

The Roberge-Weiss transition from $N_f = 2$ QCD with Wilson fermions

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Outline

QCD phase diagram

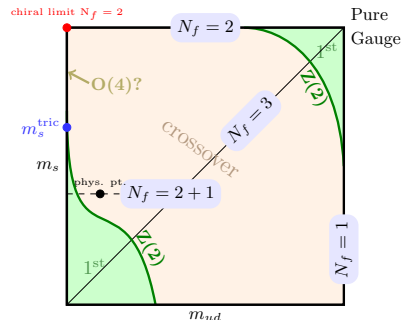
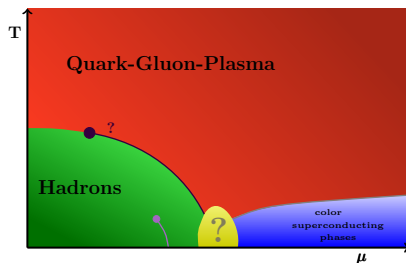
QCD at imaginary μ

- Roberge-Weiss symmetry

Endpoint of Roberge-Weiss transition for $N_f = 2$ QCD

Summary & Perspectives

QCD phase diagram



Sign-Problem:

$$\det(D + \mu)^* = \det(D - \mu^*)$$

Solutions: Reweighting, Taylor-Series, Complex Langevin...

Imaginary μ

Roberge-Weiss symmetry

Roberge & Weiss, Nucl. Phys. B 275, 734 (1986)

At imaginary μ , QCD has the symmetries:

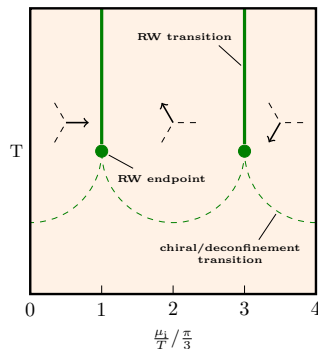
$$Z(\mu) = Z(-\mu)$$

$$Z(\mu/T) = Z(\mu/T + i \frac{2\pi n}{N_c}), n \in \mathbb{N}$$

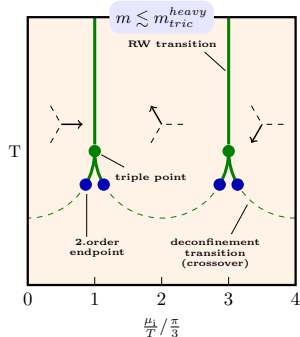
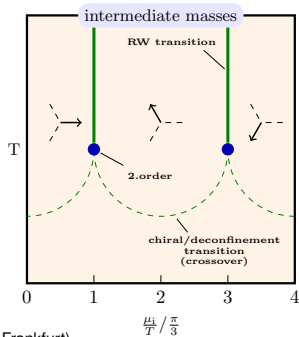
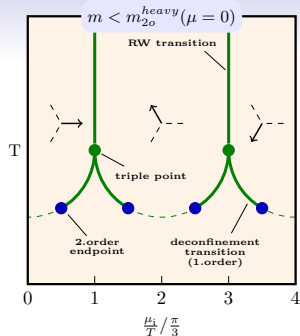
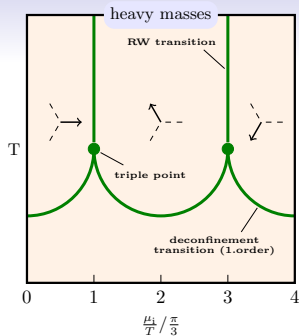
Phases of Polyakov-Loop cycle through different $\mathbb{Z}(N_c)$ sectors:

$$L(x) = \frac{1}{N_c} \text{Tr} \prod_{\tau=1}^{N_\tau} U_0(x) = |L| e^{-i\varphi}$$

$$\langle \varphi \rangle = n (2\pi/N_c), n = 0, 1, \dots, N_c-1$$



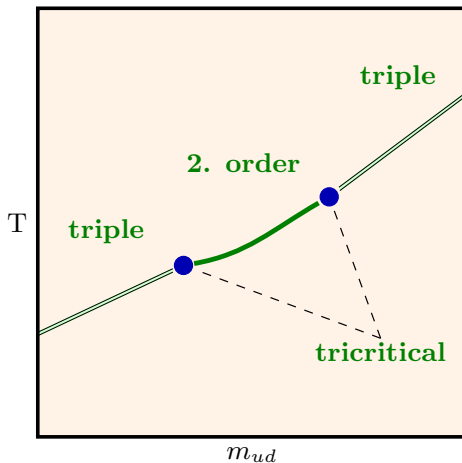
de Forcrand & Philipsen, PRL 105 152001 (2010), D'Elia & Sanfilippo, PRD 80 (2009)



Nature of RW endpoint at $\mu = i\pi T$

based on staggered studies, $N_\tau = 4$

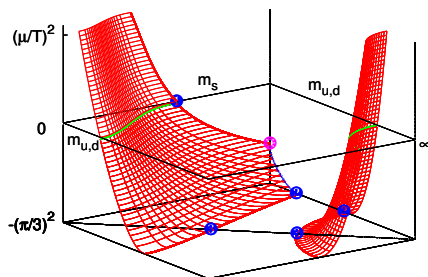
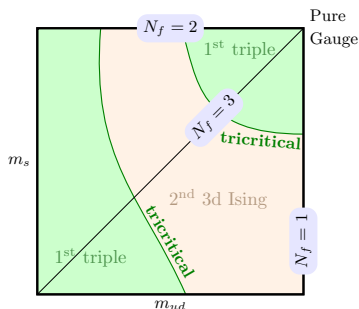
de Forcrand & Philipsen, PRL 105 152001 (2010), D'Elia & Sanfilippo, PRD 80 (2009)



Extended Columbia Plot

based on staggered studies, $N_\tau = 4$

de Forcrand & Philipsen, PRL 105 152001 (2010), D'Elia & Sanfilippo, PRD80 (2009)



Rich phase structure
Constraints on QCD phase diagram

→ Talk by O. Philipsen (8A) ($N_f = 2$)

RW endpoint with Wilson fermions

- Studies done at $\mu = i\pi T$ and $N_\tau = 4$ for various κ and N_σ
- $\mathcal{O}(10)$ β for each (κ, N_σ)
- Employ reweighting [Ferrenberg & Swendsen, PRL 63, 1195 \(1989\)](#)
- Simulations carried out with OpenCL-based CL²QCD
[Bach, Lindenstruth, Philipsen & Pinke \[arXiv:1209.5942\]](#)
→ talk by M. Bach (4G)

Related work:

- Heavy-Quark effective lattice theory predicts:
[Fromm, Langelage, Lottini & Philipsen \[arXiv:1111.4953\]](#)
 $\kappa_{tric}^{heavy} = 0.1048 \pm 0.0008$
→ Talks by J. Langelage & M. Neuman (7A)
- Study by Wu & Meng [\[arXiv:1303.0336\]](#): Triple points for $\kappa \geq 0.155$

RW endpoint with Wilson fermions

Binder-Cumulant:

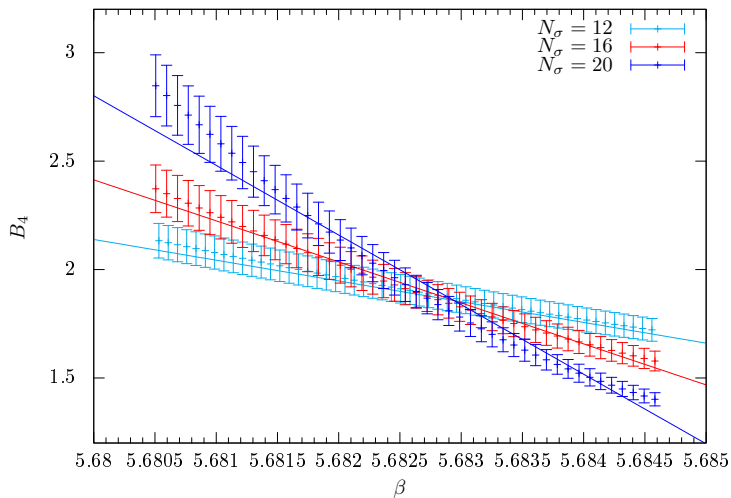
$$B_4(\text{L.Im}) = \frac{\langle (\text{L.Im} - \langle \text{L.Im} \rangle)^4 \rangle}{\langle (\text{L.Im} - \langle \text{L.Im} \rangle)^2 \rangle^2} \xrightarrow{V \rightarrow \infty} \left\{ \begin{array}{ll} 1.5 & \text{1. order triple} \\ 1.604 & \text{2. order (3D Ising)} \\ 2 & \text{tricritical} \\ 3 & \text{crossover} \end{array} \right.$$

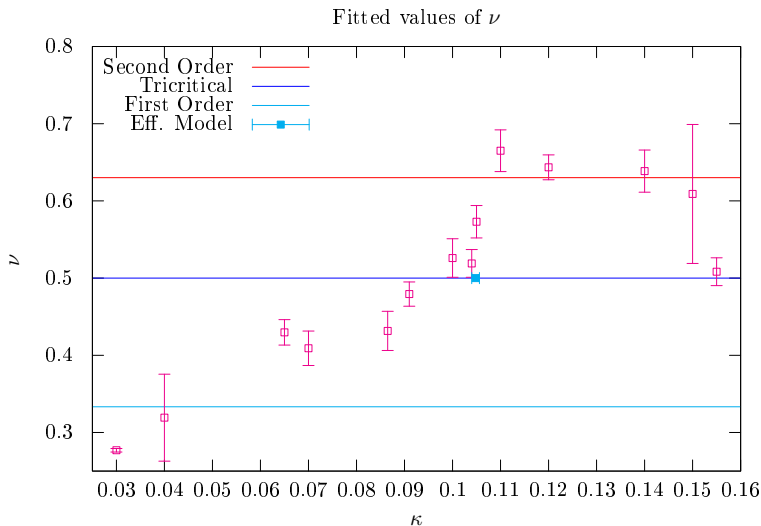
Finite Size scaling:

Close to the RW-endpoint, B_4 scales with critical exponent ν :

$$B_4(\beta, N_\sigma) = B_4(\beta, \infty) + a_1(\beta - \beta_c)N_\sigma^{1/\nu} + a_2((\beta - \beta_c)N_\sigma^{1/\nu})^2 + \dots$$

Finite size scaling analysis





Summary

- No sign problem at imaginary μ , HMC applicable
- QCD phase diagram constrained by imaginary μ region
- Started studies to map out $N_f = 2$ phase diagram at Roberge-Weiss value of μ using Wilson fermions
- Qualitative agreement with staggered results
- Quantitative agreement of κ_{tric}^{heavy} with effective theory

Perspectives

- Simulate at smaller masses
- Aoki phase at imaginary μ ?