Momentum broadening of partons on the light cone from the lattice

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[Based on arXiv:1307.5850]

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Outline

- Motivation
- Theoretical approach
- Soft physics contribution from a Euclidean setup
- Lattice implementation
- Results
- Discussion and conclusions

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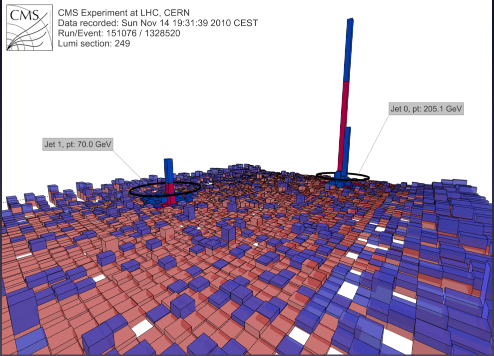
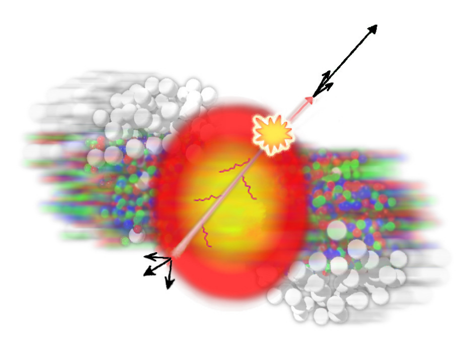
Jet quenching

Jet quenching: suppression of high-p_T particles and back-to-back correlations in A-A collisions



Jet quenching

Provides experimental evidence for strongly coupled quark-gluon plasma (QGP)



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Theory overview

A *hard probe* to heavy-ion collisions, involving large momentum transfer Q

QCD factorization theorems:

$$\sigma_{(\mathrm{M}+\mathrm{N}\rightarrow \text{hadron})} = \mathrm{f_M}(\mathrm{x}_1, \mathrm{Q}^2) \otimes \mathrm{f_N}(\mathrm{x}_2, \mathrm{Q}^2) \otimes \sigma(\mathrm{x}_1, \mathrm{x}_2, \mathrm{Q}^2) \otimes \mathrm{D}_{\text{parton}\rightarrow\text{hadron}}(\mathrm{z}, \mathrm{Q}^2)$$

$\mathrm{f_A}(\mathrm{x}, \mathrm{Q}^2)$: parton distribution functions

$\sigma(\mathrm{x}, \mathrm{y}, \mathrm{Q}^2)$: short-distance cross-section

$\mathrm{D}_{\text{parton}\rightarrow\text{hadron}}(\mathrm{z}, \mathrm{Q}^2)$: fragmentation function

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$f_A(x, Q^2)$: parton distribution functions

$\sigma(x, y, Q^2)$: short-distance cross-section

$D_{\text{parton}\to\text{hadron}}(z, Q^2)$: fragmentation function

Here: Focus on propagation of a light-cone parton in QGP

Hard parton propagation in QGP

- Multiple soft-scattering, eikonal approximation
- Transverse momentum broadening described by jet quenching parameter:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}$$

- Can be evaluated in terms of a *collision kernel* $C(p_\perp)$ (differential parton-plasma constituents collision rate)

$$\hat{q} = \int \frac{d^2 p_\perp}{(2\pi)^2} p_\perp^2 C(p_\perp)$$

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Hard parton propagation in QGP

$C(p_\perp)$ related to a two-point correlator of *light-cone Wilson lines*

Benzke et al. arXiv:1208.4253, Laine arXiv:1208.5707, Laine and Rothkopf arXiv:1304.4443, Rothkopf's talk, Cherednikov et al. arXiv:1307.5518

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Key idea

Energy scale hierarchy in high-temperature, perturbative QCD:

$$g^2 T/\pi \text{ (ultrasoft)} \ll gT \text{ (soft)} \ll \pi T \text{ (hard)}$$

IR divergences accounted for by 3D effective theories:

- electrostatic QCD (3D Yang-Mills + adjoint scalar field) for soft scale
- magnetostatic QCD (3D pure Yang-Mills) for ultrasoft scale

Large NLO corrections related to soft, *classical* fields

Observation: Soft contributions to physics of light-cone partons *insensitive* to parton velocity $\longrightarrow$ Turn the problem Euclidean!

Caron-Huot arXiv:0811.1603

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Proof

Spatially separated ($|t| < |z|$) light-like Wilson lines:

$$G^{<}(t, x_\perp, z) = \int d\omega d^2p_\perp dp^z \tilde{G}^{<}(\omega, p_\perp, p^z) e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

$$= \int d\omega d^2p_\perp dp^z \left[\frac{1}{2} + n_B(\omega)\right] \left[\tilde{G}_R(\omega, p_\perp, p^z) - \tilde{G}_A(\omega, p_\perp, p^z)\right] e^{-i(\omega t - x_\perp \cdot p_\perp - z p^z)}$$

Shift $p'^z = p^z - \omega t/z$, integrate over frequencies by analytical continuation into upper (lower) half-plane for retarded (advanced) contribution

Caron-Huot arXiv:0811.1603, Ghiglieri et al. arXiv:1302.5970

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Proof

Result: Sum over Matsubara frequencies

$$G^{<}(t, x_\perp, z) = T \sum_{n \in \mathbb{Z}} \int d^2p_\perp dp'^z \tilde{G}_E(2\pi nT, p_\perp, p'^z + 2\pi i nTt/z) e^{i(x_\perp \cdot p_\perp + zp'^z)}$$

- $n \neq 0$ contributions: exponentially suppressed at large separations
- Soft contribution: from $n = 0$ mode. Time-independent: evaluate in EQCD

Caron-Huot arXiv:0811.1603, Ghiglieri et al. arXiv:1302.5970

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Electrostatic QCD on the lattice

Super-renormalizable EQCD Lagrangian

$$\mathcal{L} = \frac{1}{4} F^a_{ij} F^a_{ij} + \mathrm{Tr}\left((D_i A_0)^2 \right) + m_E^2 \mathrm{Tr}\left(A_0^2 \right) + \lambda_3 \left(\mathrm{Tr}\left(A_0^2 \right) \right)^2$$

Matching parameters to high-T QCD Braaten and Nieto arXiv:hep-ph/9501375

- 3D gauge coupling: $g_E^2 = g^2 T$
- Debye mass parameter: $m_E^2 = \left(1 + \frac{n_f}{6}\right) g^2 T$
- 3D quartic coupling: $\lambda_3 = \frac{9 - n_f}{24\pi^2} g^4 T$

Our setup: $n_f = 2$, $T \simeq 398$ MeV and 2 GeV, Wilson regularization

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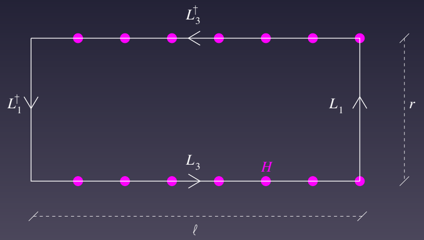
Operator implementation

Light-cone Wilson line correlator

$$\langle W(\ell, r)\rangle = \left\langle \mathrm{Tr}\left(L_3 L_1 L_3^\dagger L_1^\dagger\right)\right\rangle \sim \exp\left[-\ell V(r)\right]$$

with

$$L_3 = \prod U_3 H \qquad L_1 = \prod U_1 \qquad H = \exp(-a g_E^2 A_0)$$



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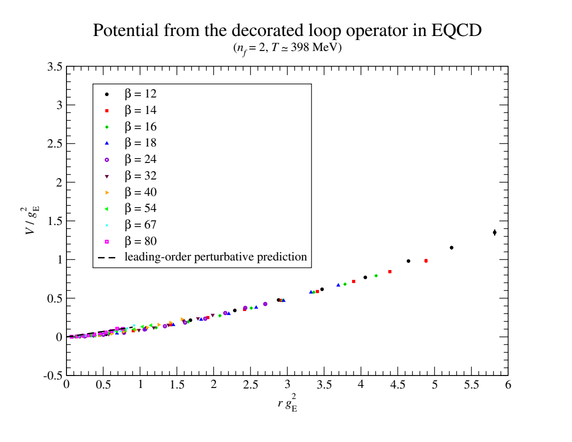
Light-cone loops: Lattice *versus* PT



$\hat{q}$ estimate

Soft NLO contribution to $\hat{q}$ quite large:

$$\hat{q}_{\text{EQCD}} \simeq \begin{cases} 0.55(5) g_E^6 & \text{for } T \simeq 398 \text{ MeV} \\ 0.45(5) g_E^6 & \text{for } T \simeq 2 \text{ GeV} \end{cases}$$

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Approximate estimate: $\hat{q} \sim 6$ GeV^2/fm at RHIC temperatures

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- Lattice approach possible for certain real-time problems (see also Ji arXiv:1305.1539 and Lin's talk)
- Here: focus on soft physics in thermal QCD
- Outlined approach is *systematic*
- Tentative estimate of jet quenching parameter
- Results in ballpark of
  - holographic estimates Liu, Rajagopal and Wiedemann arXiv:hep-ph/0605178 ✓
  - experimental estimates Eskola et al. arXiv:hep-ph/0406319 ✓

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