Lattice QCD at Finite Isospin Chemical Potential

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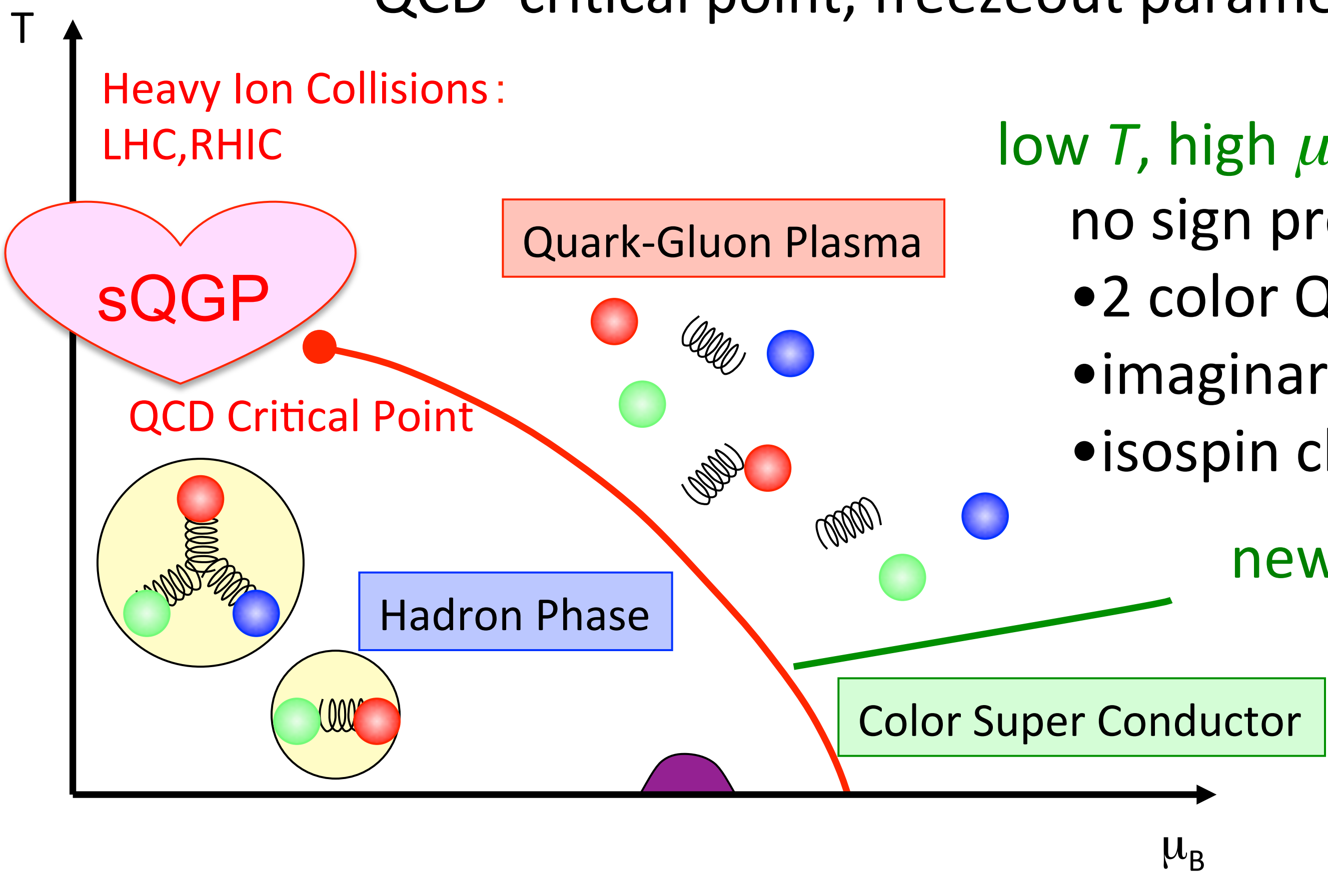
July 29, 2013@Lattice2013, Mainz, Germany

Introduction

- Finite density lattice QCD - sign problem

high T, low μ : reweighting method, Taylor expansion…
QCD critical point, freezeout parameters, fluctuations…



low T, high μ: QCD like theories
no sign problem
- 2 color QCD
- imaginary chemical potential
- isospin chemical potential

new development :
complex Langevin
etc.

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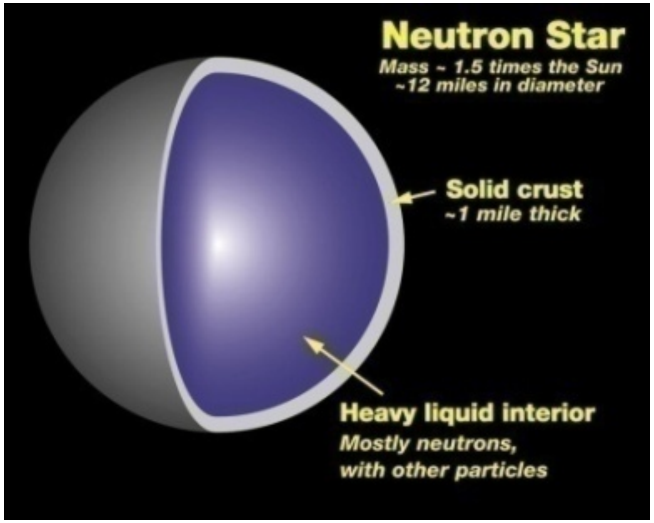
Finite Isospin Chemical Potential

- Core of neutron stars ?

$$\mu_u = \mu + \mu_I$$
$$\mu_d = \mu - \mu_I$$

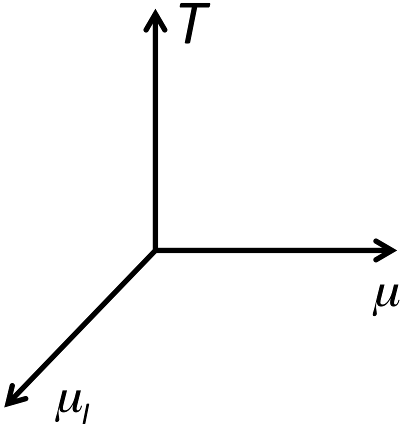


$\mu_I > 0 : \mu_u > \mu_d$, positive charge
$u_I < 0 : \mu_u < \mu_d$, negative charge

pion condensation occurs at $\mu_{IC} = \frac{1}{2} m_\pi$ (lowest meson mass)

rho condensation ?
strangeness: kaon condensation? hyperons?

- Insight of finite chemical potential
  - Phase diagram as a function of T, μ and μ_I



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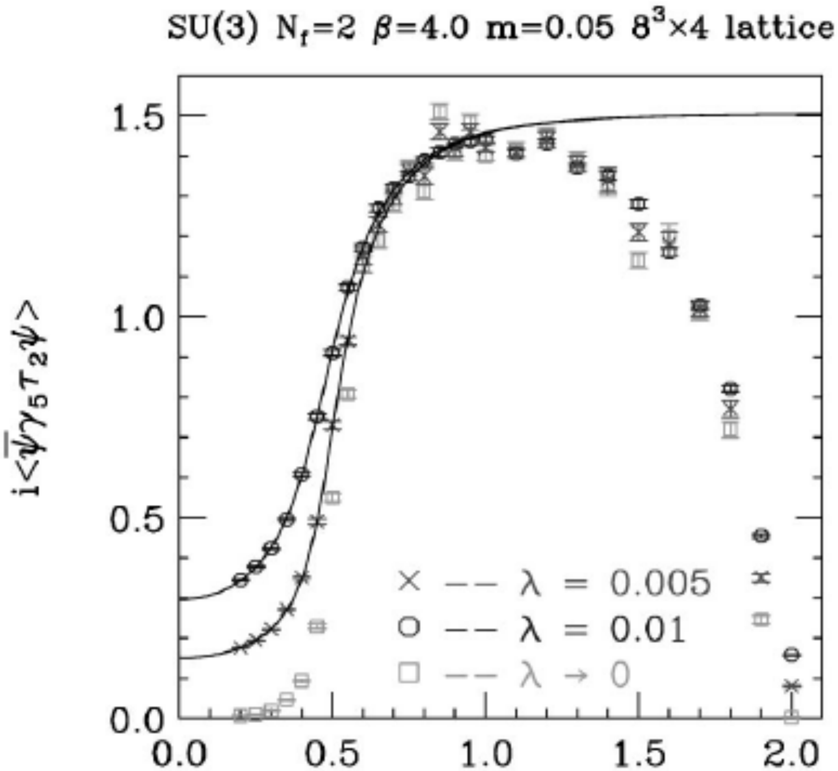
Pion Condensation on the lattice

charged pion condensation



KS fermion
J.B.Kogut,D.K.Sinclair
Phys rev.D66,034505(2002)

Questions?

- μ_I dependence of m_{π}, m_{ρ}?
- π condensation?
- ρ condensation?

Introduction of μ_I

- 2 flavor fermion action (Wilson fermion)

$$S_F = \bar{\Psi}[\gamma_\mu D_\mu + m_q + \mu\gamma_4\frac{\tau^3}{2} + i\lambda\gamma_5\frac{\tau^2}{2}]\Psi$$

$$= \bar{\Psi}\begin{pmatrix} D(\mu) & \lambda\gamma_5 \\ -\lambda\gamma_5 & D(-\mu) \end{pmatrix}\Psi \qquad D(\mu) = \gamma_\mu D_\mu + m_q + \frac{\mu}{2}\gamma_4$$

$$= \bar{\Psi}D(U)\Psi$$

τ^2, τ^3: Pauli matrix

- $\bar{\Psi}[\, i\lambda\gamma_5\frac{\tau^2}{2}]\Psi$
  - λ: explicit I_3 breaking parameter
  - positivity of $\det D(U)$ ⟺ sign problem at finite μ
    $\det D(U) = \det\left[D^\dagger(\mu)D(\mu) + \lambda^2\right]$
    Hybrid Monte Carlo method
  - observables: λ ⟶ 0

Hybrid Monte Carlo

- Hybrid Monte Carlo method

Partition function

$$Z = \int D\phi^* D\phi DU e^{-S_G + S_F}$$

$$S_F = \phi^\dagger D^{-1} (D^\dagger)^{-1} \phi$$

ϕ : pseudo fermion field

P : conjugate momentum of U

$$Z = \int DPDU \exp\left\{ -\left(\frac{1}{2} P^2 + S_G + S_F \right) \right\}$$

Hamiltonian: H

Hamilton equation

$$\begin{cases} \dfrac{dA_l}{dt} = \dfrac{\partial H}{\partial P_l} \\ \dfrac{dP_l}{dt} = -\dfrac{\partial H}{\partial A_l} \end{cases} \Rightarrow \begin{cases} \dfrac{dU}{dt} = iPU \\ \dfrac{dP_l}{dt} = -\dfrac{\partial S_G}{\partial A_l} - \dfrac{\partial S_F}{\partial A_l} \end{cases}$$

$$\frac{\partial S_F}{\partial A_l} = -\eta^\dagger \frac{\partial D}{\partial A_l} X - X^\dagger \frac{\partial D^\dagger}{\partial A_l} \eta$$

where

$$\eta = \left(D^\dagger\right)^{-1} \phi \, , X = D^{-1} \eta$$

Inverse of fermion matrix

HMC with μ_I

- Fermion matrix: $D = D^{\dagger}(\mu)D)\mu) + \lambda^2$

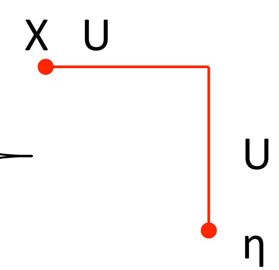
$$
\begin{cases}
\dfrac{dU}{dt} = iPU \\
\dfrac{dP_l}{dt} = -\dfrac{\partial S_G}{\partial A_l} - \dfrac{\partial S_F}{\partial A_l}
\end{cases}
$$

$$\frac{\partial S_F}{\partial A_l} = -\eta^{\dagger}\frac{\partial D}{\partial A_l}X - X^{\dagger}\frac{\partial D^{\dagger}}{\partial A_l}\eta$$

where

$$\eta = \left(D^{\dagger}\right)^{-1}\phi\,,\, X = D^{-1}\eta$$

$$
\begin{aligned}
\frac{\partial S_F}{\partial A} = & -\frac{1}{2}\Big[-i\kappa_\lambda^{+}T\left\{\left(r-\gamma_\lambda\right)U_\lambda(x)Q(x,x+\lambda)\right\} + i\kappa_\lambda^{-}T\left\{Q(x+\lambda,x)\left(r+\gamma_\lambda\right)U_\lambda^{\dagger}(x)\right\} \\
& +i\kappa_\lambda^{+}T\left\{Q(x+\lambda,x)\left(r-\gamma_\lambda\right)U_\lambda^{\dagger}(x)\right\} - i\kappa_\lambda^{-}T\left\{\left(r+\gamma_\lambda\right)U_\lambda(x)Q(x,x+\lambda)\right\} \\
& -i\kappa_\lambda^{+}\kappa_\nu^{+}T\left\{\left(r-\gamma_\nu\right)U_\nu(x)Q(x+\lambda,x+\nu)\left(r-\gamma_\lambda\right)U_\lambda^{\dagger}(x)\right\} \\
& +i\kappa_\mu^{+}\kappa_\lambda^{+}T\left\{\left(r-\gamma_\lambda\right)U_\lambda(x)Q(x+\mu,x+\lambda)\left(r-\gamma_\mu\right)U_\mu^{\dagger}(x)\right\} \\
& -i\kappa_\lambda^{+}\kappa_\nu^{-}T\left\{\left(r+\gamma_\nu\right)U_\nu^{\dagger}(x-\nu)Q(x+\lambda,x-\nu)\left(r-\gamma_\lambda\right)U_\lambda^{\dagger}(x)\right\} \\
& -i\kappa_\mu^{+}\kappa_\lambda^{-}T\left\{Q(x+\mu+\lambda,x)\left(r-\gamma_\mu\right)\left(r+\gamma_\lambda\right)U_\mu^{\dagger}(x+\lambda)U_\lambda^{\dagger}(x)\right\} \\
& +i\kappa_\lambda^{-}\kappa_\nu^{+}T\left\{\left(r+\gamma_\lambda\right)\left(r-\gamma_\nu\right)U_\lambda(x)U_\nu(x+\lambda)Q(x,x+\lambda+\nu)\right\} \\
& +i\kappa_\mu^{-}\kappa_\lambda^{+}T\left\{\left(r-\gamma_\lambda\right)U_\lambda(x)Q(x-\mu,x+\lambda)\left(r+\gamma_\mu\right)U_\mu(x-\mu)\right\} \\
& +i\kappa_\lambda^{-}\kappa_\nu^{-}T\left\{\left(r+\gamma_\lambda\right)\left(r+\gamma_\nu\right)U_\lambda(x)U_\nu^{\dagger}(x+\lambda-\nu)Q(x,x+\lambda-\nu)\right\} \\
& -i\kappa_\mu^{-}\kappa_\lambda^{-}T\left\{Q(x-\mu+\lambda,x)\left(r+\gamma_\mu\right)\left(r+\gamma_\lambda\right)U_\mu(x-\mu+\lambda)U_\lambda^{\dagger}(x)\right\}\Big]
\end{aligned}
$$



Observables

- Propagators of π and ρ

pion operator: $\pi^a = \bar{\psi}\gamma_5\tau^a\psi \qquad a = 0, +, -$

pion propagator:

$$\langle \pi^a(x)\pi^b(y)\rangle = \langle \bar{\psi}(x)\gamma_5\tau^a\psi(x)\bar{\psi}(y)\gamma_5\tau^b\psi(y)\rangle$$
$$= \int dU \det D(U) e^{-S_{\rm gauge}(U)}[-{\rm Tr}\{\gamma_5\tau^a D^{-1}(U)_{xy}\gamma_5\tau^b D^{-1}(U)_{yx}\}$$
$$+{\rm Tr}\{\gamma_5\tau^a D^{-1}(U)_{xx}\}\cdot {\rm Tr}\{\gamma_5\tau^b D^{-1}(u)_{yy}\}]$$

Tr: color, dirac

Rho operator: $\rho^a = \bar{\psi}\gamma_\mu\tau^a\psi$ rho propagator: $\langle\rho^a(x)\rho^b(y)\rangle$

- $D^{-1}(U)$ ⇐ source term for isospin chemical potential

$$D^{-1} = \begin{pmatrix} \left(D^\dagger(\mu)D(\mu)+\lambda^2\right)^{-1} D^\dagger(\mu) & -\lambda\left(D^\dagger(\mu)D(\mu)+\lambda^2\right)^{-1}\gamma_5 \\ \lambda\gamma_5 D(\mu)\left(D^\dagger(\mu)D(\mu)+\lambda^2\right)^{-1} D^{-1}(\mu) & \gamma_5 D(\mu)\left(D^\dagger(\mu)D(\mu)+\lambda^2\right)^{-1}\gamma_5 \end{pmatrix}$$

Isospin chemical potential affects propagators of π and ρ.

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Propagators for π and ρ

- Pion

$$\langle \pi^-(x)\pi^+(y)\rangle = -\mathrm{Tr}\left[\left\{D(\mu)(D(\mu)D^\dagger(\mu)+\lambda^2)^{-1}\right\}_{xy}\left\{(D(\mu)D^\dagger(\mu)+\lambda^2)^{-1}D^\dagger(\mu)\right\}_{yx}\right]$$

$$\langle \pi^+(x)\pi^-(y)\rangle = -\mathrm{Tr}\left[\left\{(D(\mu)D^\dagger(\mu)+\lambda^2)^{-1}D^\dagger(\mu)\right\}_{xy}\left\{D(\mu)(D(\mu)D^\dagger(\mu)+\lambda^2)^{-1}\right\}_{yx}\right]$$

$$\begin{aligned}\langle \pi^0(x)\pi^0(y)\rangle = -\frac{1}{2}\mathrm{Tr}\Big[&\gamma_5\left\{(D^\dagger(\mu)D(\mu)+\lambda^2)^{-1}D^\dagger(\mu)\right\}_{xy}\gamma_5\left\{(D^\dagger(\mu)D(\mu)+\lambda^2)^{-1}D^\dagger(\mu)\right\}_{yx}\\ &+\left\{D(\mu)(D^\dagger(\mu)D(\mu)+\lambda^2)^{-1}\right\}_{xy}\gamma_5\left\{D(\mu)(D^\dagger(\mu)D(\mu)+\lambda^2)^{-1}\ \gamma_5\right\}_{yx}\Big]\\ +\frac{1}{2}&\mathrm{Tr}\left[\gamma_5(D^\dagger(\mu)(D(\mu)+\lambda^2)^{-1}D^\dagger(\mu)-D(\mu)(D^\dagger(\mu)(D(\mu)+\lambda^2)^{-1}\gamma_5\right]_{xx}\\ &\times\mathrm{Tr}\left[\gamma_5(D^\dagger(\mu)(D(\mu)+\lambda^2)^{-1}D^\dagger(\mu)-D(\mu)(D^\dagger(\mu)(D(\mu)+\lambda^2)^{-1}\gamma_5\right]_{yy}\end{aligned}$$

Disconnected diagram

At finite isospin chemical potential

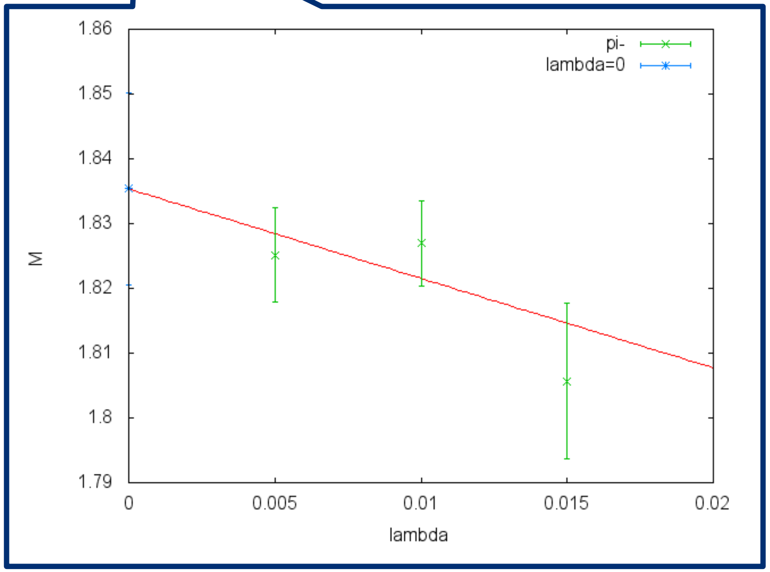
π^+, π^- :different response

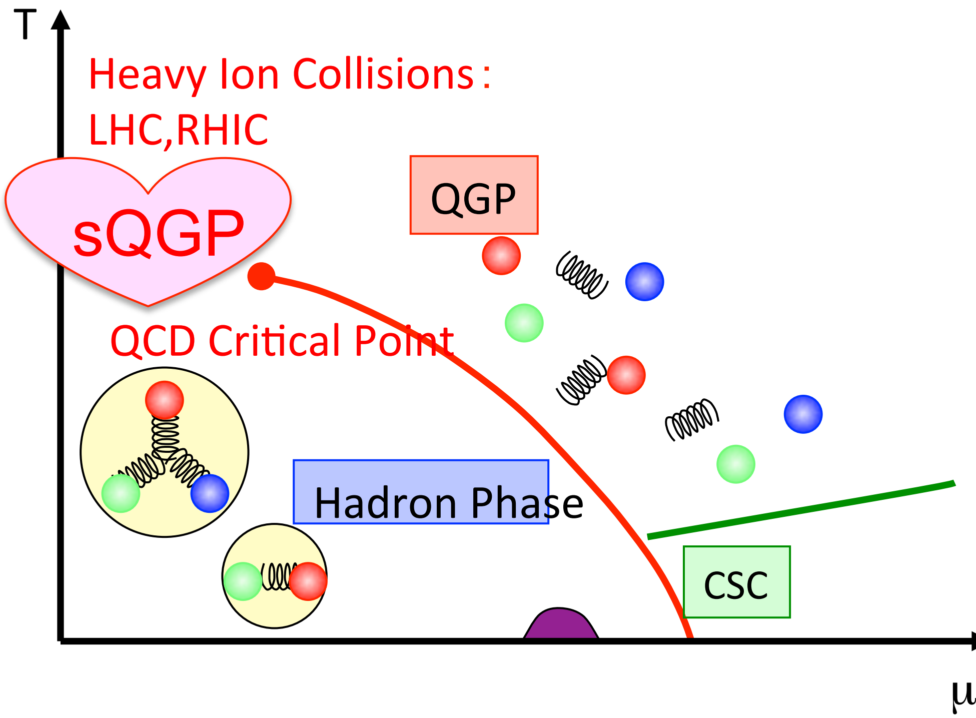
π^0 :contribution from disconnected diagrams appear.

Parameters

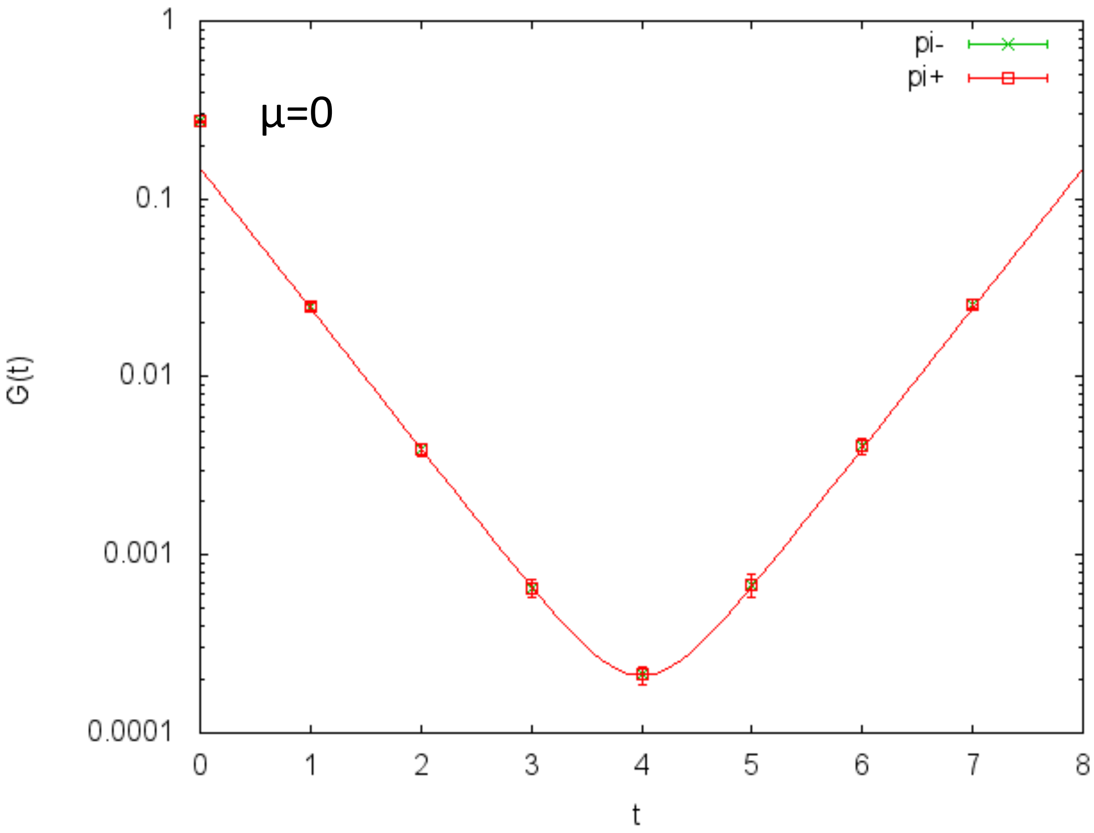
- Wilson fermion

▪lattice size:4^3×8
▪κ=1.6
▪β=4.0
▪μ=0〜0.8
▪λ=0.005〜0.015($\mu \geqq 0.3$)





μ_I Effect on Pion Correlators



- μ_I=0
  The propagator of π^+ is identical with that of π^-.

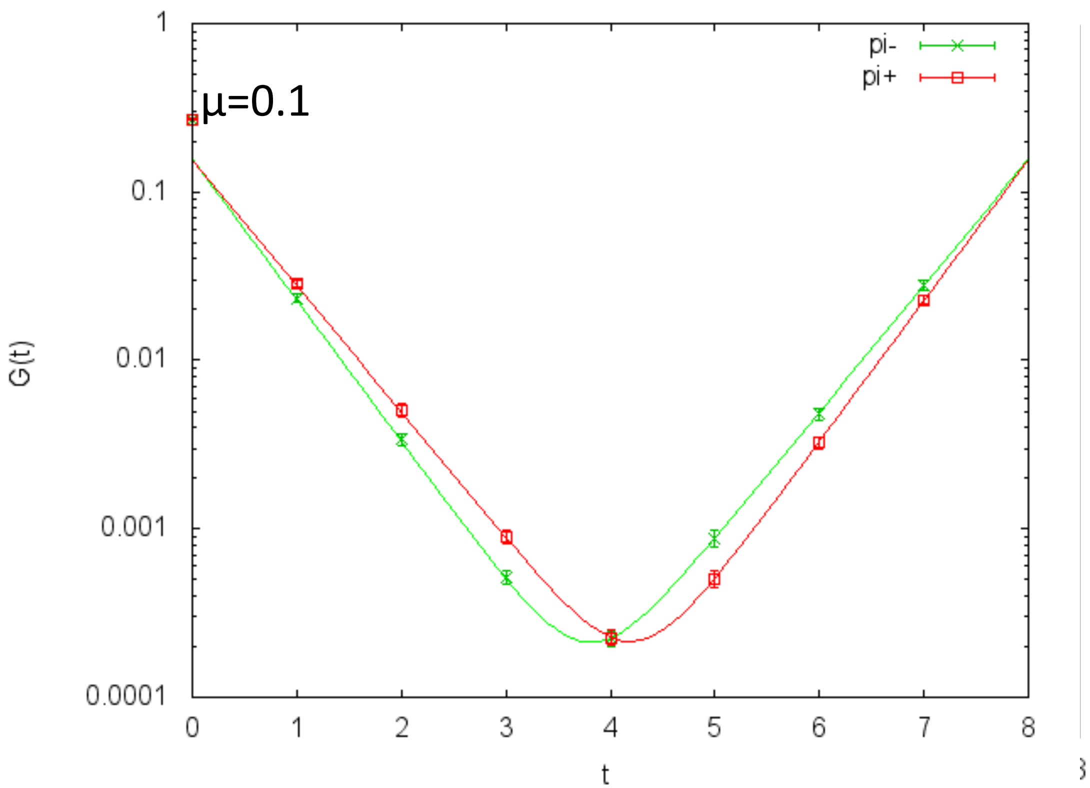
$$\pi^-(x)\pi^+(y) : C_+\left(e^{-(m_0-\mu)t} + e^{-(m_0+\mu)(T-t)}\right)$$

$$\pi^+(x)\pi^-(y) : C_-\left(e^{-(m_0+\mu)t} + e^{-(m_0-\mu)(T-t)}\right)$$

→ m_π

$$C_a \propto |\langle \pi^a | \pi^a \, |0\rangle|^2$$

μ_I Effect on Pion Correlators



t

- $\mu_I=0$
  The propagator of π^+ is identical with that of π^-.
- At finite μ_I
  μ_I affects propagators of π^+ and π^- differently.

$$\pi^-(x)\pi^+(y) : C_+\left(e^{-(m_0-\mu)t} + e^{-(m_0+\mu)(T-t)}\right)$$

$$\pi^+(x)\pi^-(y) : C_-\left(e^{-(m_0+\mu)t} + e^{-(m_0-\mu)(T-t)}\right)$$

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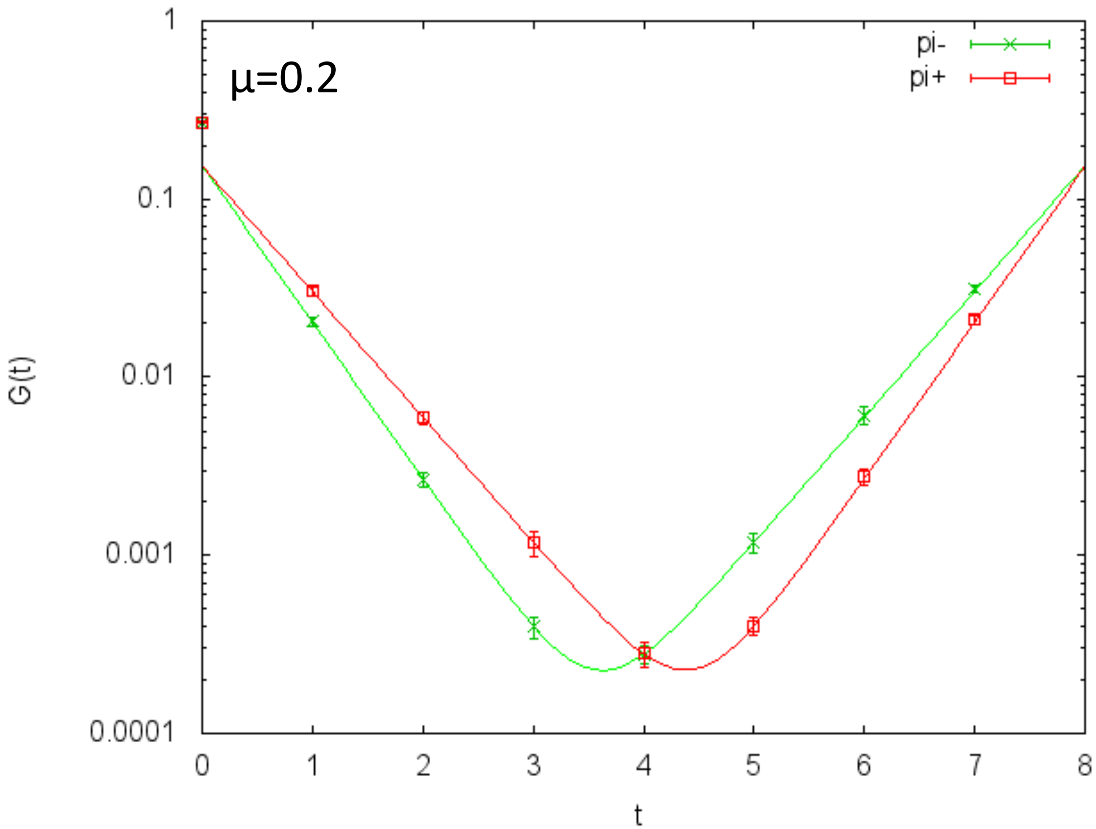
μ_I Effect on Pion Correlators



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→ m_π

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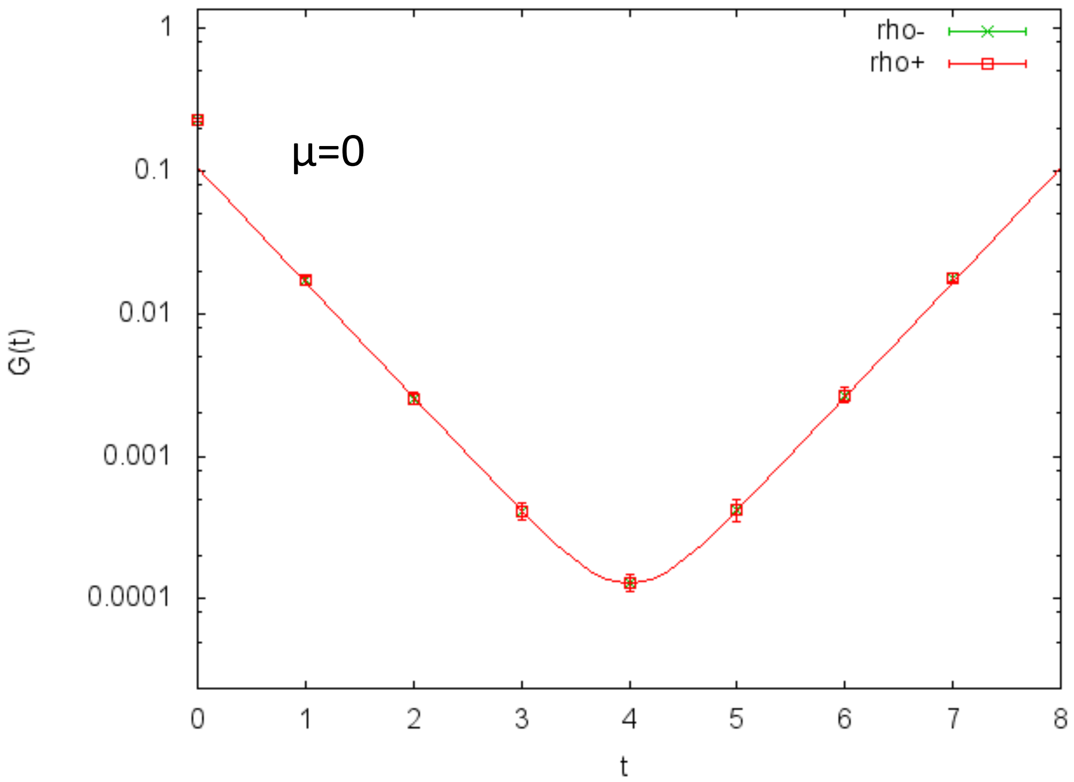
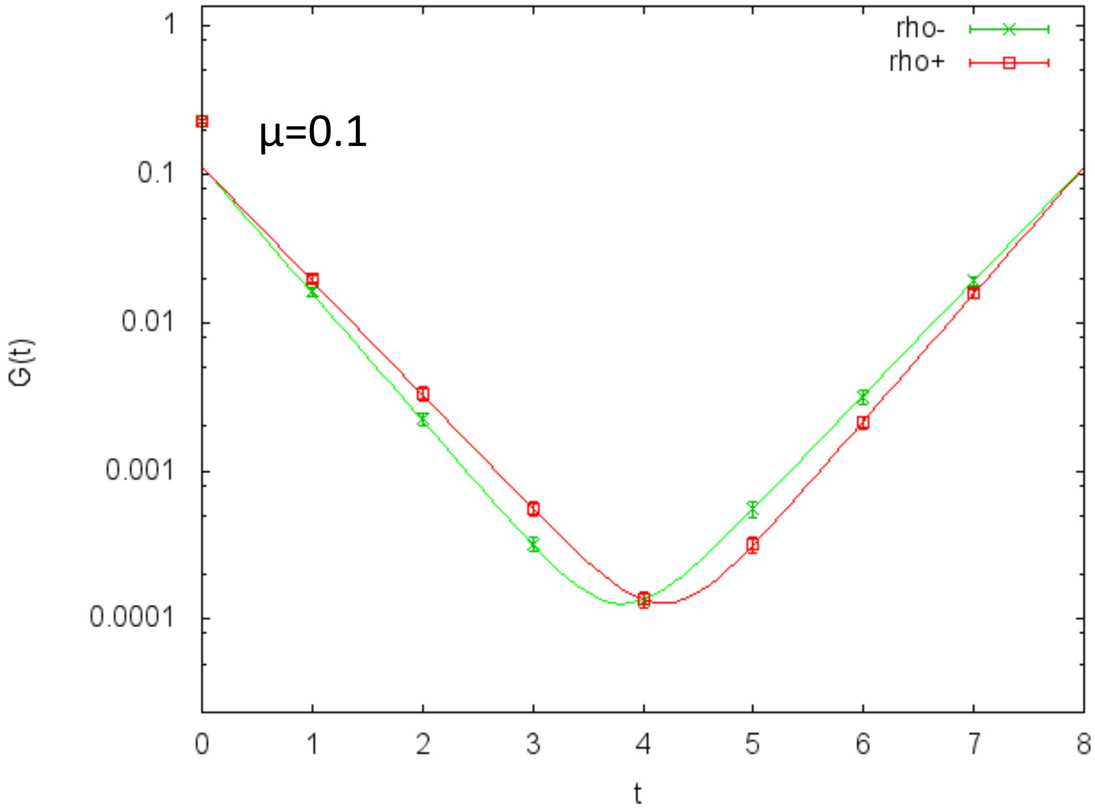
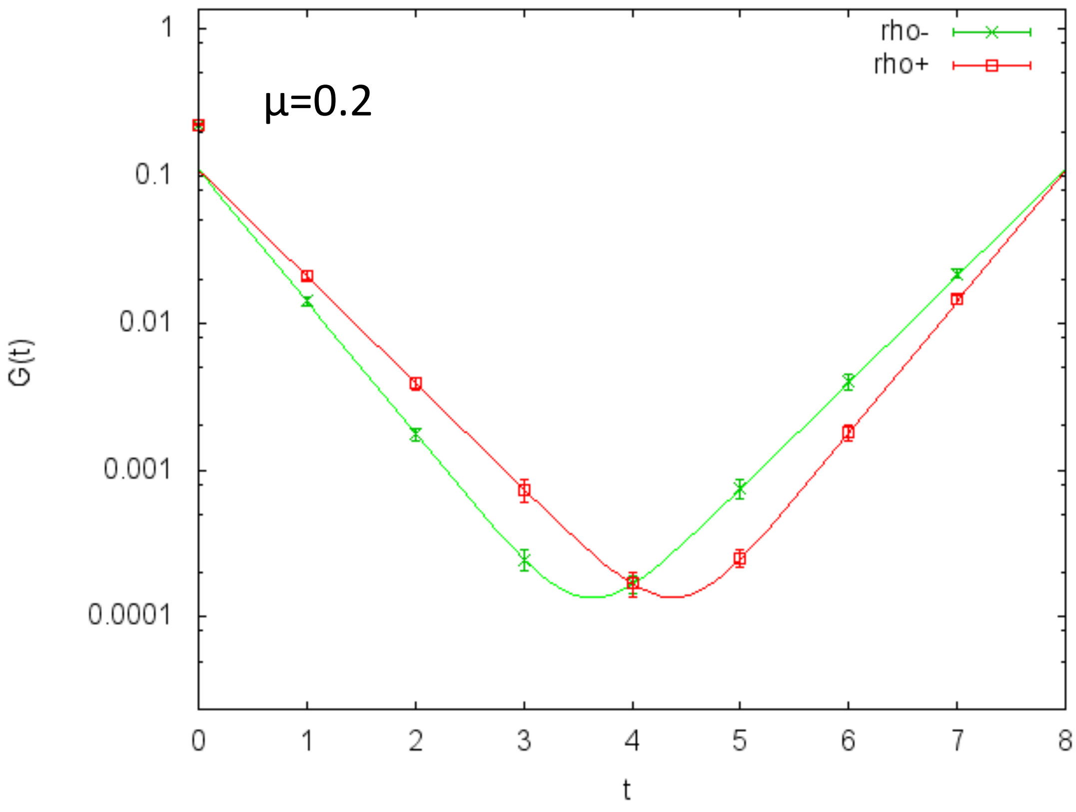
μ_I Effect on Rho Correlators



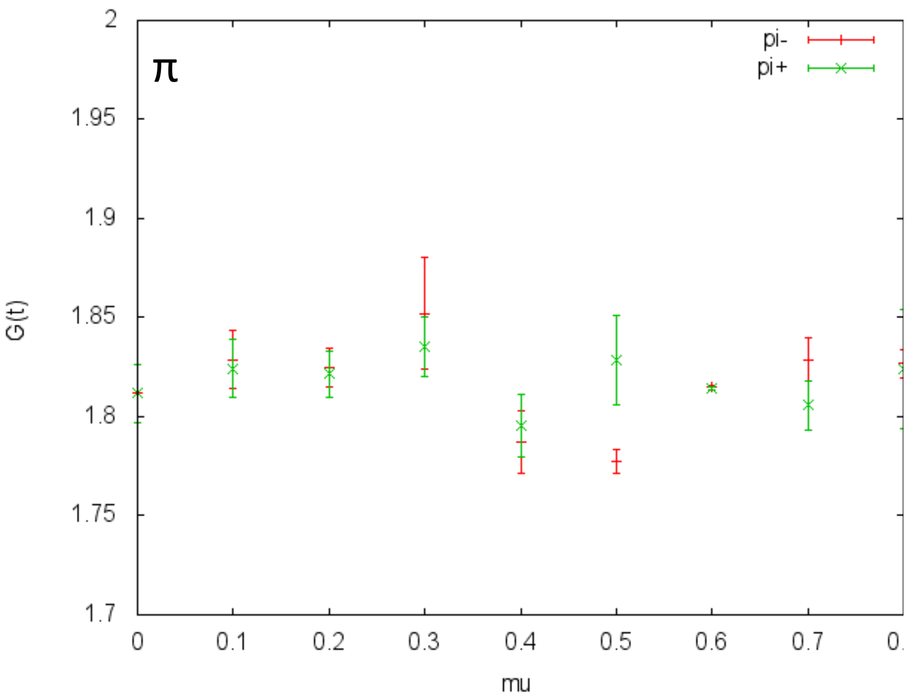
μ_I Effect on Rho Correlators

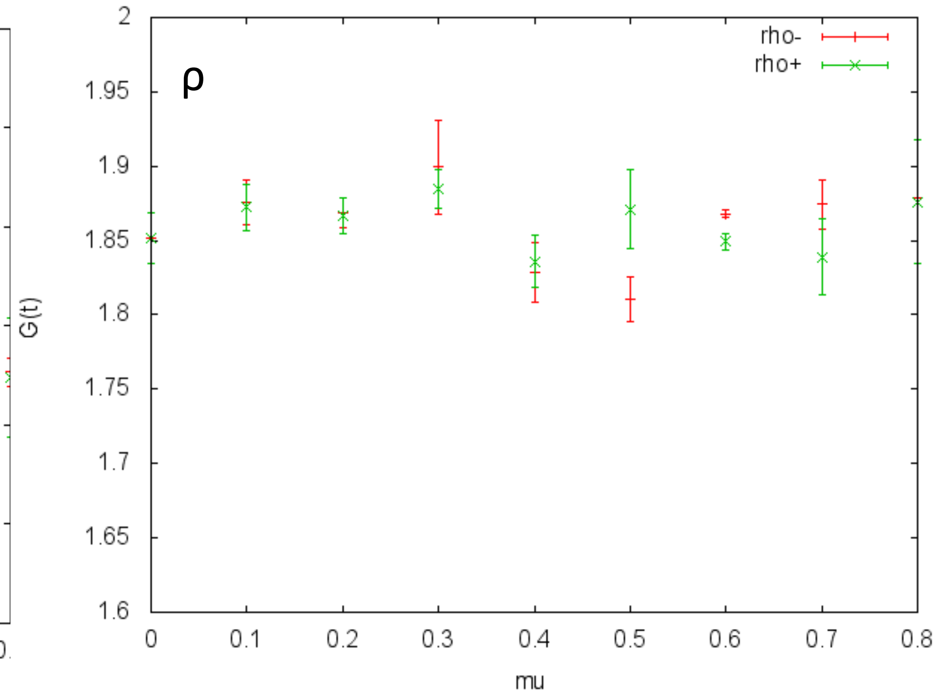


μ_I Effect on Rho Correlators



Isospin Chemical Potential Dependence





- heavy quark mass
- higher μ_I

Summary

- Isospin chemical potential dependence of m_π and m_ρ
  - Introduction of isospin chemical potential to HMC
  - Pion and rho propagators with finite μ_I
  - Isospin chemical potential dependence of m_π and m_ρ
  - pion (rho) condensation?
- Work in progress
  - Larger lattice size, high μ_I and light quark mass
  - π condensation, ρ condensation ⟺ m_π, m_ρ
  - Temperature dependence