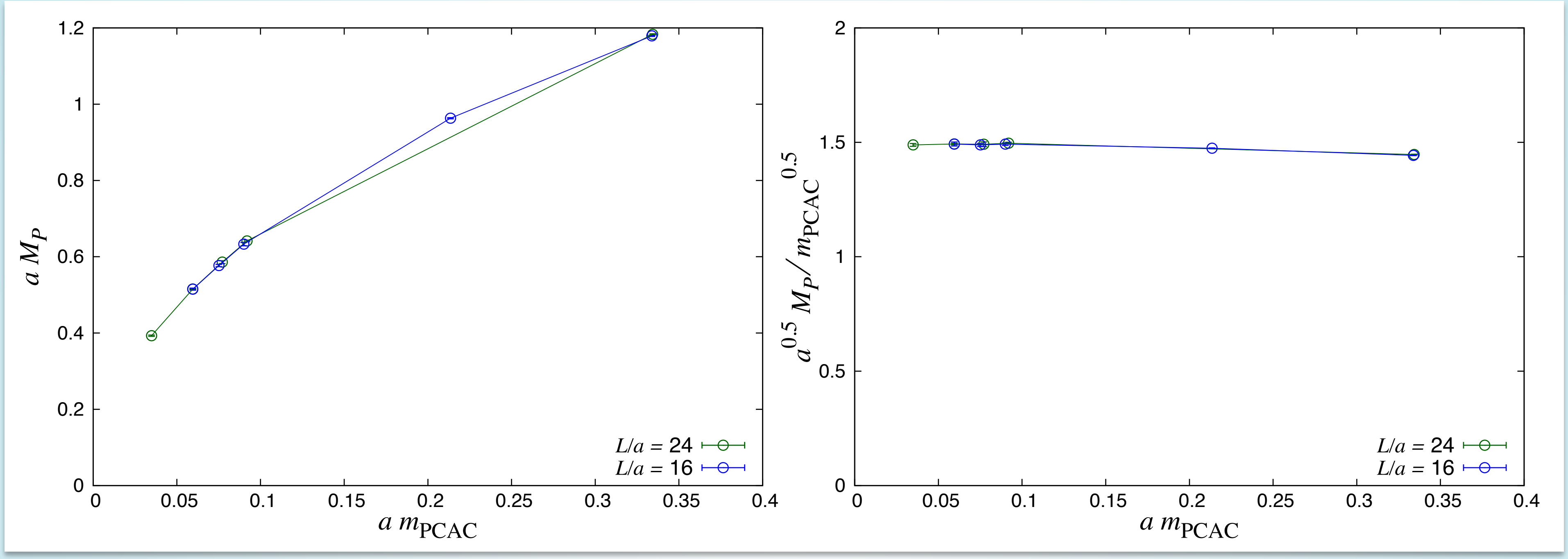


- $M_P \not\propto \sqrt{m_{\mathrm{PCAC}}}$ in the region $am_{\mathrm{PCAC}} \lesssim 0.3$.
- If the system breaks the chiral symmetry
  $\Rightarrow$ $M_P \propto \sqrt{m_q}$ for smaller m_q, predicted by ChPT.

➢ It implies the system does not break the chiral symmetry.

$\Rightarrow$ conformal dynamics in the IR limit!

Mass of P-channel



The quark mass dependence of aM_P (left panel), $a^{0.5}M_P/\sqrt{m_{\mathrm{PCAC}}}$ (right panel) in two-flavor theory

- All data seem not to suffer FSE.
- The value of $a^{0.5}M_P/\sqrt{m_{\mathrm{PCAC}}}$ is approximately constant.

$\Rightarrow$ This result is compatible with the expectation from ChPT.

Decay Constant

- Next, we focus on the decay constant f_P of the lightest pseudo-scalar meson.
- We define f_P by considering PCAC relation :

$$f_P = \frac{2\kappa \cdot 2m_{\mathrm{PCAC}}}{\sinh M_P}\sqrt{\frac{2A^{PP}}{M_P}}$$

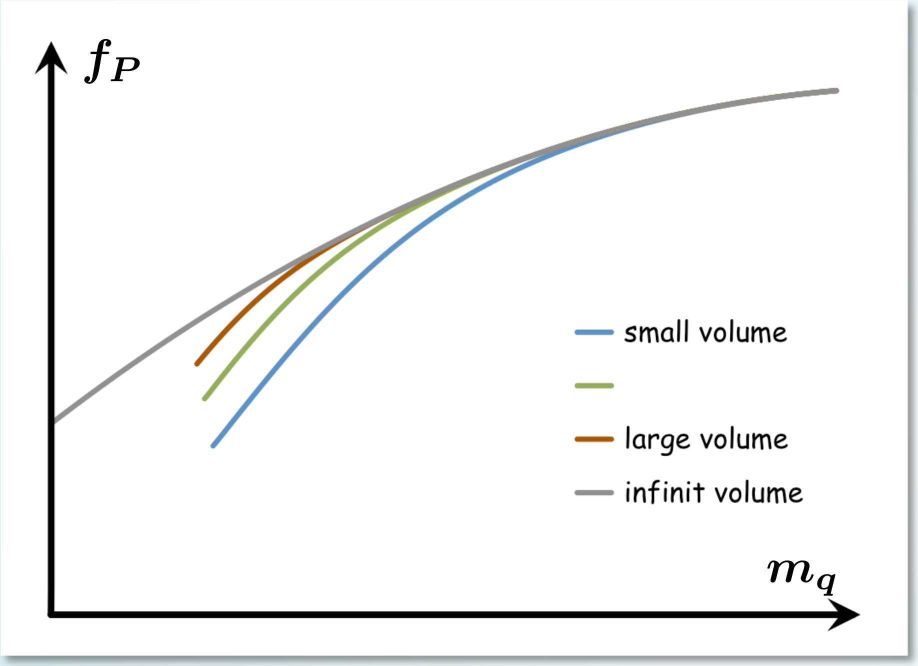
A^{PP} is amplitude of space summed PP correlator :

$$\langle P(t)P(0)\rangle \xrightarrow{\text{large } t} A^{PP}\left(e^{-M_P t} - e^{-M_P(T-t)}\right)$$

- According to this definition, f_P must vanish in the massless quark limit even if the system breaks the chiral symmetry because of the finite size effect on M_P.

Decay Constant

- If the system breaks the chiral symmetry, FSE on f_P behaves like below, expected by ChPT.



In the case of $\cancel{\chi}$,
FSE decreases f_P

- This FSE is demonstrated for the case of $\mathbf{SU(3)_C, SU(4)_C, \cdots}$

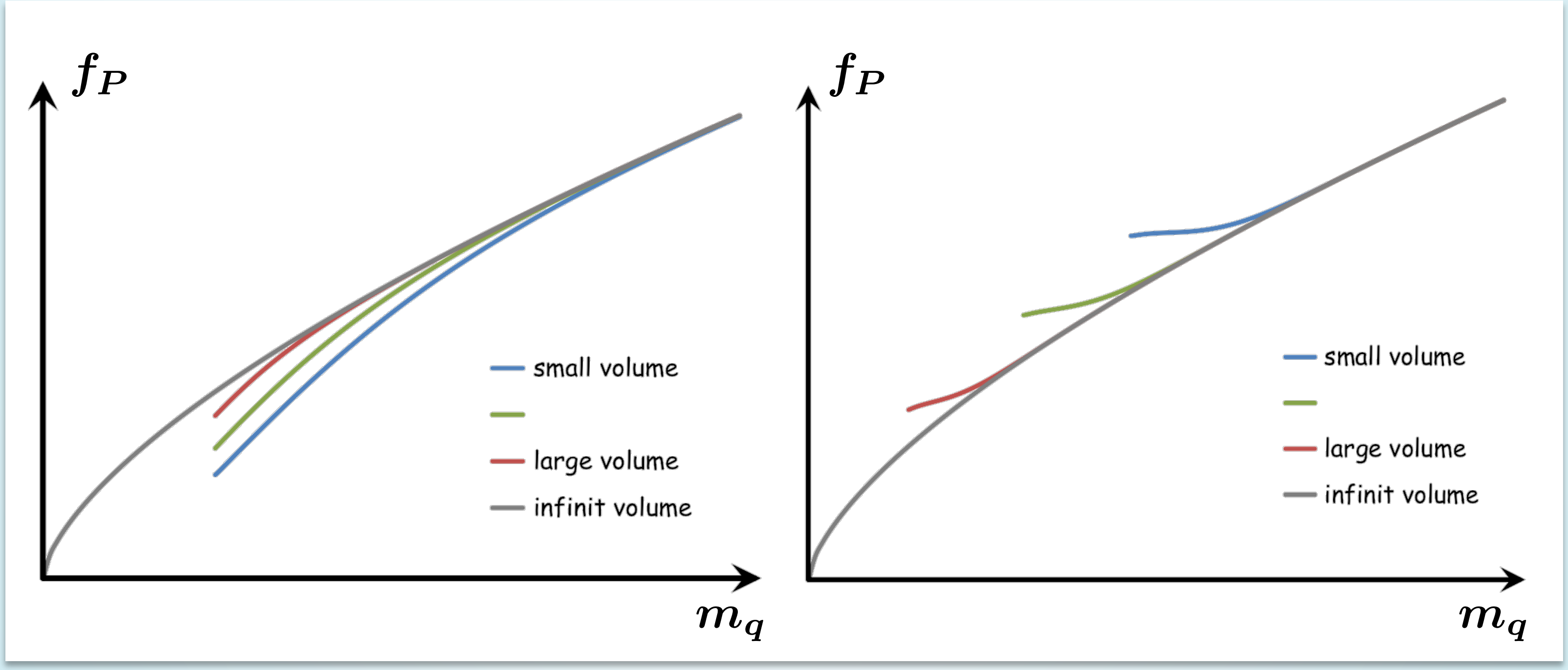
Gilberto Colangelo, Nucl. Phys. B (Proc. Suppl.) 140 (2005) 120

S. Aoki and H. Fukaya, Phys. Rev. D84 (2011) 014501

- We presume this FSE is true even for $\mathbf{SU(2)_C}$.

Decay Constant

- For the conformal dynamics with IR fixed point, there are two possibilities about FSE on f_P. One decreases f_P and another increases f_P.



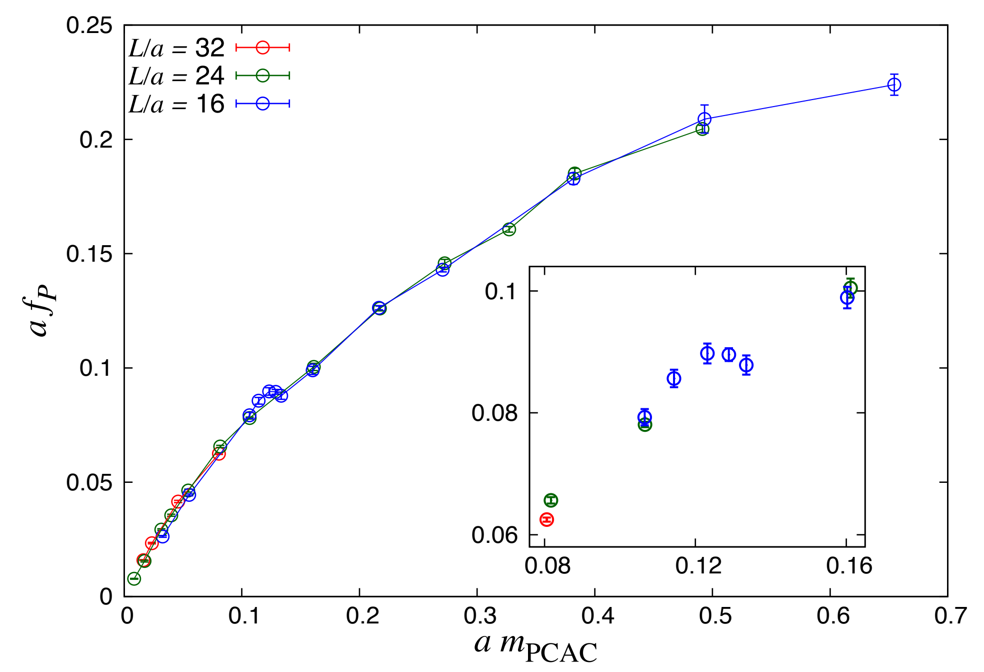
- If we observed FSE as right panel, we can judge the system is conformal dynamics in IR limit.

Decay Constant

The quark mass dependence of af_P

$$f_P = \frac{2\kappa \cdot 2m_{\rm PCAC}}{\sinh M_P}\sqrt{\frac{2A^{PP}}{M_P}}$$

- In this definition, FSE on M_P decreases f_P.
- But, the data of $L/a = 16$ around $am_{\rm PCAC} \simeq 0.12$ are seen to be increased by the other FSE.



- This FSE is seen to be FSE on f_P itself.
- This FSE is opposite to the expectation in χ case.

➢ It also implies the system does not break the chiral symmetry.

⇒ conformal dynamics in IR limit.

Subtracted Chiral Condensate

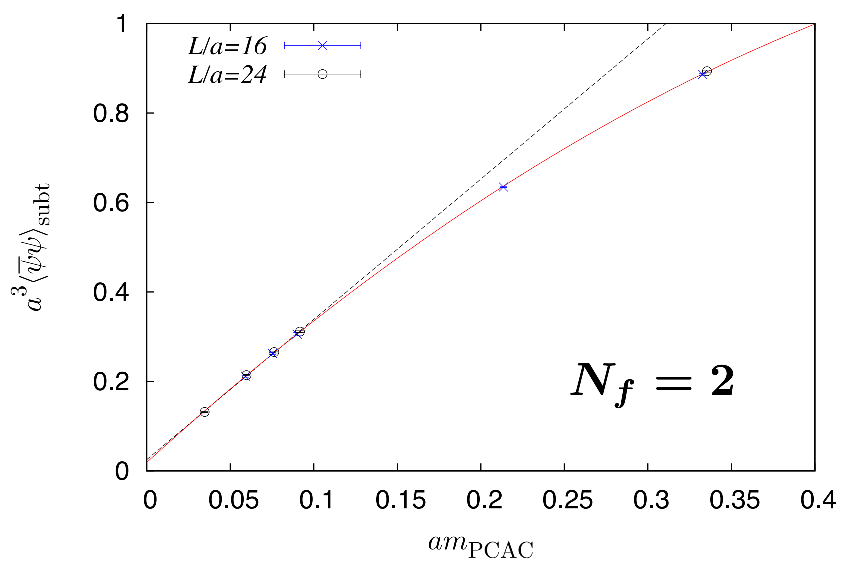
- Lastly, we focus on the order parameter of chiral symmetry breaking “chiral condensate”.
- Since we use Wilson fermion, VEV of $\overline{\psi}\psi$ suffers hard UV divergence $\sim O(a^{-3})$.

⇒ We employ “subtracted chiral condensate”
calculated by time-space summed Word-Takahashi identity:

$$\langle\overline{\psi}\psi\rangle_{\mathrm{subt}} = 2m_{\mathrm{PCAC}} \cdot (2\kappa)^2 \sum_t \langle P(t)P(0)\rangle$$

Subtracted Chiral Condensate

- We survey the behavior of $\langle\bar{\psi}\psi\rangle_{\mathbf{subt}}$ in two-flavor theory



- The data in the figure do not suffer the visible FSE.
- By extrapolating from these data, we confirm the chiral limit of $a^3\langle\bar{\psi}\psi\rangle_{\mathbf{subt}}$ is nonzero and well below $O(1)$ while chiral limit of ordinal condensate of Wilson fermion $a^3\langle\bar{\psi}\psi\rangle$ is $O(1)$ in lattice unit.

➢ We judge $\langle\bar{\psi}\psi\rangle_{\mathbf{subt}}$ can be regarded to as the counterpart of the chiral condensate.

Subtracted Chiral Condensate

- We extrapolate the chiral limit of $\langle\overline{\psi}\psi\rangle_{\mathbf{subt}}$ by performing various fit.

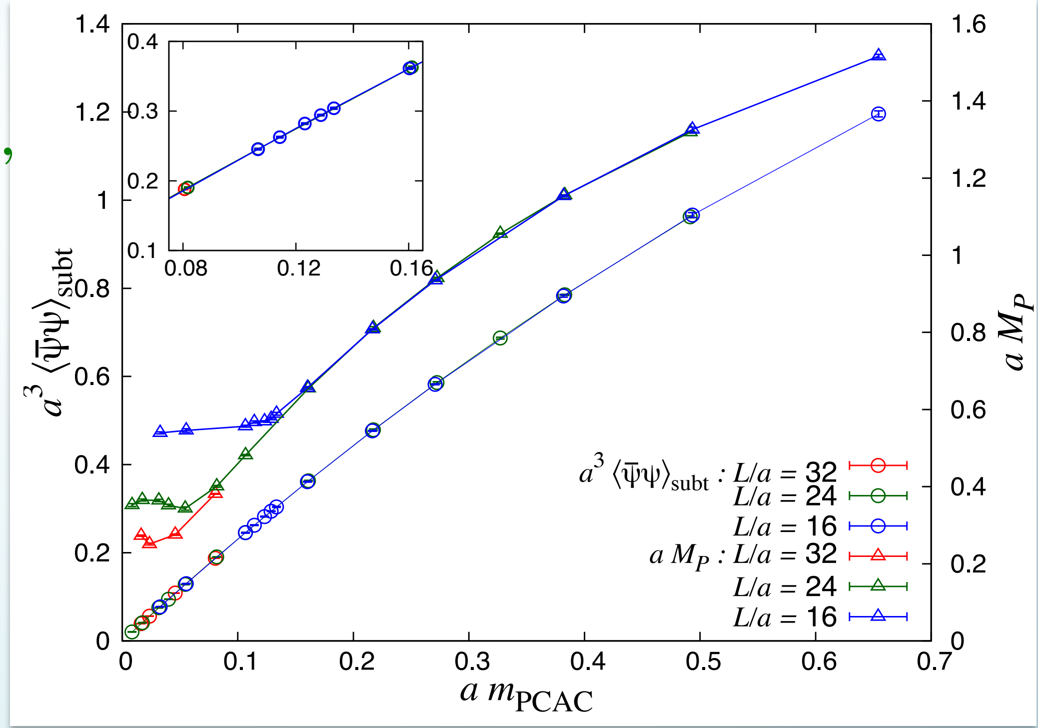
$$\langle\overline{\psi}\psi\rangle_{\mathbf{subt}} = 2m_{\mathbf{PCAC}} \cdot (2\kappa)^2 \sum_t \langle P(t)P(0)\rangle$$

- The eq. implies FSE vanishes $\langle\overline{\psi}\psi\rangle_{\mathbf{subt}}$ linearly in the chiral limit.

➢ We should not extrapolate from the data which suffer FSE.

- M_P of $L/a = 32,\ am_{\mathbf{PCAC}} \simeq 0.08,$ & $L/a = 24,\ am_{\mathbf{PCAC}} \simeq 0.16$ do not suffer FSE.

- M_P of $L/a = 16$, $am_{\mathbf{PCAC}} \simeq 0.1$ suffers FSE sufficiently. But $\langle\overline{\psi}\psi\rangle_{\mathbf{subt}}$ does not.



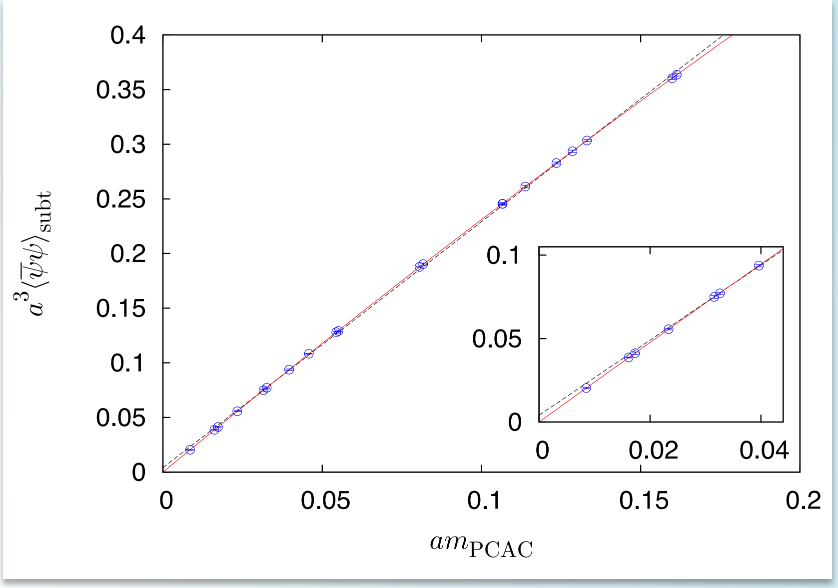
➢ We decided to extrapolate chiral limit of $\langle\overline{\psi}\psi\rangle_{\mathbf{subt}}$ from data including the smallest quark mass.

Subtracted Chiral Condensate

The result of the chiral extrapolation by linear fit and quadratic fit

✧ data set to be fit

(S1) data with $am_{\mathrm{PCAC}} \leq \mathbf{0.2}$
(S2) data with $am_{\mathrm{PCAC}} \leq \mathbf{0.1}$



◆ Linear fit $\boldsymbol{f_1(x) = a_0 + a_1 x}$

data set	a_0	a_1
S1	0.00406 (78)	2.2499 (86)
S2	0.00137 (27)	2.3191 (58)

Linear fit will overestimate $\boldsymbol{a_0}$ because the quark mass dependence of $\langle\overline{\psi}\psi\rangle_{\mathrm{subt}}$ seems to be convex upward.

◆ Quadratic fit $\boldsymbol{f_1(x) = b_0 + b_1 x + b_2 x^2}$

data set	b_0	b_1	b_2
S1	−0.00003 (12)	2.4030 (35)	−0.927 (21)
S2	0.00017 (20)	2.3897 (97)	−0.76 (10)

- $\boldsymbol{b_0}$ is consistent with 0 within available precision.

➢ It implies the system is a conformal dynamics with IR limit.

Summary

- We performed lattice simulation for the system with two-color, six fundamental fermions.

- The fact $M_P \not\propto \sqrt{m_{\mathrm{PCAC}}}$ for small m_{PCAC} is not compatible with the expectation by χ-theory.

- The fact finite size effect on f_P seems to increase f_P is not consistent with the expectation by χ-theory.

- Chiral limit of $\langle\overline{\psi}\psi\rangle_{\mathrm{subt}}$ seems to be consistent with 0 within available precision

☆ From these results, we conclude this system is a conformal dynamics with IR limit.