Correlation functions of atomic nuclei in Lattice QCD I

Jana Günther Balint Toth Lukas Varnhorst

Bergische Universität Wuppertal

2013/July/29

Correlation functions of atomic nuclei

The Correlation function of a nucleus with N_p protons and N_n neutrons is defined as

$$C_{N_p,N_n}(\vec{x},t) = \left\langle \prod_{i=1}^{N_p} P_{\alpha_i}(\vec{x},t) \prod_{j=1}^{N_n} N_{\alpha_j}(\vec{x},t) \prod_{k=1}^{N_p} \overline{P}_{\overline{\alpha}_k}(\vec{0},0) \prod_{l=1}^{N_n} \overline{N}_{\overline{\alpha}_l}(\vec{0},0) \right\rangle ,$$

$$P_\alpha = \varepsilon^{abc} u^a_\alpha (u^b_\beta (C\gamma_5)_{\beta\gamma} d^c_\gamma) \quad \text{and} \quad N_\alpha = \varepsilon^{abc} d^a_\alpha (u^b_\beta (C\gamma_5)_{\beta\gamma} d^c_\gamma).$$

The effort for the evaluation via Wick contractions scales as $6^{N_p+N_n}(2N_p+N_n)!(2N_n+N_p)!$.

For $N_p = N_n = 4$ there are $4 \cdot 10^{23}$ contributions.

Other publications:

Unified contraction algorithm for multi-baryon correlators on the lattice

- by Takumi Doi and Michael Endres
- on 2nd May 2012, arXiv:1205.0585
- Published in Comput. Phys. Commun. 184 (2013)

Nuclear correlation functions in lattice QCD

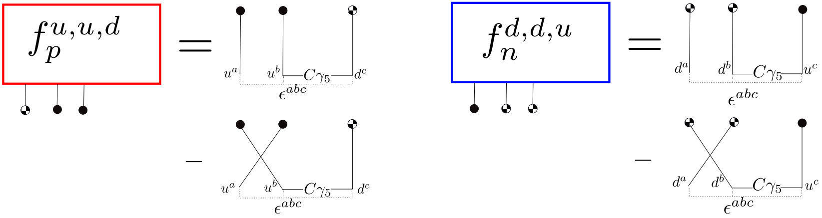
- by William Detmold and Kostas Orginos
- on 5th July 2012, arXiv:1207.1452

General idea:

- using the unified contraction algorithm by *Doi* and *Endres*
- exploring some more symmetries, only calculating the independent components
- constructing the components for $C^{(N)}$ recursively from $C^{(N-1)}$

Defining blocks:

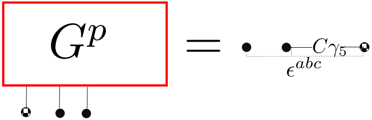
$$f_B^{q_1,q_2,q_3} = \sum_{\vec{x}} \left\langle B_\delta(\vec{x},t) \cdot \overline{q}_1^{\alpha;a} \overline{q}_2^{\beta;b} \overline{q}_3^{\gamma,c} \right\rangle$$

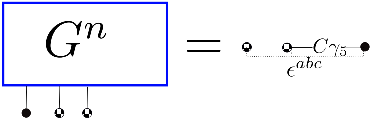


Rewriting C:

Writing C as:

$$C^{(N)} = \sum_{\sigma\in\Sigma} f_{B_1}^{q_1,q_2,q_3} \dots f_{B_N}^{q_1,q_2,q_3} \cdot G^{B_1} \dots G^{B_N} \ \mathrm{sgn}(\sigma)$$



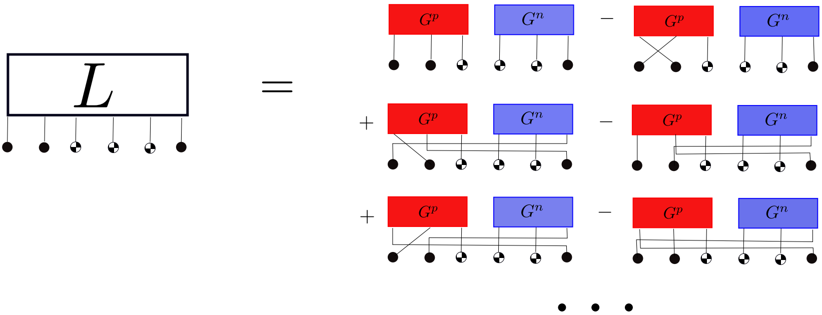


With G containing the appropriate combination of γ-matrices and ϵ-tensors for the baryon.
Only the indices of G are permuted.

$$C^{(N)} = f_{B_1}^{q_1,q_2,q_3} \dots f_{B_N}^{q_1,q_2,q_3} \cdot \sum_{\sigma\in\Sigma} G^{B_1} \dots G^{B_N} \ \mathrm{sgn}(\sigma)$$

The object L:

$$C^{(N)} = f_{B_1}^{q_1,q_2,q_3} \dots f_{B_N}^{q_1,q_2,q_3} \cdot \underbrace{\sum_{\sigma \in \Sigma} G^{B_1} \dots G^{B_N} \operatorname{sgn}(\sigma)}_{L^{(N)}}$$



Antisymmetric tensors

Storing an antisymmetric tensor:

- $X(a, b, c)$ is antisymmetric in a, b and c
- $a, b, c \in \{1 \ldots 4\}$
- each independent component can be identified by a tuple $\mathbf{A} = (\alpha, \beta, \gamma, \delta)$
- Example: $A = (1, 0, 1, 1) \longrightarrow X(1, 3, 4)$

In general:

- $X(\xi_1, \xi_2, \ldots, \xi_l)$ is antisymmetric in $\xi_1, \xi_2, \ldots, \xi_l$
- $\xi_1, \xi_2, \ldots, \xi_l \in \{1 \ldots k\}$
- can be represented by $A\{\xi\} = (n(1), n(2), \ldots, n(k))$

The antisymmetric product

An antisymmetric product $Z = X \bullet Y$ can be defined:

$$(X \bullet Y)(\mathbf{z}) := Z(\mathbf{z}) = \sum_{\mathbf{z}=\mathbf{x}+\mathbf{y}} X(\mathbf{x})Y(\mathbf{y})\,\mathrm{sgn}(\mathbf{x}|\mathbf{y}),$$

where

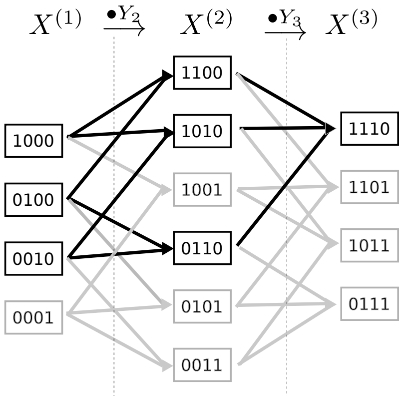
$$\begin{aligned}\mathbf{z} &= \mathbf{A}\{\xi_1, \ldots, \xi_{k+l}\}\\ \mathbf{x} &= \mathbf{A}\{\xi_1, \ldots, \xi_k\}\\ \mathbf{y} &= \mathbf{A}\{\xi_{k+1}, \ldots, \xi_{k+l}\}\end{aligned}$$

and

$$\mathrm{sgn}(\mathbf{x}|\mathbf{y}) = \prod_{\substack{i>j\\ y_j=1}} (-1)^{x_i}$$

Often the computational cost can be reduced by calculating $X^{(n)} = Y_1 \bullet Y_2 \bullet \cdots \bullet Y_n$ recursively as $X^{(i)} = X^{(i-1)} \bullet Y_i$

Calculating only some components



The object L:

From the the unified contraction algorithm:

$$C^{(N)} = f_{B_1}^{q_1,q_2,q_3} \dots f_{B_N}^{q_1,q_2,q_3} \cdot \underbrace{\sum_{\sigma\in\Sigma} G^{B_1} \dots G^{B_N} \operatorname{sgn}(\sigma)}_{L^{(N)}}$$

The indices of L:

$$L^{(N)}(\alpha_1, \dots, \alpha_N; \xi_1^{(q_1)}, \xi_2^{(q_2)}, \xi_3^{(q_3)}, \dots, \xi_{3N-2}^{(q_1)}, \xi_{3N-1}^{(q_2)}, \xi_{3N}^{(q_3)})$$

- α_i: Spin index of the baryon B_i
- $\xi_k^{(q_l)}$: combined spinor-color indices for one quark

Symmetries of L

$$L^{(N)}(\alpha_1, \dots, \alpha_N; \xi_1^{(q_1)}, \xi_2^{(q_2)}, \xi_3^{(q_3)}, \dots, \xi_{3N-2}^{(q_1)}, \xi_{3N-1}^{(q_2)}, \xi_{3N}^{(q_3)})$$

Symmetries:

- antisymmetric under exchange of to ξ belonging to the same quark flavor
- antisymmetric under exchange of to α belonging to the same type of baryon

Rewriting:

$$L(\mathbf{A}^{(B_a)}\{\alpha\}, \mathbf{A}^{(B_b)}\{\alpha\}, \dots, \mathbf{A}^{(u)}\{\xi\}, \mathbf{A}^{(d)}\{\xi\}, \mathbf{A}^{(s)}\{\xi\})$$

Recursive construction

L was defined as:

$$L^{(N)} = \sum_{\sigma \in \Sigma} G^{B_1} \dots G^{B_N} \operatorname{sgn}(\sigma)$$

G has the same indices and symmetries as L:

$$G^B(\alpha, \mathbf{A}^{(u)}\{\xi\}, \mathbf{A}^{(d)}\{\xi\}, \mathbf{A}^{(s)}\{\xi\}).$$

L can be calculated recursively:

$$L^{(n+1)} = L^{(n)} \bullet G_{B_{n+1}}$$

The object F_{-}

The correlator:

$$C^{(N)} = \underbrace{f_{B_1}^{q_1,q_2,q_3} \dots f_{B_N}^{q_1,q_2,q_3}}_{F^{(N)}} \cdot \underbrace{\sum_{\sigma \in \Sigma} G^{B_1} \dots G^{B_N} \operatorname{sgn}(\sigma)}_{L^{(N)}}$$

- F has to be contracted with L in the antisymmetric form
- Projecting F to an antisymmetric tensor F_{-}
- Only F_{-} contributs to C
- calculating contraction between F_{-} and L

Also F_{-} can be calculated recursively:

$$F_{-}^{(n+1)} = F_{-}^{(n)} \bullet f_{B_{n+1}}^{q_1,q_2,q_3}$$

Summary

- Starting from the unified contraction algorithm we have explored the antisymmetry of the correlation function.
- A way to to deal only with the independent components of antisymmetric tensors has been introduced.
- The algorithm is only valid for a single quark source in the form presented to this point.
- This limits the the correlation function to 4 protons and 4 neutrons.
- Extensions to circumvent this restriction are possible.

Thank you!