Two-Color Schrödinger Functional with Six Flavors of Stout Smeared Wilson Fermions

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Lattice Strong Dynamics (LSD) Collaboration

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Motivation

- A technicolor model based on a walking gauge theory may explain electroweak symmetry breaking while avoiding experimental constraints for SM fermion masses and flavor-changing neutral currents. Such a theory is expected to reside just below the conformal window.
- $SU(2)$ gauge theories are promising class of theories for realizing a composite Higgs model due to an an enhanced $SU(2N_f)$ global chiral symmetry[1].
- Evidence for IRFP at 8[2] and 10 flavors and χSB at 4 flavors[3]. Several inconclusive 6 flavors calculations.
- Tension between continuum estimates on the edge of the $N_c = 2$ conformal window may be resolved by examining $N_f = 6$ theory.

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[1] Peskin
[2] Itou, et al.
[3] Karavirta, et al

Ladder Gap Analysis

- Ladder gap equation lets one estimate critical value of $\bar{g}_c^2$ required to trigger χSB[4].
- Basic idea: find value of $\bar{g}^2$ at which solutions to the rainbow approximated Schwinger-Dyson equation are consistent with spontaneous χSB.
- $\bar{g}_c^2 = \frac{4\pi^2}{3C_2(R)} \approx 17.5$.
- By combining this estimate with the two-loop IRFP value, can get an estimate on the edge of the conformal window N_f^c with condition $\bar{g}_*^2(N_c, N_f) = \bar{g}_c^2(N_c)$, yielding

$$N_{f,\mathrm{LG}}^c = N_c \left(4 + \frac{6}{15 - 25N_c^2}\right) \approx 8.$$

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[4] Cohen and Georgi

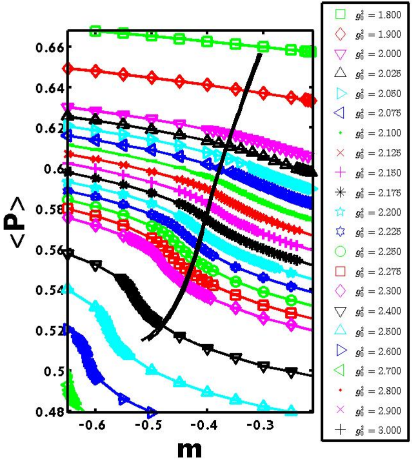
Thermal Inequality

- Another way to estimate N_f^c is conjectured thermal inequality[5].
- Basic idea: postulate that the massless degrees of freedom in the UV should be greater than or equal to the massless degrees of freedom in the IR.
- In an asymptotically free theory that undergoes χSB, it's easy to calculate the degrees of freedom:
  - in UV (free theory of gauge bosons and fermions): $f_{\text{UV}} = 2\left(N_c^2 - 1\right) + \frac{7}{2} N_c N_f$.
  - In IR (free theory of NGBs):
    - $N_c \geq 3$: $f_{\text{IR}} = N_f^2 - 1$.
    - $N_c = 2$: $f_{\text{IR}} = 2N_f^2 - N_f - 1$.
- For $N_c \geq 3$ ACS conjecture implies $N^c_{f,\text{ACS}} < \frac{1}{4}\left(7N_c + \sqrt{81N_c^2 - 16}\right)$ and this bound is slightly larger than the LG estimate.
- For $N_c = 2$, thermal inequality implies $N_f^c \lesssim 4.7$ which is substantially below the LG estimate.

[5] Appelquist, et al.

Our Action

- We use the two-color Wilson gauge action and stout-smeared Wilson fermion action.
- Enforce no smearing of boundary links with bulk links and conversely no smearing of bulk links by boundary links.



Interpolating Functions

- We fit an interpolating function to

$$\frac{1}{g_0^2} - \frac{1}{\bar{g}_{\mathrm{SF}}^2}.$$

- Try various fits:
  - Fit each lattice volume with a piecewise linear (connect-the-dots) function.
  - Fit each lattice volume independently to function

$$\frac{1}{g_0^2} - \frac{1}{\bar{g}_{\mathrm{SF}}^2\left(g_0^2, \frac{a}{L}\right)} = \alpha_{1,L/a} + \alpha_{2,L/a} g_0^2 + \alpha_{3,L/a} e^{\alpha_{4,L/a} g_0^2}$$

Step Scaling Function

- Quantity of interest is the continuum step scaling function $\sigma(u, s)$ arrived at by integrating the continuum beta function

$$\int_u^{\sigma(u,s)} \frac{d\bar{g}^2}{\beta(\bar{g}^2)} = 2\log(s).$$

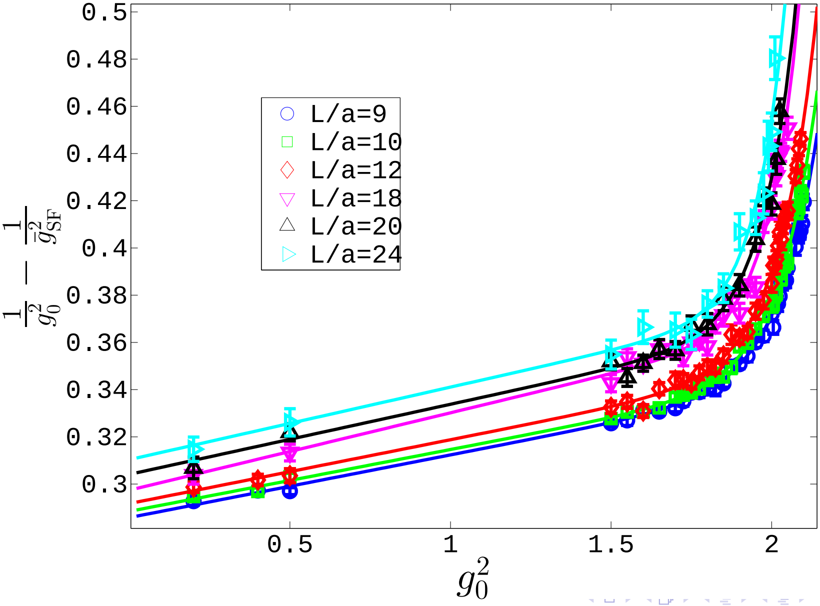
- On the lattice, we have the discrete step scaling function

$$\Sigma_{\mathrm{SF}}\left(u, s, \frac{a}{L}\right) \equiv \bar{g}^2_{\mathrm{SF}}\left(g_{0*}, \frac{sL}{a}\right)\Bigg|_{\bar{g}^2_{\mathrm{SF}}\left(g_{0*}, \frac{L}{a}\right)=u}.$$

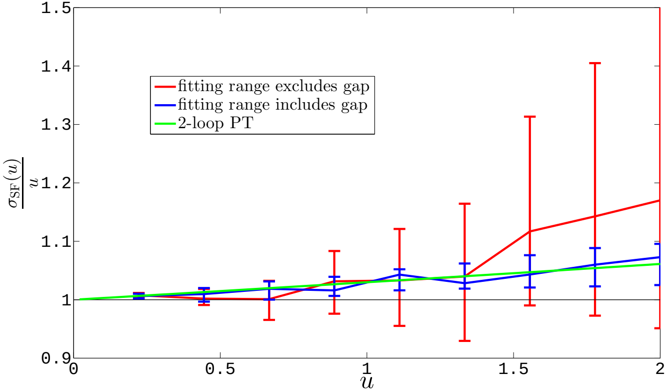
- Taking the continuum limit,

$$\lim_{\frac{a}{L}\to 0} \Sigma_{\mathrm{SF}}\left(u, s, \frac{a}{L}\right) = \sigma_{\mathrm{SF}}(u, s).$$

Plot of all Data Alongside a Linear + Exponential Fit

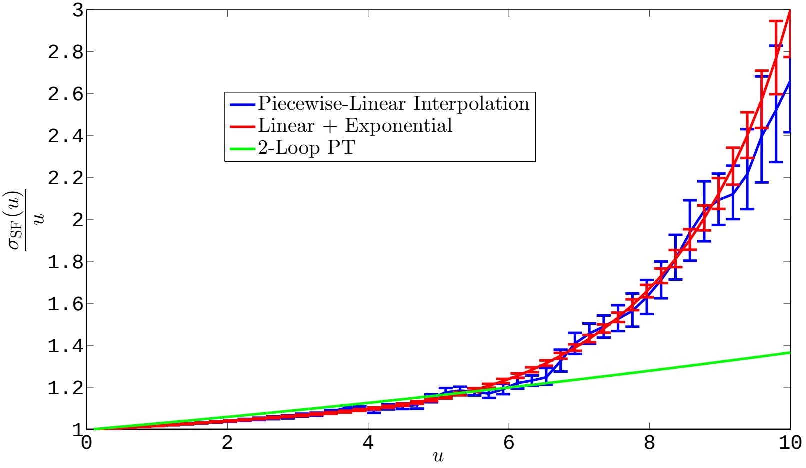


Fit $\frac{1}{g_0^2} - \frac{1}{\bar{g}_{\mathrm{SF}}^2}$ Using $g_0^2 \leq 1.6$ to linear function.

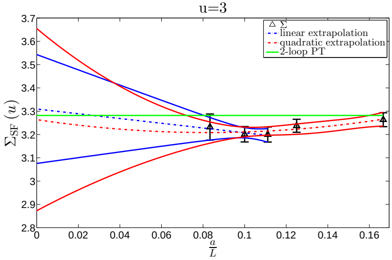


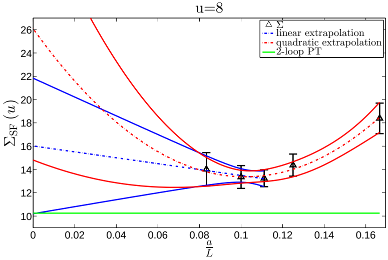
- Fit, with a linear function, all renormalized coupling data using only $g_0^2 \leq 0.5$ and alternatively $g_0^2 \leq 1.6$.

Comparing Piecewise-Linear and Linear + Exponential Interpolation

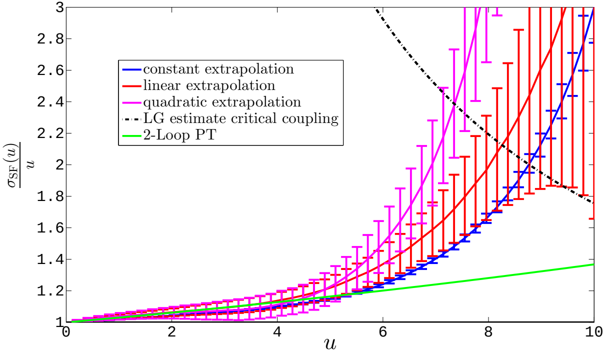


Continuum Extrapolations at weak and strong coupling



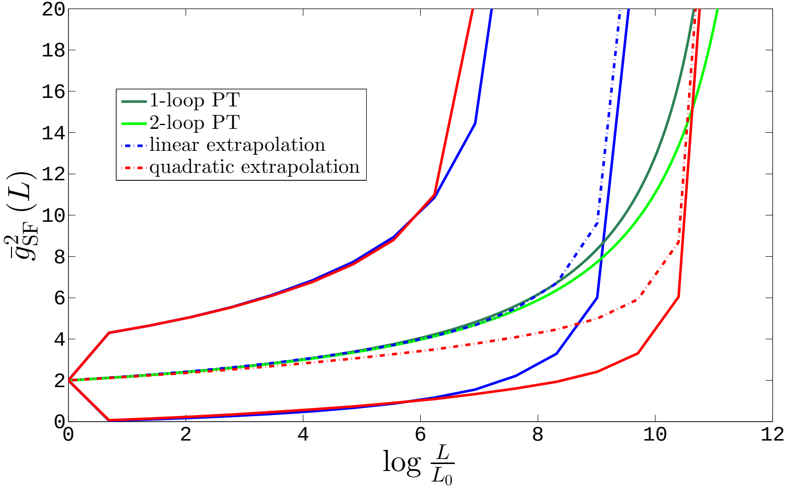


Discrete Beta Function for Various Continuum Extrapolations



- Quadratic extrapolation uses steps $6 \to 12$, $8 \to 16$, $9 \to 18$, $10 \to 20$, and $12 \to 24$.
- Constant and Linear extrapolations only uses steps $9 \to 18$, $10 \to 20$, and $12 \to 24$.

Running Coupling



Conclusions and Outlook

- No evidence for IRFP below $\bar{g}^2_{\rm SF} \lesssim 30$, well above LG estimate of $\bar{g}^2_c \approx 17.5$. Suggests that the $N_f = 6$ theory is outside the conformal window.
- Getting to larger renormalized couplings via SF method is not feasible.
- Zero-temperature studies should be performed to see if the spectrum looks confining or conformal.
- If this theory is really confining and chirally broken, finite temperature studies of this theory should demonstrate novel phenomena.