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Singularities around the QCD critical point in the complex chemical potential plane

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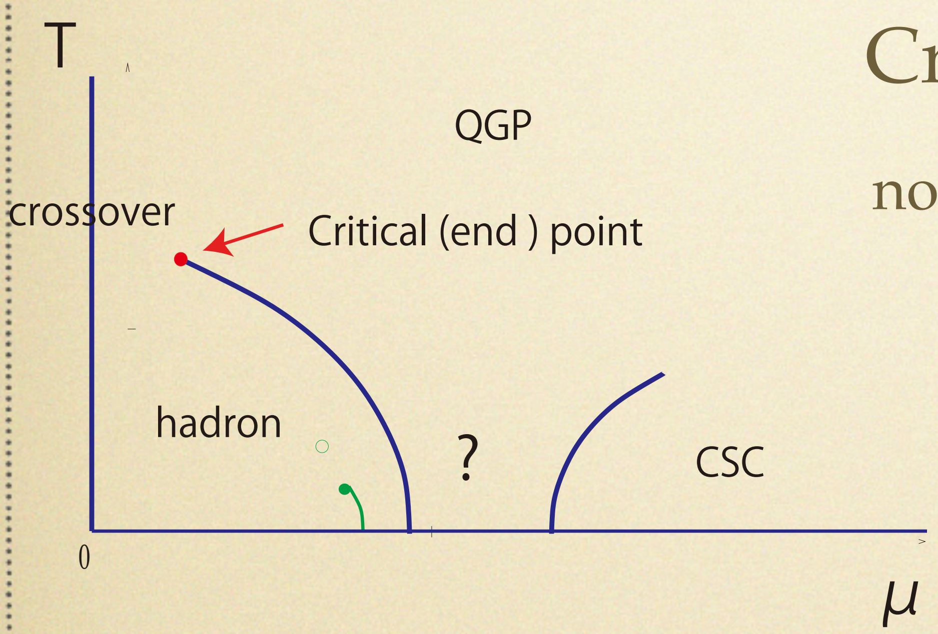
1. Introduction

QCD phase diagram



Complex chemical potential plane

Critical (end) point ?

notorious sign problem for $\mu \neq 0$

- Taylor series
- imaginary chemical potential
- reweighting
- strong coupling expansion

etc

- Partition function zeroes: complementary view of critical phenomena
  Lee-Yang (1952)
- scaling in the complex plane
  Itzykson-Pearson-Zuber (1983)
- critical point and singularities in QCD
  Stephanov (2006)
- in connection with experimental data
  Nakamura-Nagata (2013)

In this talk, we have a look at complex effective potential in terms of complex order parameter for complex μ.

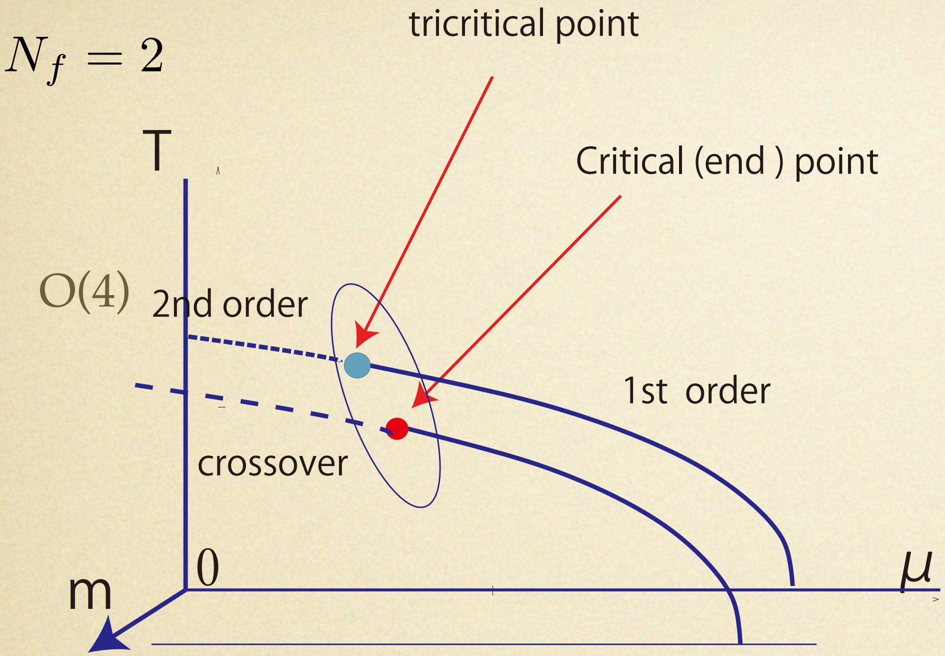
By using an effective theory based on a mean field theory, we study

- singularities in the complex μ plane
- extrema of real part of the effective potential
- the Stokes lines

Plan

- 1. Introduction (✓)
- 2. a QCD effective theory
- 3. Singularities in the complex μ plane
- 4. Stokes lines
- 5. Summary

2. a QCD effective theory: mean field approach



$m = 0$ tricritical point: upper critical dimension

d=3

mean field description is expected to be valid (up to log corrections) ,

because the system is effectively in three dimensions at finite T

$m \neq 0$ critical point= liquid-gas phase transition

σ massless

=3d Ising model

π massive

$$M = \langle \bar{q_L} q_R \rangle = \sigma + i\vec{\tau} \cdot \vec{\pi}$$

a QCD effective theory based on mean field approach

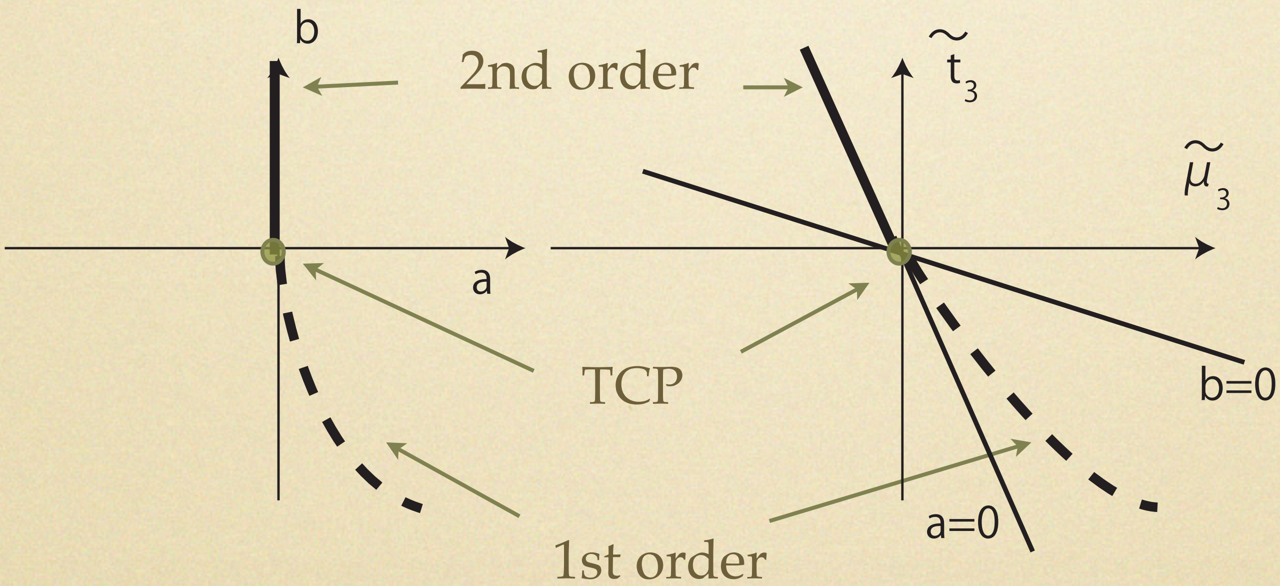
Hatta - Ikeda (2003)

$$\Omega = -m\sigma + \frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 + \frac{c}{6}\sigma^6,$$

expands around the tricritical point (TCP) $a = 0, b = m = 0$

$$\begin{aligned} a(T,\mu) &= C_a\ (T - T_3) + D_a\ (\mu - \mu_3) \\ b(T,\mu) &= C_b\ (T - T_3) + D_b\ (\mu - \mu_3), \end{aligned}$$

$$C_b D_a - C_a D_b > 0.$$



$$\tilde{t}_3 \equiv (T - T_3)$$
$$\tilde{\mu}_3 \equiv (\mu - \mu_3)$$

$$m = 0$$

By switching on m , the condition for the critical end point

$$\frac{\partial\Omega(T_E,\mu_E,\sigma_0)}{\partial\sigma}=\frac{\partial^2\Omega(T_E,\mu_E,\sigma_0)}{\partial\sigma^2}=\frac{\partial^3\Omega(T_E,\mu_E,\sigma_0)}{\partial\sigma^3}=0$$

$\Rightarrow$

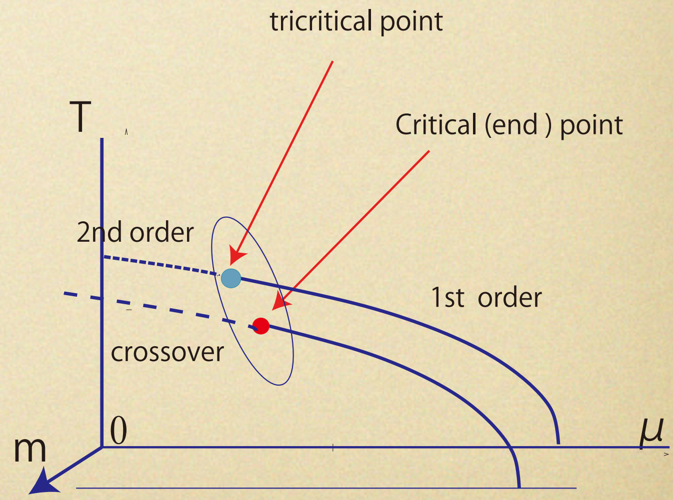
$$a(T_E,\mu_E)=\frac{9b(T_E,\mu_E)^2}{20c},\quad -b(T_E,\mu_E)=\frac{5}{54^{1/5}}c^{3/5}m^{2/5},\quad \sigma_0=\sqrt{\frac{-3\,b(T_E,\mu_E)}{10c}}.$$

thermodynamic potential around the critical end point

$$\Omega(T,\mu,\sigma)=\Omega(T_E,\mu_E,\sigma_0)+A_1\hat{\sigma}+A_2\hat{\sigma}^2+A_3\hat{\sigma}^3+A_4\hat{\sigma}^4,$$

$$\begin{aligned}
A_1 &= \left(C_a\sigma_0+C_b\sigma_0^3\right)\tilde{t}_E+\left(D_a\sigma_0+D_b\sigma_0^3\right)\tilde{\mu}_E\\
A_2 &= \frac{1}{2}\left(C_a+3C_b\sigma_0^3\right)\tilde{t}_E+\frac{1}{2}\left(D_a+3D_b\sigma_0^3\right)\tilde{\mu}_E\\
A_3 &= \left(C_b\tilde{t}_E+D_b\tilde{\mu}_E\right)\sigma_0\\
A_4 &= -\frac{b(T_E,\mu_E)}{2}+\frac{1}{4}\left(C_b\tilde{t}_E+D_b\tilde{\mu}_E\right),
\end{aligned}$$

$$\tilde{t}_E=T-T_E,\ \tilde{\mu}_E=\mu-\mu_E$$

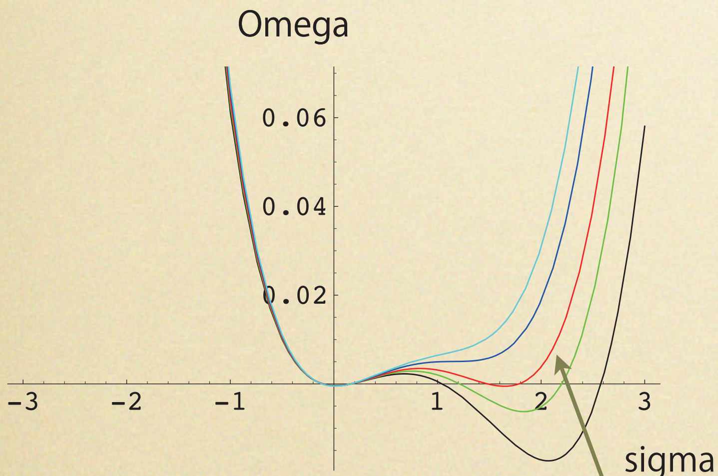


stability condition

$$A_4 > 0 \quad \Longrightarrow \quad \tilde{\mu}_E > \frac{2b(T_E, \mu_E) - C_b \tilde{t}_E}{D_b}.$$

typical behaviors of Ω at temperature around the CEP.

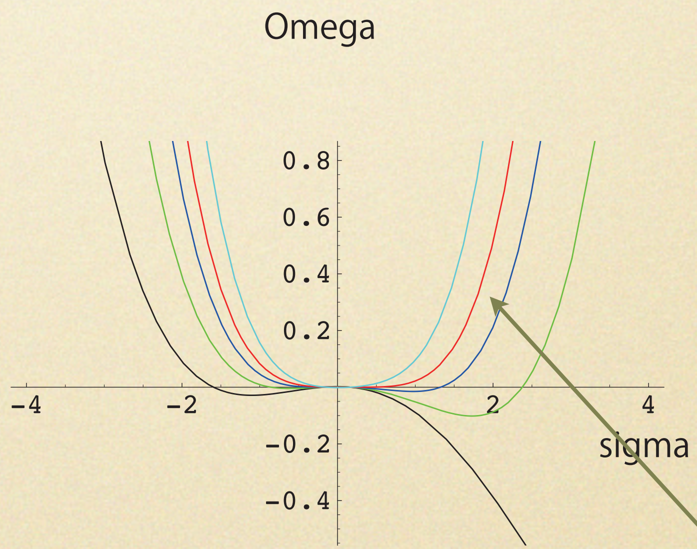
first order phase transition for $\tilde{t}_E < 0$



$$\tilde{t}_E = -0.2$$

$\tilde{\mu}_E \equiv \mu - \mu_E = 0.052, 0.054, 0.0555, 0.058, 0.06$

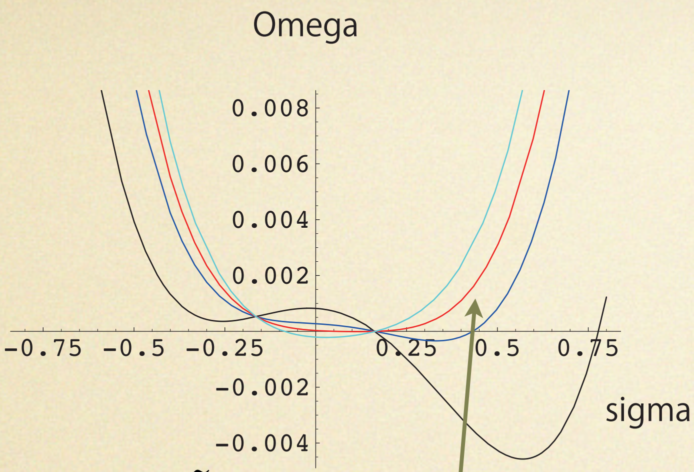
crtical point at $\tilde{t}_E = 0$

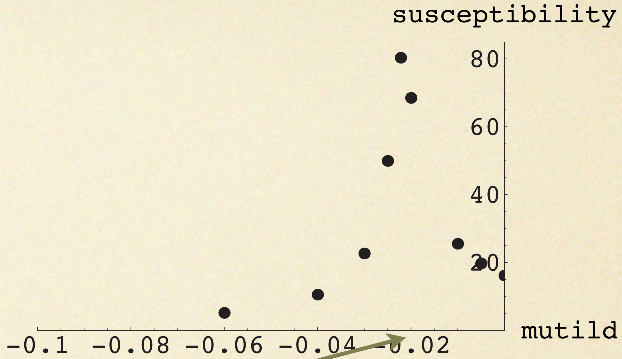


$$\tilde{t}_E = 0$$

$\tilde{\mu}_E$= -0.15, -0.1, -0.05, 0, 0.1

crossover for $\tilde{t}_E > 0$





$\tilde{t}_E = 0.2$

chiral susceptibility at $\tilde{t}_E = 0.2$

$\tilde{\mu}_E$ = -0.08, -0.04, -0.0222 (crossover), -0.005.

3. Singularities in the complex μ plane

$$\frac{\partial \Omega}{\partial \sigma} = 0, \quad \frac{\partial^2 \Omega}{\partial \sigma^2} = 0 \quad (*)$$

$$A_1 + 2A_2\sigma + 3A_3\sigma^2 + 4A_4\sigma^3 = 0, \quad 2A_2 + 6A_3\sigma + 12A_4\sigma^2 = 0.$$

Example: for $x_m = 0.2$ and $\tilde{t}_E = 0.2$

solutions to $(*)$

$$\tilde{\mu}_E = \cancel{-0.3857}\ \text{(i)}, \quad -0.0538\ \text{(ii)}, \quad -0.0222 \pm 0.00254\, i\ \text{(iii)}.$$

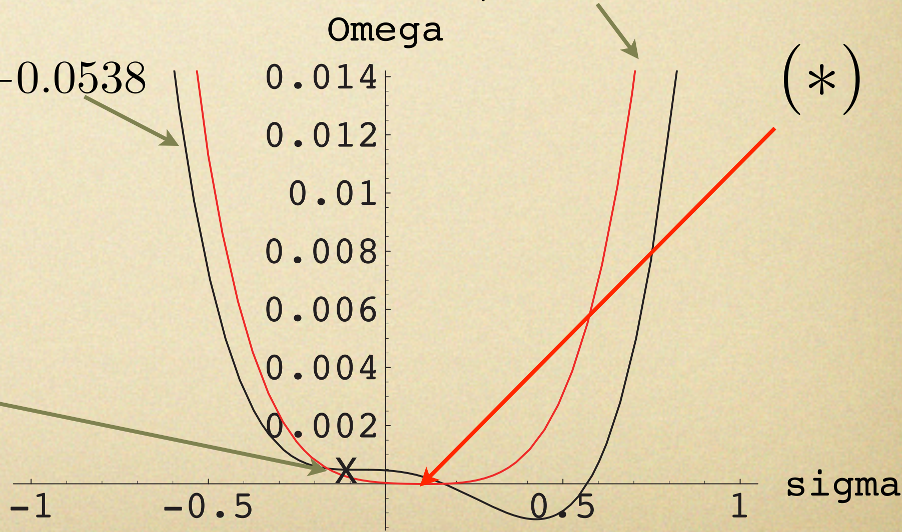
The stability condition of $\Omega \rightarrow \tilde{\mu}_E > -0.3801$ for $\tilde{t}_E = 0.2$

$\tilde{\mu}_E = -0.0222$

singularity is located at

$\tilde{\mu}_E = -0.0538$

$$\text{Re } \tilde{\mu}_E^* = -0.0222 \pm 0.00254i$$

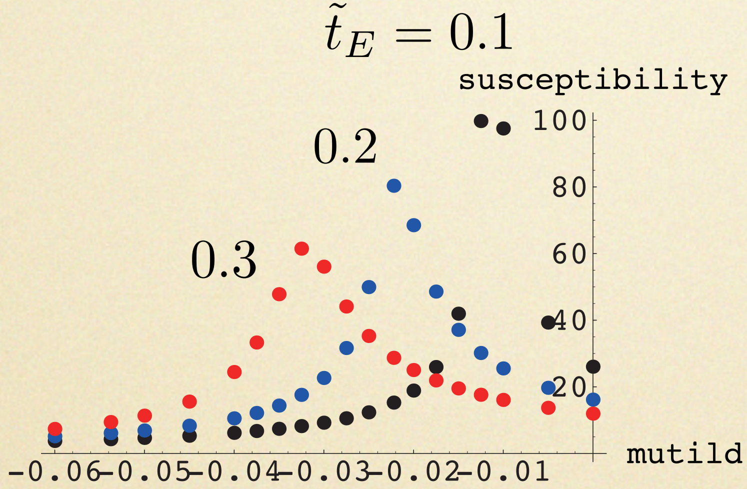


$(*)$

$(*)$

at $\sigma = -0.07602$

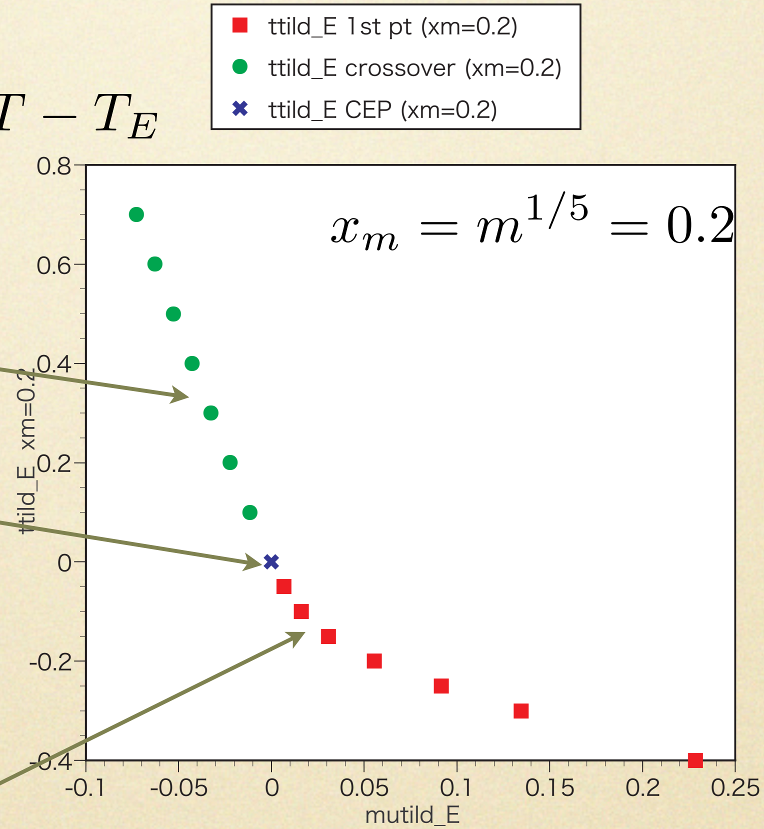
Susceptibility peak is attained at Re $\tilde{\mu}_E^*$



chiral susceptibility at $\tilde{t}_E = 0.1, 0.2, 0.3$

phase diagram

$\tilde{t}_E = T - T_E$



crossover

$\tilde{t}_E > 0, \tilde{\mu}_E < 0$

CEP

$\tilde{t}_E = 0, \ \tilde{\mu}_E = 0$

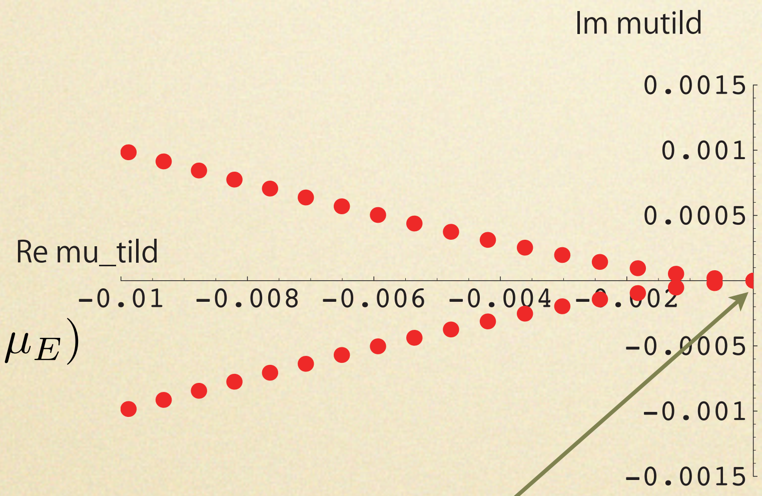
1st order

$\tilde{t}_E < 0, \tilde{\mu}_E > 0$

$\tilde{\mu}_E = \mu - \mu_E$

Locations of singularities type (iii) for $\tilde{t}_E > 0$

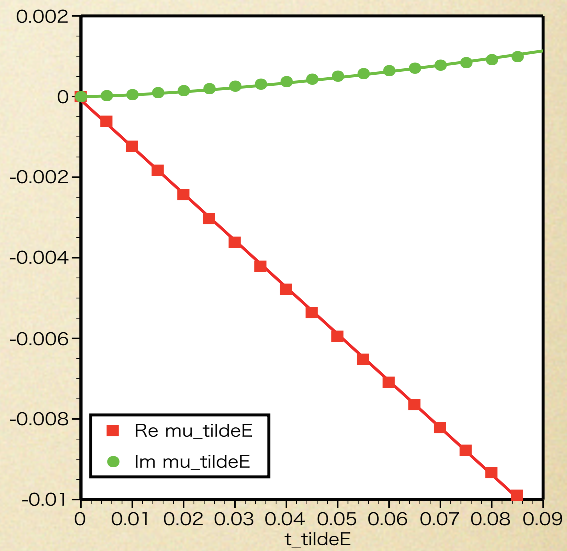
$\mathrm{Im}\ \tilde{\mu}_E = \mathrm{Im}\ (\mu - \mu_E)$



$\mathrm{Re}\ \tilde{\mu}_E = \mathrm{Re}\ (\mu - \mu_E)$

$\mathrm{Re}\ \tilde{\mu}_E = 0,\ \mathrm{Im}\ \tilde{\mu}_E = 0$

$\mathrm{Re}\ \tilde{\mu}_E$ and $\mathrm{Im}\ \tilde{\mu}_E$ v.s. $\tilde{t}_E$



3. Stokes lines

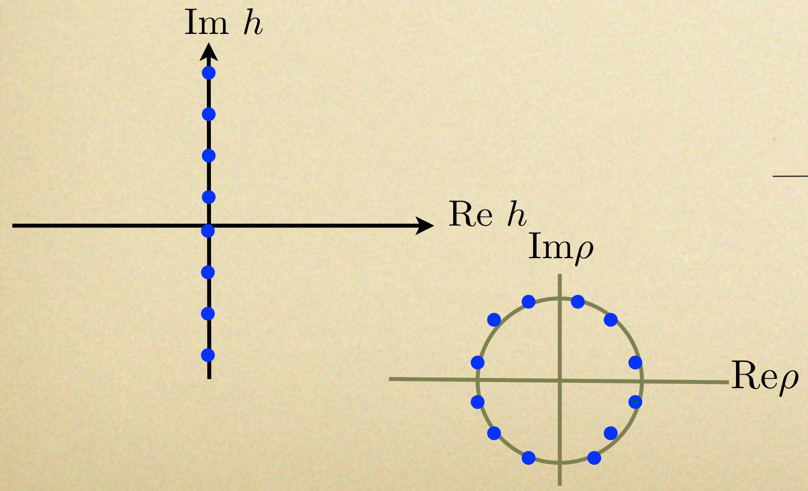
reflect the analytic structure around the branch points.

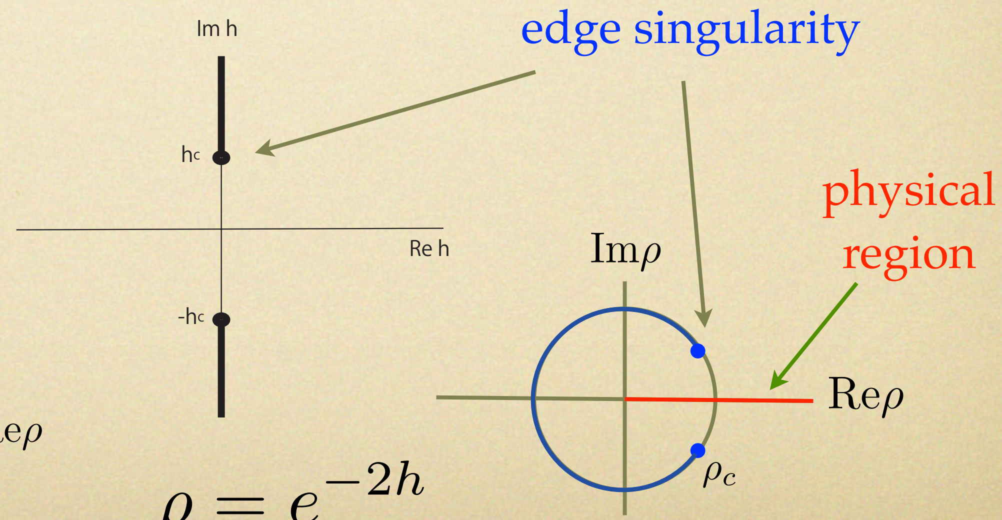
in the vicinity of the CEP

The Stokes line is understood as the curve to which the Lee-Yang zeros accumulate.

Lee-Yang theorem:

zeros on the imaginary h axis (d-dimensions)

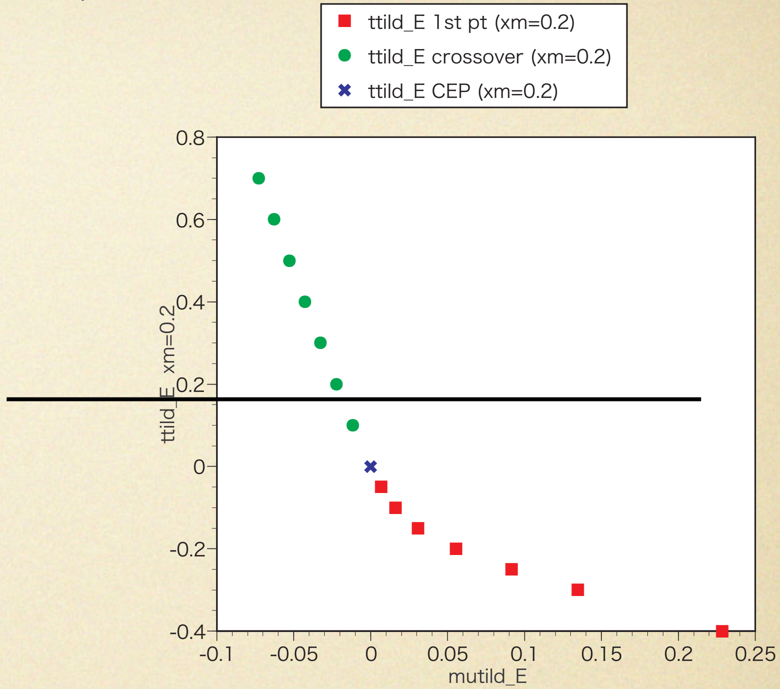




$$\rho = e^{-2h}$$

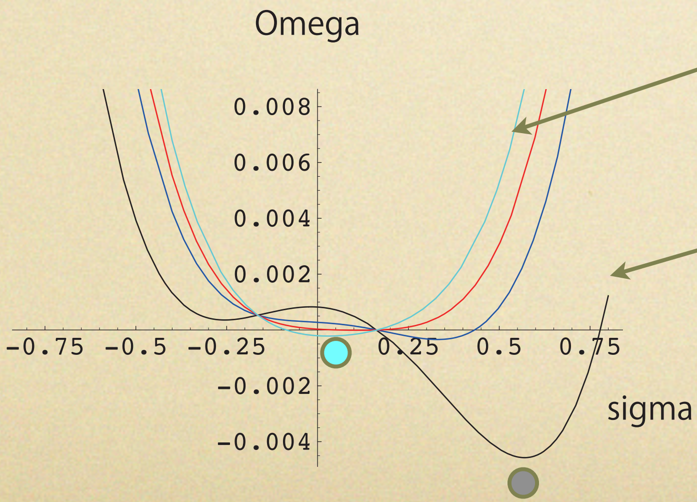
Stokes lines in the complex $\tilde{\mu}_E$ plane

$(I)\ \tilde{t}_E > 0$



$$\tilde{\mu}_E \equiv \rho\ e^{i\theta}$$

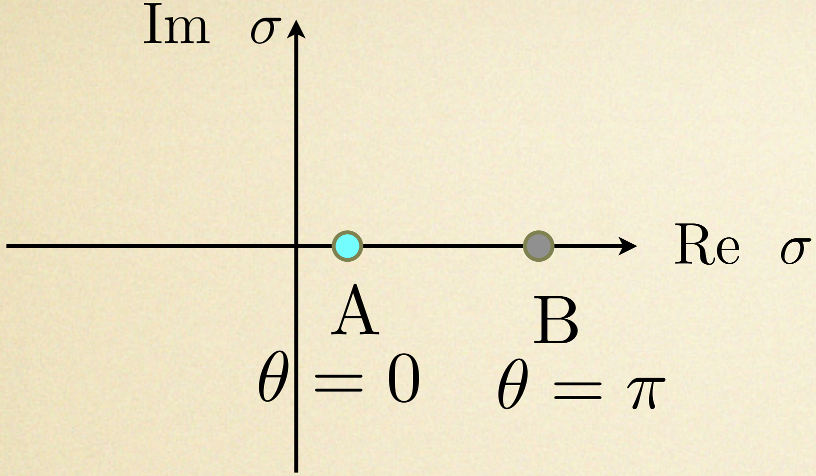
$$\Omega(T, \mu, \sigma)$$



$$\tilde{\mu}_E > 0 \quad (\theta = 0)$$

$$\tilde{\mu}_E < 0 \quad (\theta = \pi)$$

locations of the global minimum of Ω



Analytical continuation $(\theta \neq 0, \pi)$ $\quad \tilde{\mu}_E \equiv \rho \, e^{i\theta}$

vary θ 0 to π

singular point exits at $\tilde{\mu}_E^* \equiv \rho^* \, e^{i\theta^*}$

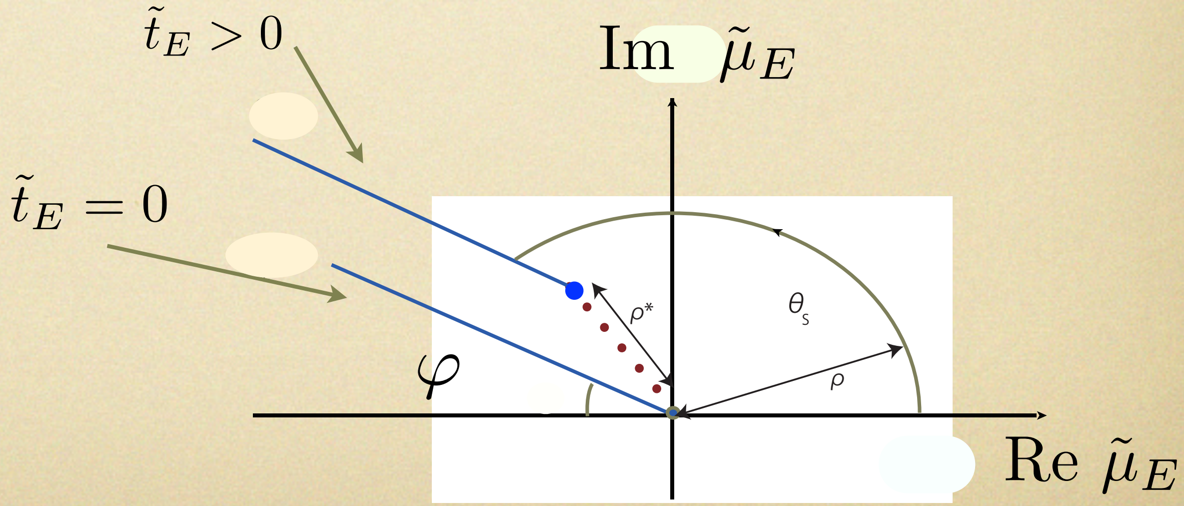
$(\rho^* > 0)$

consider three cases

(i) $\rho > \rho^*$

(ii) $\rho < \rho^*$

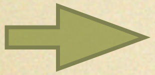
(iii) $\rho = \rho^*$



Summary

By using an effective theory based on a mean field theory, we studied

- singularities in the complex μ plane
- extrema of real part of the effective potential
- the Stokes lines



- the location of the crossover is regarded as a real part of the singularity $\tilde{\mu}_E^*$
- the Stokes lines are located by explicitly looking at Re Ω as a function of the complex order parameter σ

- ? in connection with Ejiri-H.Y. (2009)
- baryon number distribution and complex μ, Morita et. al. (2013)