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Towards understanding thermal jet quenching via lattice simulations

Mikko Laine and Alexander Rothkopf

Albert Einstein Center for Fundamental Physics

Institute for Theoretical Physics

University of Bern

Based on: M. Laine, A.R.

JHEP 1307 (2013) 082 (arXiv:1304.4443)

LATTICE 13

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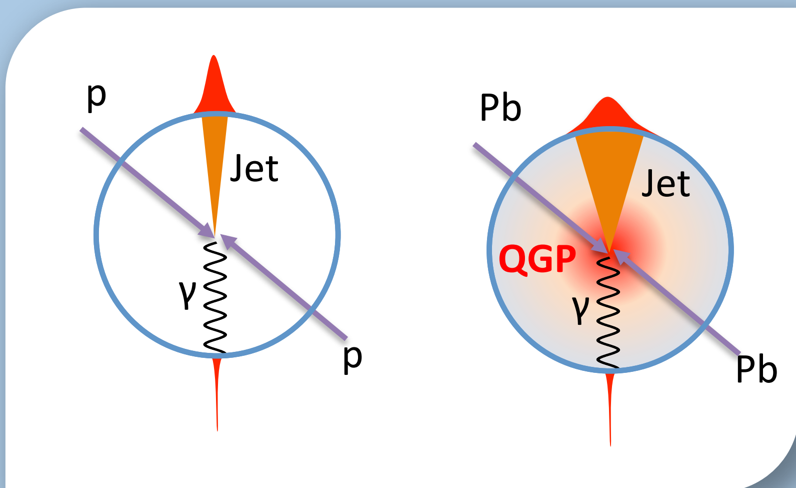
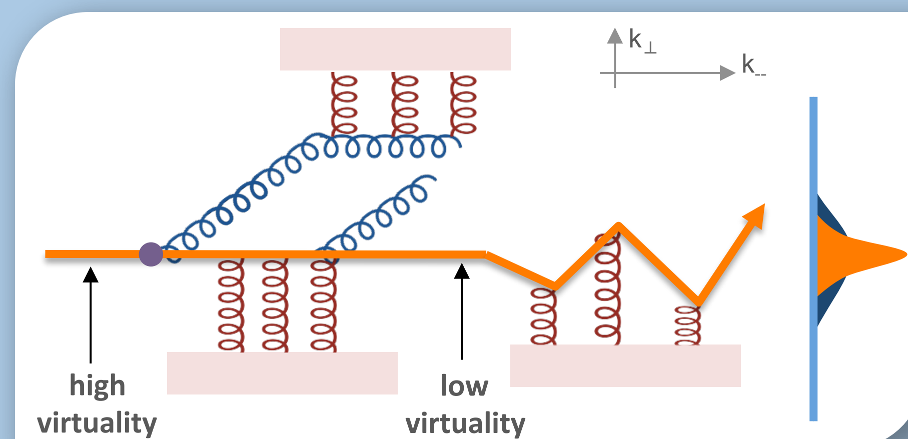
Physics motivation: Jets as QGP probes



- Tightly collimated shower of light hadrons
- Theoretically interesting since seeding parton momentum $Q \gg \Lambda_{QCD}, T$ (initially hard process)
- In the presence of QGP observed broadening is much stronger than perturbative estimates

C.-Solana, Salgado arXiv:0712.3443
U. Wiedemann arXiv:09082306

- Medium modifies:
  - Gluon emission probability
  - Gluon momentum distribution
  - seeding parton $k_\perp$ distribution
- Collisional momentum broadening only
- **$P(k_\perp)$**: $k_\perp$ gained by seeding parton after traversing a path of length L



light cone coordinates:
$x_+=(x_0+x_3)/\sqrt{2}$, $x_-=(x_0-x_3)/\sqrt{2}$, $x_\perp=(x_1,x_2)/\sqrt{2}$, $A_+=(A_0+A_3)/\sqrt{2}$

The light-cone Wilson loop

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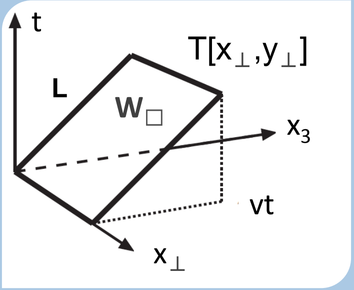
- Separation of scales allows EFT treatment: Soft collinear effective theory
  D'Eramo et. al. Phys.Rev. D84 (2011) 065015
  Benzke et. al. JHEP 1302 (2013) 129
  - Seeding quark along x_{-} and medium gluons (Glauber) couple via A_{+}
  - Multiple scatterings:

$$W[y^+, y_\perp] = \mathrm{P}\exp\left[ig\int_0^L dy^- A^+(y^+, y^-, y_\perp)\right]$$



$$P(k_\perp) = \int d^2x_\perp e^{ik_\perp x_\perp}\frac{1}{N_c}\left\langle \mathrm{Tr}\left[W^\dagger[0, x_\perp]T[x_\perp, y_\perp]W[0, y_\perp]T[0, 0]\right]\right\rangle$$

  - For t->∞: $\langle W_\Box\rangle \sim \mathrm{Exp}[-C(x_\perp)t]$

$$\hat{q} = \frac{1}{L}\int_0^{q_{max}}\frac{d^2k_\perp}{(2\pi)^2}k_\perp^2 P(k_\perp) \simeq \int_0^{q_{max}}\frac{d^2k_\perp}{(2\pi)^2}k_\perp^2 C(k_\perp)$$

  - Connection to Euclidean domain? Sensitivity of the Wilson loop to further tilting?
    S. Caron-Huot PRD79 065039 (2009)
- If formulated in Euclidean time (starting point for lattice QCD or hard-thermal loops)

$$W[(\tau, \mathbf{r}_\perp + \mathbf{v}_E\tau); (0, \mathbf{r}_\perp)] = \mathbb{1} - ig_0\int_0^\tau d\tau_1\,(A_0 + \mathbf{v}_E\cdot\mathbf{A})(\tau_1, \mathbf{r}_\perp + \mathbf{v}_E\tau_1) + \ldots$$

  - Two analytic continuations necessary for comparison: τ -> it and v_E -> -iv

The classical limit and the lattice

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- Avoid analytic continuations by real-time investigation in the classical limit
  - A single parameter remains: $\beta_G \equiv \frac{2N_c}{g^2 T a}$
- Classical Lattice Gauge Theory (Hamiltonian formulation)

Grigriev, Rubakov Nucl. Phys. B299 1988
Ambjorn et. al. Nucl. Phys. B353 1991

  - A_0=0 gauge decouples time slices: SU(3) spatial links U_i and su(3) electric fields E^b_i

$$H_{\rm cl} = \sum_{\mathbf{x}} \left\{ \sum_{i=1}^{3} {\rm Tr}\,[\mathcal{E}_i^2(\mathbf{x})] + \frac{1}{2N_c} \sum_{i,j=1}^{3} {\rm Tr}\,[\mathbb{1} - P_{ij}(\mathbf{x})] \right\}$$

P_{ij} plaquette

$$G(\mathbf{x},t) \equiv \sum_{\mathbf{x}} \left[E_i(\mathbf{x},t) - U_{-i}(\mathbf{x},t) E_i(\mathbf{x}-i,t) U^{\dagger}_{-i}(\mathbf{x},t) \right] = 0$$

Gauss constraint

  - The Hamiltonian equations of motion

$$\begin{aligned} a\,\partial_t U_i(\mathbf{x},t) &= i\,(2C_A)^{\frac{1}{2}}\, E_i(\mathbf{x},t) U_i(\mathbf{x},t)\ , \\ a\,\partial_t E_i^b(\mathbf{x},t) &= -\left(\frac{2}{C_A}\right)^{\frac{1}{2}} {\rm Im\,Tr}\left[T^b U_i(\mathbf{x},t) \sum_{|j|\neq i} S^{\dagger}_{ij}(\mathbf{x},t) \right] \end{aligned}$$

- t remains continuous
- S denotes staple
- CA=Nc color factor

Numerical implementation

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- Strategy: Thermal average as ensemble average over stochastic initial conditions
  - Hypercubic 3d lattice N^3 with spacing a, time discretization dt << a

Initial set of U and E:

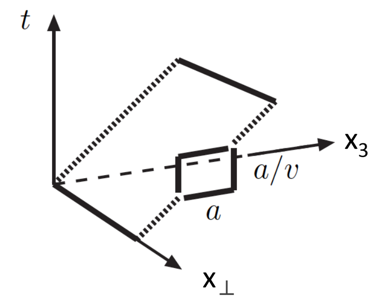
$$\exp[-\beta_G H_{cl}] \prod_{\mathbf{x}} \delta(G(\mathbf{x}))$$

- Prethermalize U with 3d Monte Carlo (not exact)
- Since $P(E)=\exp[-E^2]$ draw from Gaussian
- Project onto G=0 surface
- Mix U and E via e.o.m

M. Laine, O. Philipsen, M. Tassler
JHEP 0709 (2007) 066

Evolve e.o.m.

- Naive forward Euler



- Save copy of U at each t = n a/v
- Tilted path from average of upper and lower link

Measure the tilted Wilson loop

- Caveat: captures IR physics at $k_\perp \sim g^2T/\pi$ but not at $k_\perp \sim m_E \sim gT/a$ due to lattice artifacts

See e.g. D. Bödecker, L. McLerran, A. Smilga PRD52 4675 1995

The Light-cone Wilson loop in CLGT

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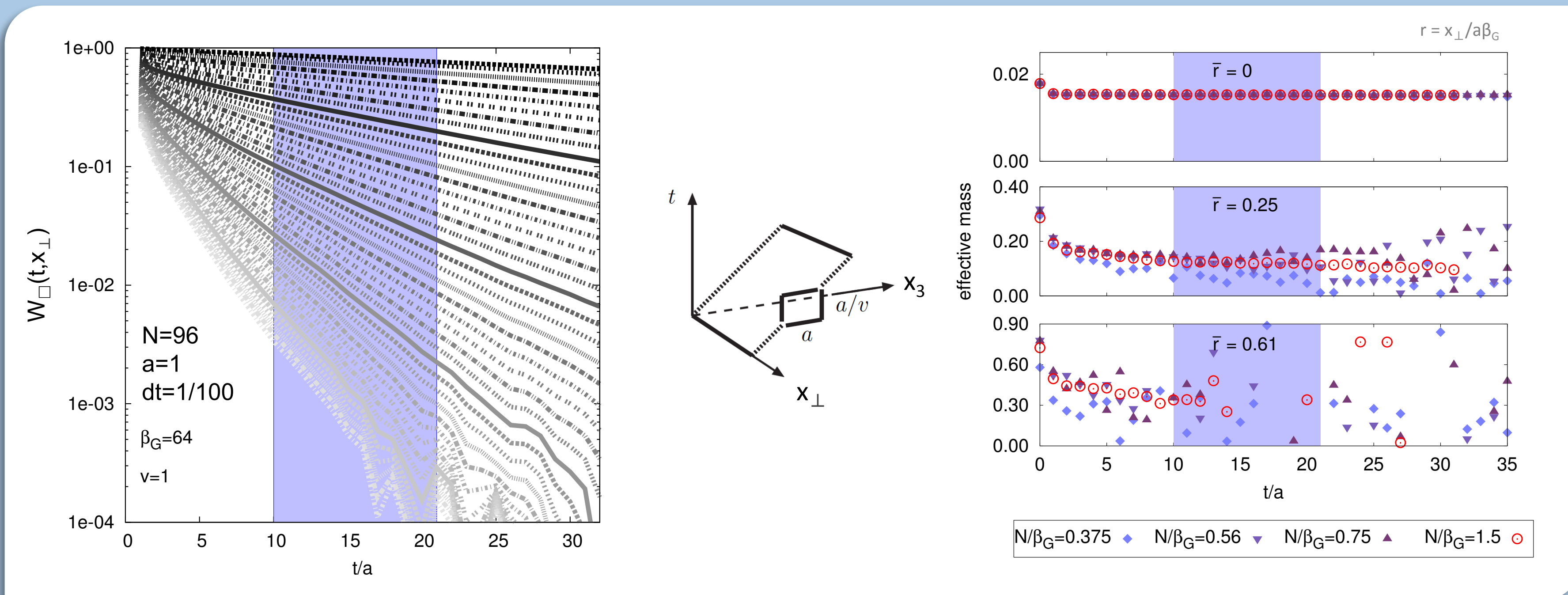

- Classical light-cone Wilson loop is purely real and shows exponential falloff at large t
- Fitting range as compromise between signal strength and single exponential regime
- Decay rate $C(x_\perp,v)$ extracted from single exponential fit

Crosscheck: Intercept velocity dependence

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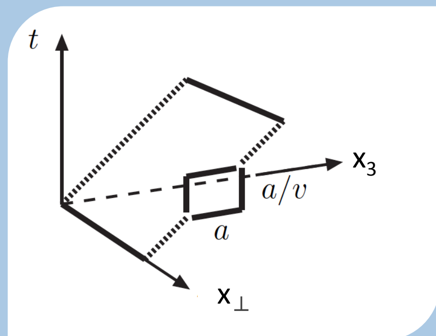
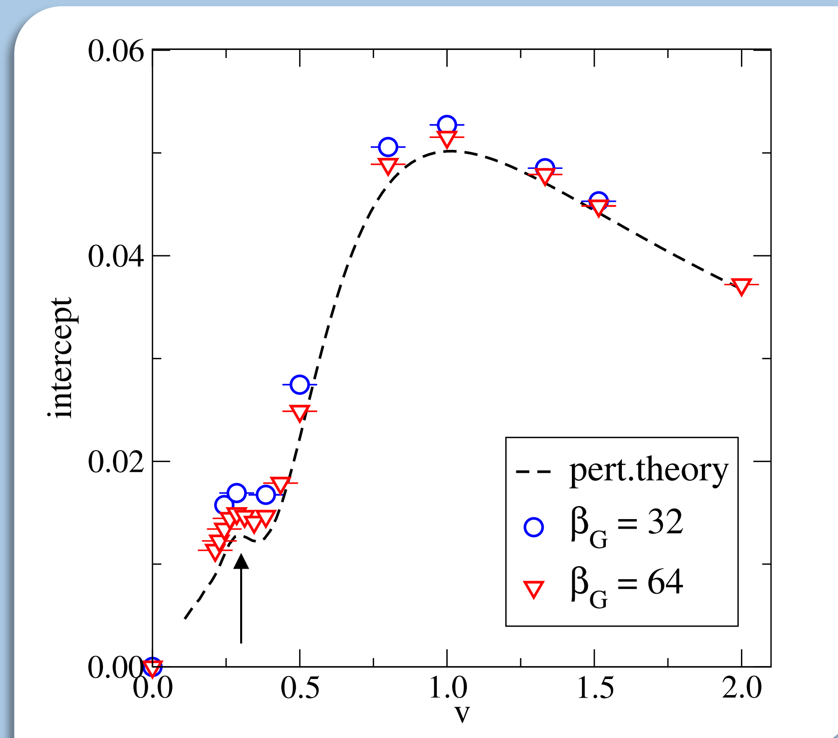
- Tilted Wilson lines do not cancel for $C(x_\perp=0,v)$



- Classical HTL on a finite lattice "intercept":

$$C(x_\perp = 0, v) = I(v) \equiv \frac{2va}{3} \int_{\mathbf{k}} \frac{\sin^2\left(\frac{a\tilde{k}}{2v}\right)}{\tilde{k}^2}$$



- For $x_\perp \ll 1/m_E$ and large β_G perturbative regime, agreement improves with larger β_G
- Structure around v=0.3 arises from lattice discretization dependent contribution

Numerical and analytical results for C(x$_\perp$,v)

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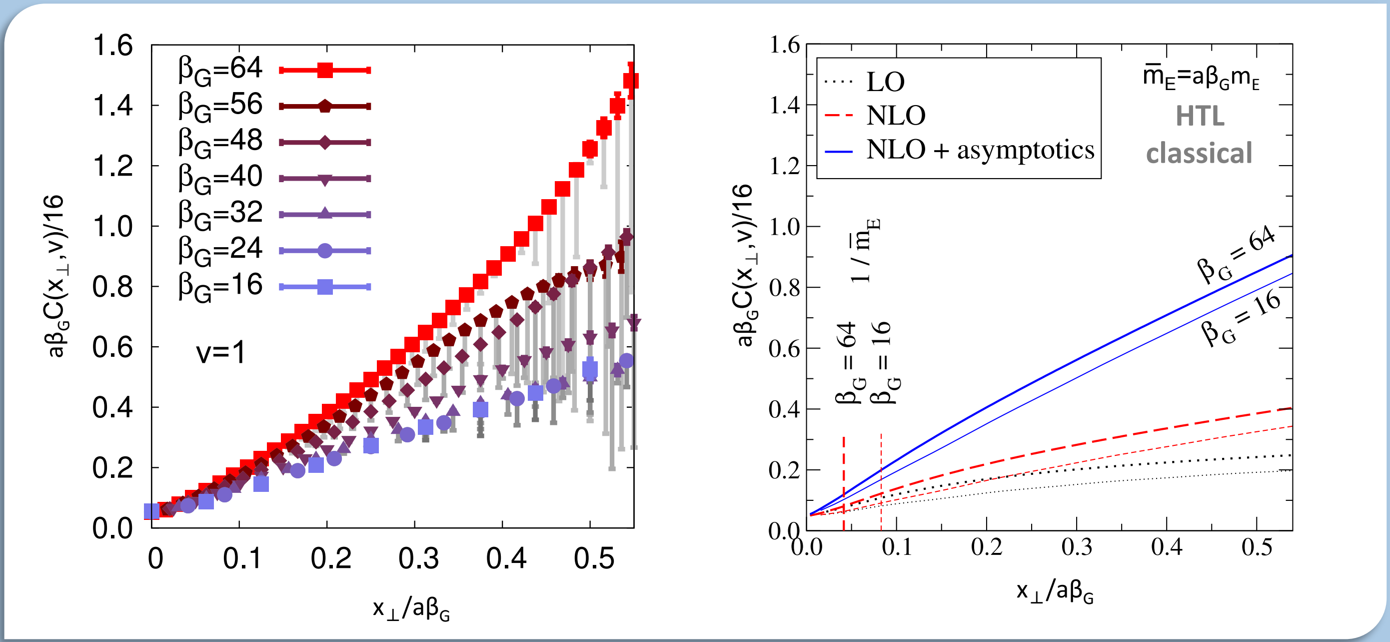

- At $x_\perp << 1/m_E$ non-zero intercept
- At $x_\perp \sim 1/m_E$ C($x_\perp$,v) rises monotonously BUT cutoff dependent m_E leads to artifacts
- At $x_\perp >> 1/m_E$ color magnetic scale g^2T/π dominates, LO/NLO result weaker than lattice
- Asymptotics from pure 3d Yang-Mills simulations quite close to lattice result at β_G=64

see also M. Laine EPJ C72 2233 (2012)

Velocity dependence of $C(x_\perp>0,v)$

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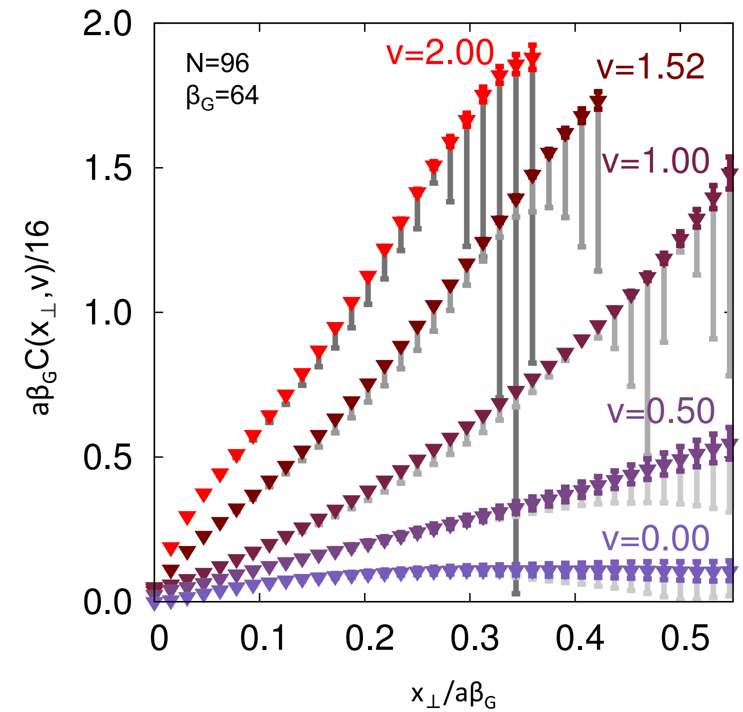
- Towards a quantum evaluation
  - $C(x_\perp,v)$ insensitive to small change in v ?
  - Tilting into space-like regime possible ?
  - Qualitatively different: Space-like Wilson loop can be captured by EQCD simulation. S. Caron-Huot PRD79 065039 (2009)
  - Classical lattice: No change as v>1
  - See talk by M. Panero (Mo 16:30h) for an EQCD based evaluation of $\hat{q}$. Panero, Rummukainen, Schäfer arXiv:1307.4850



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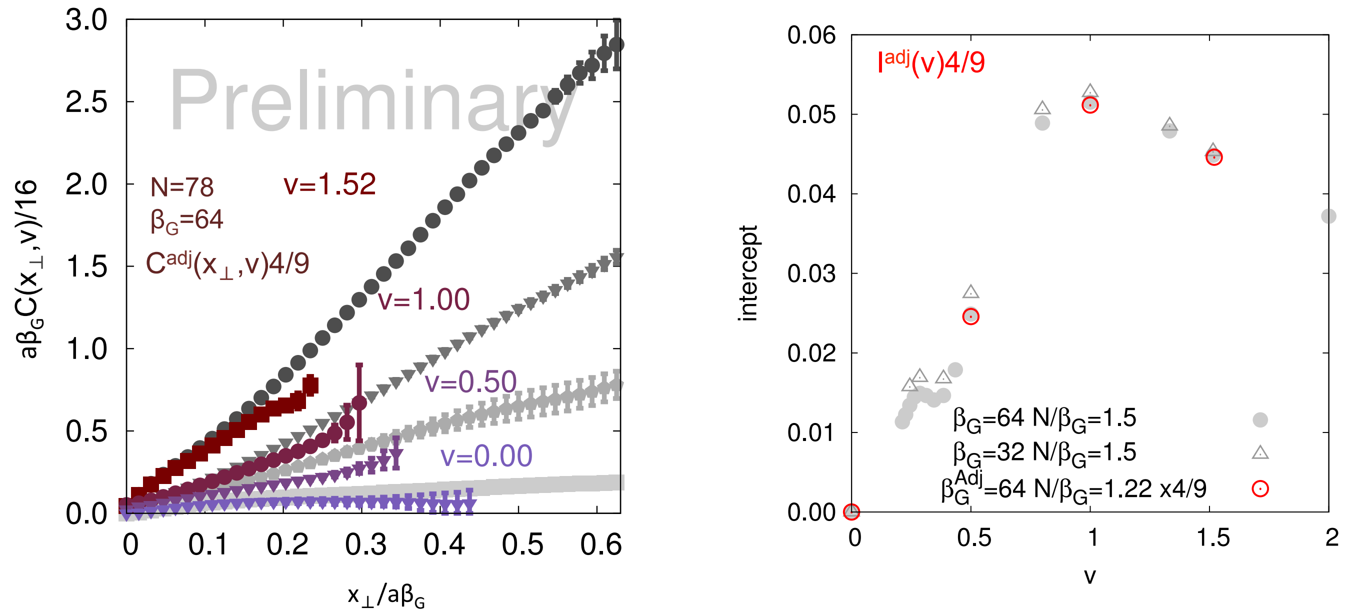
The adjoint representation



- If jet is seeded by high momentum gluon: Wilson loop in the adjoint representation
- From fundamental to adjoint representation: $$U^{adj}_{ab}(x) = \frac{1}{2}\mathrm{Tr}\left[T^a U(x) T^b U^\dagger(x)\right]$$
- From LO HTL calculation: C(x) to scale with color factor $C^{adj}(x_\perp,v)=9/4\, C(x_\perp,v)$?

Conclusion and Outlook

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- Investigation of the tilted real-time Wilson loop in the classical approximation
  - Classical Lattice Gauge Theory allows direct and non-perturbative estimation of $C(x_\perp,v)$
  - At small $x_\perp<<1/m_E$ and the largest β_G: good agreement with classical HTL on a lattice
  - For $x_\perp>>1/m_E$ and $\beta_G=64$ $C(x_\perp,v)$ larger than LO/NLO HTL but close to asymptotics
  - Tilting the Wilson loop into the space-like domain: No qualitative changes

- Need larger volumes to go to $t/a>>\beta_G$ for a more accurate extraction of $C(x_\perp,v)$
- Systematically investigate the adjoint representation Wilson loop
- Instead of tilting the Wilson loop, how about boosting the medium?

Thank you for your attention