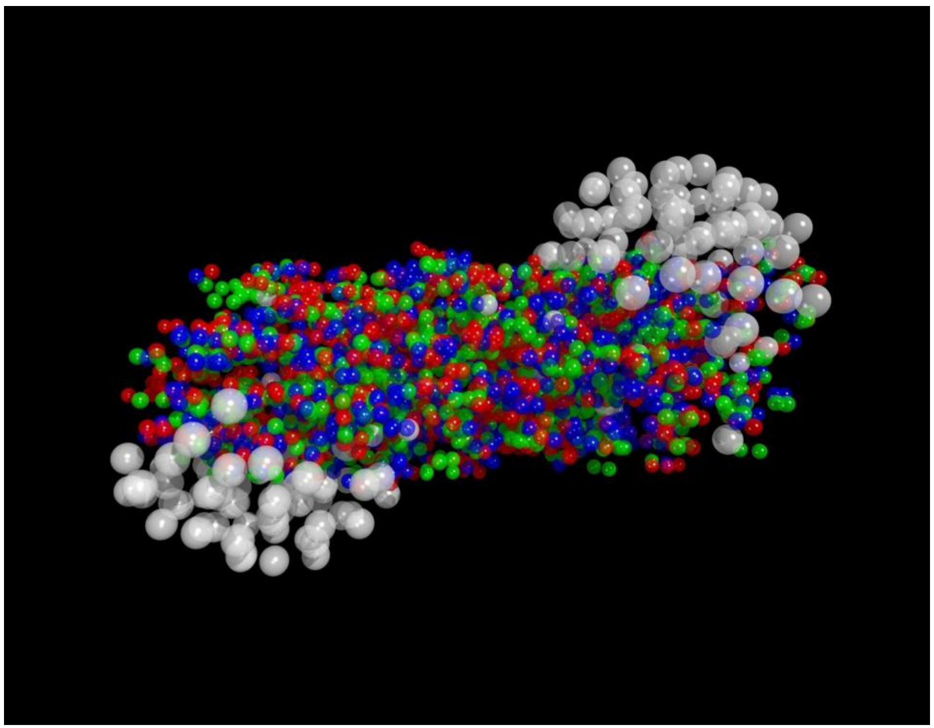
Non-equilibrium fermion production on the lattice

Daniil Gelfand | University of Heidelberg |



J. Berges, D. G., J. Pruschke, PRL 107 (2011) 061301

J. Berges, D. G., D. Sexty, hep-ph/1308.xxxx

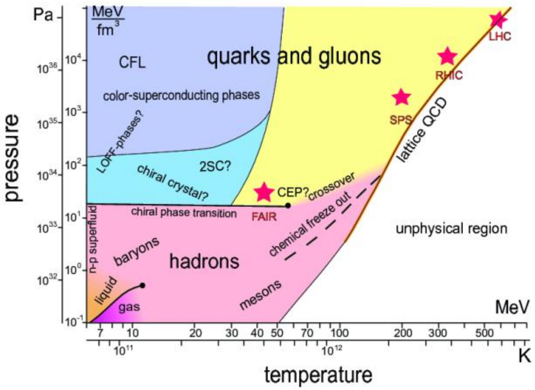
F. Hebenstreit, J. Berges, D. G., PRD 87 (2013) 105006

F. Hebenstreit, J. Berges, D. G., hep-ph/1307.4619

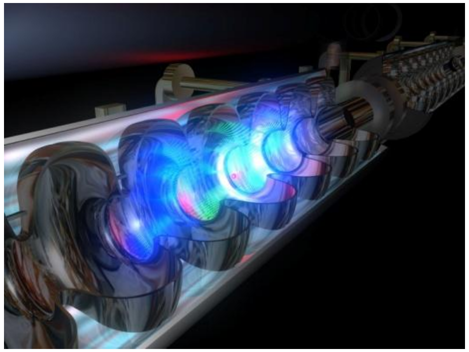
Introduction

Fermion production important for:

- **Heavy-ion collisions**
  - Production of quarks from highly occupied gauge fields
  - String breaking in QCD
- **Intense laser beams**
  - Vacuum pair production of electron-positron pairs
- **Non-equilibrium processes!**
  - Real-time description necessary
  - Initial value problems



Wambach, J. et al. AIP Conf.Proc. 1441 (2012) 794-796



http://scr3.golem.de/screenshots/1206/XFEL/xfel_5.jpg

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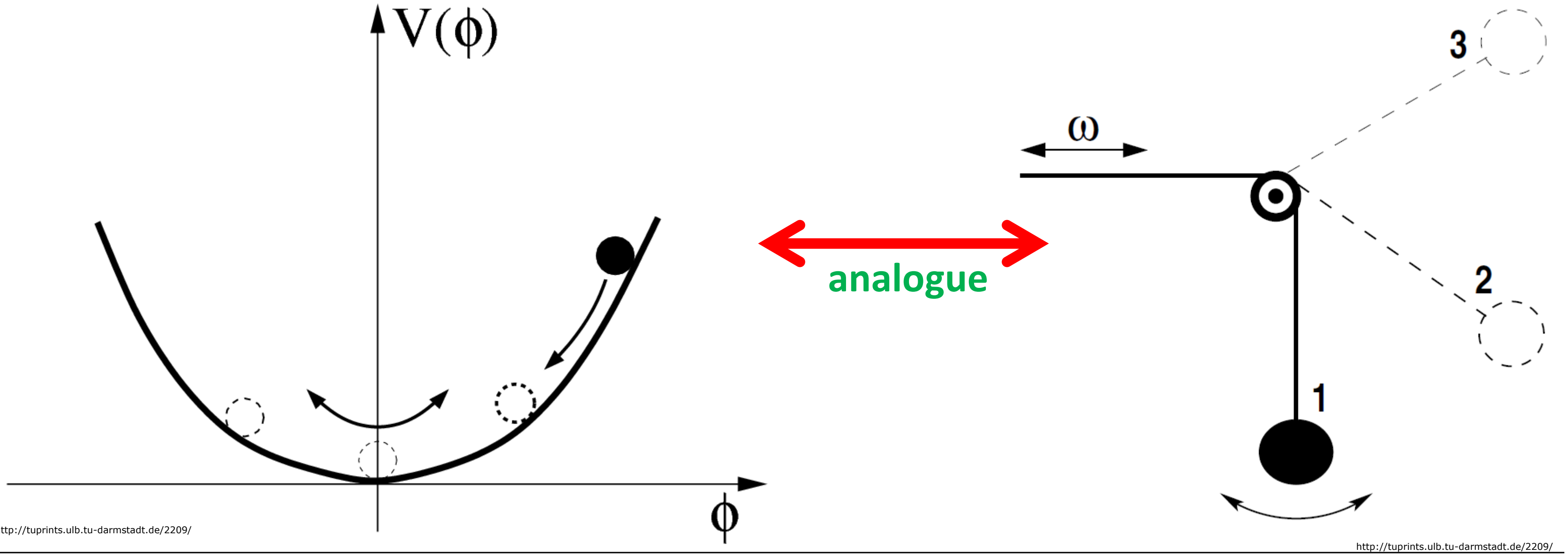
Introduction

Parametric resonance

- Enhancement of boson fluctuations through instabilities
- Strong and weak wave turbulence
  - Dual cascade

R. Micha, I. I. Tkachev, Phys.Rev. D70 (2004) 043538; J. Berges, A. Rothkopf, J. Schmidt, Phys.Rev.Lett. 101 (2008) 041603; J. Berges, D. Sexty, Phys. Rev. Lett. 108 (2012) 161601



http://tuprints.ulb.tu-darmstadt.de/2209/

http://tuprints.ulb.tu-darmstadt.de/2209/

Introduction

Quark-meson model

- 2 flavours of quarks coupled to mesons

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu\right)\psi + \frac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \frac{1}{2}m^2\phi_a\phi_a - \frac{\lambda}{4!\cdot N_s}\left(\phi_a\phi_a\right)^2 - \frac{g}{N_f}\bar{\psi}\left(\sigma + i\gamma_5\vec{\tau}\vec{\pi}\right)\psi$$

  - O(4) self-interacting meson field $\phi = \{\sigma, \pi^1, \pi^2, \pi^3\}$ $\phi(t) = \langle\sigma(t, \mathbf{x})\rangle$
  - 3+1 dimensions

Schwinger model

- QED in 1+1 dimensions $\mathcal{S} = \int d^2x \left(\bar{\psi}[i\gamma^\mu D_\mu - m]\psi - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu}\right)$
  - Fermion pair production from electric fields

$$\frac{\Delta n^\pm}{LT} = \frac{eE_0}{2\pi}\exp\left(-\frac{\pi m^2}{eE_0}\right) = \frac{m^2\epsilon}{2\pi}\exp\left(-\frac{\pi}{\epsilon}\right) \qquad E_c = \frac{m^2}{e} \qquad \epsilon = \frac{E_0}{E_c}$$

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Implementation

Schwinger model

- Linear potential between charges
  - String formation/breaking similar to QCD

- *No magnetic fields!*
- *No propagating photons!*

Lattice approach

- Plaquette formulation: $$\mathcal{S}_g[U] = \frac{1}{e^2 a_s a_t} \sum_{\mathbf{x}} \mathrm{Re}\,[1 - U_{01}(\mathbf{x})]$$
- Temporal axial gauge: $\mathcal{A}_0 = 0 \;\; U_0(\mathbf{x}) = 1$
- Classical-statistical bosonic fields

$$\langle O \rangle = \int D\mathcal{A}_{t_0} DE_{t_0}\, W[\mathcal{A}_{t_0}, E_{t_0}]\, O_{\mathrm{cl}}[\mathcal{A}_{t_0}, E_{t_0}]$$

$$O_{\mathrm{cl}}[\mathcal{A}_{t_0}, E_{t_0}] \;=\; \int D\mathcal{A}\, O[\mathcal{A}]\, \delta(\mathcal{A} - \mathcal{A}_{\mathrm{cl}}[\mathcal{A}_{t_0}, E_{t_0}])$$

$$\partial_\mu \mathcal{F}^{\mu\nu}(x,t) \;=\; -e\,\mathrm{Tr}\,[\gamma^\nu F(x,x;t)]$$

Fermion backreaction from statistical propagator:

$$F(x,y;t) \equiv \frac{1}{2}\langle [\psi(x,t), \bar{\psi}(y,t)] \rangle$$

G. Aarts and J. Smit, Nucl. Phys. B 555 (1999) 35

Implementation

Lattice approach

- Quantum fermions using ♂/♀ - method

S. Borsanyi and M. Hindmarsh, Phys. Rev. D 79 (2009) 065010

- Set of auxiliary spinor fields $\psi_{M/F}(x)$ is introduced
- Initialization

$$\psi_{M,F}(t=0,\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} e^{-i\mathbf{px}} \frac{1}{\sqrt{2}} \sum_s \left(\xi_s(\mathbf{p}) u_s(\mathbf{p}) \pm \eta_s(\mathbf{p}) v_s(\mathbf{p})\right)$$

  - With random numbers:

$$\langle \xi_s(\mathbf{p}), \xi^*_{s'}(\mathbf{q}) \rangle = (2\pi)^3 \delta_{ss'} \delta(\mathbf{p}-\mathbf{q}) \left(1 - 2n^s_+(\mathbf{p})\right)$$
$$\langle \eta_s(\mathbf{p}), \eta^*_{s'}(\mathbf{q}) \rangle = (2\pi)^3 \delta_{ss'} \delta(\mathbf{p}-\mathbf{q}) \left(1 - 2n^s_-(\mathbf{p})\right)$$

- Evolved in time using EoM

$$i\partial_\mu \gamma^\mu \psi_g(x) - \frac{g}{2} \left(\sigma(x) + i\gamma_5 \vec{\tau}\vec{\pi}(x)\right) \psi_g(x) = 0$$

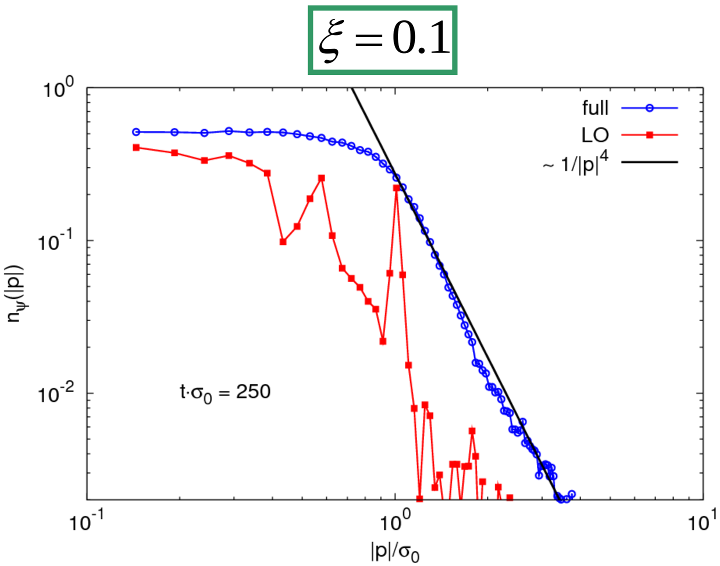
- Average over all pairs:

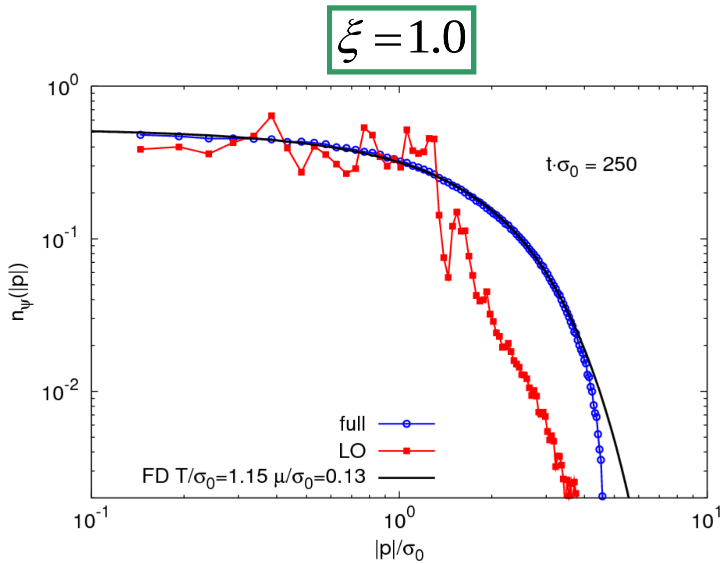
$$F_{\mathrm{sto}}(x,y;t) \equiv \langle \psi_M(x,t) \bar{\psi}_F(y,t) \rangle = \langle \psi_F(x,t) \bar{\psi}_M(y,t) \rangle$$

$$F_{\mathrm{sto}}(x,y;t) \stackrel{!}{=} F(x,y;t)$$

Results

Quark-meson model:





- *Total number of produced fermions strongly enhanced*
- *Quantum effects important, even at weak couplings*

LO:

$$\left[i\partial_\mu \gamma^\mu - \frac{g}{N_f}\phi(t) \right] D(x,y) = 0$$

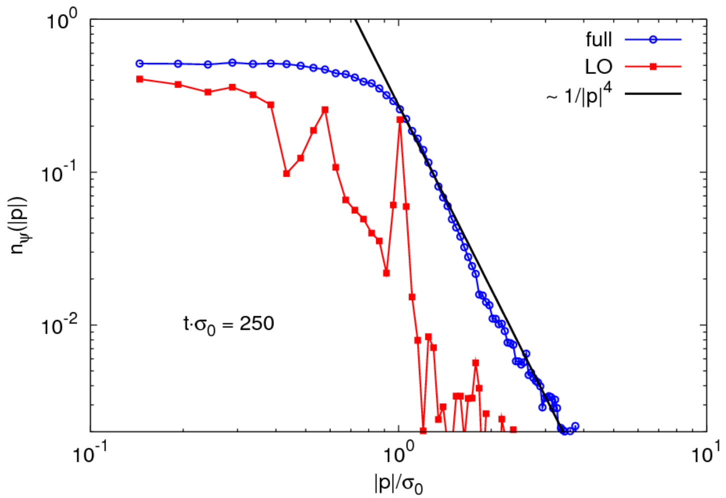
No fluctuations!

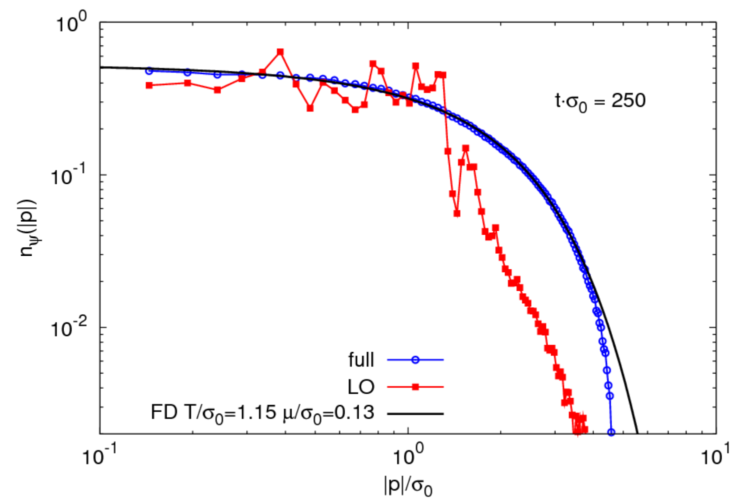
Baacke, Heitmann, Pätzold, PRD 58 (1998) 125013; Greene, Kofman, PLB 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, JHEP 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP 0002(2000) 034; …

Results

Quark-meson model:

Results consistent with functional methods (2PI)!





$\xi = 0.1$

- *Unexpected power-law dependence in the UV*
- *Exact value of exponent varies in time, stays* ≈ 4

$\xi = 1.0$

- *Full dynamics shows higher occupancy in UV*
- *Quarks seem to take on a Fermi-Dirac distribution*

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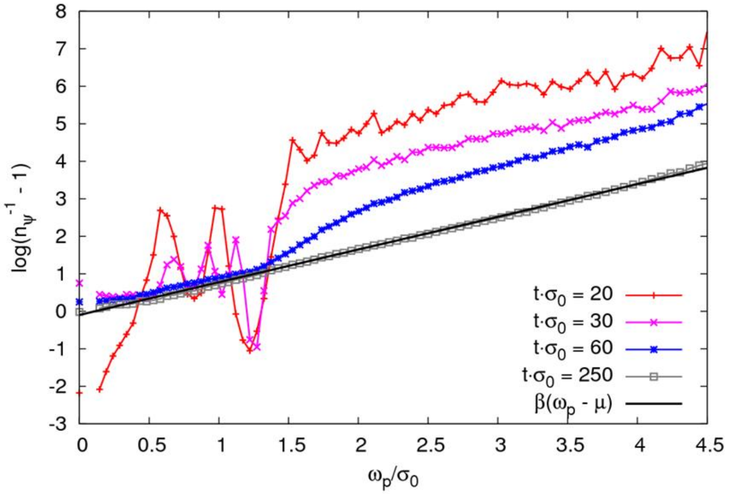
Results

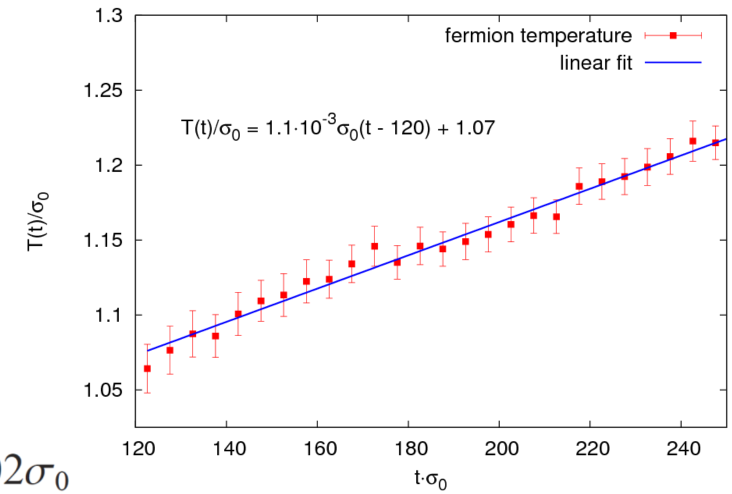
Quark-meson model:



- *Similar quasi-equilibration already observed in a strongly coupled Abelian Higgs model in 1+1d*

G. Aarts, J. Smit, Phys. Rev. D61 (2000) 025002

- *Time dependent non-equilibrium temperature*



Stefan-Boltzmann limit:

$$T_{eq} = \sigma_0 \left(\frac{45 N_s}{\pi^2 \left(N_s + \frac{7}{2} N_f \right) \lambda} \right)^{\frac{1}{4}} \simeq 2.02 \sigma_0$$

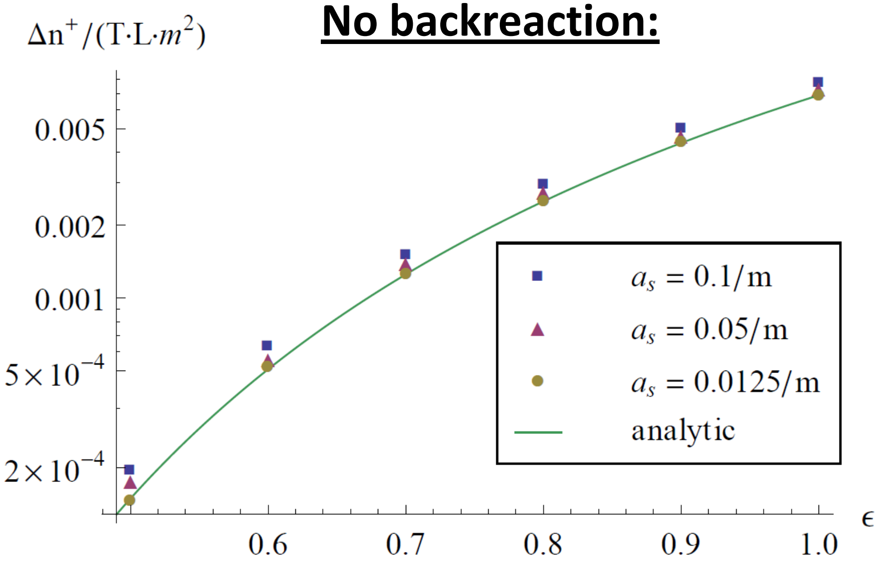
Results

Schwinger model:

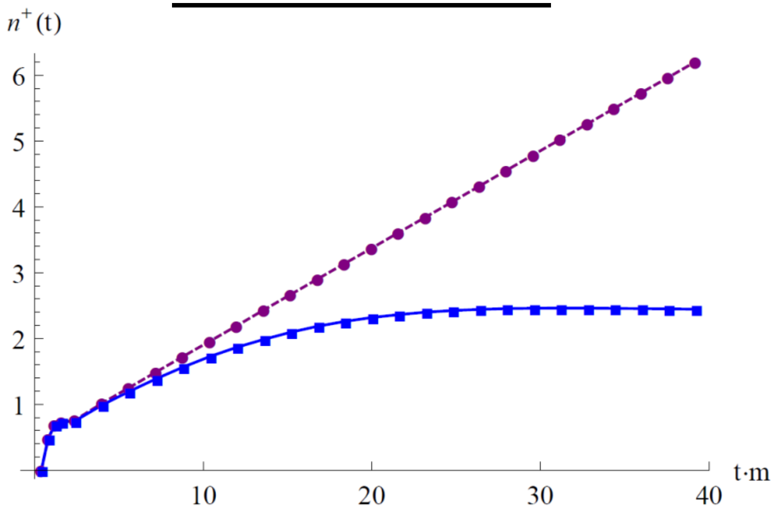
- Homogeneous field

$$E(x,t) = E_0$$

No backreaction:



With backreaction:



- *Convergence to analytic prediction in continuum limit*
- *Backreaction leads to saturation*
- *Earlier results successfully confirmed*

F. Hebenstreit, R. Alkofer and H. Gies, Phys. Rev. Lett. 107 (2011) 180403; F. Hebenstreit, PhD thesis, arXiv:1106.5965 [hep-ph]

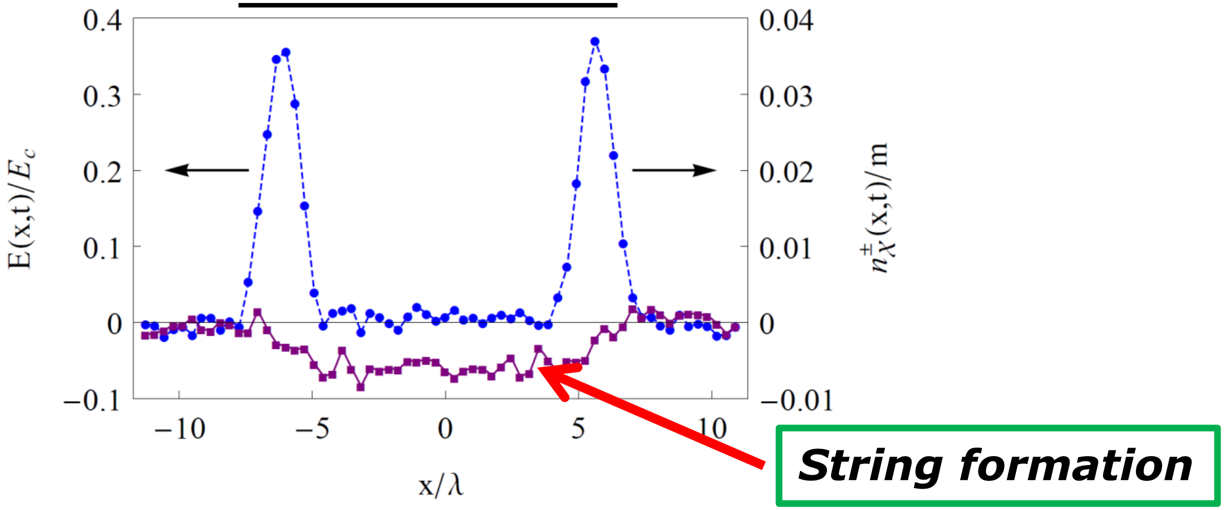
Results

Schwinger model:

- Inhomogeneous field

$$E(x,t) = E_0 \,\mathrm{sech}^2(\omega t) \exp\left(-\frac{x^2}{2\lambda^2}\right)$$

With backreaction:



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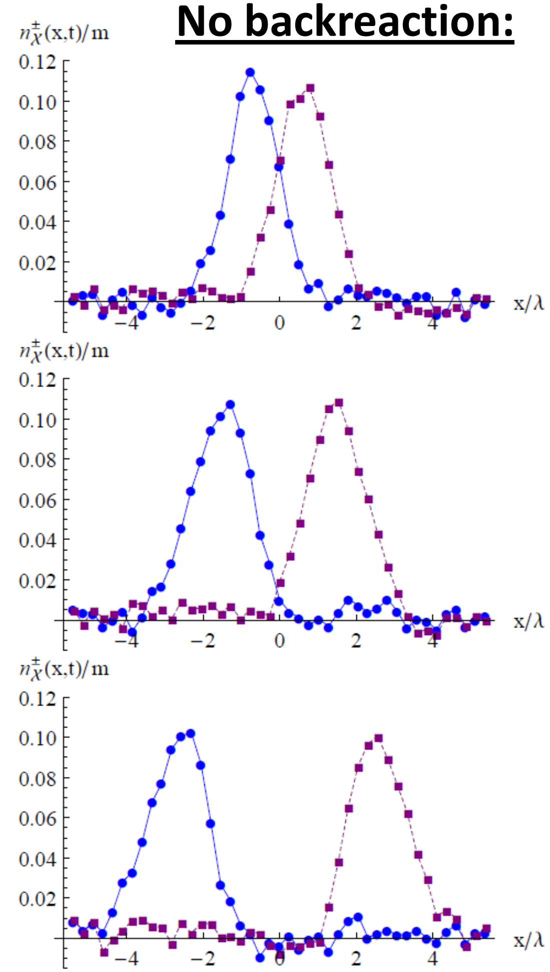

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Results

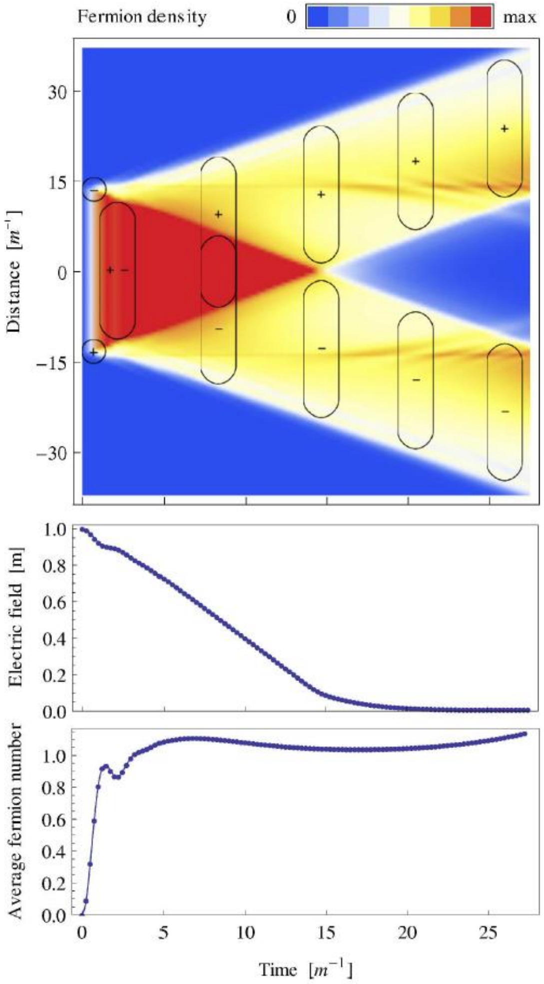
String breaking:

- Pair of static charges

$$\partial_x E = e\left[\delta(x + d/2) - \delta(x - d/2)\right]$$

- Dynamical two-stage process:
  - First production of overlapping oppositely charged fermion pairs
  - No screening
  - Then charges are separated by the external field
- Condition for string breaking:

$$V_{\text{str}} \gtrsim 2m + W \longrightarrow d_{\mathbf{c}} \simeq \frac{24m}{e^2}$$



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Summary & Outlook

Summary:

- Real-time simulations of fermion dynamics in 3+1d feasible with modern lattice techniques
- Quantum fluctuations strongly enhance quark production
- In strongly coupled systems a Fermi-Dirac distribution emerges
- Dynamical picture of string formation/breaking established

Outlook:

- Fermion production in “real” QED (3+1d) and QCD (ongoing)
- Consider other instabilities (Nielsen-Olesen, Weibel, etc.)
- Generalization to expanding systems

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The End

Thank you for your attention!

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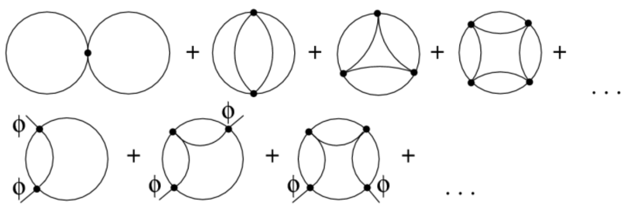
Implementation

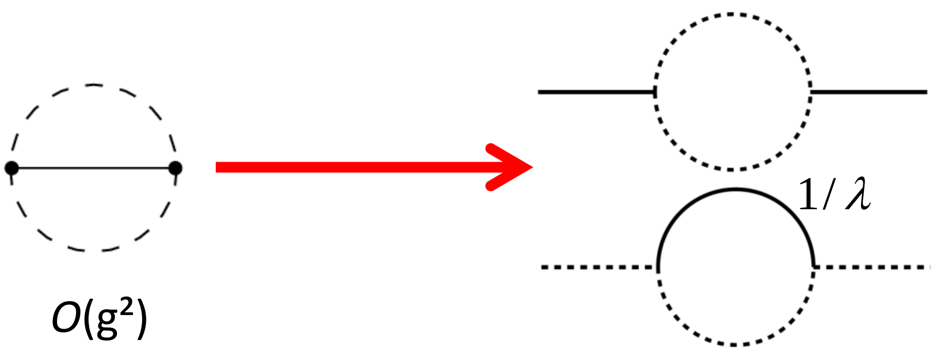
2PI effective action: J. Berges, AIP Conf. Proc. 739, 3 (2005)

- Different truncation schemes for the action
  - 1/N expansion to NLO in the number of scalar fields



J. Berges, Nucl. Phys. A 699 (2002) 847
G. Aarts et al, Phys. Rev. D 66 (2002) 045008

  - Coupling expansion to NLO in the Yukawa coupling



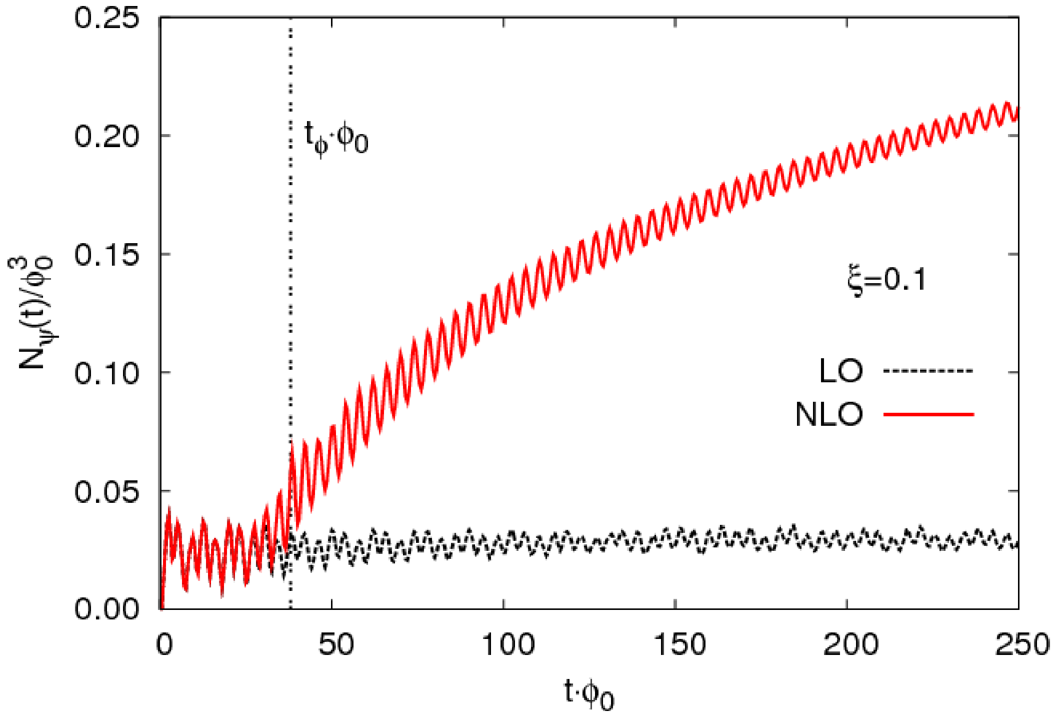
Contribution to meson self-energy

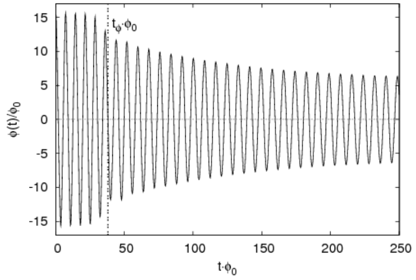
Quark self-energy

Results

Quark-meson model:

- Total number of produced fermions strongly enhanced
- Quantum effects important, even at weak couplings
- Quark production rate $\sim \xi = g^2/\lambda$





LO: $$\left[i\partial_\mu \gamma^\mu - \frac{g}{N_f}\phi(t) \right] D(x,y) = 0$$

No fluctuations!

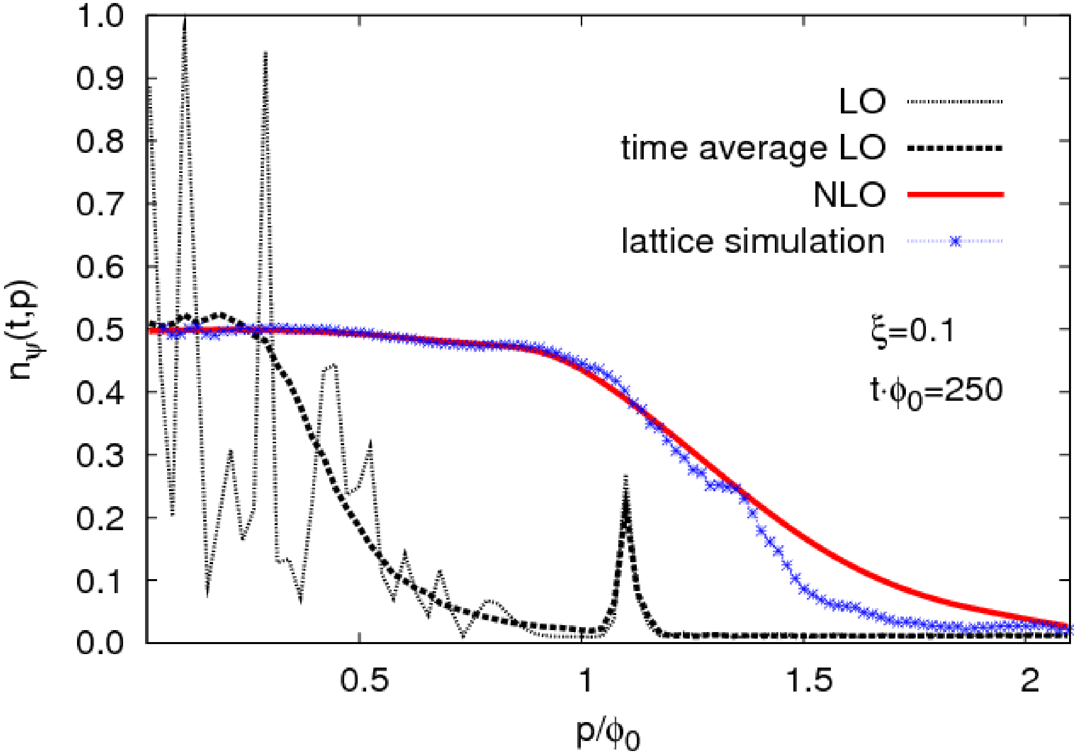
Baacke, Heitmann, Pätzold, PRD 58 (1998) 125013; Greene, Kofman, PLB 448 (1999) 6; Giudice, Peloso, Riotto, Tkachev, JHEP 9908 (1999) 014; Garcia-Bellido, Mollerach, Roulet, JHEP 0002(2000) 034; ...

Results

Weak coupling:

- Qualitative and quantitative difference between LO and NLO
- Good agreement between lattice and 2PI
- Particle numbers drop at the rescaled initial field amplitude:

$$\phi_0 = \phi(t=0)/\sqrt{6N_s/\lambda}$$



LO:

$$\left[i\partial_\mu \gamma^\mu - \frac{g}{N_f}\phi(t) \right] D(x,y) = 0$$

Supplement

Initial conditions:

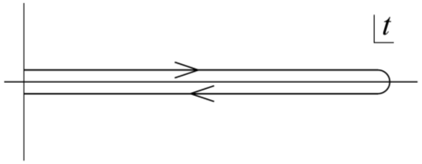
- Large sigma field expectation value $\phi(t=0) \sim \sqrt{\frac{6N_s}{\lambda}}$
- Fermionic (quantum) vacuum fluctuations
- Bosonic vacuum fluctuations for unstable modes only
  - Otherwise problems with classical Rayleigh-Jeans divergence
- Enhancement of boson fluctuations through instabilities
  - Bosonic two-point function becomes parametrically large $\langle\phi\phi\rangle \sim \frac{1}{\lambda}$
  - IR fixed point prevents early thermalization (**strong turbulence**)

J. Berges, A. Rothkopf and J. Schmidt, Phys. Rev. Lett. 101 (2008) 041603; J. Berges and G. Hoffmeister, Nucl. Phys. B 813, 383 (2009); J. Berges and D. Sexty, Phys. Rev. D 83, 085004 (2011); J. Berges, D. Sexty, arXiv:1201.068

Supplement

2PI effective action: J. Berges, AIP Conf.Proc. 739 (2005) 3–62

- Functional method to describe time evolution of quantum fields
  - Closed time path (in-in formalism)



  - Time evolution of two-point functions

$$\text{commutator:} \qquad \rho(x,y) = i\langle[\Phi(x),\Phi(y)]\rangle\,,$$

$$\text{anti-commutator:} \qquad F(x,y) = \frac{1}{2}\langle\{\Phi(x),\Phi(y)\}\rangle\,.$$

  - Kadanoff-Baym equations of motion

$$\left[\Box_x + M^2(x)\right]\rho(x,y) \;=\; -\int_{y^0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)\rho(z,y)\,,$$

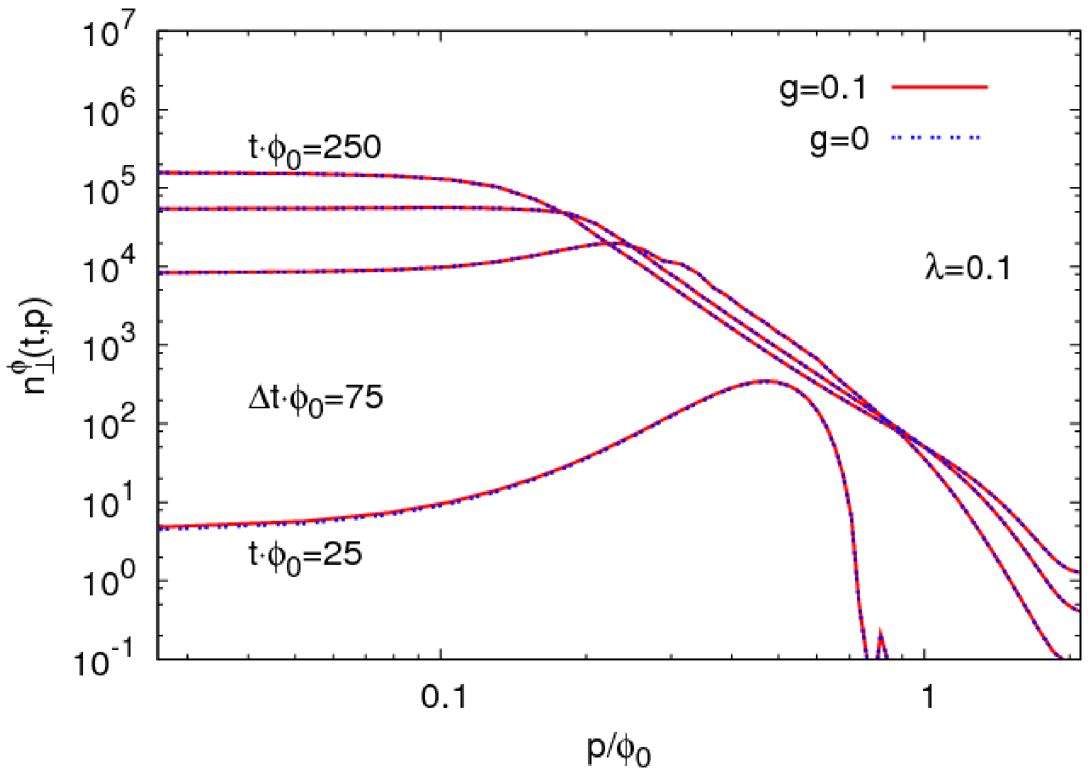
$$\left[\Box_x + M^2(x)\right]F(x,y) \;=\; -\int_{0}^{x^0} \mathrm{d}z\,\Sigma_\rho(x,z)F(z,y) + \int_0^{y^0} \mathrm{d}z\,\Sigma_F(x,z)\rho(z,y).$$

Memory integrals!

Supplement

Weak coupling:

- Bosonic dynamics unchanged for weak Yukawa couplings
- High occupation numbers in the IR (classical-statistical regime)
- Strong turbulence at later times



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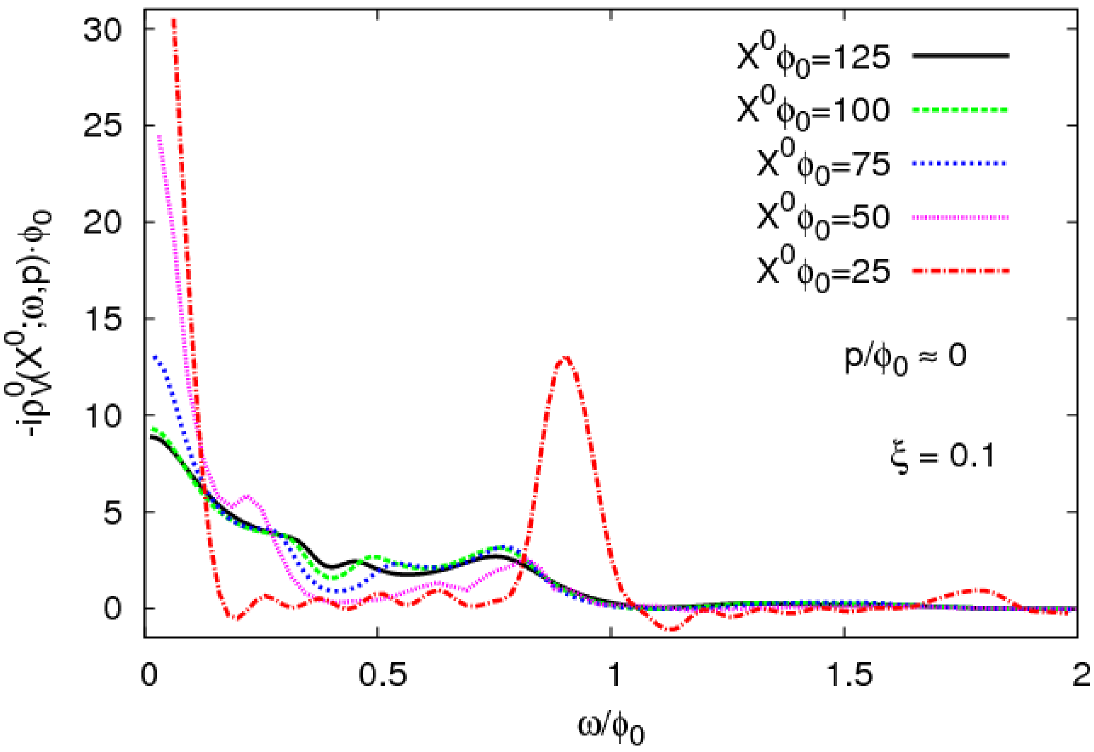
Supplement

Weak coupling:

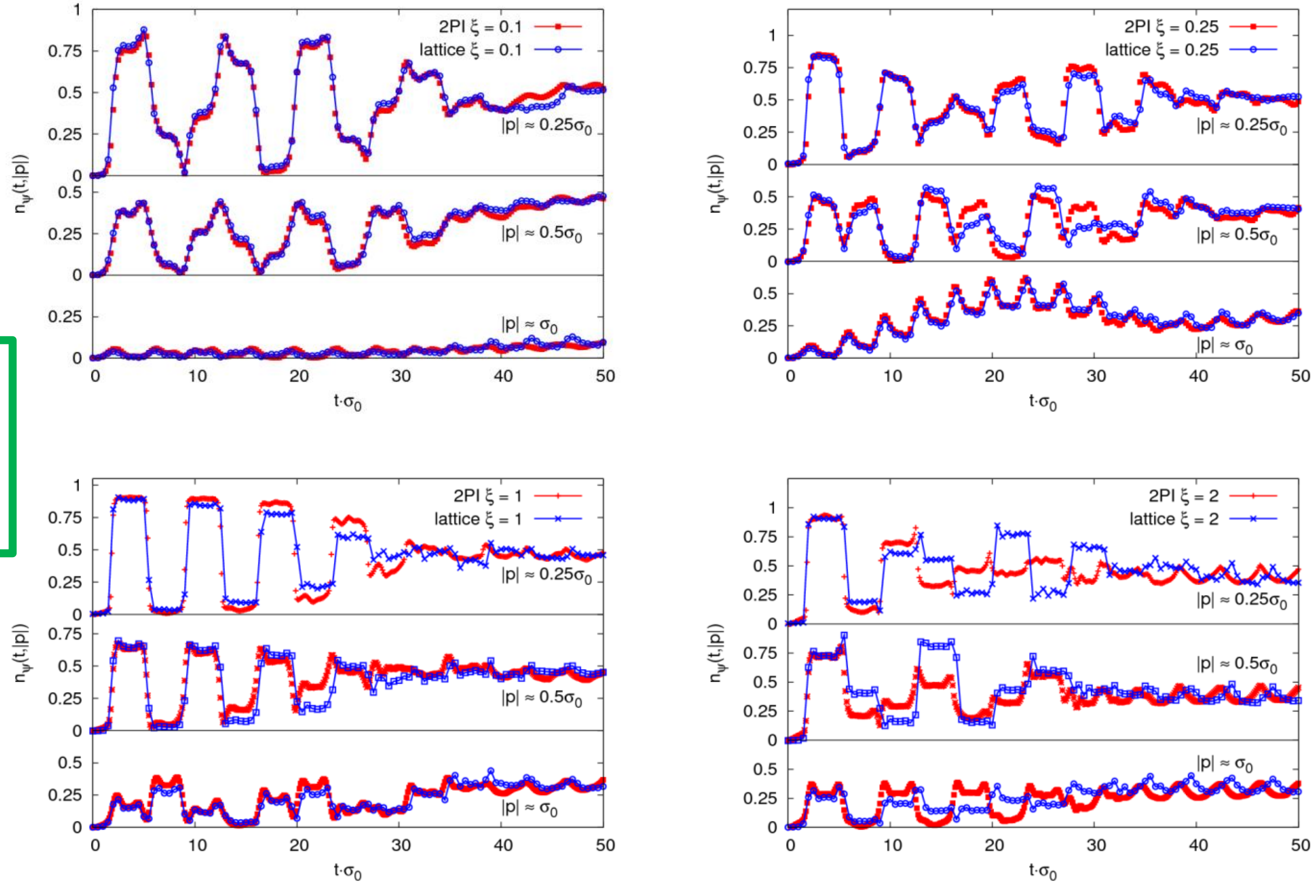
- Wigner transformed dynamical spectral function
- Quasi-particle description valid at early times
- Strongly correlated medium at later times



Supplement

Increasing coupling:

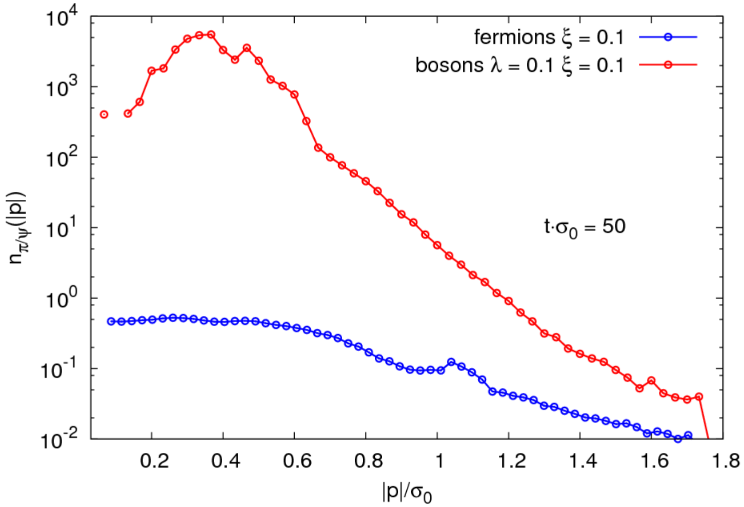
Loop expansion breaks down at stronger couplings!

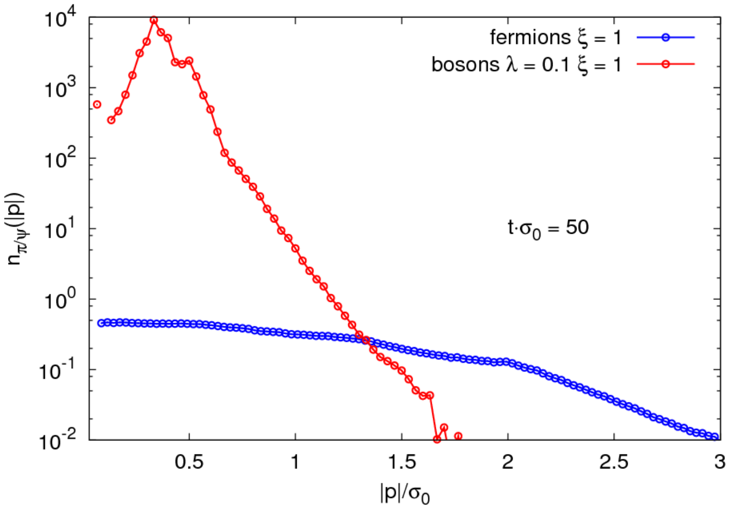


Supplement

From weak to strong:

- Production of high-momentum quarks kinematically forbidden in LO perturbation theory
- Multiple scatterings become relevant at strong coupling

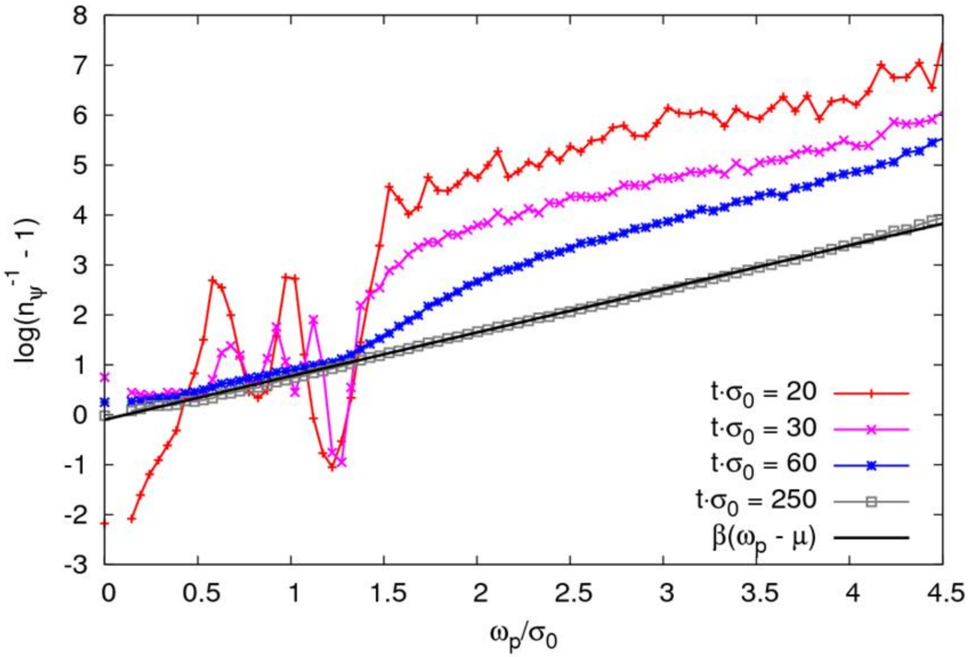




Supplement

Strong coupling:

- First scattering smoothens the IR distribution
- Occupancy in the UV is built up much later
- Lattice dispersion relation used



$$\xi = 1.0$$

Supplement

Further issues:

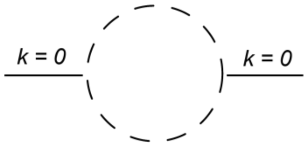
- Lattice discetization causes so-called fermion doublers
  - Spatial doublers addressed with pseudoscalar Wilson term

$$W_{PS}\psi_g(x) = i\gamma_5 \frac{ra_s}{2} \triangle_x \psi_g(x) \quad \longrightarrow \quad \omega(\mathbf{p}) = \sqrt{m_\psi^2 + \bar{p}_i \bar{p}^i + \frac{r^2 a_s^2}{4} (\mathbf{p})_{lat}^4}$$

- Quadratically divergent scalar mass has to be renormalized

$$m_{0,\sigma/\vec{\pi}}^2 + \Sigma_{\sigma/\vec{\pi}}(m_0^2, k=0) = m_R^2$$

  - Perturbative one-loop renormalization:



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Supplement

Lattice formulas and definitions:

Scalar (standard) Wilson term:

$$W_S \psi_g(x) = \frac{r a_s}{2} \triangle_x \psi_g(x)$$

$$\omega(\mathbf{p}) = \sqrt{m_\psi^2 + \bar{p}_i \bar{p}^i + r a_s m_\psi (\mathbf{p})^2_{lat} + \frac{r^2 a_s^2}{4} (\mathbf{p})^4_{lat}}$$

Free statistical propagator:

$$D(t=0,\mathbf{p}) = \frac{m_\psi - \gamma^i p_i - i\gamma_5 \frac{r a_s}{2} (\mathbf{p})^2}{2\omega(\mathbf{p})} \Big(1 - 2n_\psi(\mathbf{p})\Big)$$

Particle number definitions:

$$n_\psi(t,\mathbf{p}) = \frac{1}{2} - \frac{\left[\bar{p}_i D_V^i(t,\mathbf{p}) + m_\psi(t) D_S(t,\mathbf{p}) + i \frac{r a_s}{2} (\mathbf{p})^2_{lat} D_{PS}(t,\mathbf{p})\right]}{\sqrt{\bar{p}_i \bar{p}^i + m_\psi^2(t,\mathbf{p}) + \frac{r^2 a_s^2}{4} (\mathbf{p})^4_{lat}}}$$

$$\epsilon_a(t,\mathbf{p}) = \sqrt{\frac{\partial_t \partial_{t'} F_a(t,t',\mathbf{p})\,|_{t=t'}}{F_a(t,t',\mathbf{p})\,|_{t=t'}}}$$

$$n_\phi^a(t,\mathbf{p}) = F_a(t,t,\mathbf{p}) \epsilon_a(t,\mathbf{p}) - \frac{1}{2}$$

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