Equation of State of Nucleon Matters from Lattice QCD Simulations

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for

HAL QCD Collaboration

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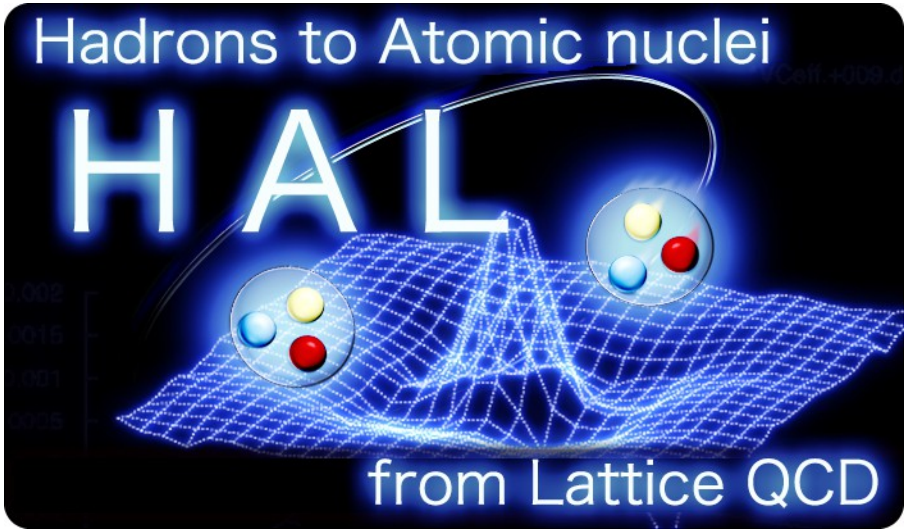


Lattice 2013, July 29 to Aug 3, Mainz

Introduction

⋆ Equation of State of matter

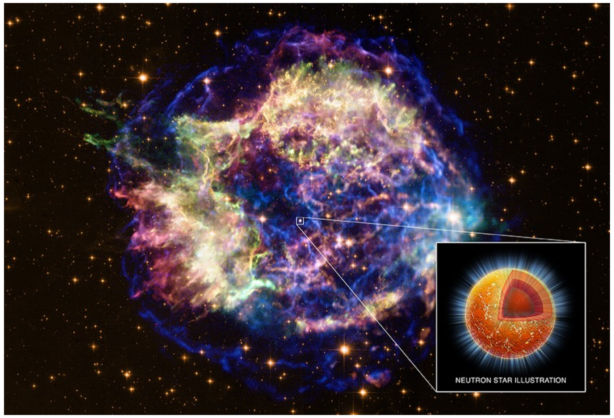
- is a relation between energy and pressure of matter

$$P(E) \text{ or } E(P) \text{ or } E(\rho) \text{ or } E(k_F) \text{ or...}$$

- EoS of dense baryonic matter is a key ingredient of many interesting physics eg. heavy ion collision supernova, neutron star.

⋆ Neutron Star

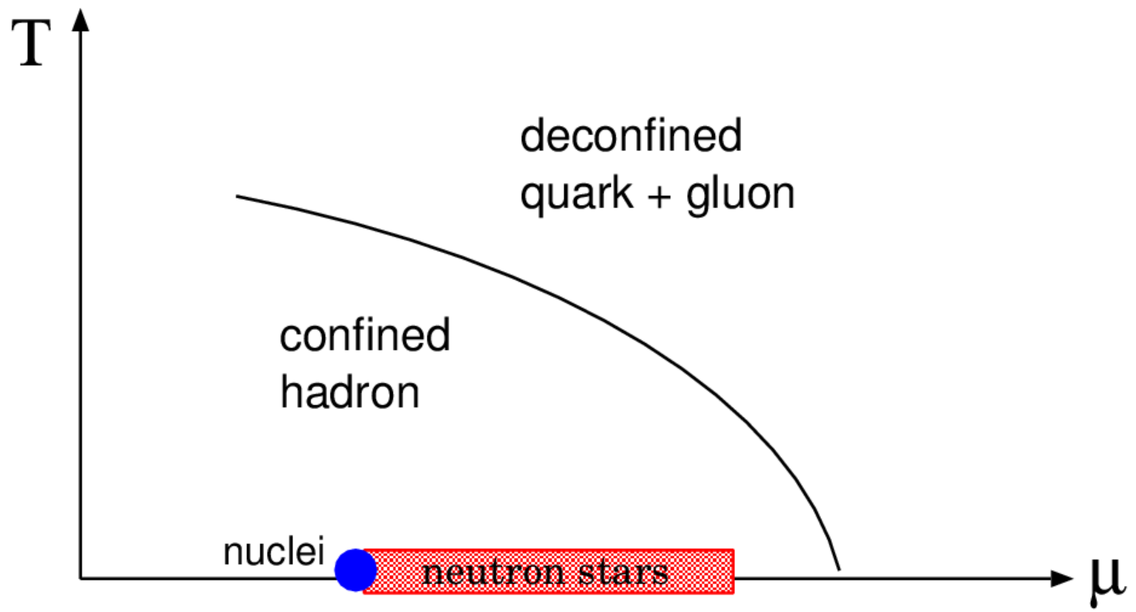
- is a compact star formed after supernova explosion of massive star.
- Typically, $M = 1.5\, M_{\odot}$, $R = 10$ km.
- Density ρ = several $\times$ ρ_0 at the center.
- Temperature T $\simeq 10^8$ [K] = 0.01 [MeV] $\simeq 0$



Mass-energy density is about 10^{15} [g/cm^3] ! Most dense in Universe!

Introduction

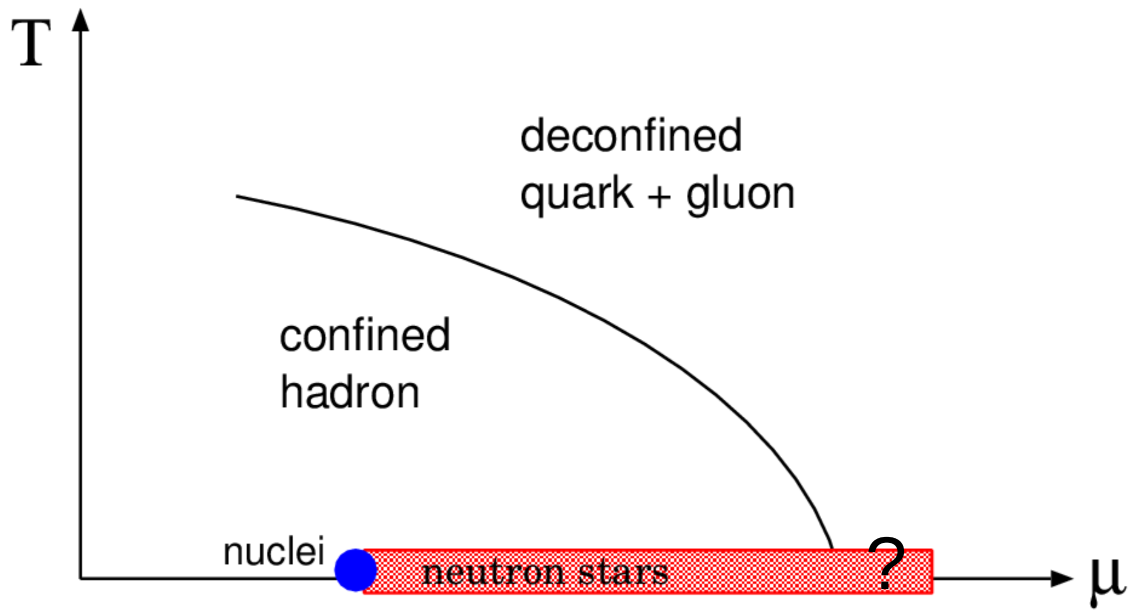
- QCD phase diagram



- NS observation provides information of EoS of baryonic matter at the  region on the QCD phase-diagram.

Introduction

⭑ QCD phase diagram



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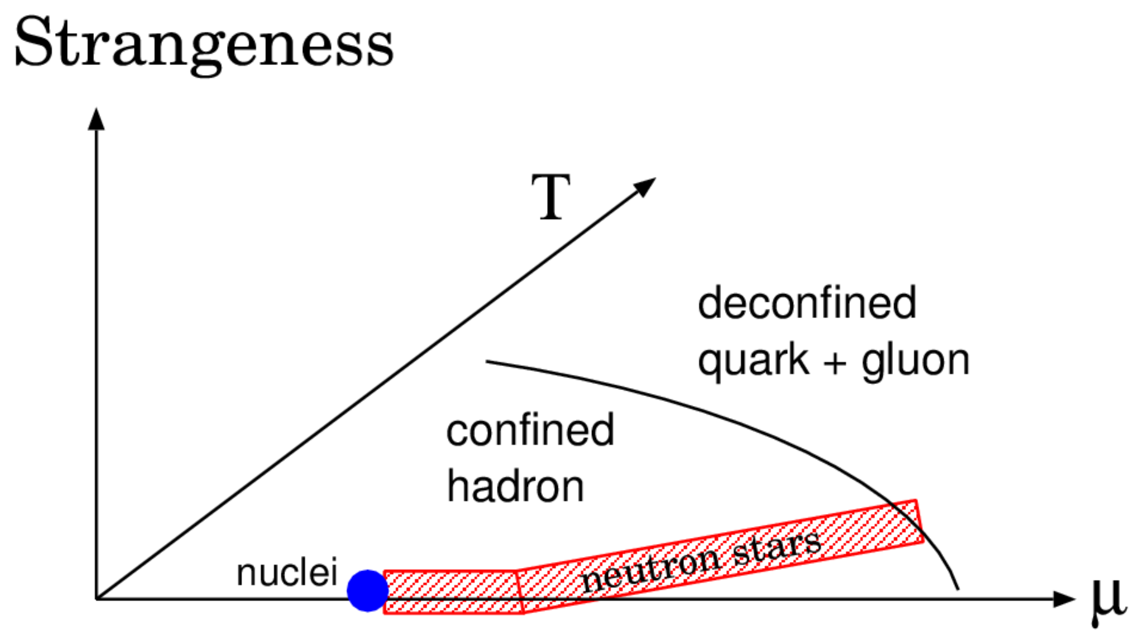
Introduction

* QCD phase diagram



- NS observation provides information of EoS of baryonic matter at the [red box] region on the QCD phase-diagram.
- Perhaps, it touches the deconfined QGP phase.
- Probably, it goes to finite strangeness direction.

Introduction

- Today, I try to

Quark + gluon (QCD) → Baryonic matter → Neutron Stars

- Our approach
  - We put one step General BB interactions , and we extract them from QCD by doing numerical simulations on the lattice.
  - At the later stages, we follow standard way.

- We want to answer:
  - Can QCD reproduce know features of nuclear matter ?
  - What does QCD predict about hyperons in medium ?
  - How about EoS of NS matter & M^{max} of neutron star ?

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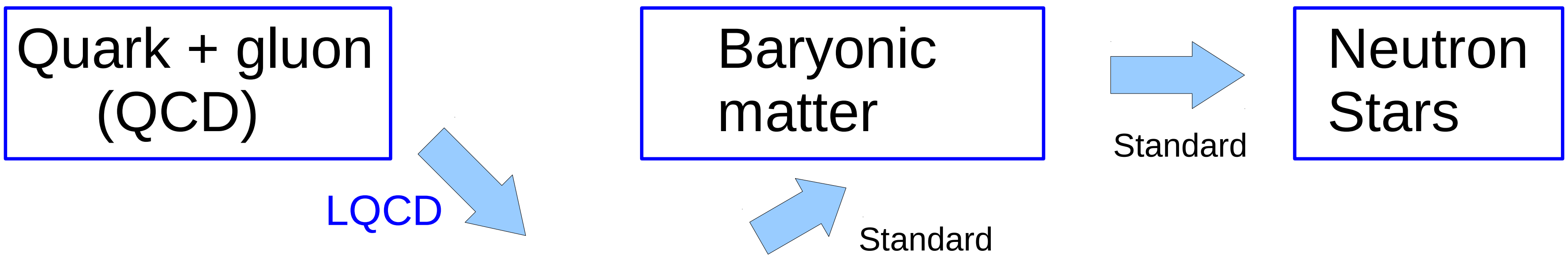

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Quark + gluon (QCD) → (LQCD) → General BB interactions → (Standard) → Baryonic matter → (Standard) → Neutron Stars

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  - How about EoS of NS matter & $M^{\rm max}$ of neutron star ?
  - We can attack and answer these questions in principle. However, we have many limitations at present. So, my talk is just a first step.

Outline

1. Introduction
2. General BB Interaction from QCD
3. Nuclear Matter EoS from QCD
4. Neutron Stars from QCD
5. Hyperon in medium from QCD
6. Summary and Plan

General BB Interaction from QCD

LQCD simulation at $SU(3)_F$ Limit

- I've carried out LQCD simulations at flavor SU(3) limits, in order to capture essential feature of BB interaction.

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

In this limit,

Irreducible multiplet = Convenient basis to describe interaction

- Any BB interaction (e.g. NN and ΛN) can be reconstructed from interactions in these six basis and C.G. coefficients.

Huge reduction of computation time

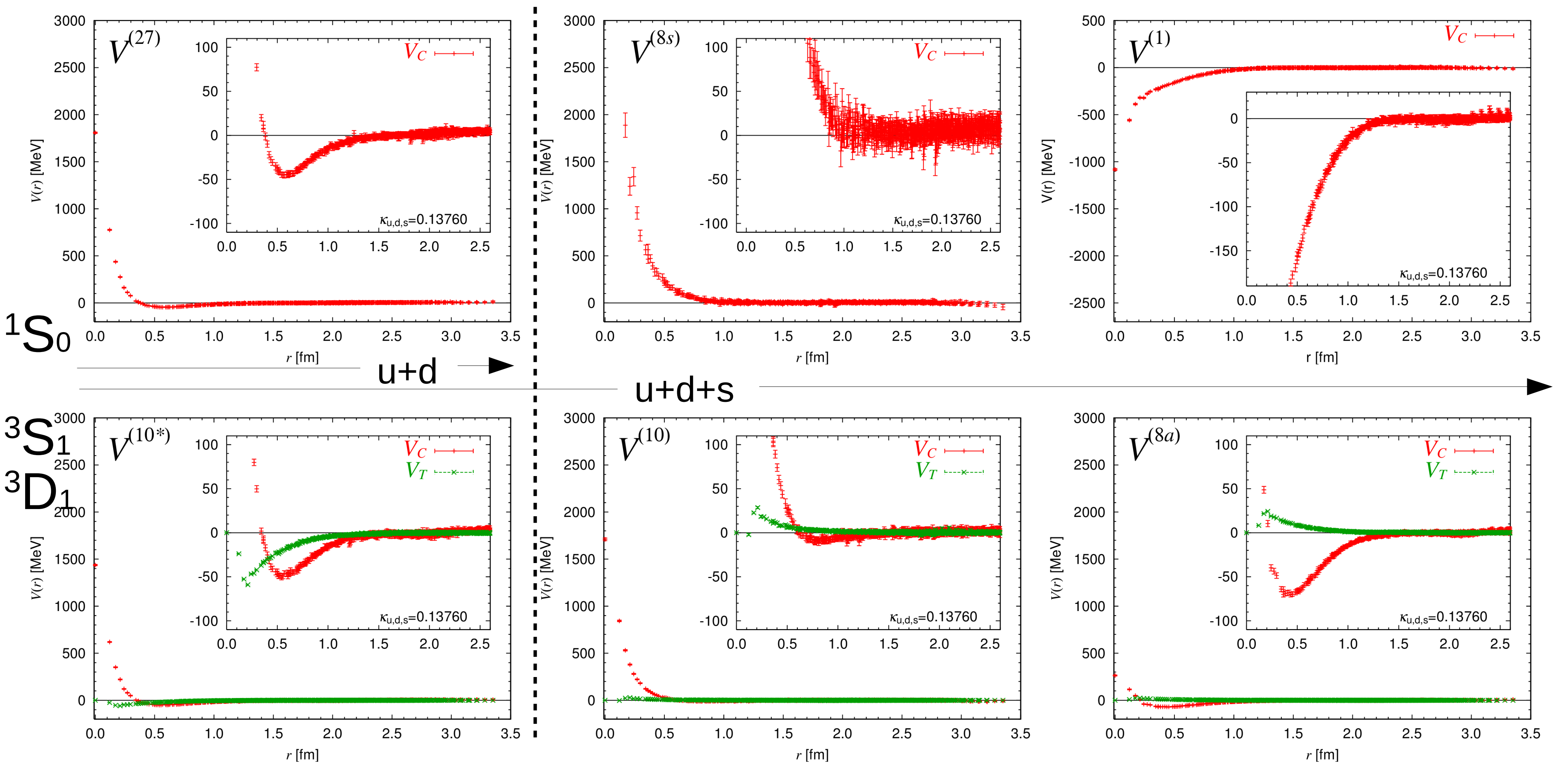
size	β	C_{SW}	a [fm]	L [fm]
32^3 x 32	1.83	1.761	0.121(2)	3.87

- Iwasaki gauge & Wilson quark.
- Thanks to PACS-CS collaboration for their DDHMC/PHMC code.

K_uds	M_P.S. [MeV]	M_Bar [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1161(2)

BB pot. in the flavor irr. basis

@SU(3)$_F$ limit
Mps = 837 MeV



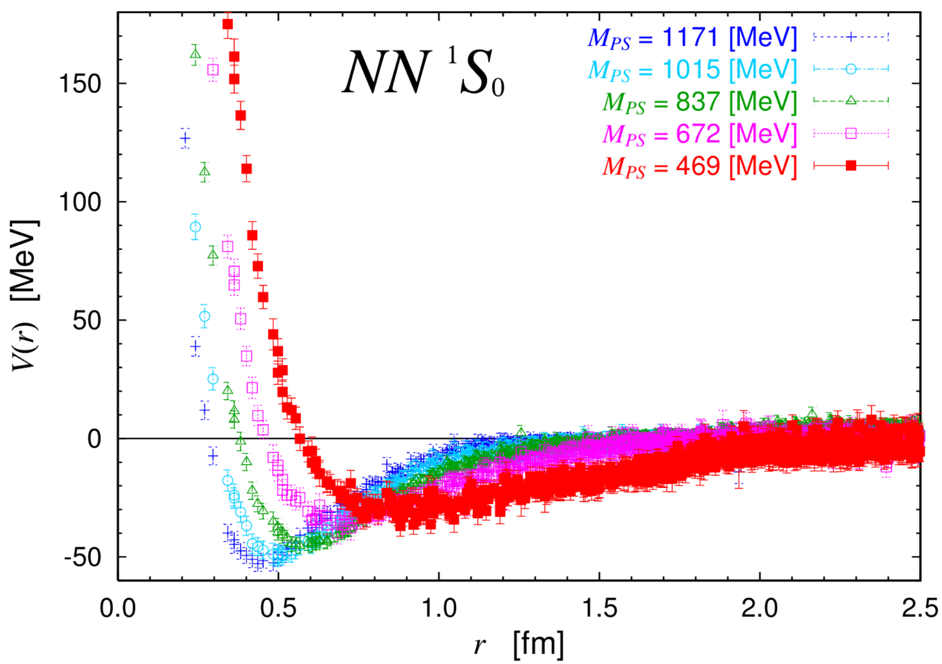
- Extracted by

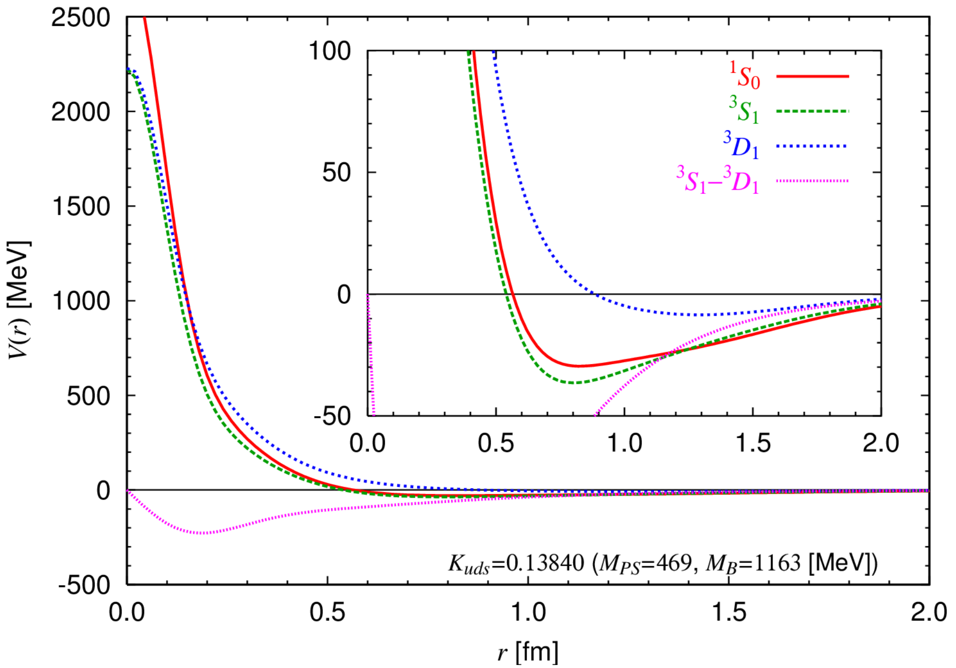
$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

HAL QCD
imaginary time method
$\psi(\vec{r},t)$: NBS W.F.

Talk by T. Doi

LQCD NN potentials



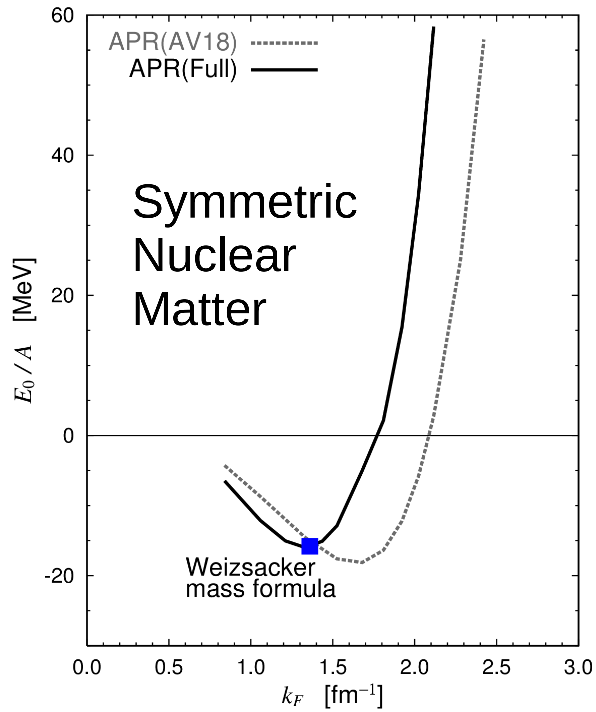


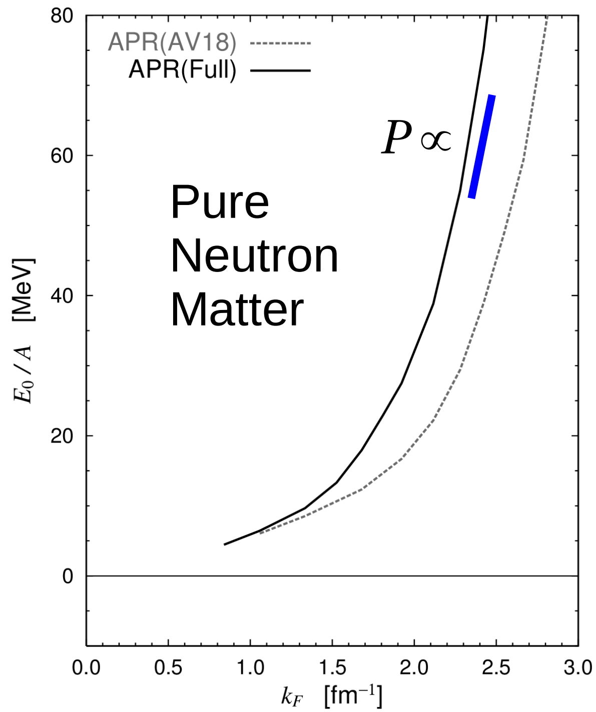
- Left: NN $^1S_0$ potential at five quark mass. (27-plet)
  - Repulsive core & attractive pocket grow as m_q decrease.
- Right: NN potential in partial waves at the lightest m_q.
  - Best χ^2 fit of data which gives <u>central value of observable</u>.

Nuclear Matter
from QCD

Equation of State

- Ground state of interacting infinite nucleon system
  - Relativistic Mean Field — J. D. Walecka, Ann. Phys. 83 (1974) 491
  - Fermi Hyper-Netted Chain — A. Akmal, V.R. Phandharipande, D.G. Ravenhall Phys. Rev. C 58 (1998) 1804





They used phenomenological Argonne NN force & "Urbana" NNN force.

- For SNM, most important feature is the saturation.
- For PNM or NSM, the slope at large k_F is important.

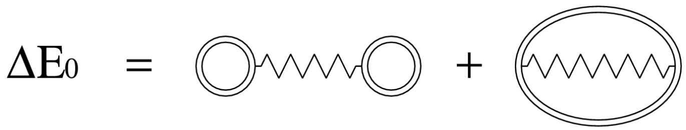
Brueckner-Hartree-Fock

K.A. Brueckner and J.L.Gammel Phys. Rev. 109 (1958) 1023

M.I. Haftel and F. Tabakin, Nucl. Phys. A158(1970) 1-42

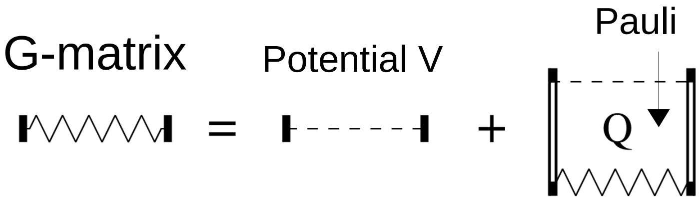
- Ground state energy in BHF framework

$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \langle G_i(e(k)+e(k')) \rangle_A$$



- G-matrix

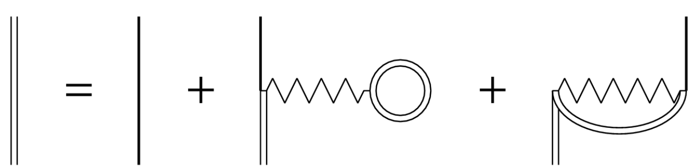
$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$



- Single particle spectrum & potential

$$e(k) = \frac{k^2}{2M} + U(k)$$

$$U(k) = \sum_{i} \sum_{k' \le k_F} \langle k k' | G_i(e(k)+e(k')) | k k' \rangle_A$$



- Angle averaged Q-operator, Continuous choice w/ parabolic approximation

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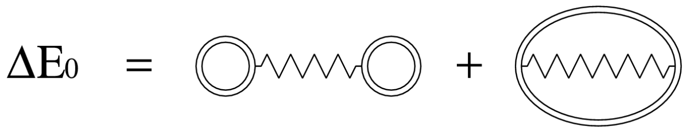

- G-matrix

LQCD V_{NN}

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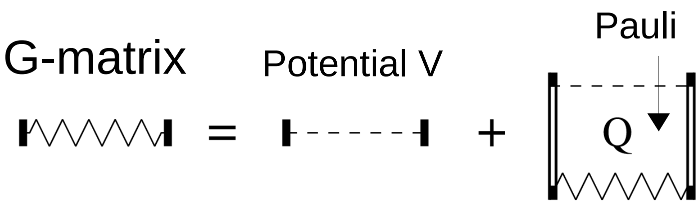

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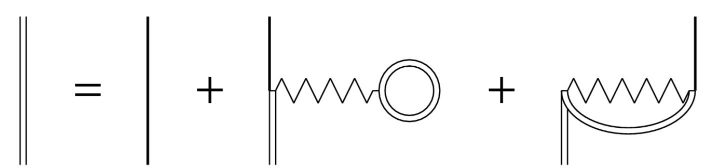

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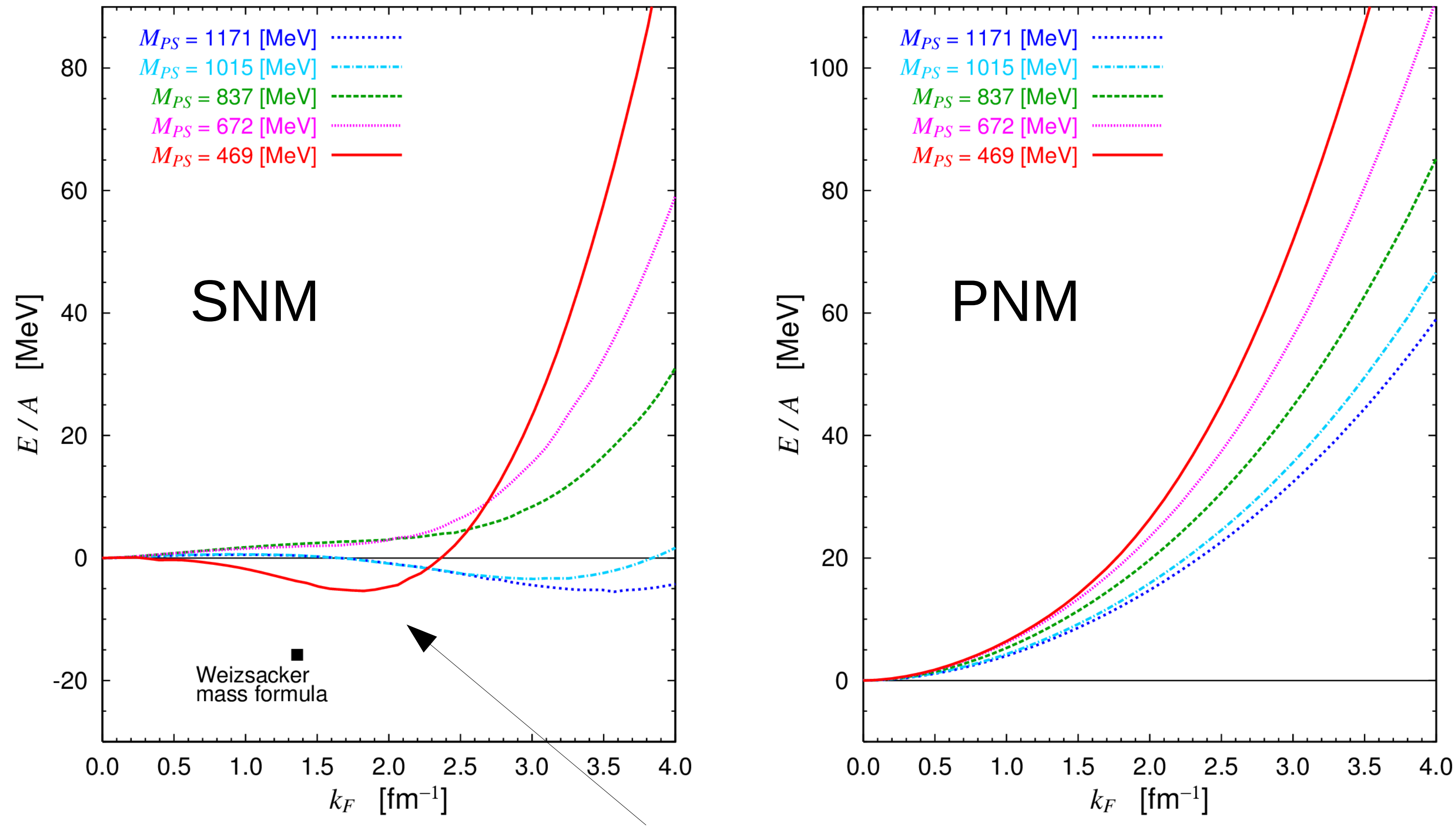
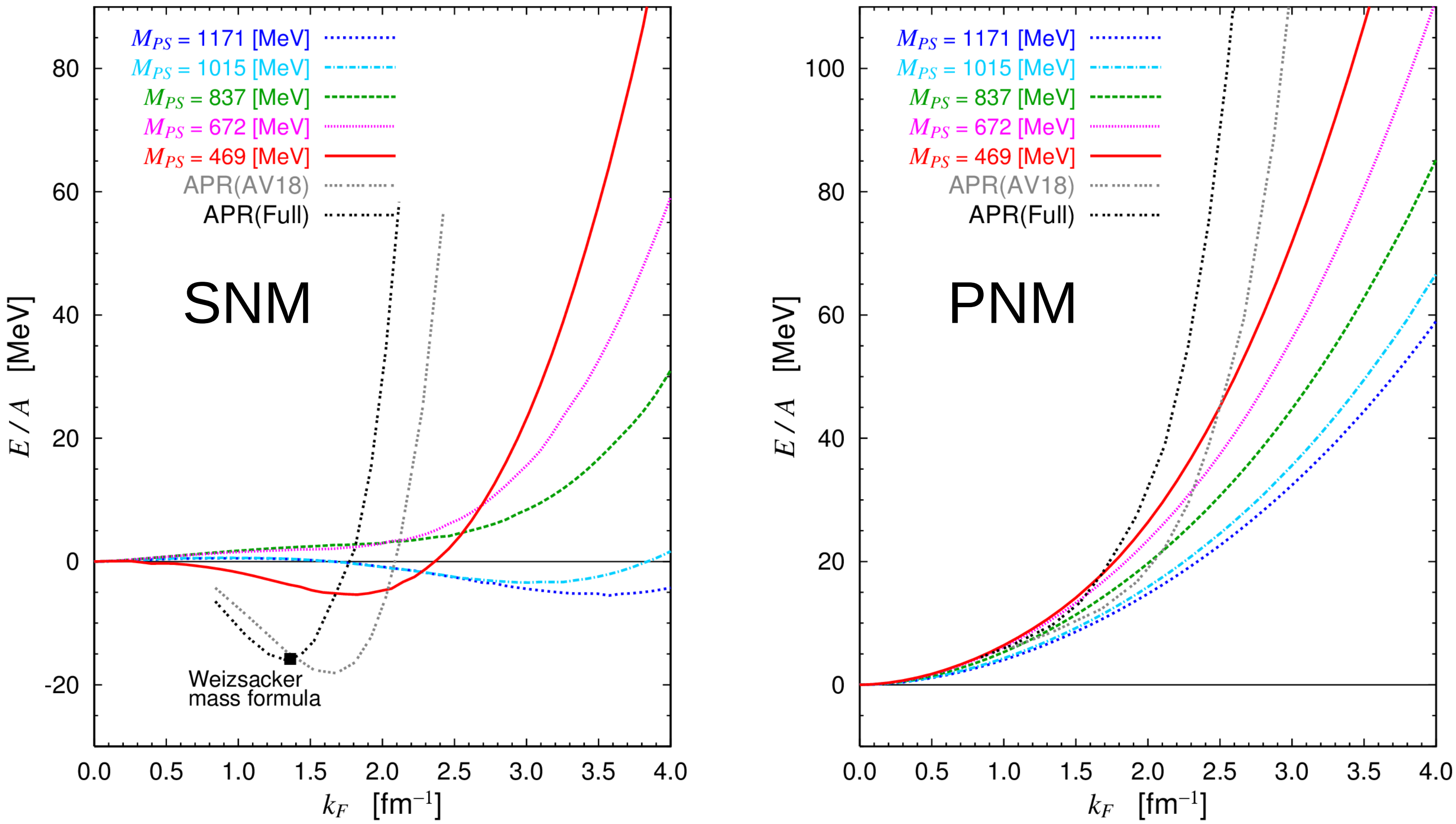
Matter EoS from QCD



- SNM is bound and the saturation occurs at M_{PS} = 469 MeV.
  - Saturation is very delicate against change of quark mass.
- PNM is unbound as normal.
  - PNM become stiff at high density as quark mass decrease.

Comparison to APR



- Deviation from empirical ones due to heavy u,d quark.
- LQCD curve approaches to empirical one as mq decrease.

optimistically?

Neutron Stars from QCD

Neutron Stars (spherical & non-rotating & stable)

- Tolman-Oppenheimer-Volkoff equation

$$\frac{dP(r)}{dr} = -\frac{G\left(E(r)+P(r)\right)\left(M(r)+4\pi r^3 P(r)\right)}{r\left(r-2GM(r)\right)}$$

R. C. Tolman, Phys Rev. 55(1939) 364

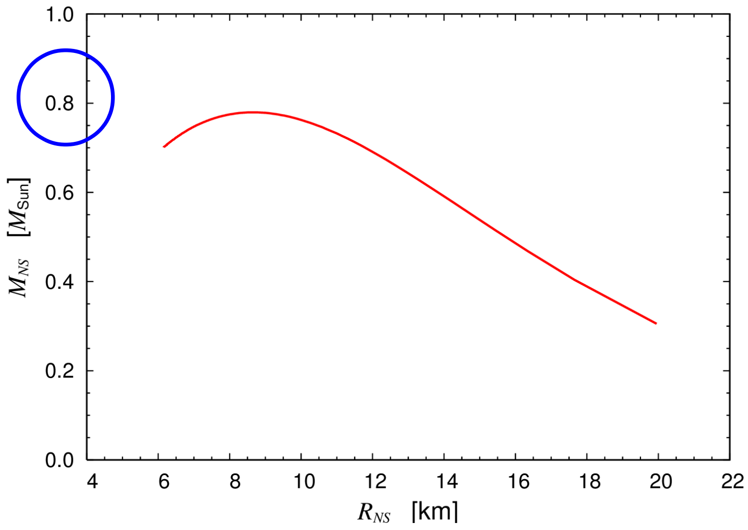
J. R. Oppenheimer and G. M. Volkoff
Phys. Rev. 55 (1939) 374

$$\frac{dM(r)}{dr} = 4\pi r^2 E(r)$$

$P(r)$: Pressure
$E(r)$: Mass-energy density
$M(r)$: Enclosed mass

Gravitational constant
$G = 6.6743$ [m^3 kg^{-1} s^{-2}]

- with *P(E)* (EoS) of a cold Fermi gas of neutrons.



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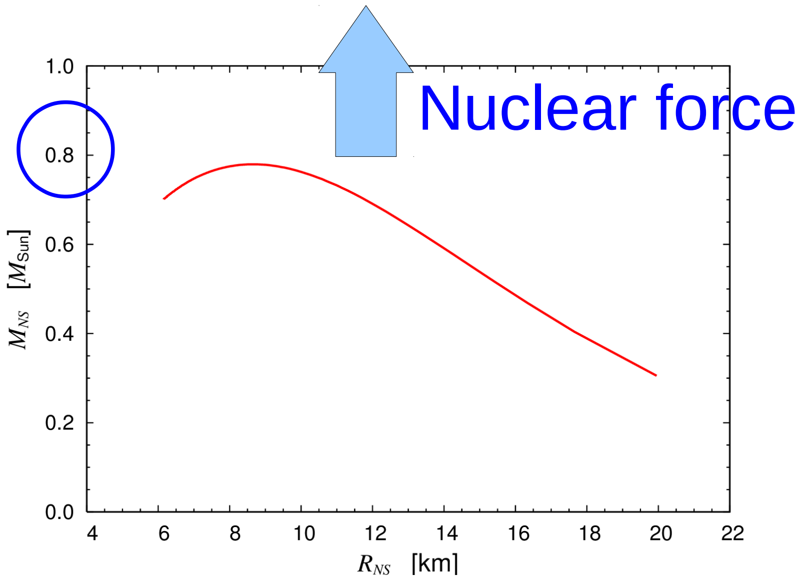


QCD is essential for NS!

Let us apply our LQCD EoS to neutron stars.

Neutron Star Matter

We consider hyperons later.

- For the moment, I restrict component to n, p, e^- and μ^-.

- Asymmetric nuclear matter with the parabolic approx.

$$\frac{E_0}{A}(\rho, x) = \frac{E_0^{\mathrm{SNM}}}{A}(\rho) + \beta^2 E_{\mathrm{sym}}(\rho)$$

$$\rho = \rho_n + \rho_p, \quad x = \rho_p / \rho$$

$$\beta = 1 - 2x$$

$$[\mu_n - \mu_p](\rho, x) = 4\beta E_{\mathrm{sym}}(\rho)$$

$$E_{\mathrm{sym}}(\rho) = \frac{E_0^{\mathrm{PNM}}}{A}(\rho) - \frac{E_0^{\mathrm{SNM}}}{A}(\rho)$$

- Cold Fermi gas for lepton

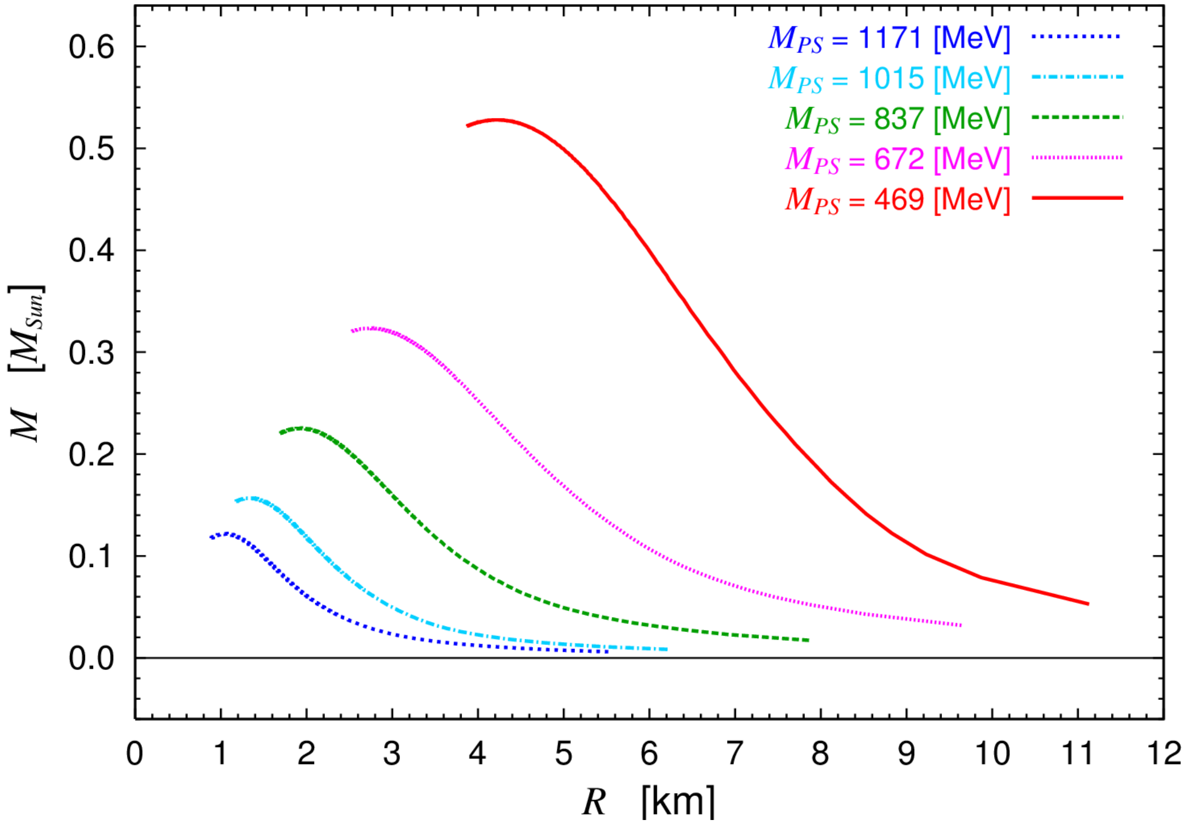
$$\mu_e = \varepsilon_F^e = k_F^e = (3\pi^2 \rho_e)^{(1/3)} \qquad \mu_\mu = \varepsilon_F^\mu = \sqrt{m_\mu^2 + (k_F^\mu)^2}$$

- Chemical equilibrium and charge neutrality

$$\mu_n - \mu_p = \mu_e = \mu_\mu \qquad \rho_p = \rho_e + \rho_\mu$$

- Particle fractions x_i and $P(E)$ are determined. → TOVeq.

Neutron Star M-R relation

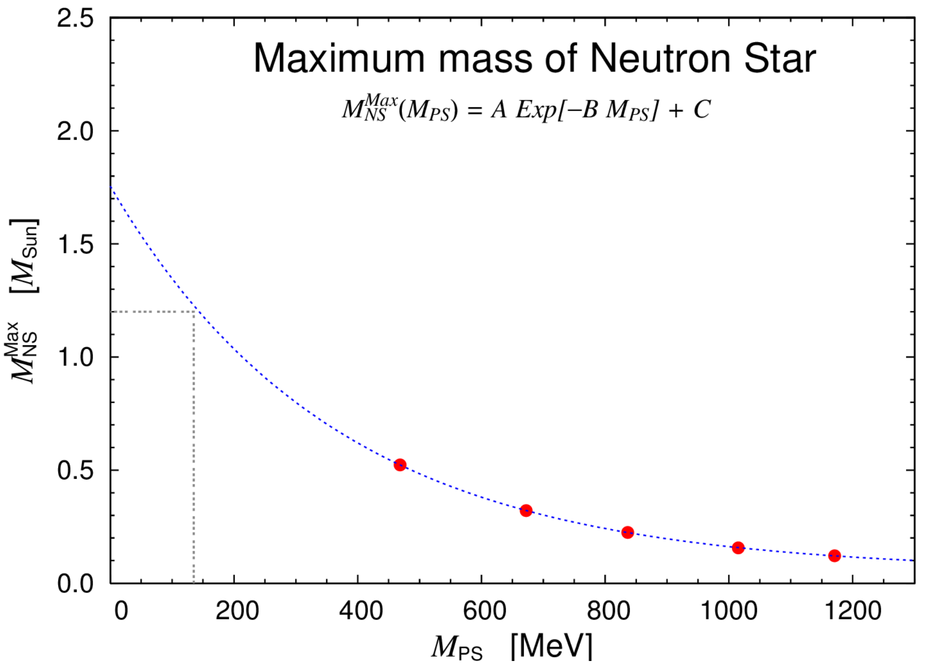


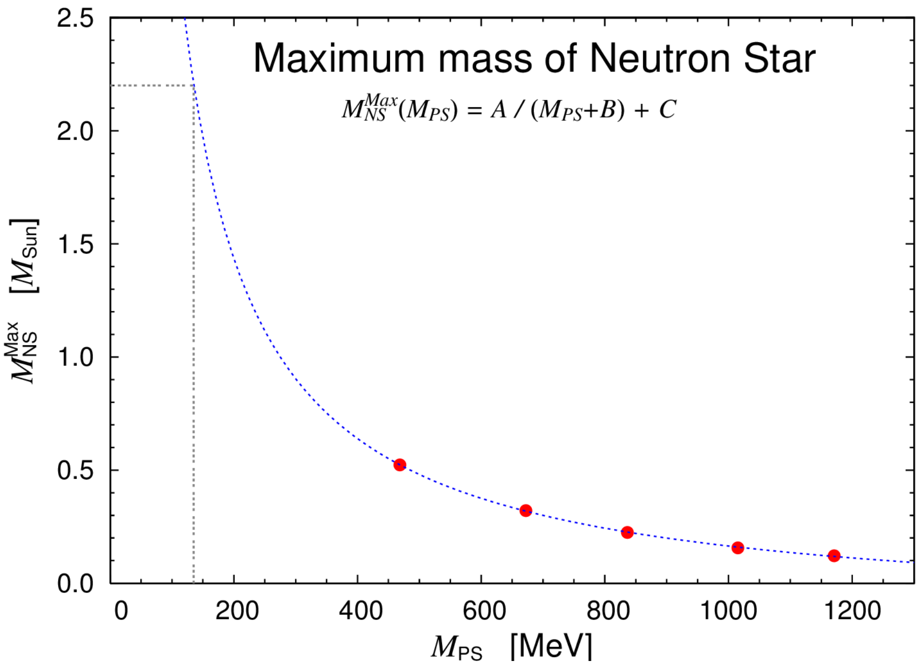
M_N = 2274 MeV
M_N = 2031 MeV
M_N = 1749 MeV
M_N = 1484 MeV
M_N = 1161 MeV

Crust part is ignored.
Uniform matter only.

- Mass-radius curve of neutron stars at five value of m_q.
  - M^{max} = 0.12 – 0.52 [M_{Sun}] for M_{ps} = 1171 – 469 [MeV].
  - due to heavy nucleon and weaker repulsion at short distance.
  - M^{max} will be much bigger with lighter u,d quark.

Maximum mass of Neutron Star





- Blue line shows a function $f(M_{PS})$ fitted to data.
- This suggests that M_{NS}^{max} = 1.2 to 2.2 M_{sun} at M_{PS} = 135 MeV.

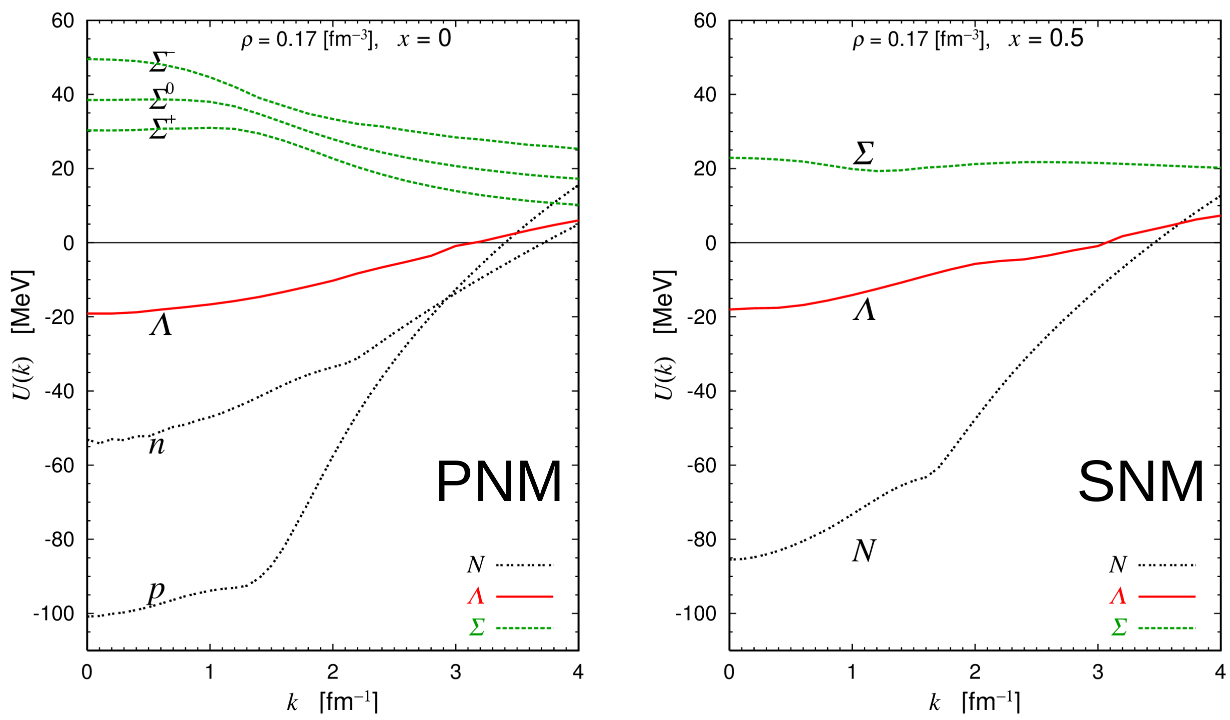
Hyperon in medium from QCD

Hyperon single particle potential

- Hyperon s.p. potential $U_Y(k)$

  Spectrum in the medium $$e_Y(k) = \frac{k^2}{2M_Y} + U_Y(k)$$

- Model prediction: AV18 NN + Nijmegen YN (ESC08c)



At the normal nuclear density

YN interactions up to $^1S_0$, $^3SD_1$ channels are included.

- $U_\Lambda(0) \approx -20$ [MeV], $U_\Sigma > 0$ in both the nuclear matter.

BHF for Hyperons

M. Baldo, G.F. Burgio, H.-J. Schulze, Phys. Rev. C58, 3688 (1998)

- Hyperon s.p. potential in the BHF framework

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k\, k' | \, G^{SLJ}_{(YN)(YN)}\left(e_Y(k) + e_N(k')\right) | k\, k' \rangle$$

- YN G-matrix   by suppressing momentum indices

Q=0
$$\begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$$

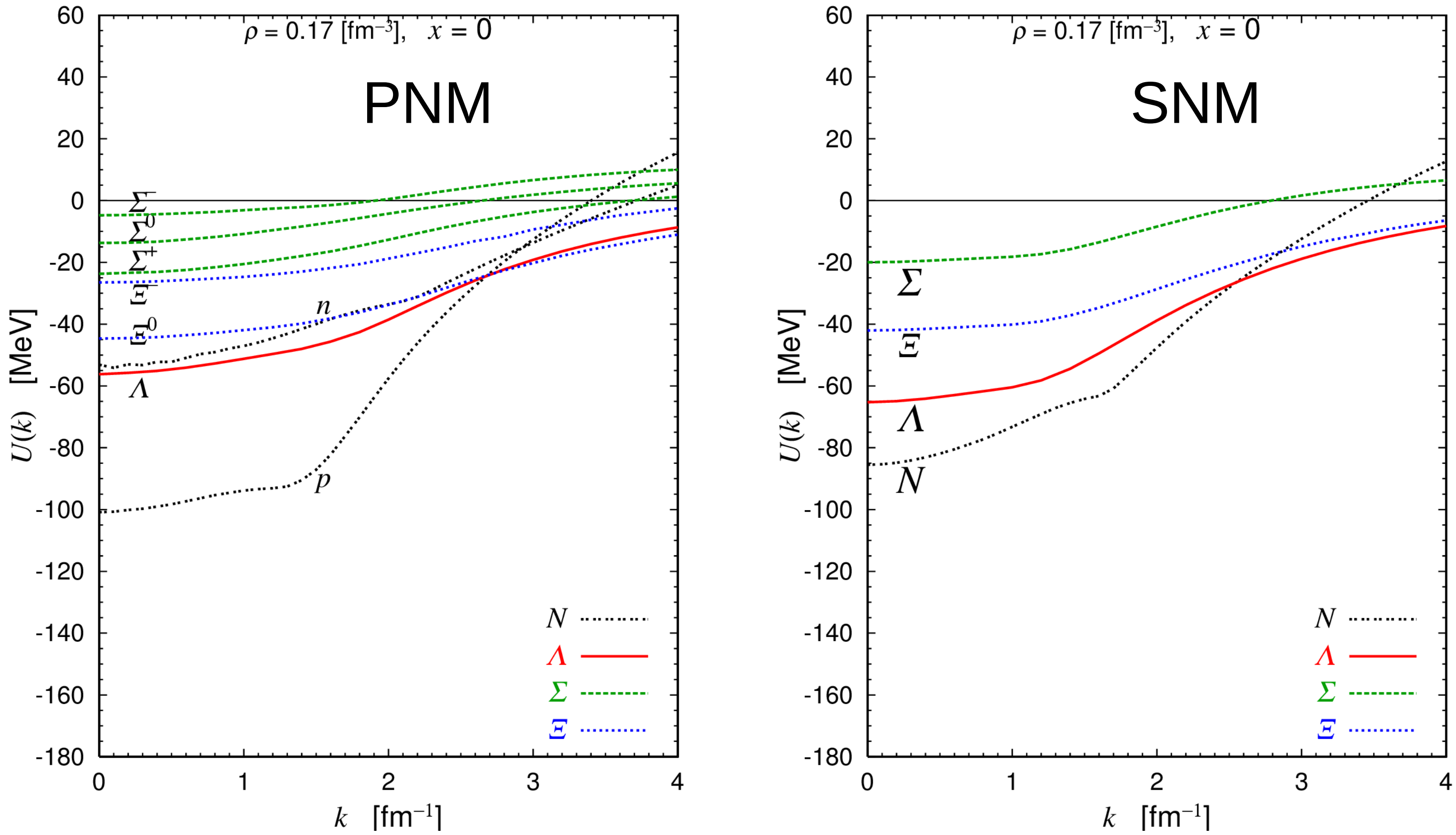
Q=+1
$$\begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

Q=−1 $G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)}$ Q = +2 $G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$

$$^{2S+1}L_J = {}^1S_0,\ {}^3S_1,\ {}^3D_1,\ \Big|\ {}^1P_1,\ {}^3P_J\ \cdots$$

in our study

M_B^{Phys} + AV18 NN + LQCD YN,YY



- with YN & YY potentials from LQCD at a $SU(3)_F$ limit.
- We see that $U_\Lambda(k) < U_\Xi(k) < U_\Sigma(k)$.
- LQCD $U_Y(k)$ are deeper than model predictions and data.

due to heavy u,d quark? Models are wrong?

Summary and Plan

- We've tried to reach Neutron Stars from QCD.
  - We studied BB interaction from LQCD.
    - We extracted BB potentials in 6 flavor irreducible basis.
  - We studied nuclear matter in the BHF theory.
    - We could reproduce the saturation feature of SNM.
  - We studied neutron stars solving TOV eq.
    - We obtained mq dependence of NS M-R relation.
  - We studied hyperons in nuclear medium.

- Plan
  - Study NS with hyperons according to QCD.
  - Inclusion of P,D-wave BB and BBB interactions.
  - LQCD simulations with realistic quark at 京-computer.

Thank you!!