QCD at imaginary chemical potential with Wilson fermions

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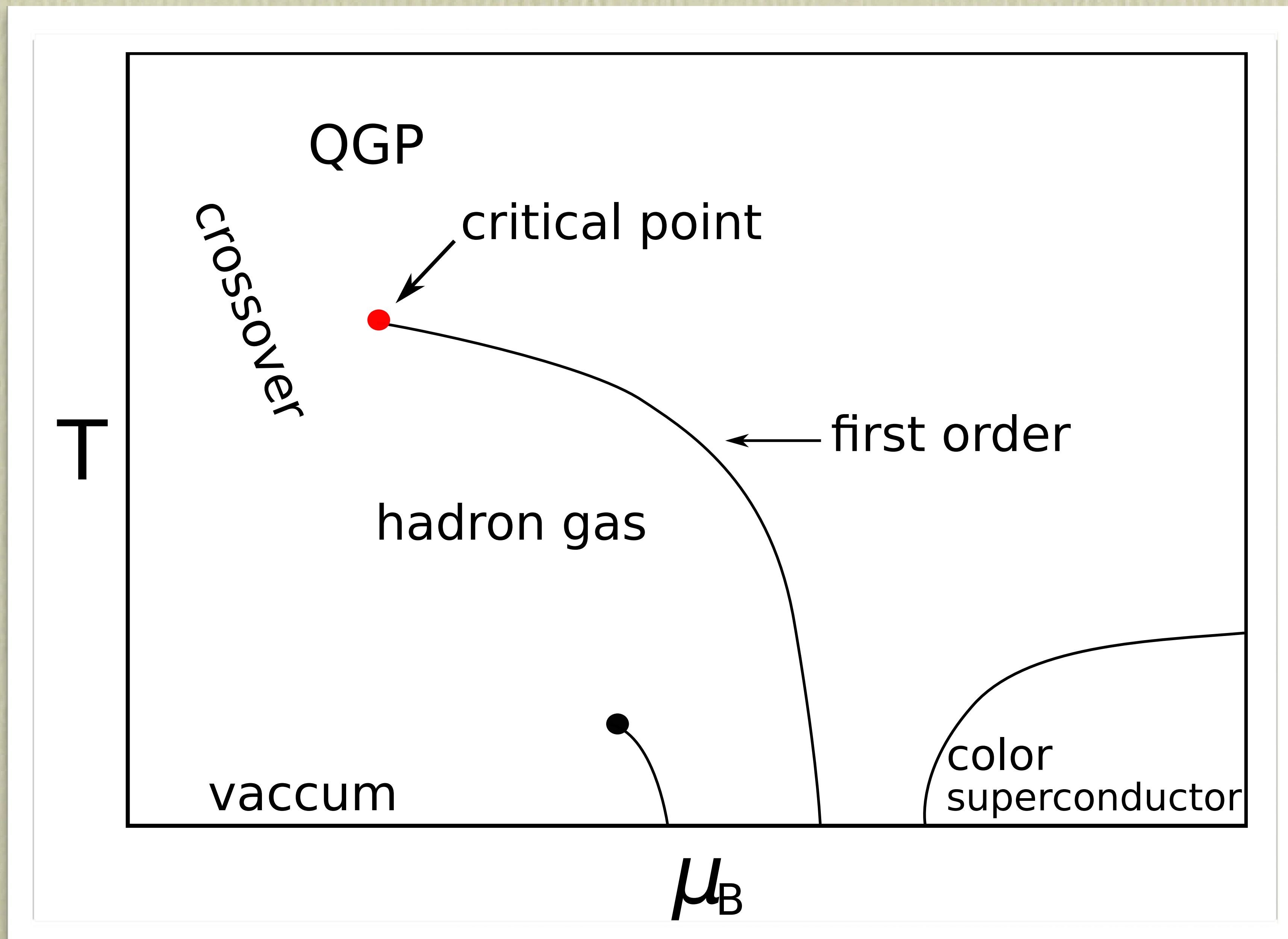
Outline

- Motivation
- Imaginary chemical potential
- Compression method and reweighting
- Numerical results
- Conclusions

Expected QCD phase diagram



complex

$$Z_{GC}(T,V,\mu) = \int \mathcal{D}U e^{-S_g(U)} \det M(U,\mu)$$

Imaginary chemical potential

For imaginary chemical potential, γ_5 symmetry insures that the determinant is real.

$$M(U,\mu)^\dagger = \gamma_5 M(U,-\mu^*)\gamma_5 \Rightarrow \det M(U, i\mu_I) \in R$$

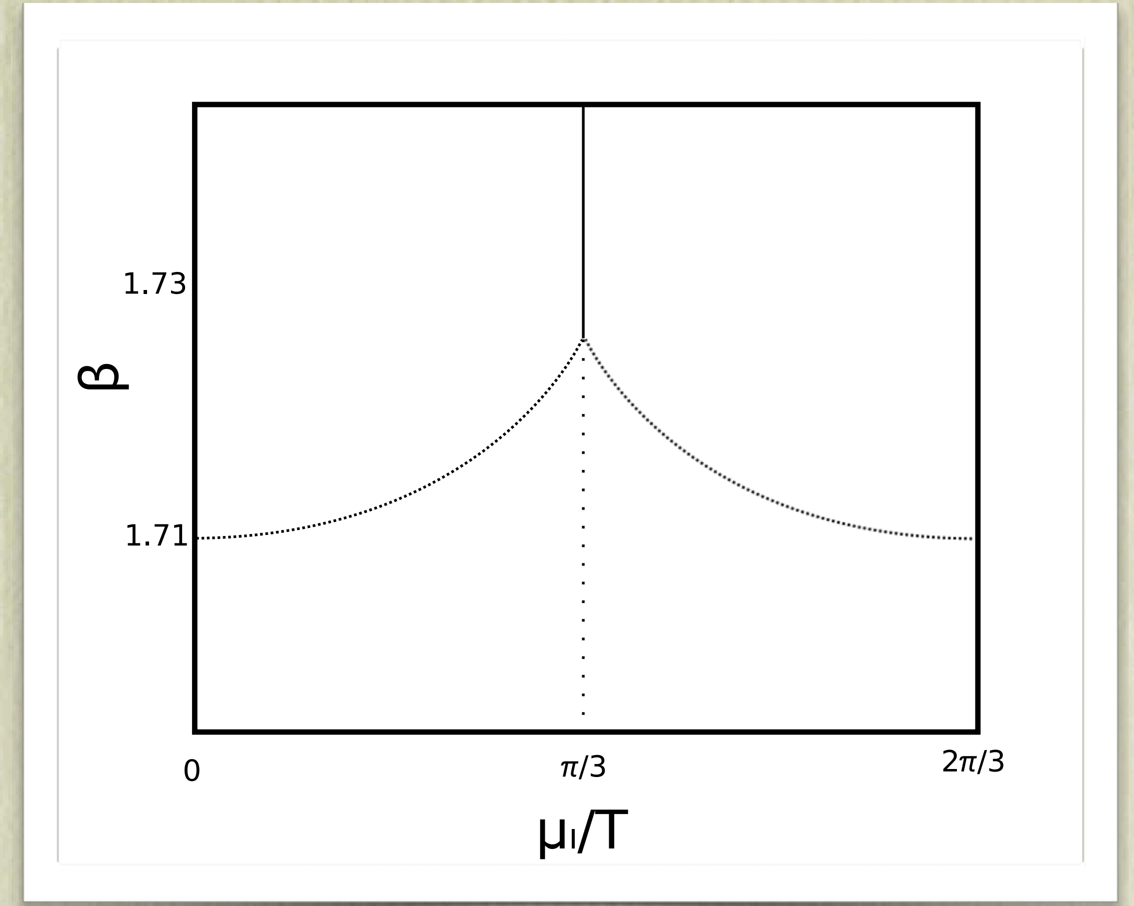
The grand canonical partition function is periodic in the complex plane due to the invariance of Haar measure and pure gauge action's invariance under the Z_3 transformations

$$[U_\mu(\boldsymbol{x},t)]_\pm = \begin{cases} U_\mu(\boldsymbol{x},t)e^{\pm i\frac{2\pi}{3}} & \text{if } t = N_t - 1 \text{ and } \mu = 4, \\ U_\mu(\boldsymbol{x},t) & \text{otherwise.} \end{cases}$$

$$Z_{GC}(T,V,\mu) = Z_{GC}(T,V,\mu \pm i\frac{2\pi}{3}T)$$

Roberge-Weiss transition

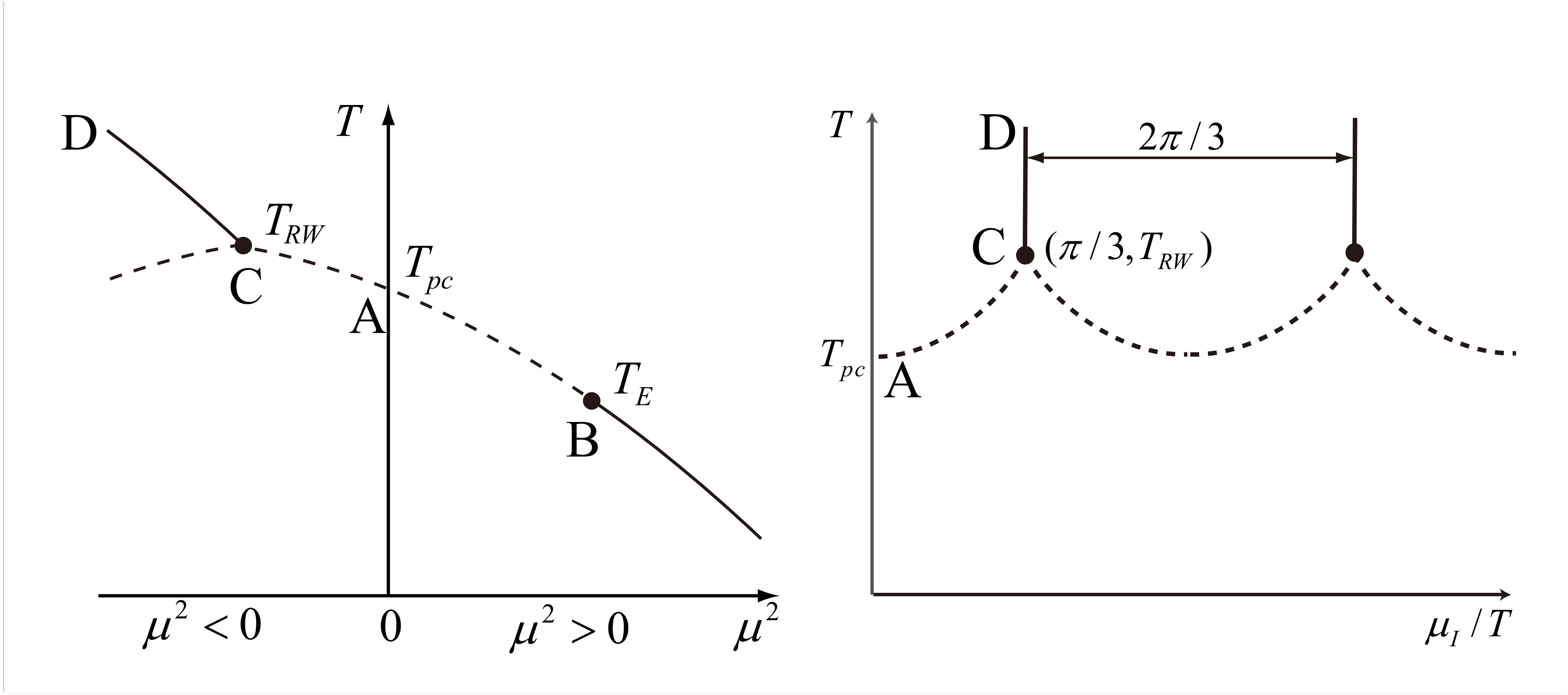
- Simulations are easy to setup since the chemical potential is introduced as a phase.
- For $\mu/T = i\pi, \pm i\pi/3$ we have a Z(2) symmetry. For example for $\mu = i\pi$, U and U* have equal probability.
- At high temperatures this symmetry is spontaneously broken and restored at low temperatures. (Roberge-Weiss transition)



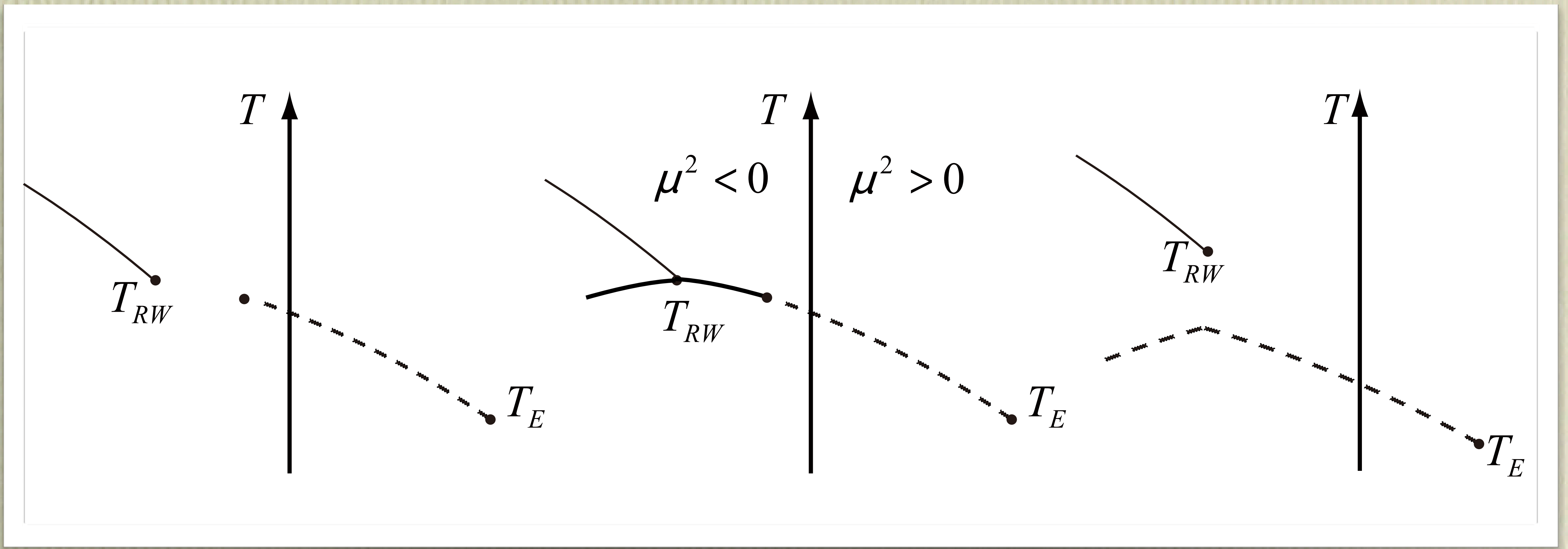
$$P_{\pm i\pi/3}(U) = P_{\pm i\pi/3}((U^*)_{\mp})$$

A. Roberge and N. Weiss, *Nucl. Phys.* **B275** (1986) 734.

Imaginary chemical potential



Possible scenarios



Previous studies

- N_f=2 P. de Forcrand and O. Philipsen 2002, M. D'Elia and F. Sanfilippo 2009 (staggered)
- N_f=3 P. de Forcrand and O. Philipsen 2010 (staggered)
- N_f=4 M. D'Elia and M.-P. Lombardo 2003, 2004, M. D'Elia, F. Di Renzo, and M. P. Lombardo 2007, P. Cea, L. Cosmai, M. D'Elia, and A. Papa 2010 (staggered)
- N_f=2 K. Nagata and A. Nakamura 2011 (wilson)

Fermion discretizations

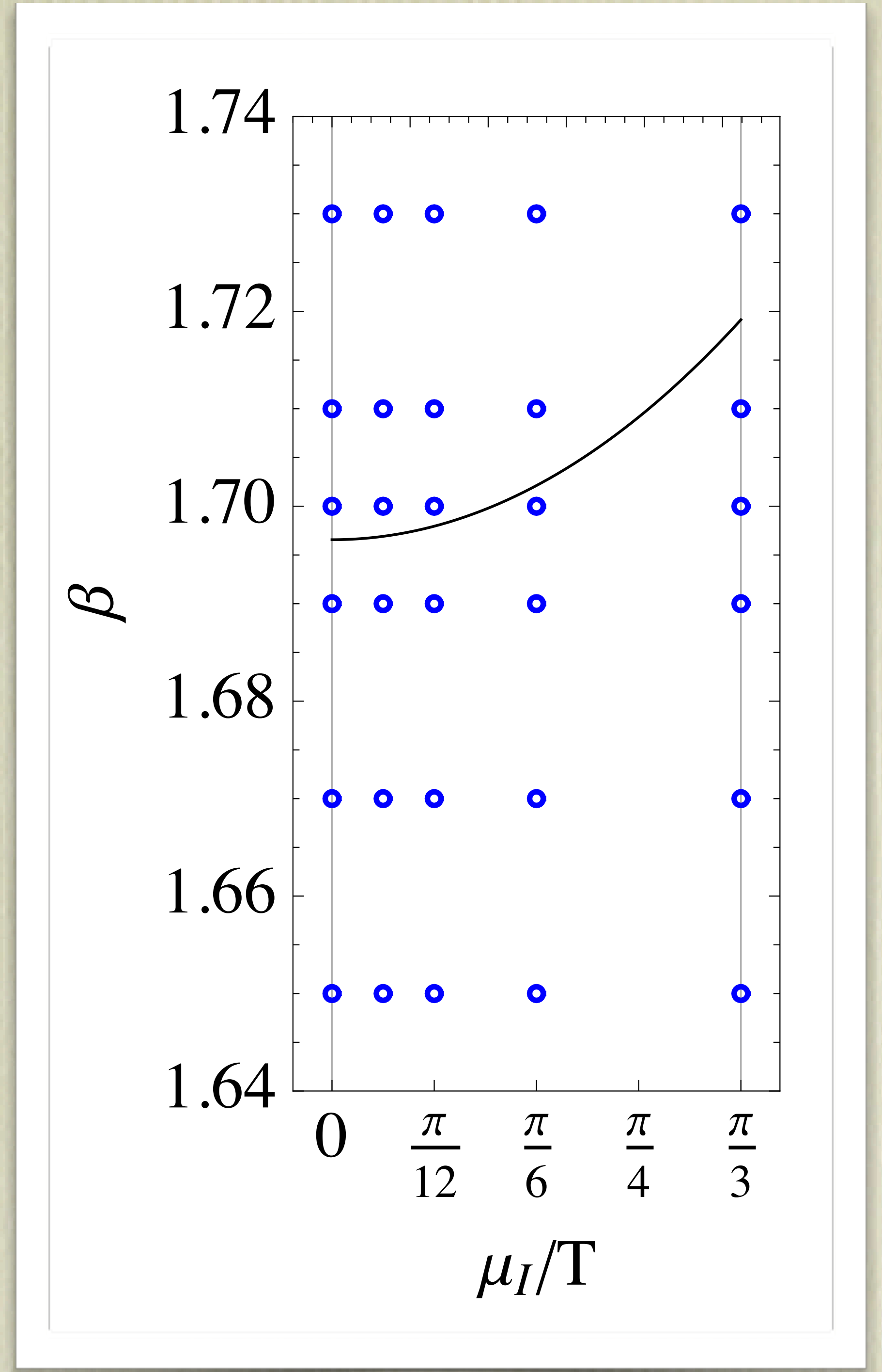
Staggered	**Wilson**
Residual chiral symmetry	No chiral symmetry
Four flavors	Any number of flavors
1 spinor components	4 spinor components

Reweighting

We want to use multi-histogram reweighting in β and μ to fill in the gaps in the scanned region.

$$\langle O(U)\rangle_{\beta,\mu} = \frac{\langle O(U)\alpha(U)\rangle_{\beta_0,\mu_0}}{\langle \alpha(U)\rangle_{\beta_0,\mu_0}}$$

$$\alpha(U) = e^{-(\beta-\beta_0)S_g(U)}\,\frac{\det M(U,\mu)}{\det M(U,\mu_0)}$$



A. M. Ferrenberg and R. H. Swendsen, *Phys. Rev. Lett.* **63** (1989) 1195–1198.

Compression method

- Using Schur complement techniques separate out the phase dependence in the determinant

$$\det M = \det \mathcal{Q} \cdot \det \left[e^{-\mu L_t/2} + T \cdot \mathcal{U} \cdot e^{+\mu L_t/2} \right]$$

- Once the eigenvalues of TU are known we can compute the determinant for any phase, hence any Fourier coefficient

$$\det M(\mu) = \det \mathcal{Q} \cdot e^{+\mu L_t \cdot 2 N_c L_s^3} \prod_{i=1}^{4 N_c L_s^3} \left(e^{-\mu L_t} + \lambda_i \right)$$

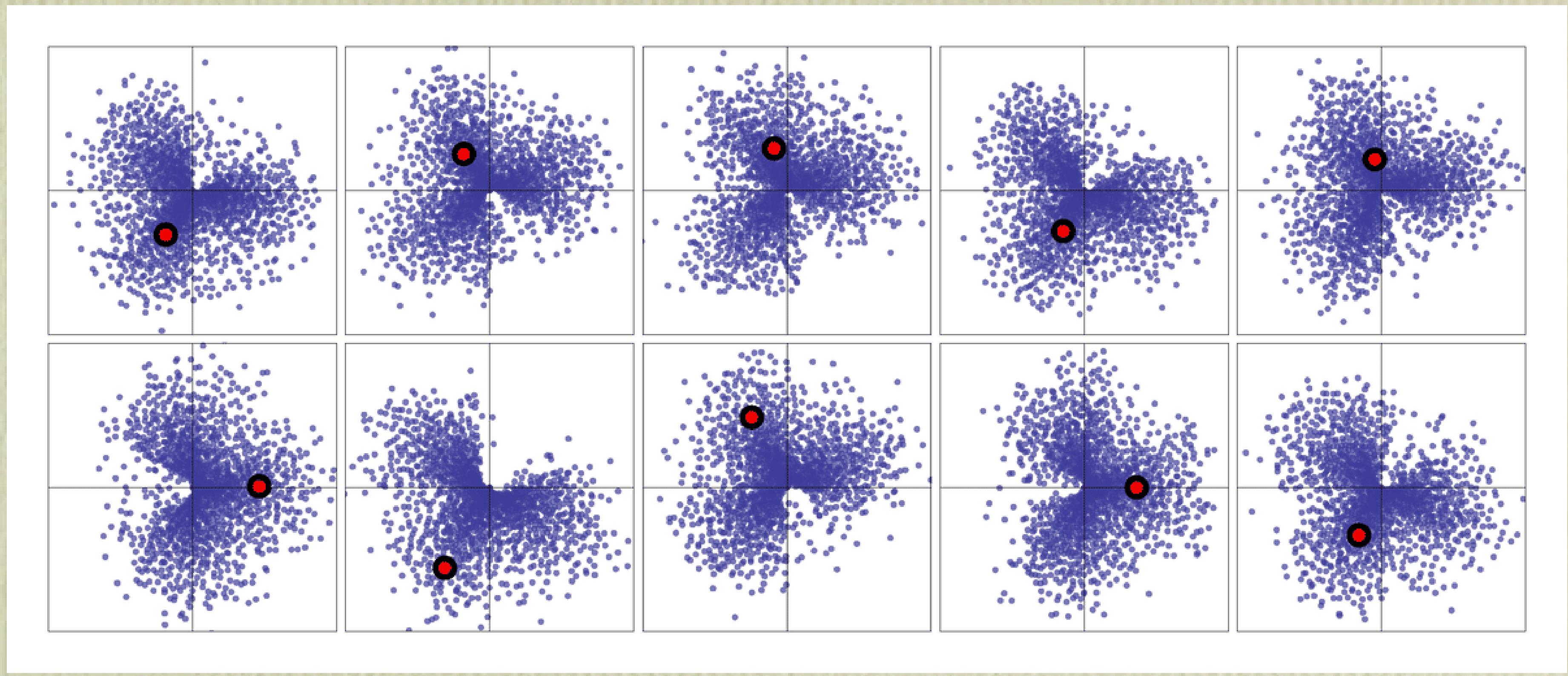
- The T and U matrices are N_t times smaller than M and the calculation is sped up considerably.

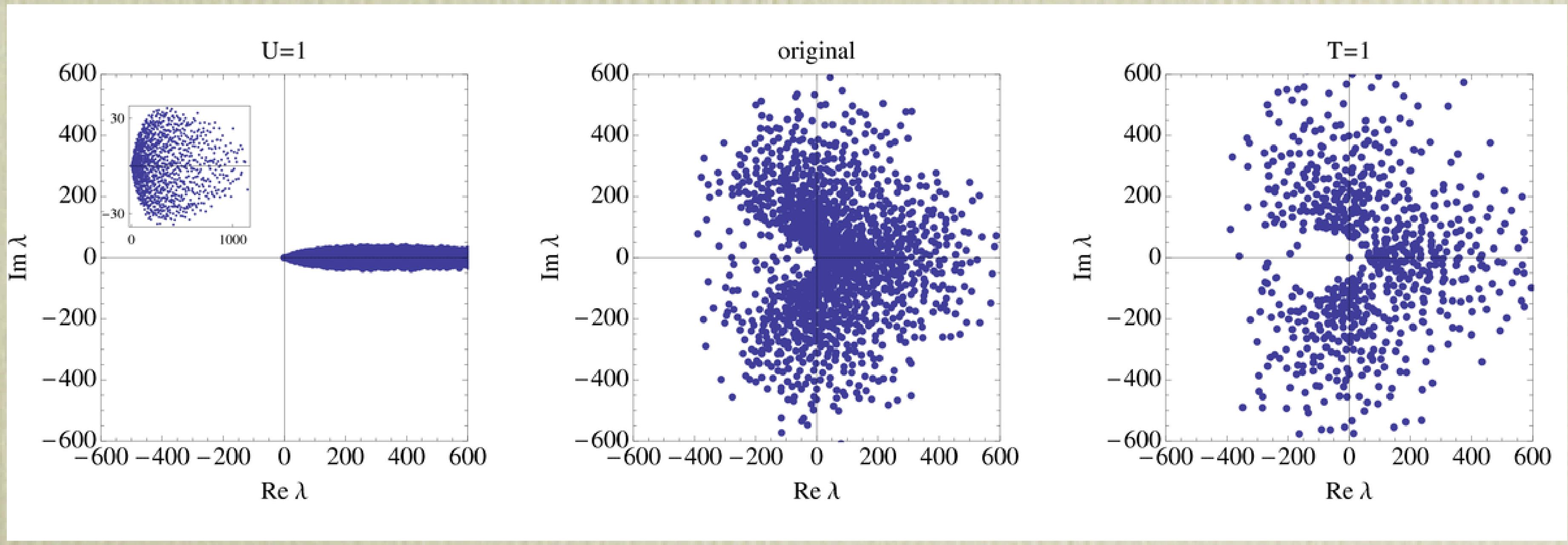
P. E. Gibbs, *Phys. Lett.* **B172** (1986) 53.

AA and U. Wenger, *Phys.Rev.* **D83** (2011) 034502, [arXiv:1009.2197].

K. Nagata and A. Nakamura, *Phys.Rev.* **D82** (2010) 094027, [arXiv:1009.2149].

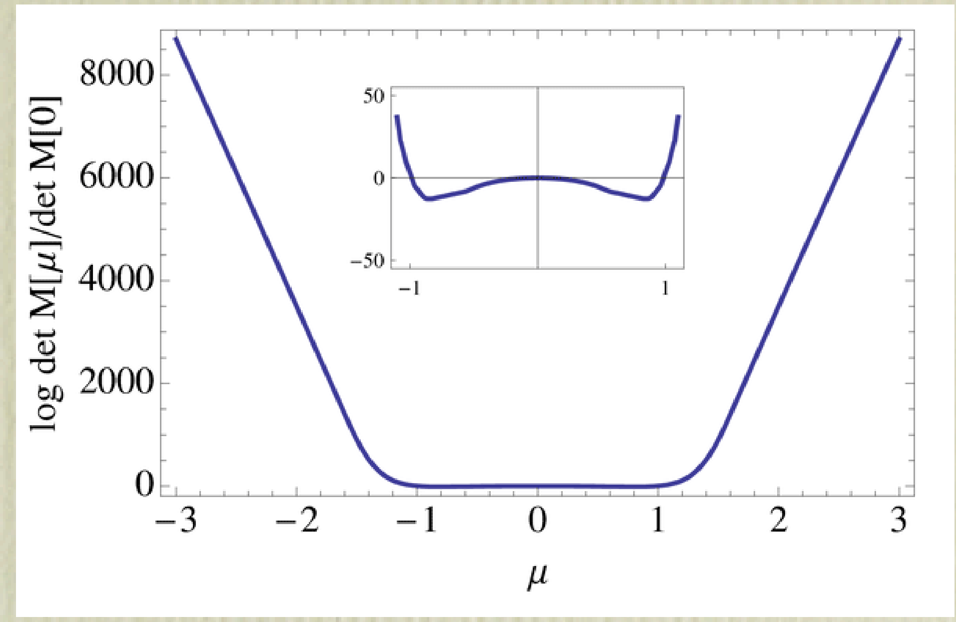
Compression method

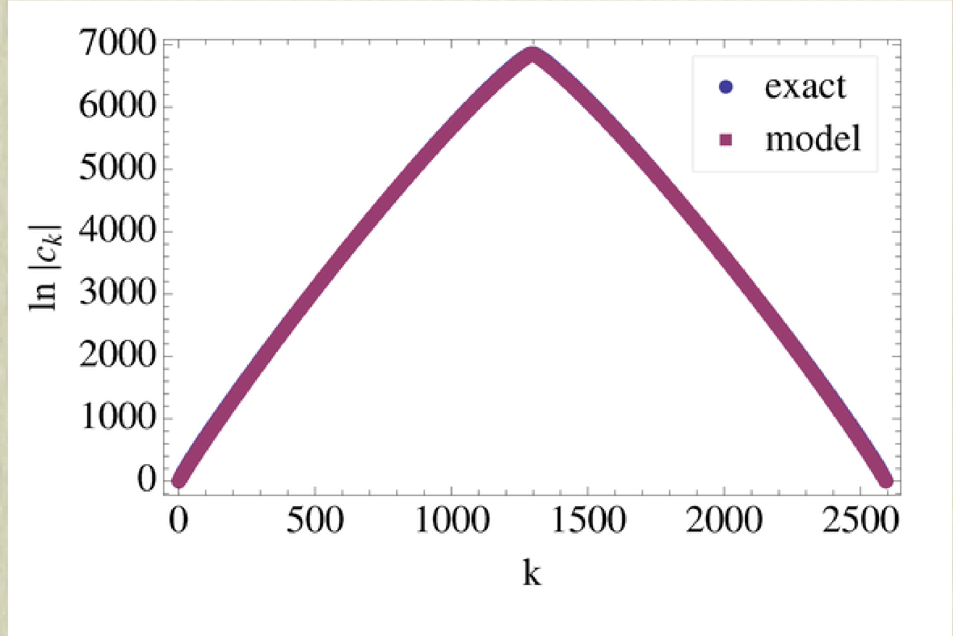




Compression method

- compute determinants for arbitrary chemical potential
- compute them "fast"
- useful both for direct simulation and reweighting
- compute projected determinants exactly

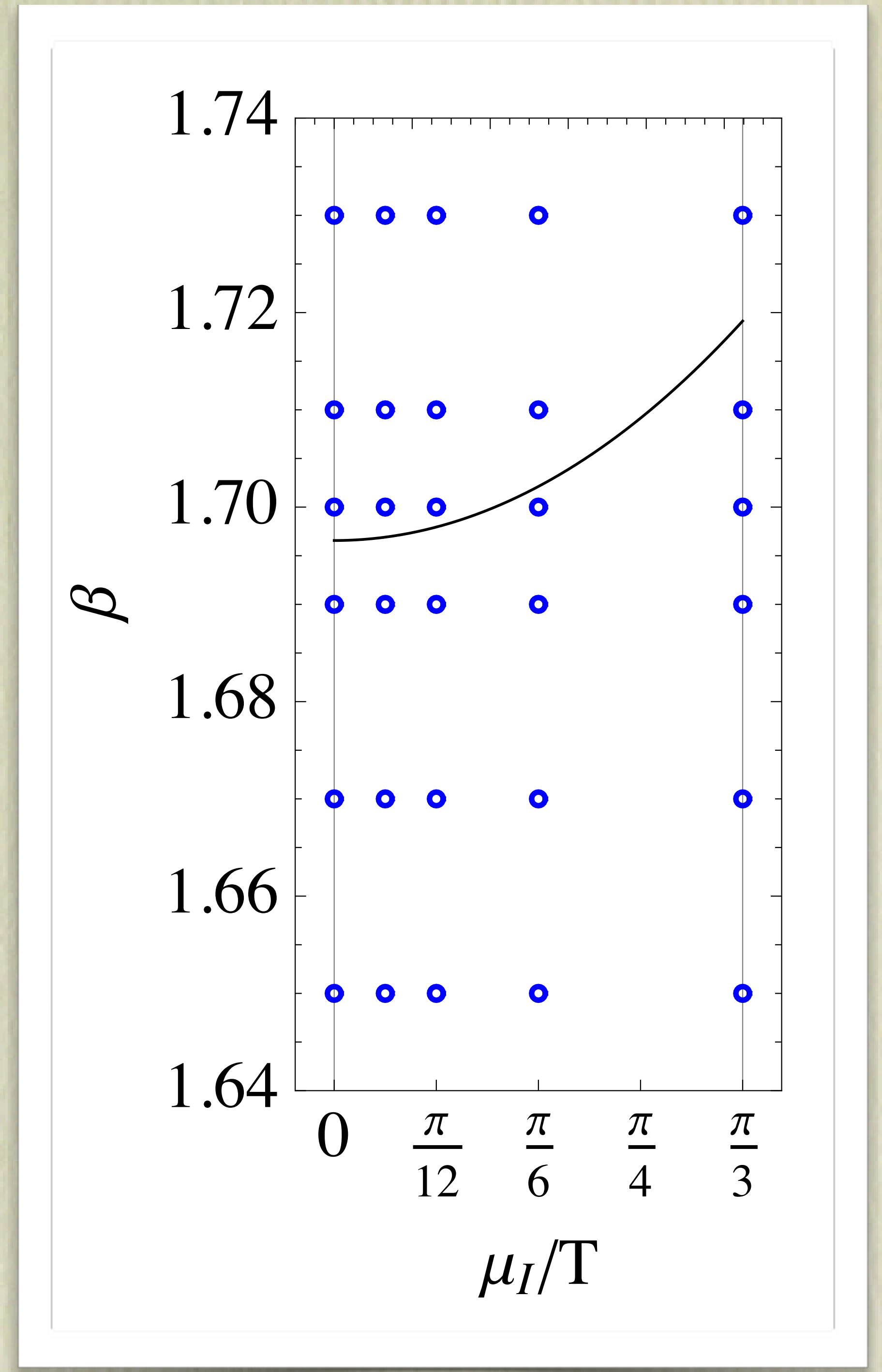




Numerical results

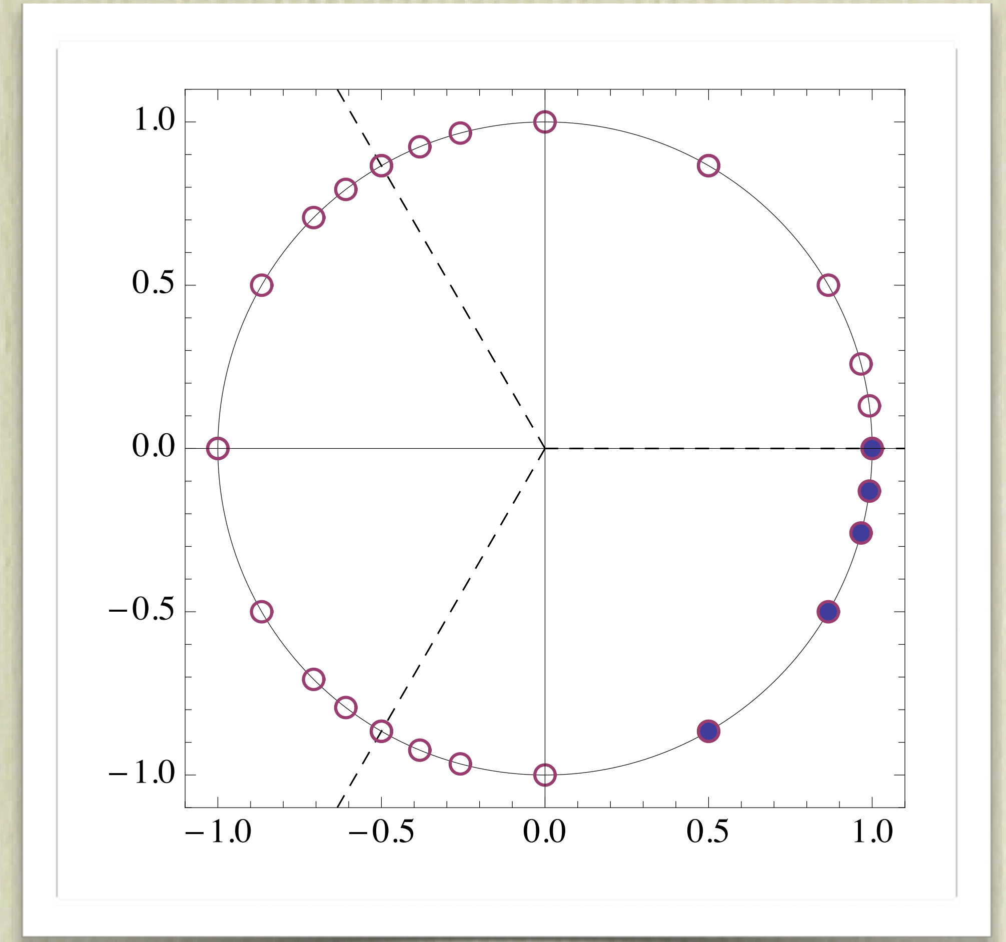
Simulation parameters

- Clover fermions with fixed c_{sw}
- Iwasaki action: β = 1.65, 1.67, 1.69, 1.71, 1.70, 1.73
- Imaginary chemical potential: μ/T = 0, i π/24, i π/12, i π/6, i π/3
- About 20,000 configs for each ensemble
- We compute the determinant compression for each config



Rotated ensembles

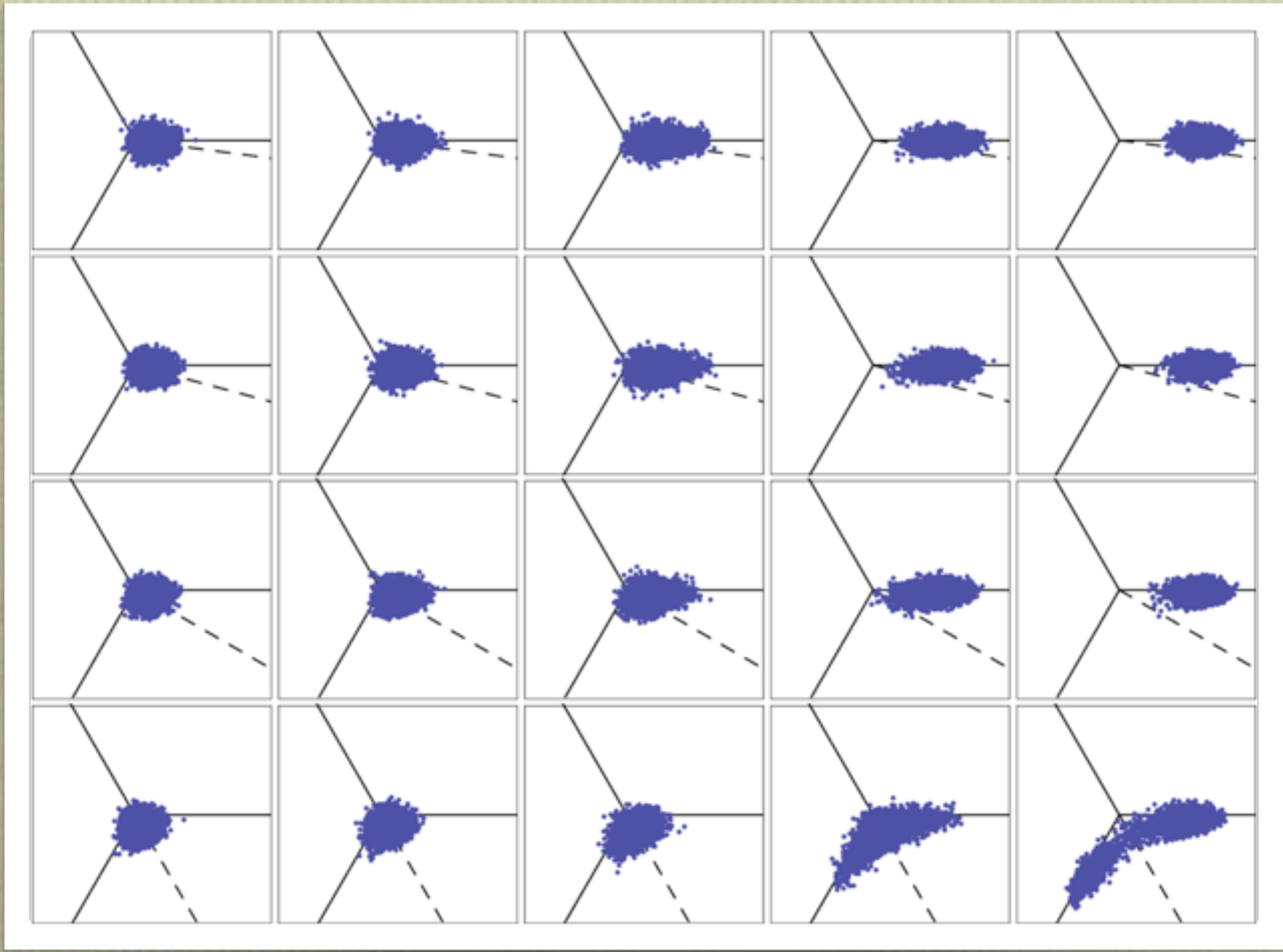
For each generated ensemble, we can use charge conjugation and Z(3) periodicity to add new simulation points

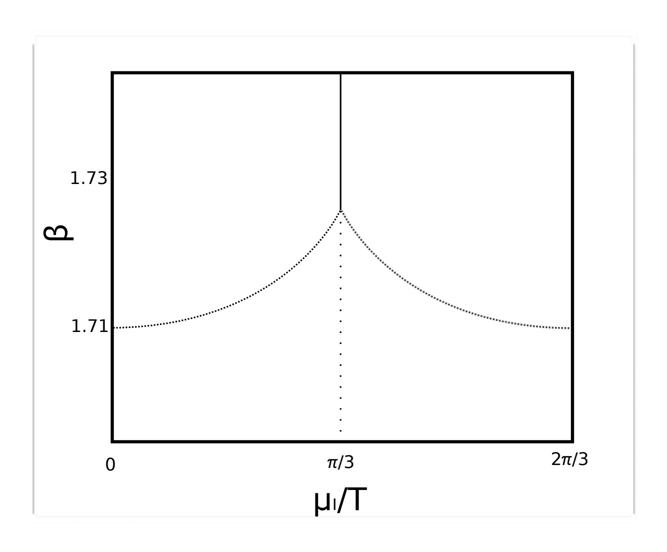


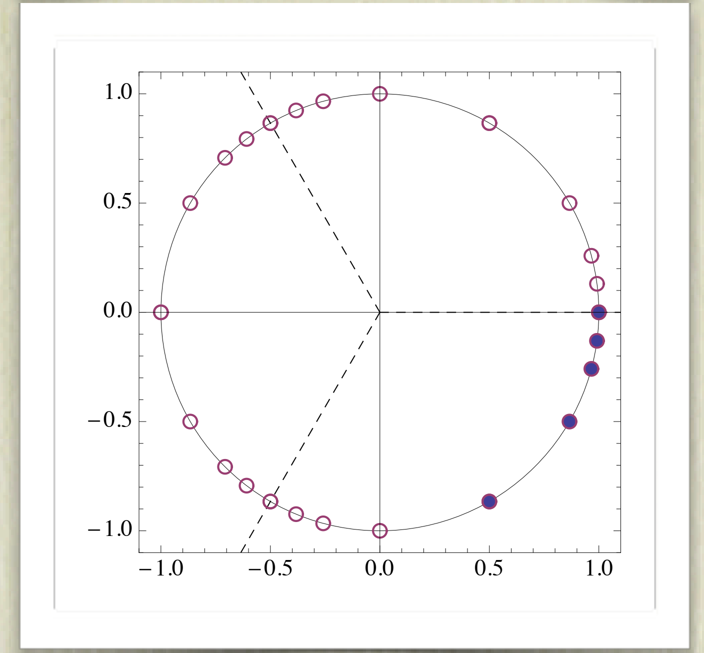
$$P_{i\mu_I}(U) = P_{-i\mu_I}(U^*)$$

$$P_{i\mu_I}(U) = P_{-i(\mu_I \pm 2\pi/3)}(U_{\mp})$$

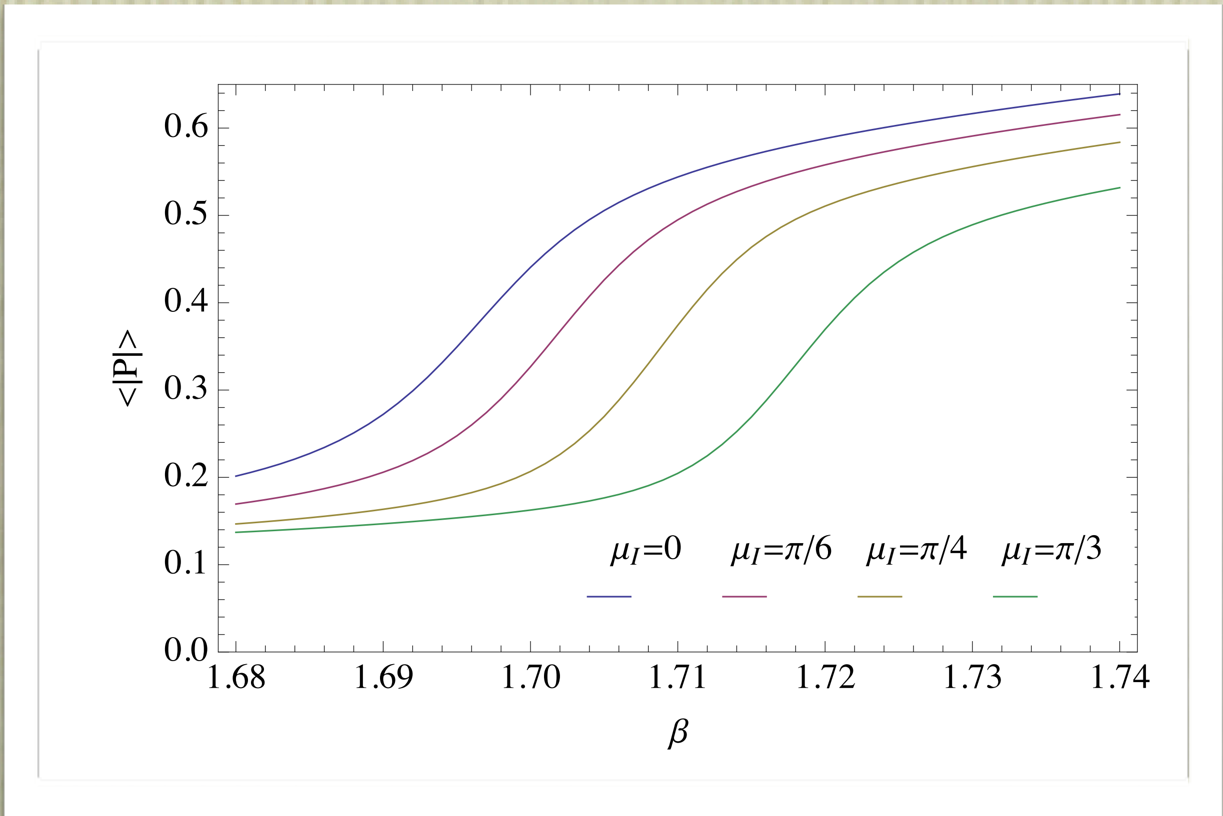
Polyakov loop distribution

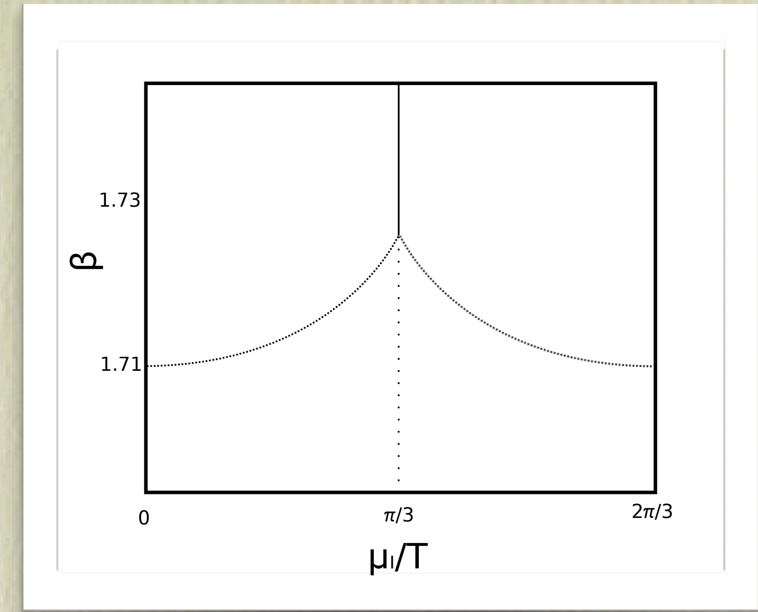


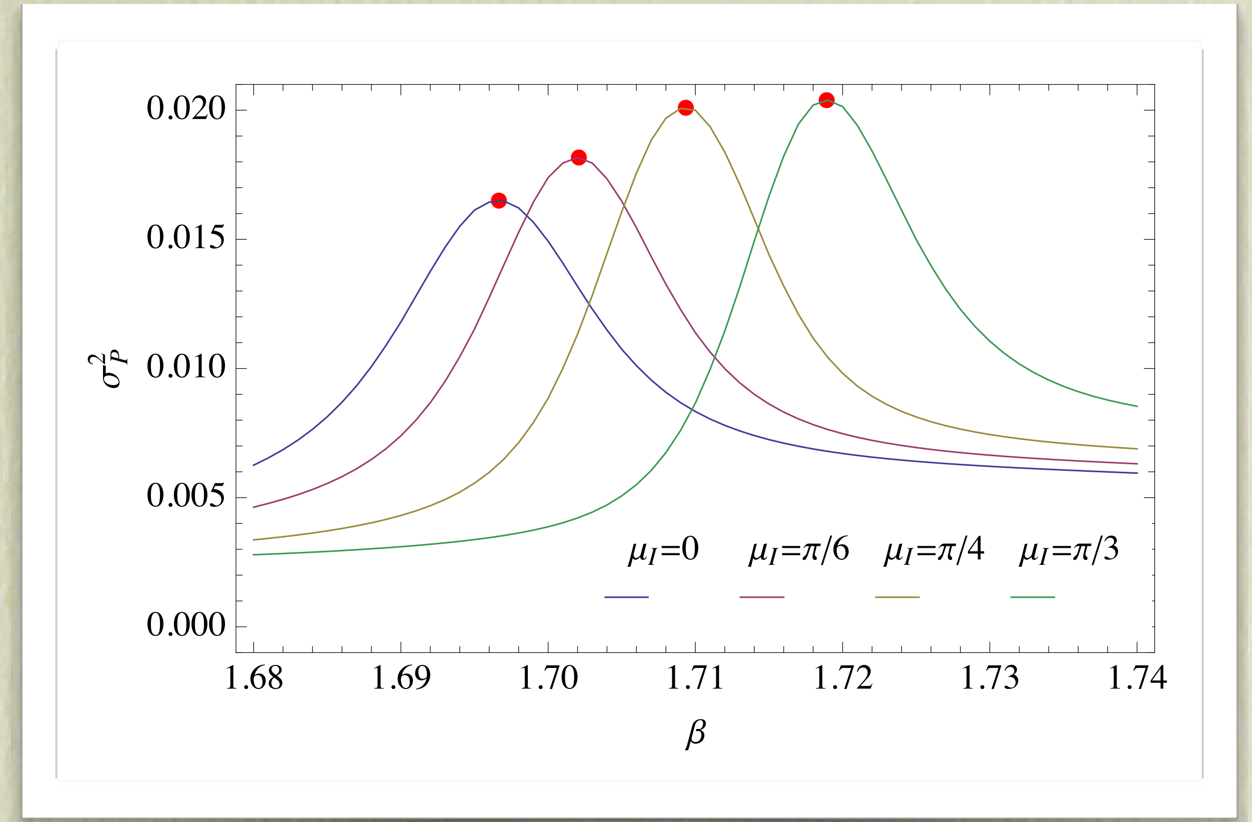




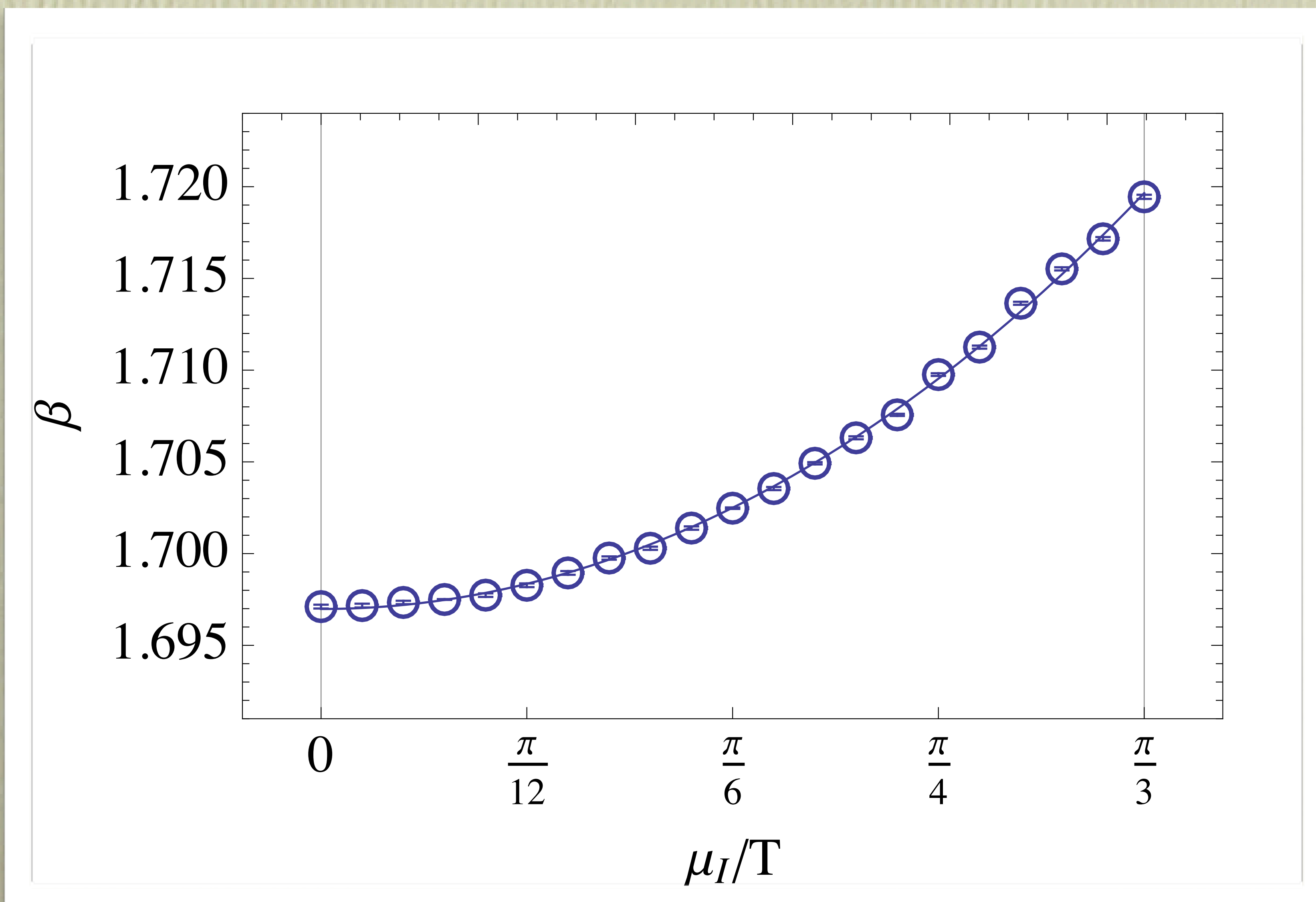
Polyakov loop susceptibility

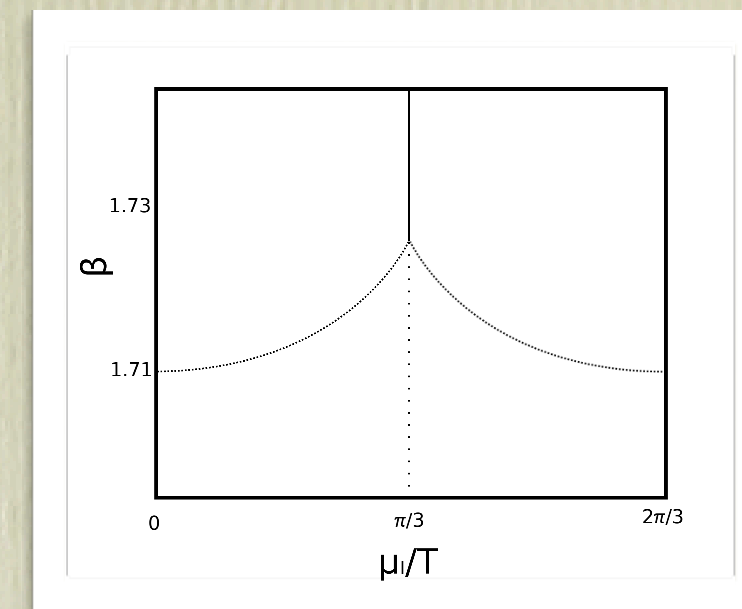


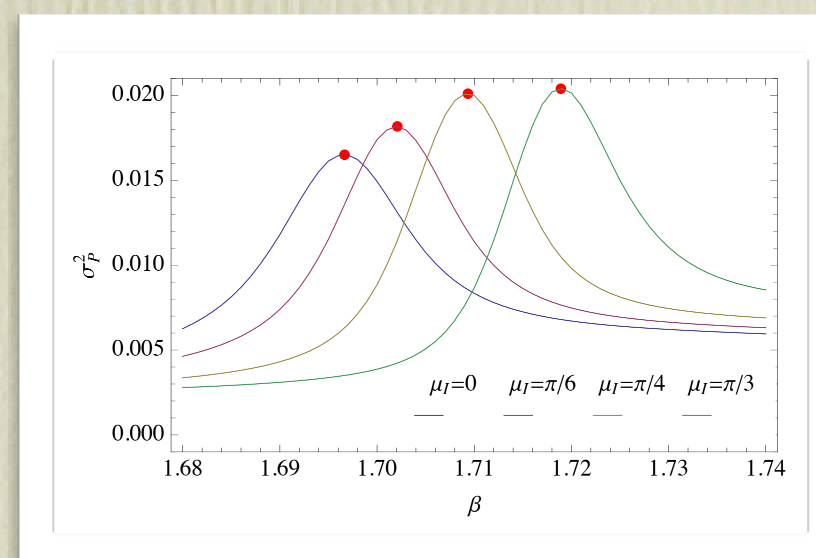




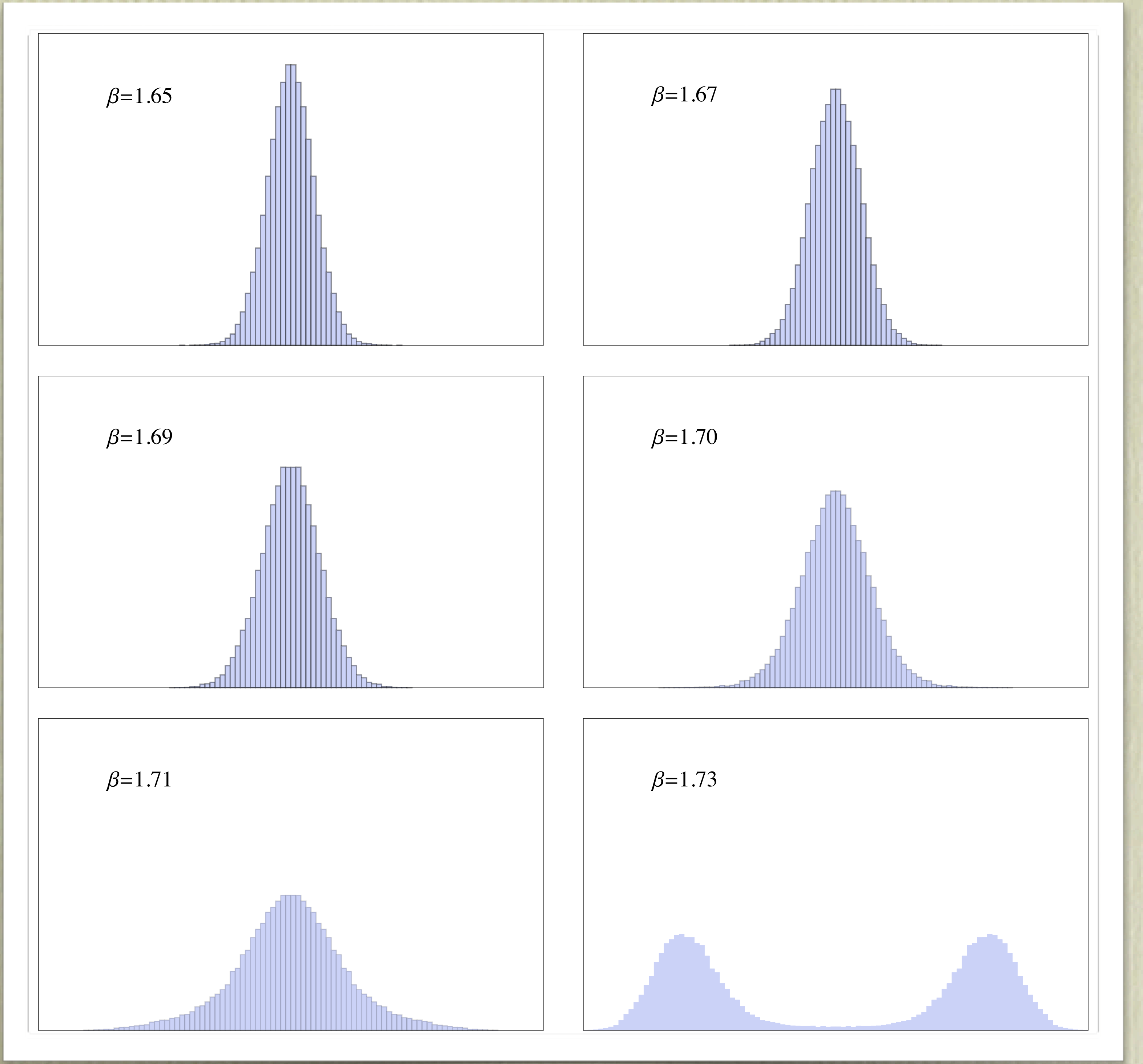
Pseudo-critical line

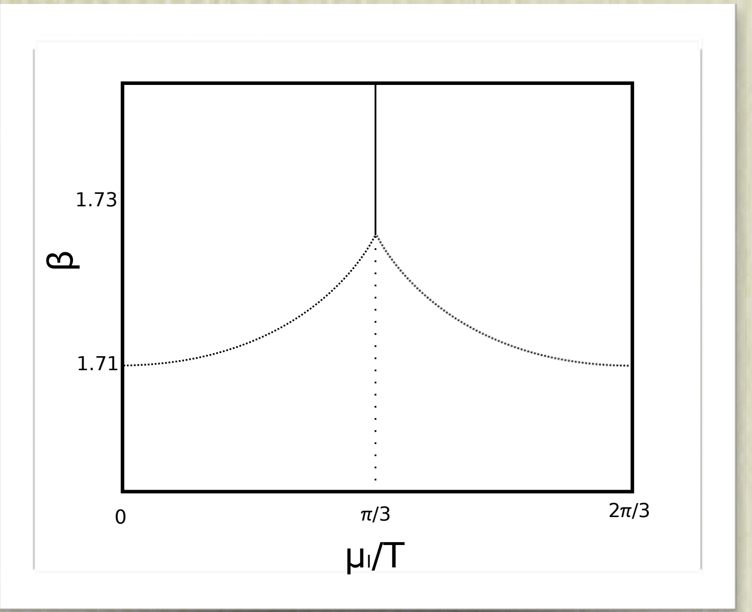




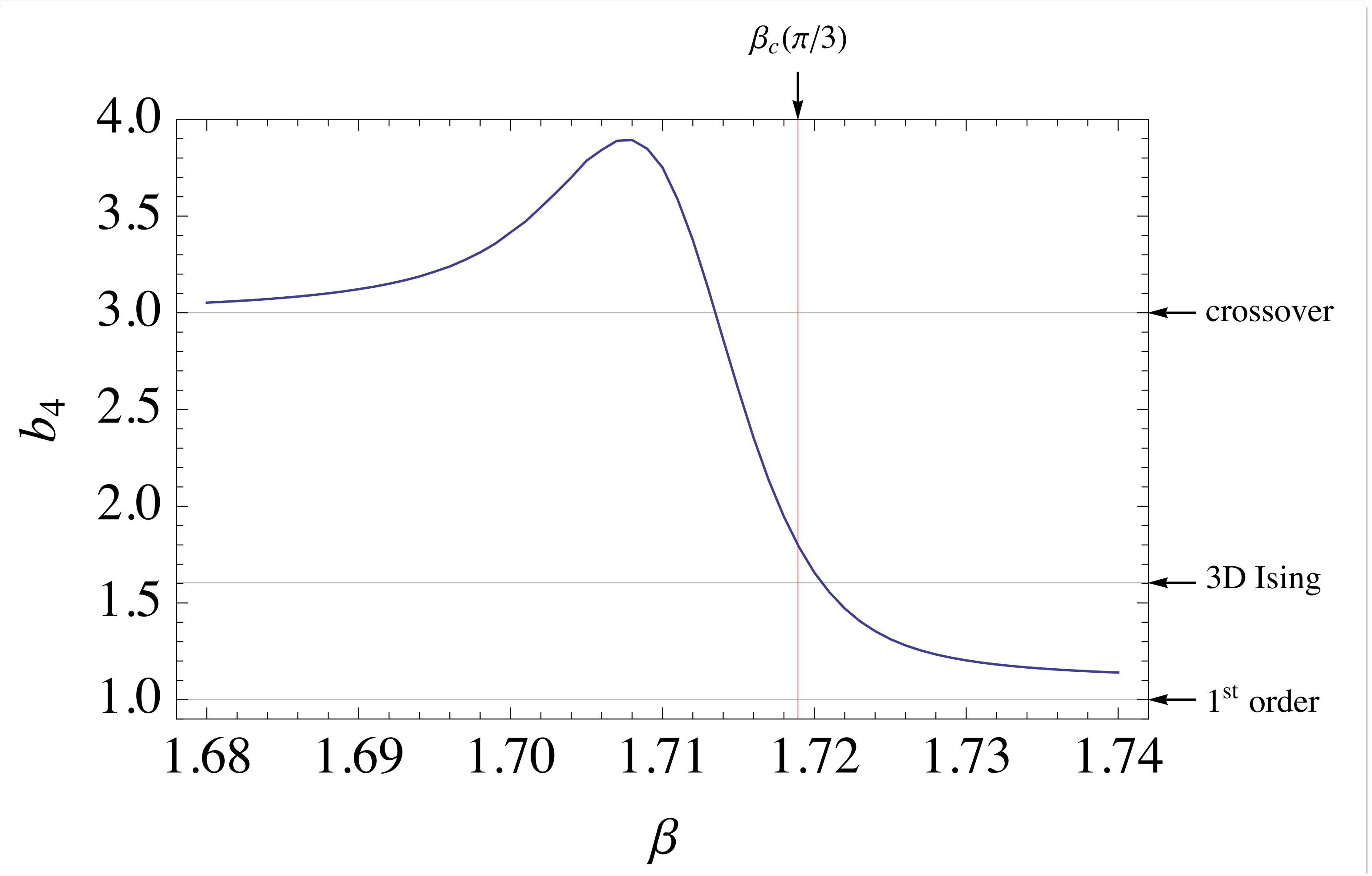


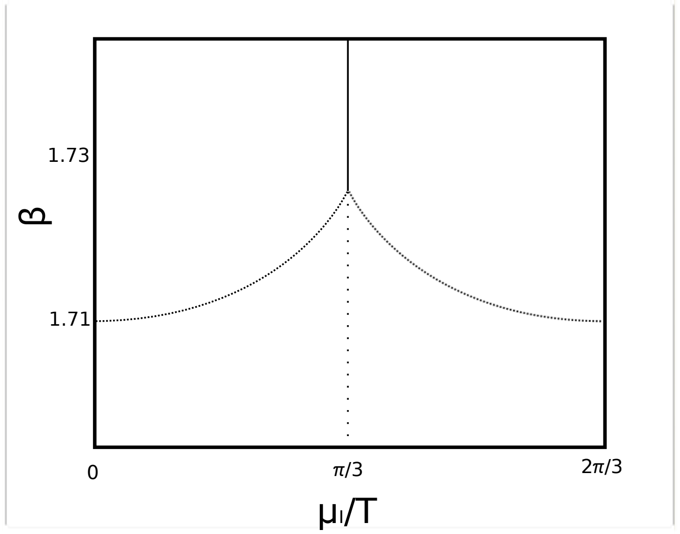
Roberge-Weiss transition



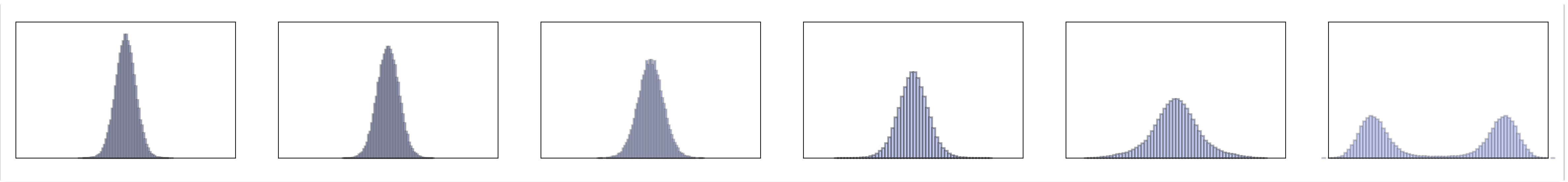


Roberge-Weiss transition

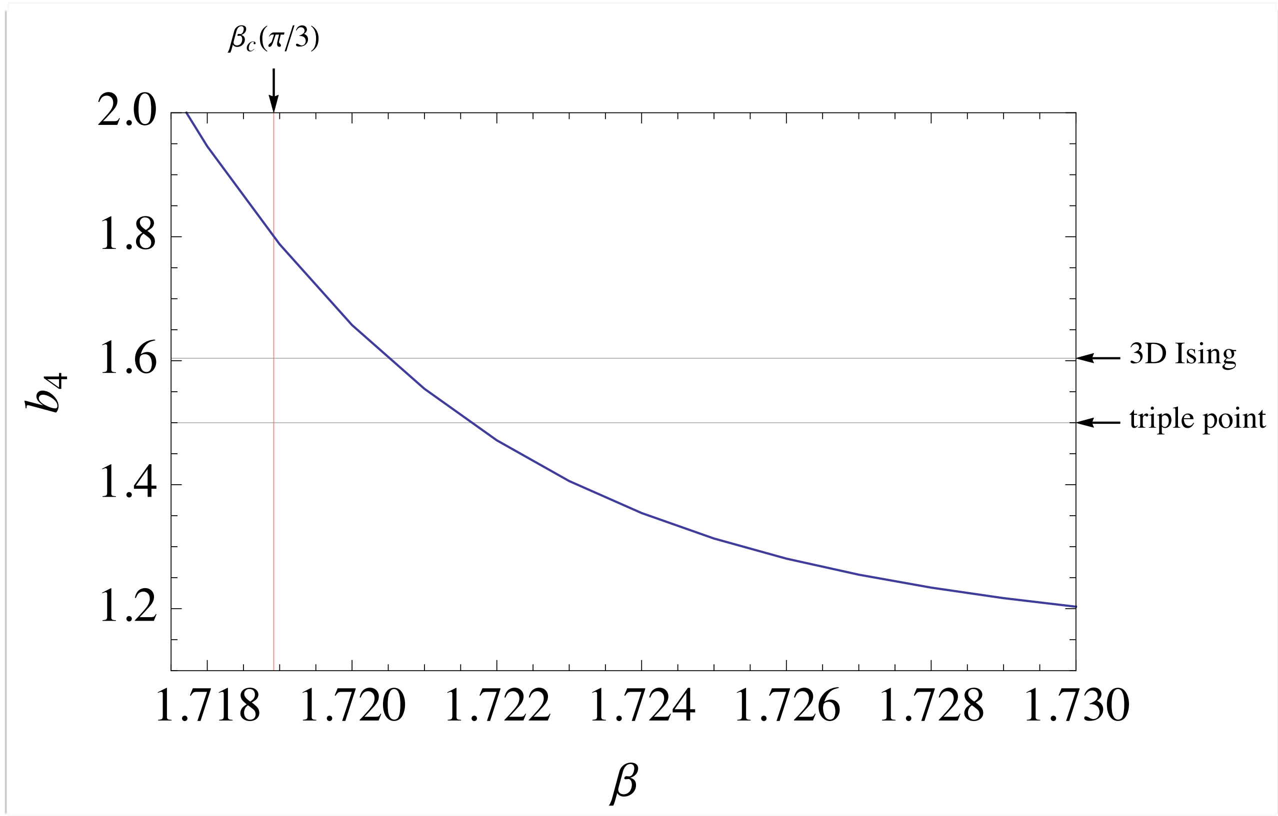


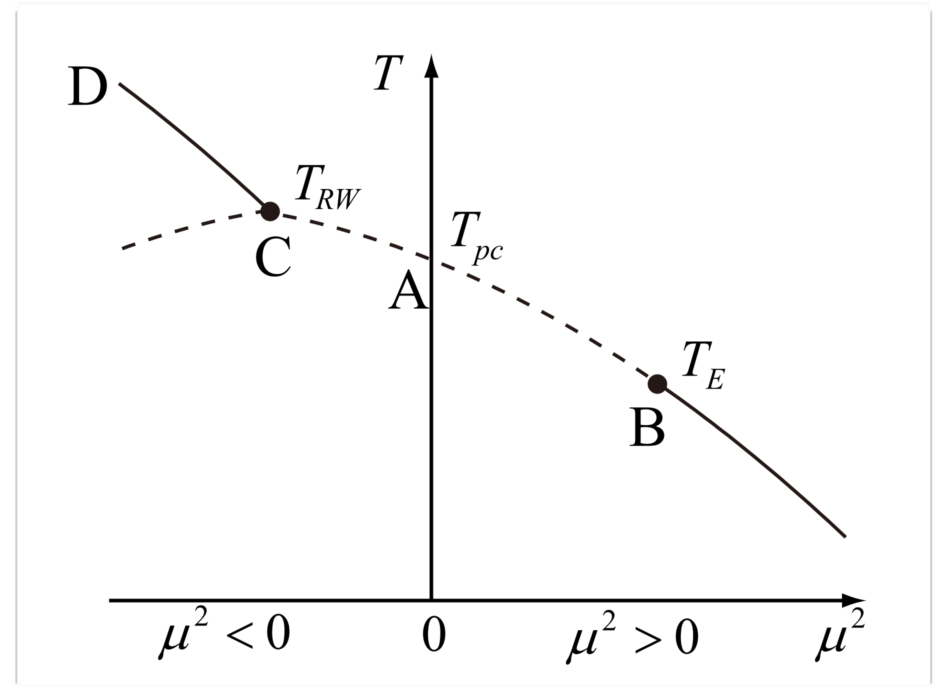


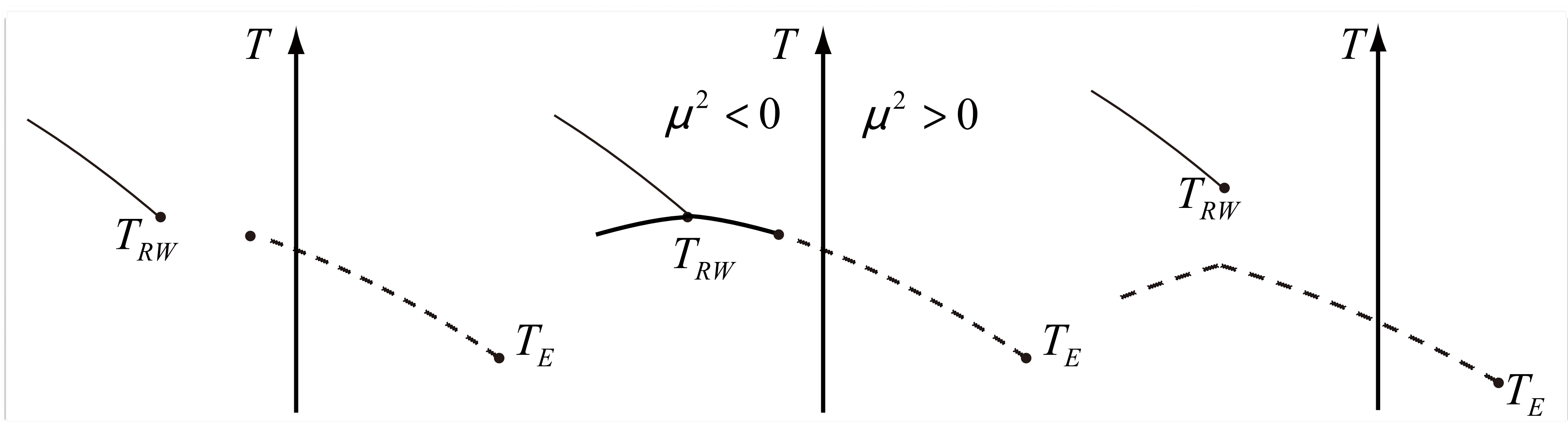
$$b_4 = \frac{\langle(\delta O)^4\rangle}{\langle(\delta O)^2\rangle^2}$$



Roberge-Weiss transition







Conclusions and outlook

- We analyzed the phase diagram of N_f=3 QCD with m_π=760MeV at imaginary chemical potential using a multi-histogram reweighting both in temperature and chemical potential.
- Our results are consistent with the Roberge-Weiss 1st transition line terminating in a second order phase transition point that sits on the pseudo-critical line.
- Other scenarios cannot yet be ruled out -- we require simulations at different volumes.
- We started generating data for larger volume and plan study the quark mass dependence of the phase transition at imaginary μ.
- Analytical continuation to real μ produces results consistent with reweighting.