Lattice study of the Boer-Mulders transverse momentum distribution in the pion

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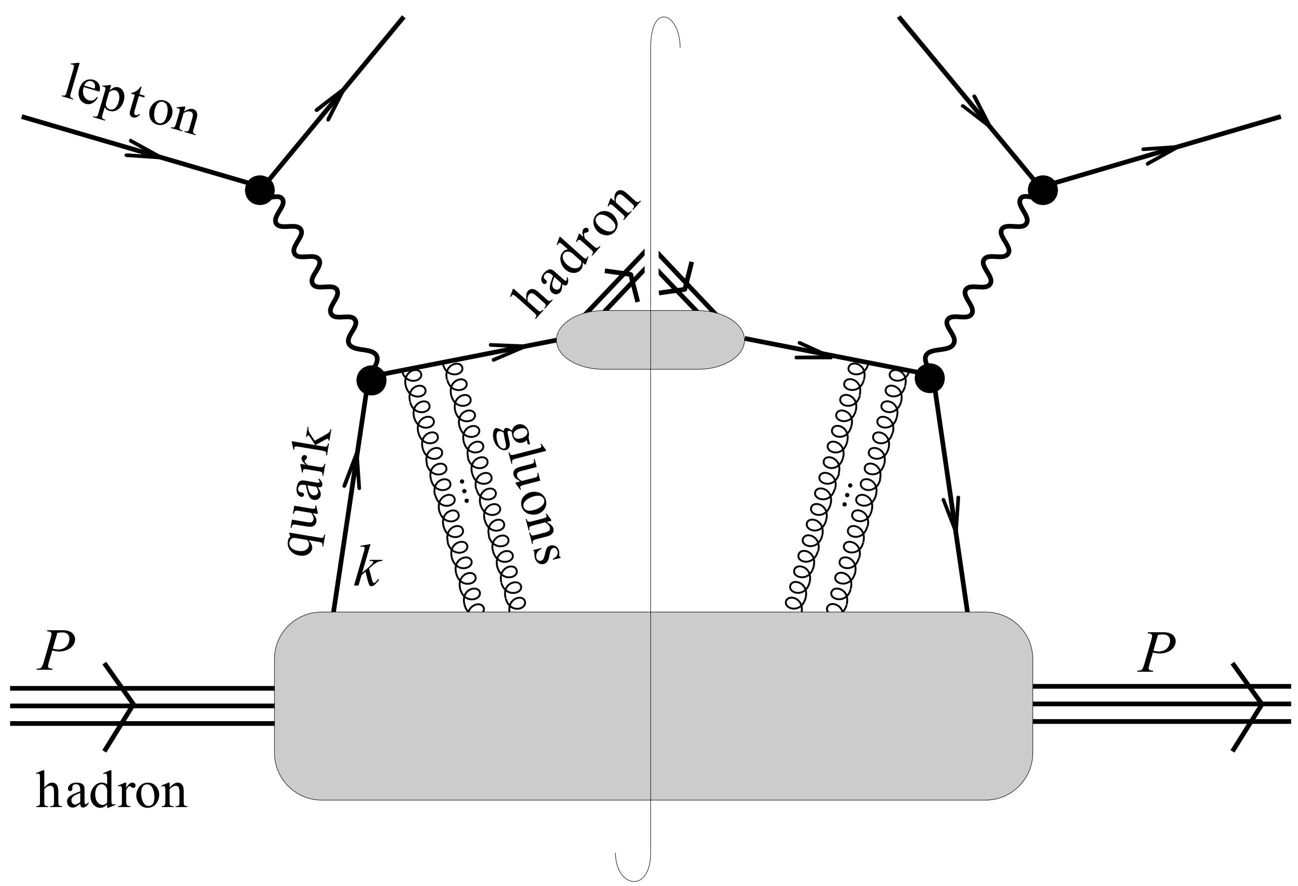
Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\Bigg|_{b^+=0}$$

- “Soft factor” $\widetilde{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\widetilde{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Gauge link structure motivated by SIDIS



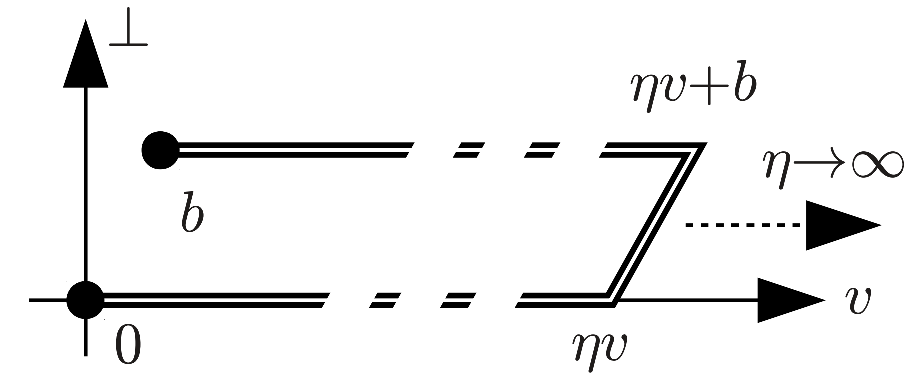
$$l + H(P) \longrightarrow l' + h(P_h) + X$$

Gauge link structure:

In matrix element $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv$

$\frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \ldots, b]\ q(b)\ |P, S\rangle$

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

Gauge link structure motivated by SIDIS

Staple-shaped links incorporate SIDIS final state effects:

- Gauge link roughly follows direction of ejected quark, (close to) light cone
- Effective resummed description of gluon exchanges between ejected quark and remainder of nucleon in evolving final state
- Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

  $$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

  Light-like staple for $\hat{\zeta} \to \infty$. Perturbative evolution equations for large $\hat{\zeta}$.
- In this approach, have "modified universality", $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$ (initial state interactions in DY case). SIDIS: $\eta v \cdot P \to \infty$, DY: $\eta v \cdot P \to -\infty$.

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\eta v,\eta v+b,b]\ q(b)\ |P,S\rangle$$

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Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+ \gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+} \gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^{\perp}\right] \text{odd}$$

TMD Classification

All leading twist structures:

H ↓ \ q →	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

⟵ Boer-Mulders (T-odd)

↑ Sivers (T-odd)

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + i m_H \epsilon_{ij} b_i S_j \widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda \widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_H(b_T \cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= i m_H \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\
&\quad - i m_H \Lambda b_i \widetilde{A}_{10B} + m_H[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)] b_i \widetilde{A}_{11B}
\end{aligned}$$

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

In limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left(\frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

ill-defined for large k_T, so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

Also, we can only access limited range of $b \cdot P$, so cannot Fourier-transform to obtain x-dependence. For now, consider only first x-moments (accessible at $b \cdot P = 0$):

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Boer-Mulders shift:

$$\langle k_y \rangle_{UT} \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}}{\tilde{f}_1^{[1](0)}} = \left. \frac{\int dx \int d^2k_T \, k_y \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)}{\int dx \int d^2k_T \, \Phi^{[\gamma^+ + s^j i\sigma^{j+}\gamma^5]}(x, k_T, P, \ldots)} \right|_{s_T=(1,0)}$$

Average transverse momentum of quarks polarized in the orthogonal transverse ("T") direction in an unpolarized ("U") hadron; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) \equiv m_H \frac{\tilde{h}_1^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = m_H \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Sivers shift (in a polarized hadron):

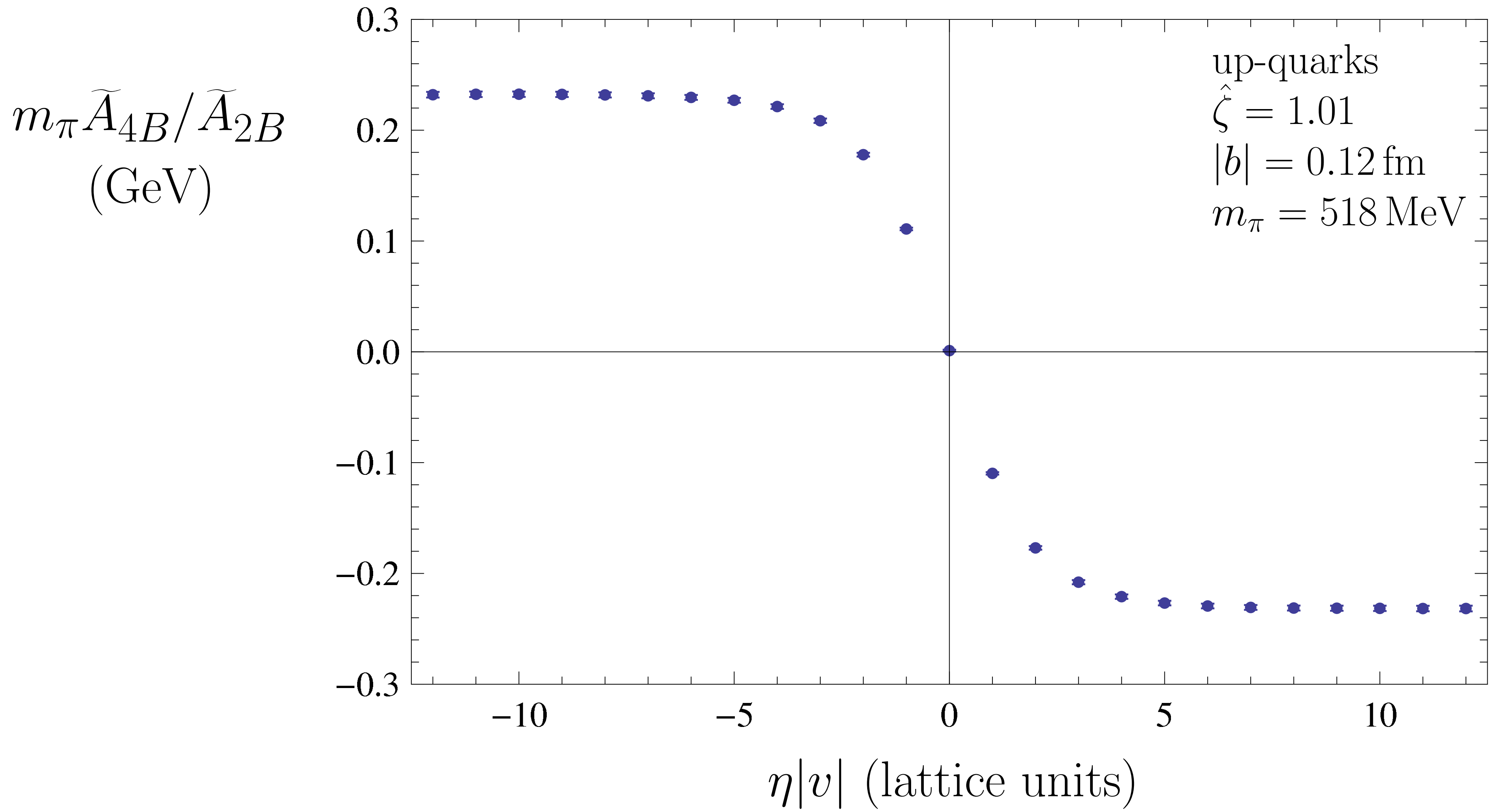
$$\langle k_y \rangle_{TU}(b_T^2, \ldots) = -m_H \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup

- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\hat{\zeta},\mu) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\eta v,\eta v+b,b]\ q(b)\ |P,S\rangle$
- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame
- Form desired ratios of $\widetilde{A}_i$ invariants
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically. Pion: largest $\hat{\zeta} = 2.03$
- Use MILC 2+1-flavor gauge ensemble with $a \approx 0.12\,\text{fm}$, $m_\pi = 518\,\text{MeV}$; $20^3 \times 64$
- Use variety of P, b, ηv; note $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)

Results: Boer-Mulders shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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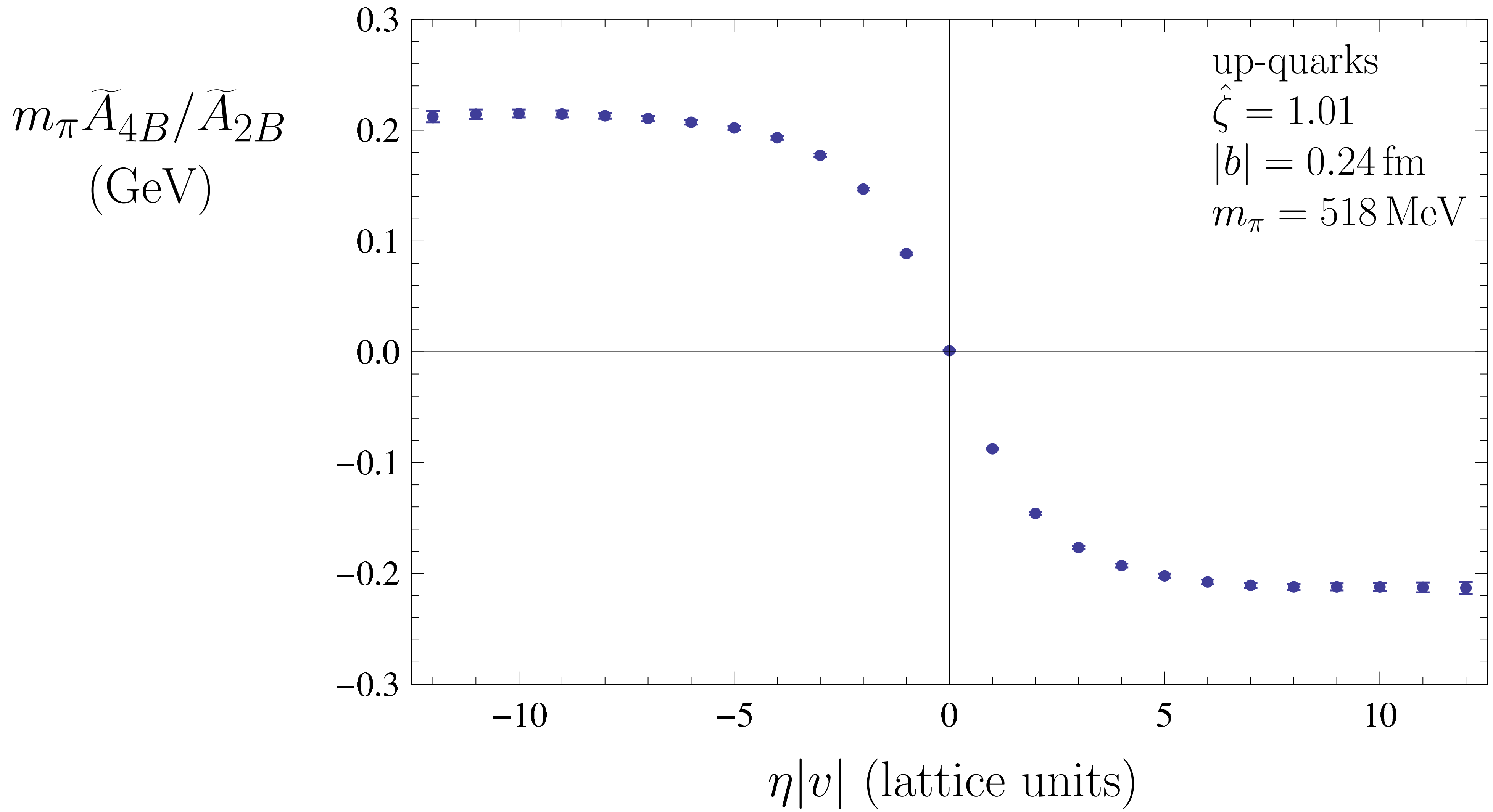

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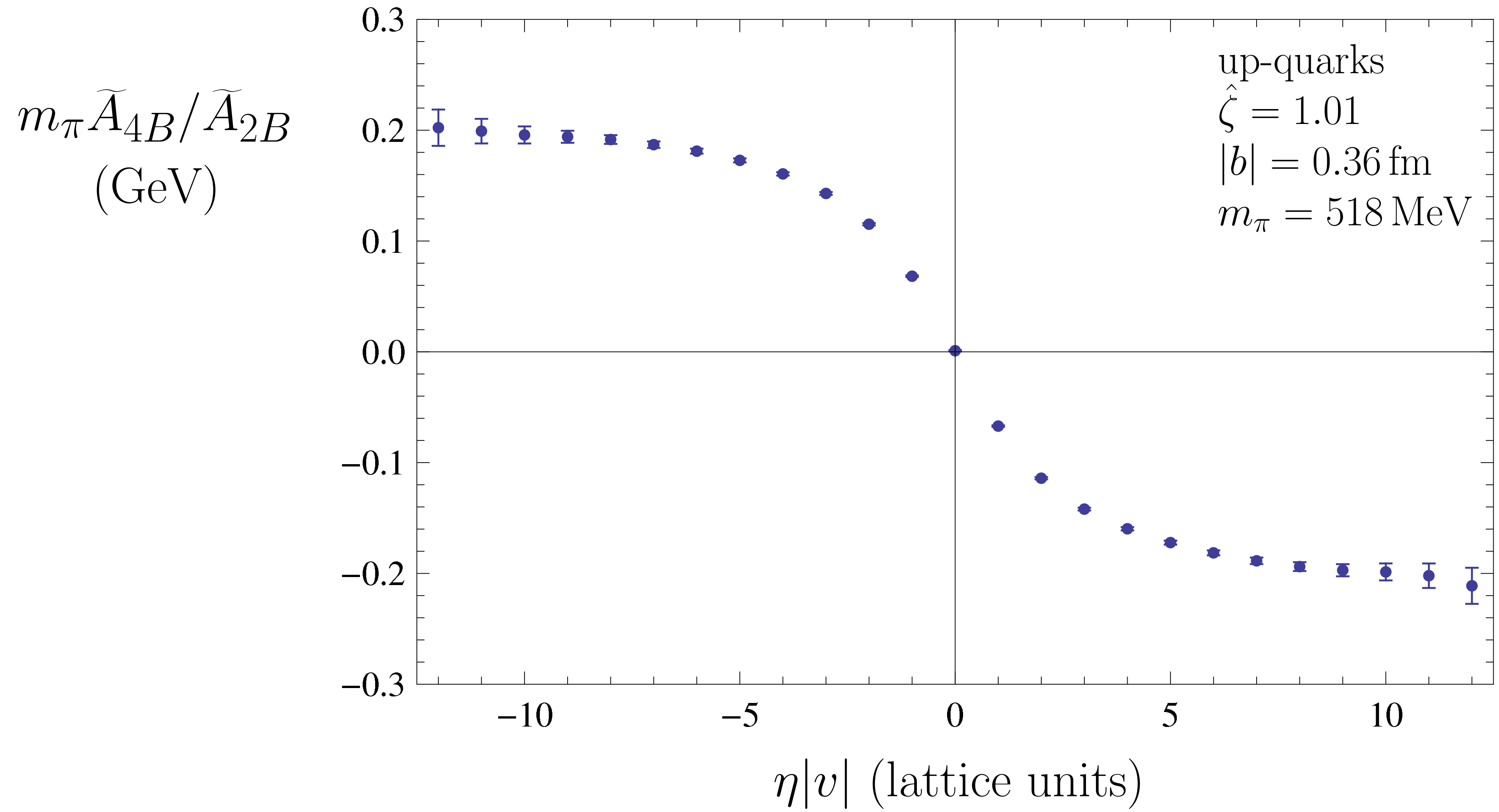

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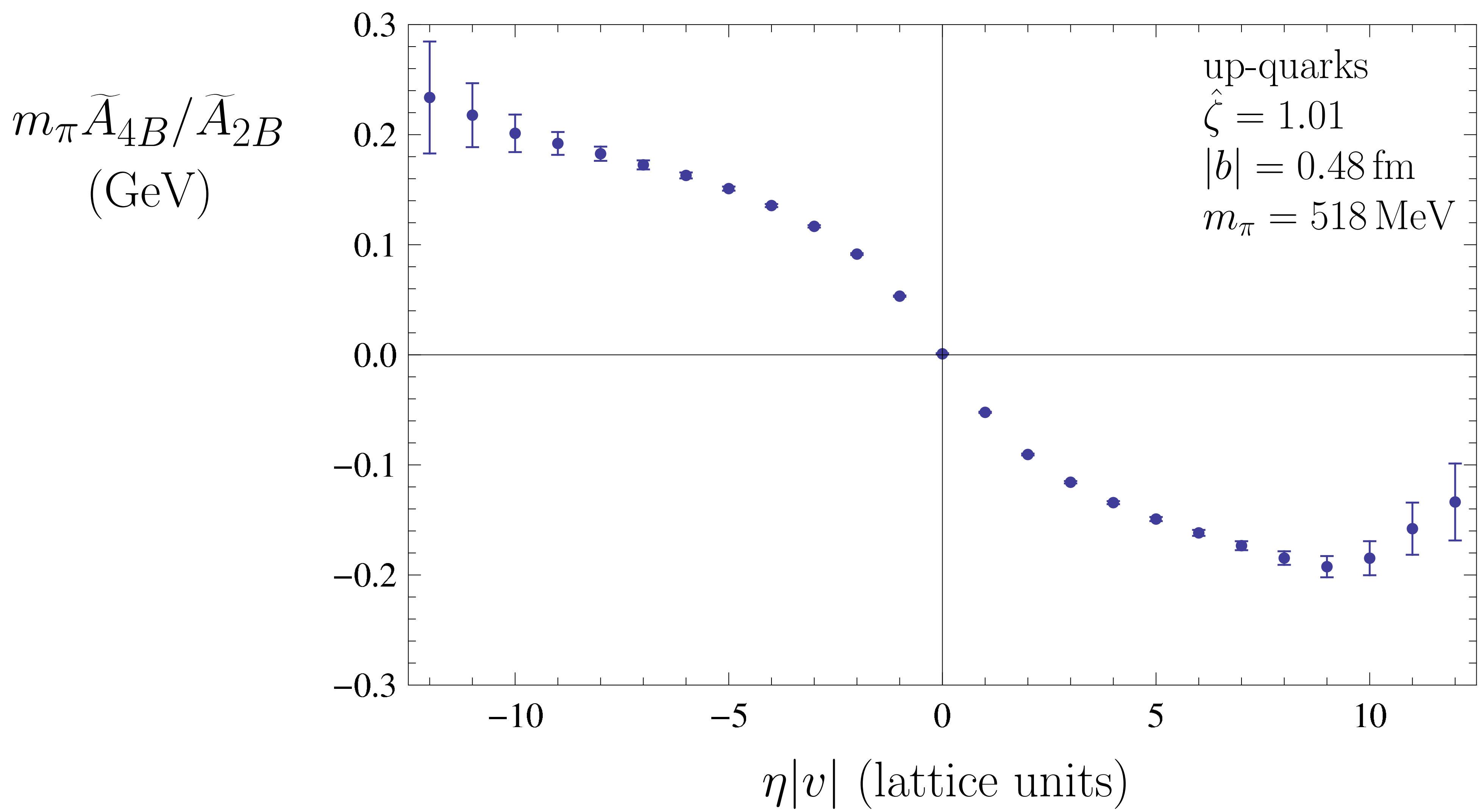
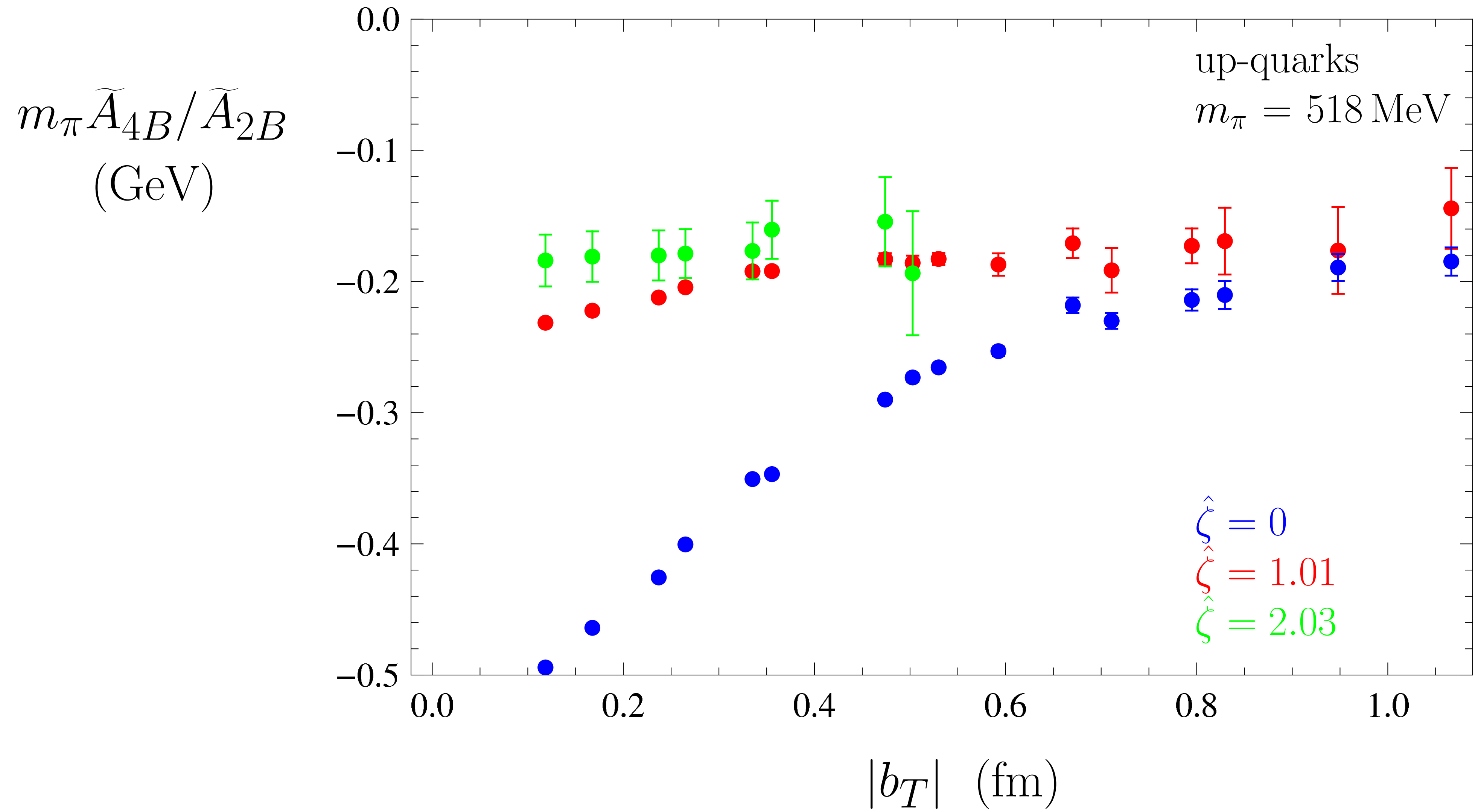
Dependence on staple extent; sequence of panels at different $|b_T|$



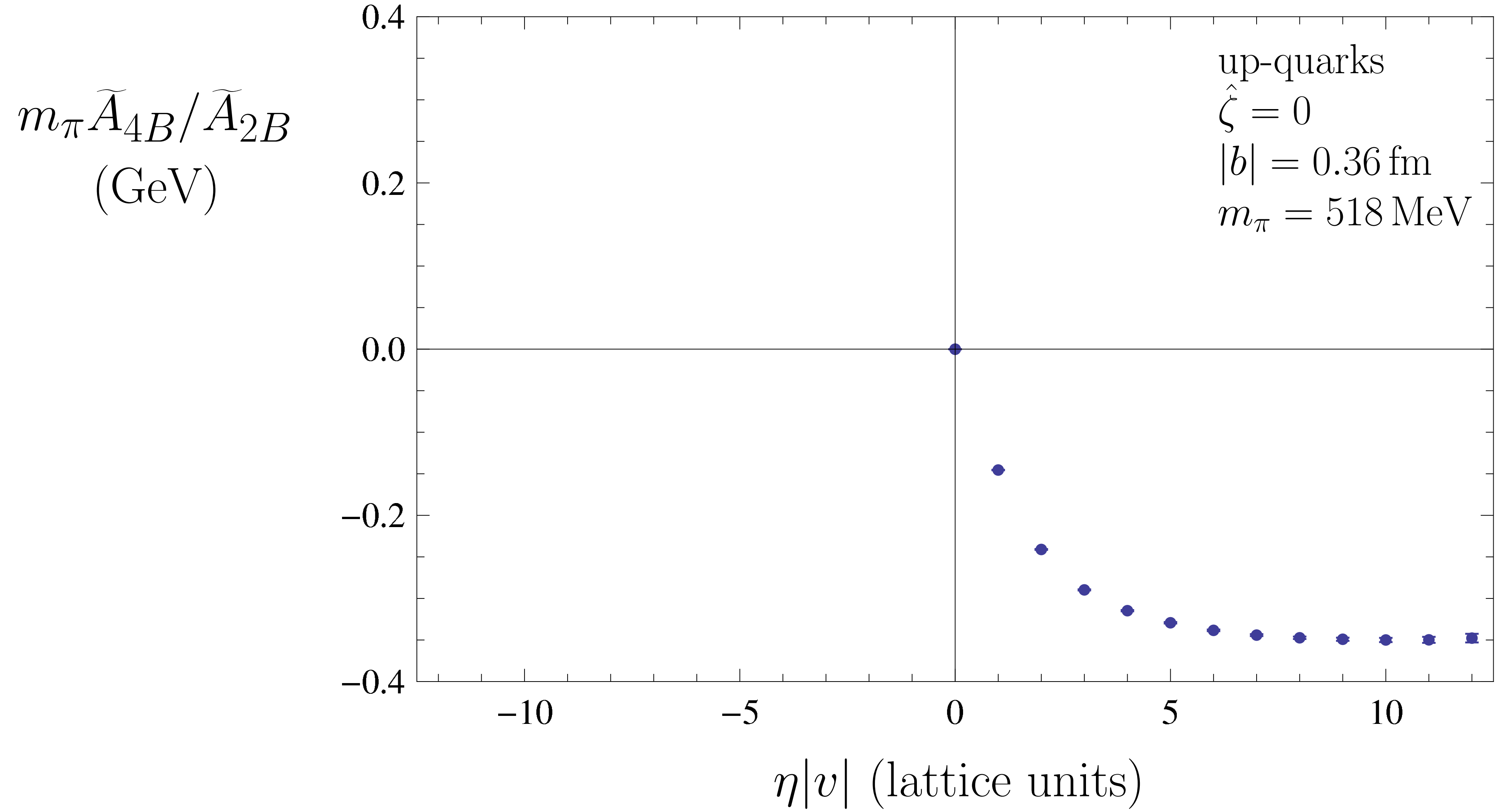
Results: Boer-Mulders shift

Dependence of SIDIS limit on $|b_T|$



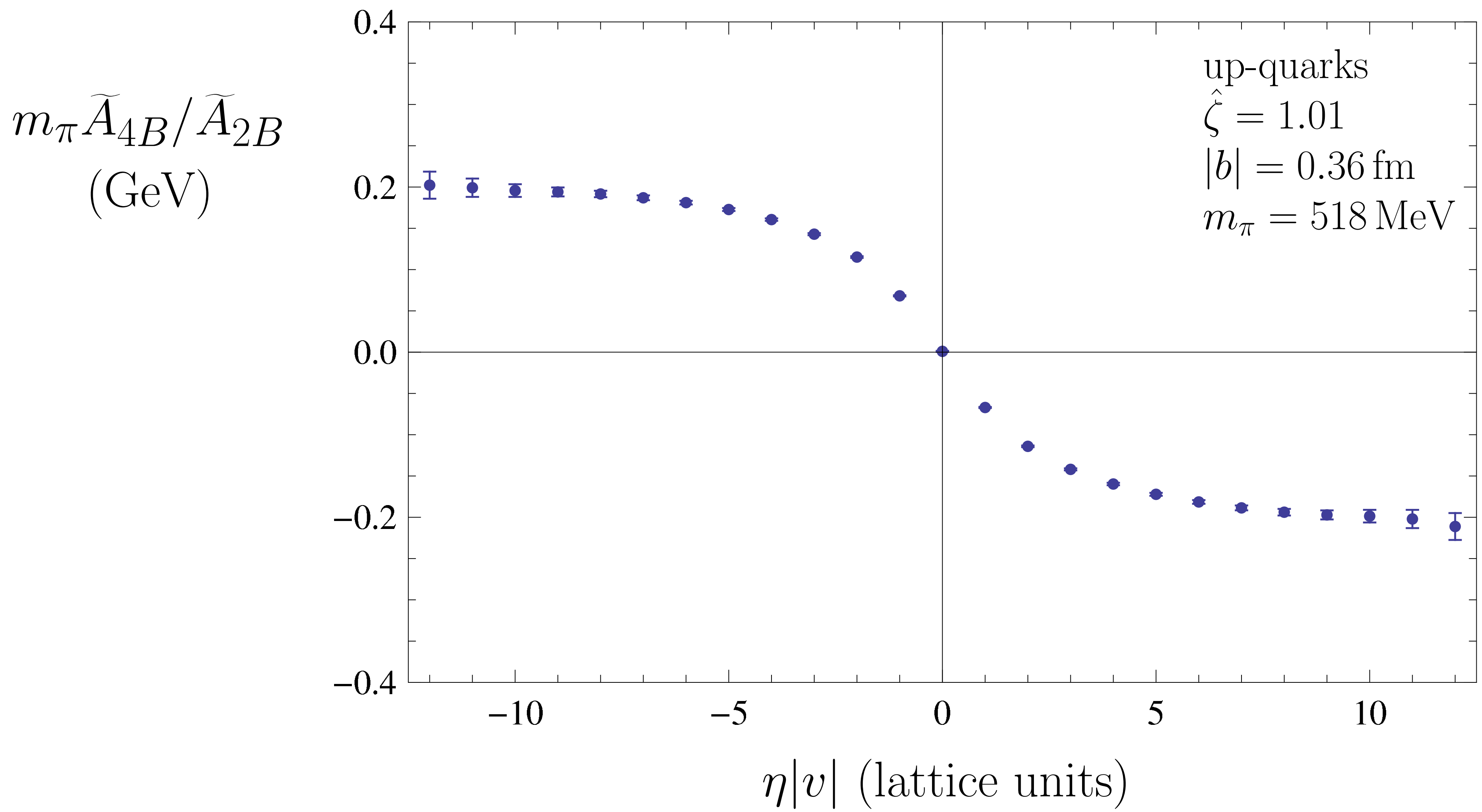
Results: Boer-Mulders shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



Results: Boer-Mulders shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



Results: Boer-Mulders shift

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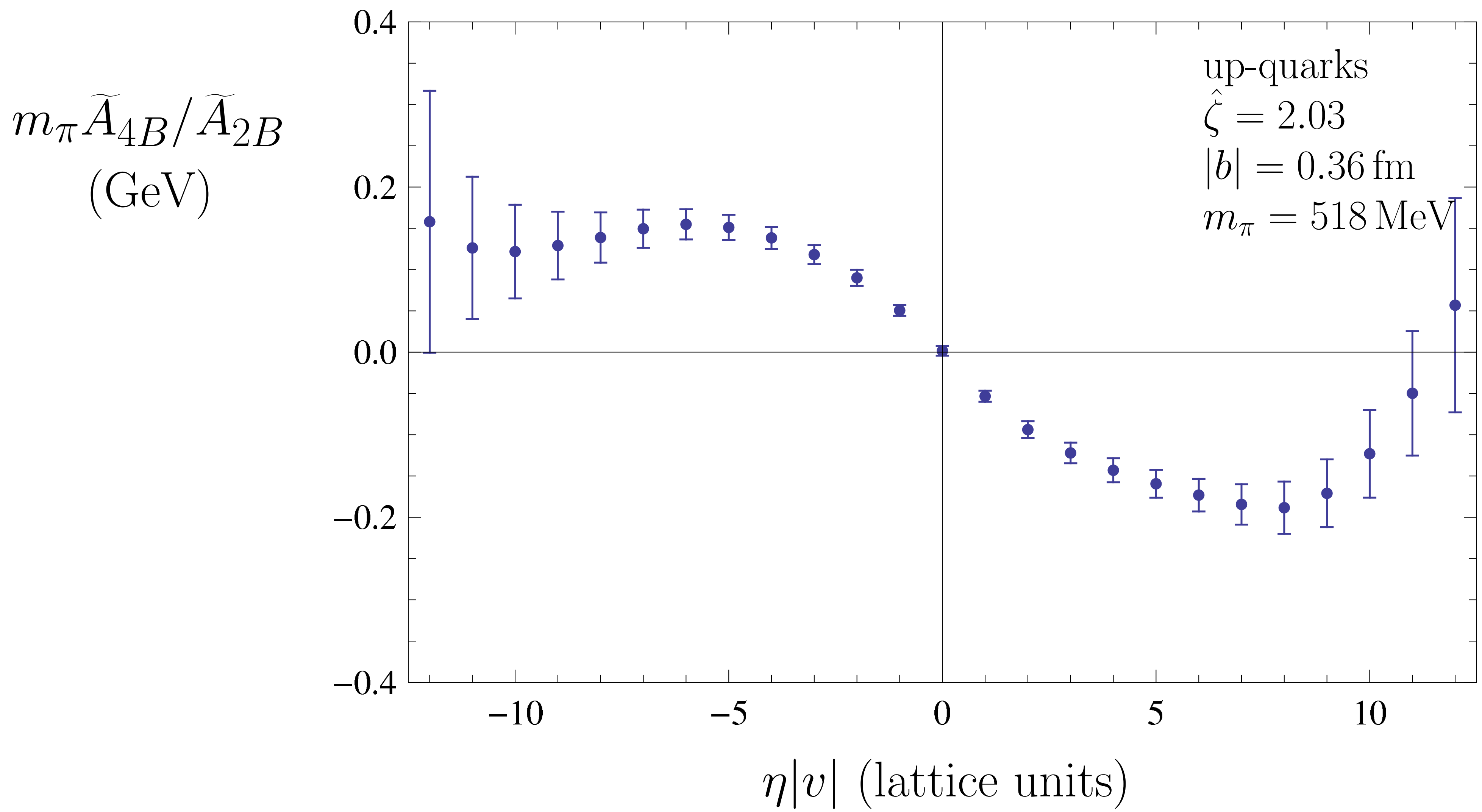
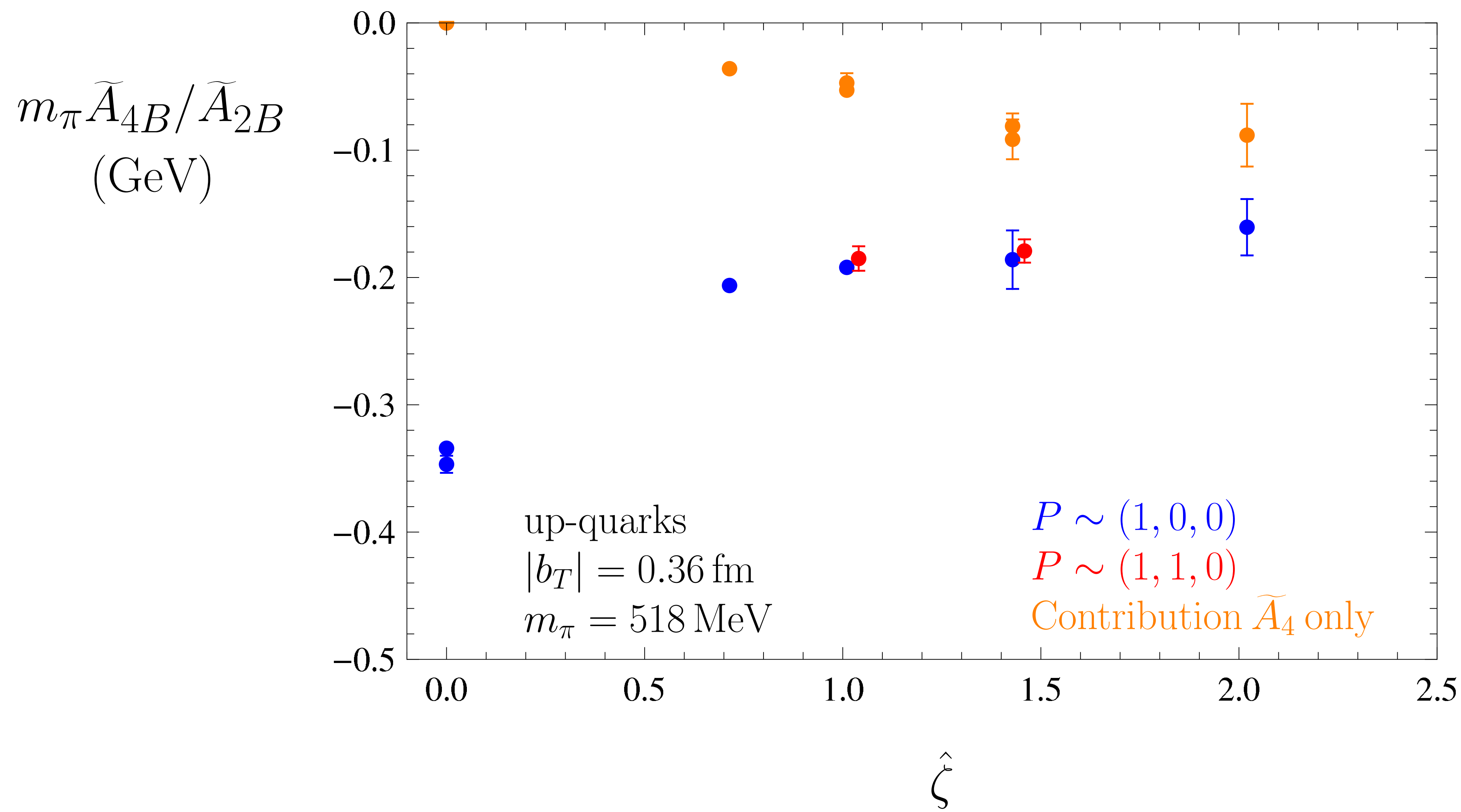


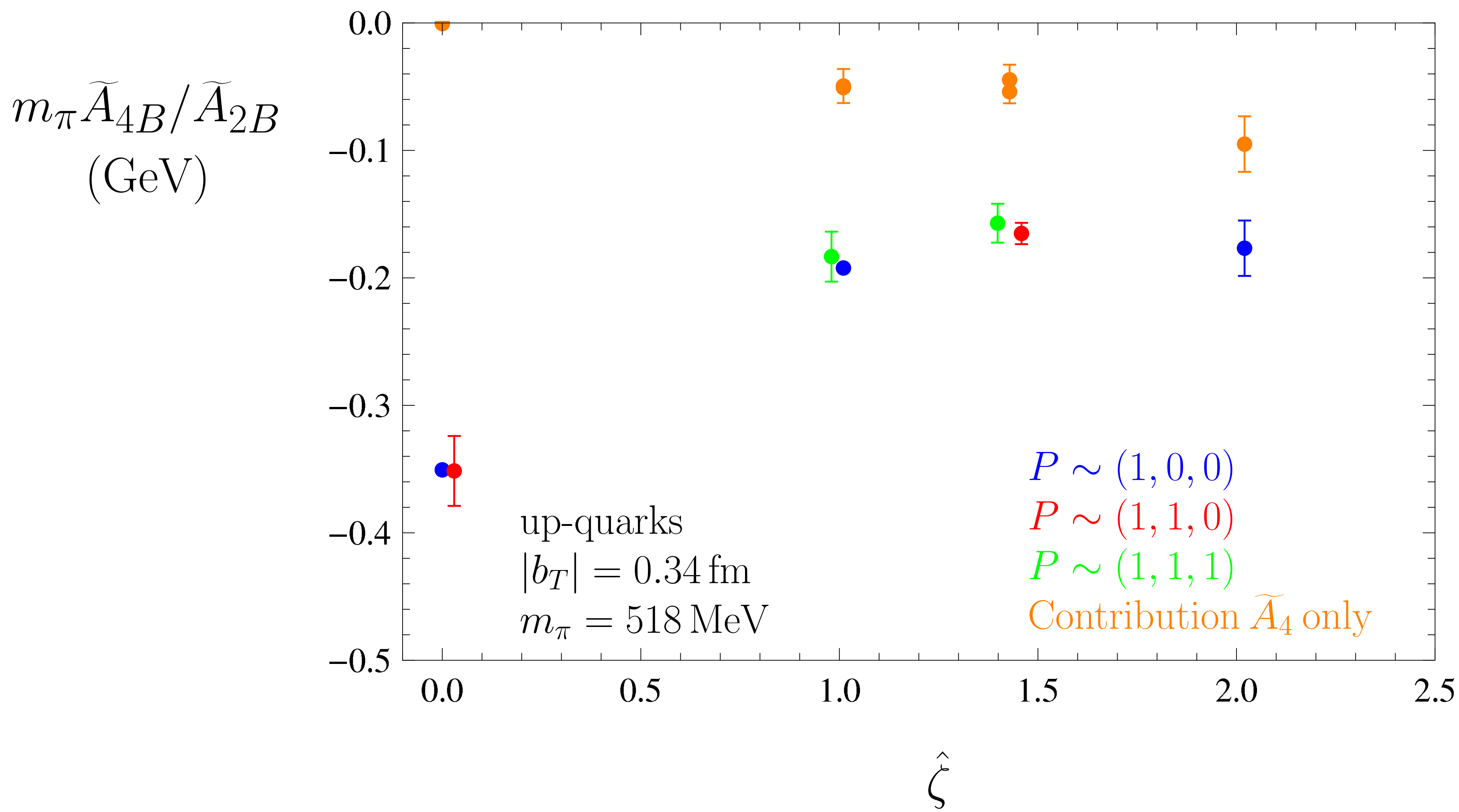
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



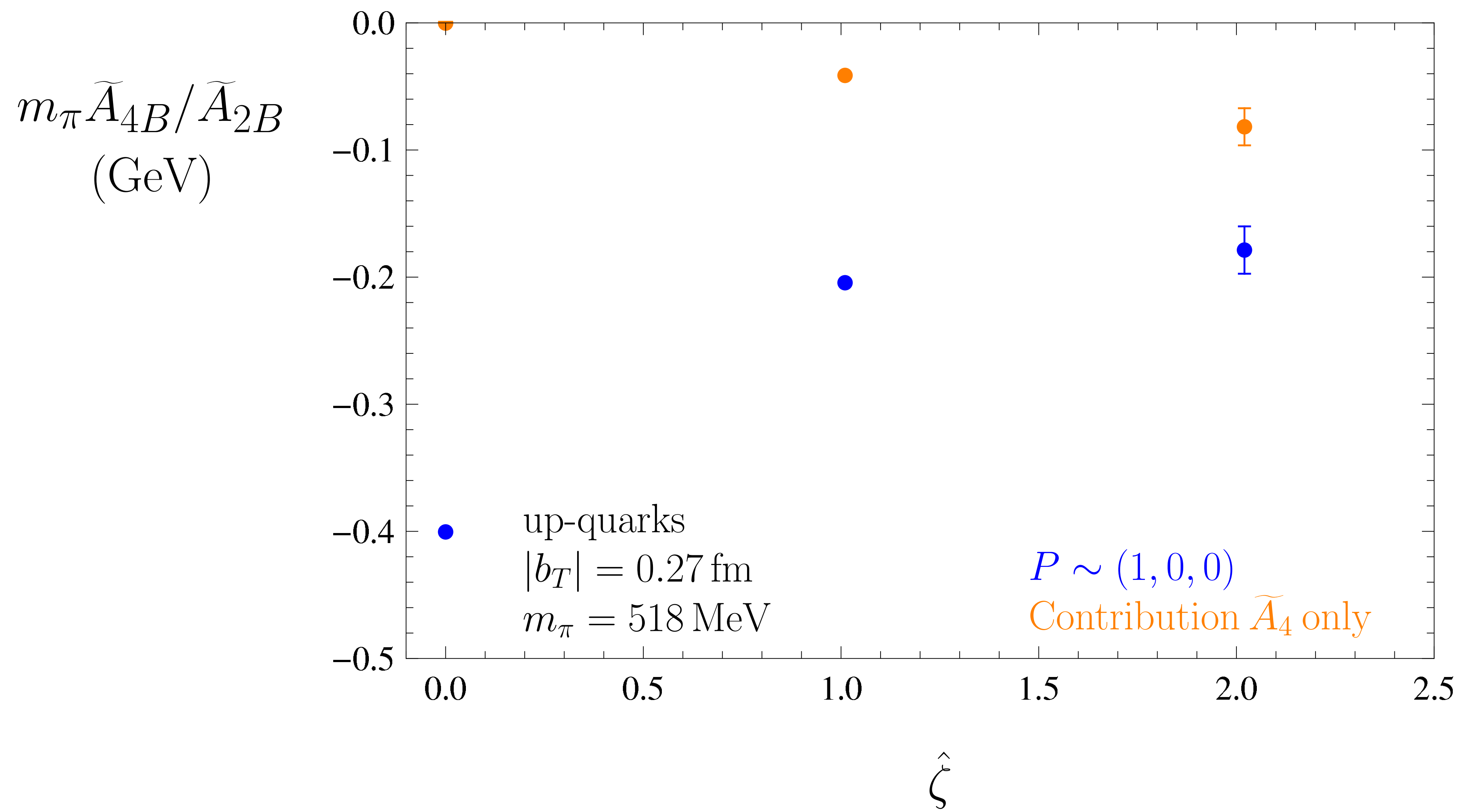
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$

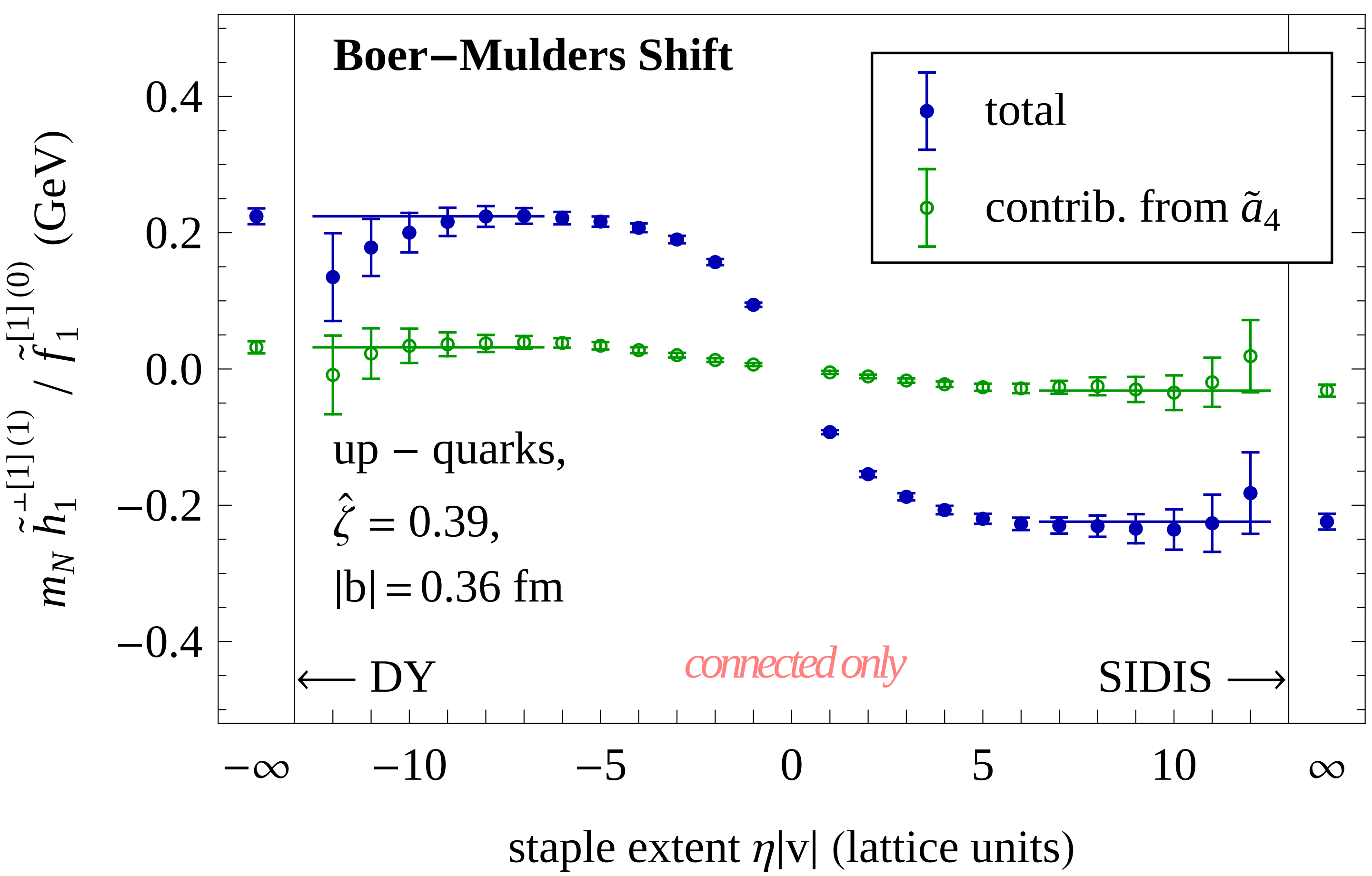


Results: Boer-Mulders shift

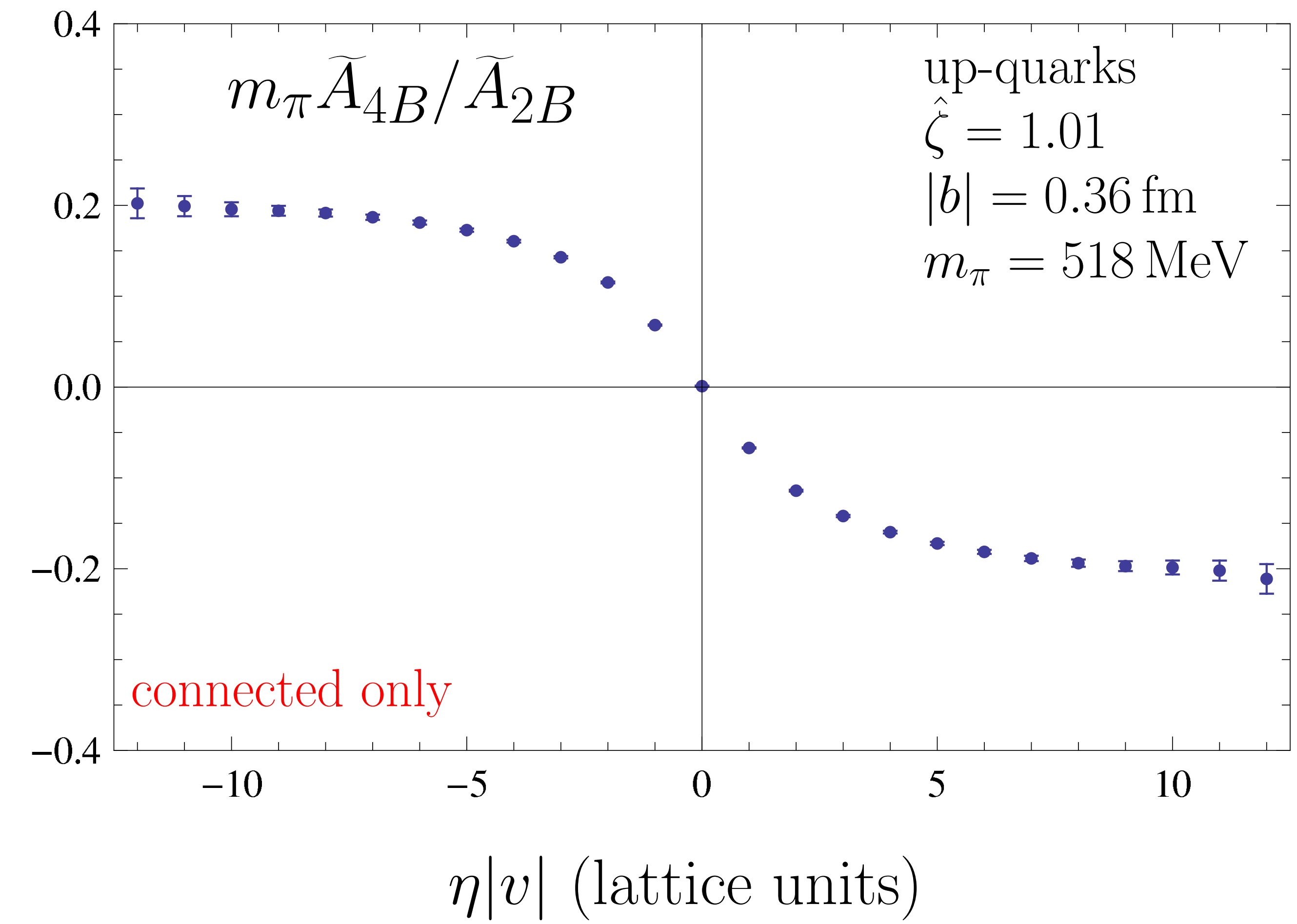
Dependence of SIDIS limit on $\hat{\zeta}$



Results: Boer-Mulders shift



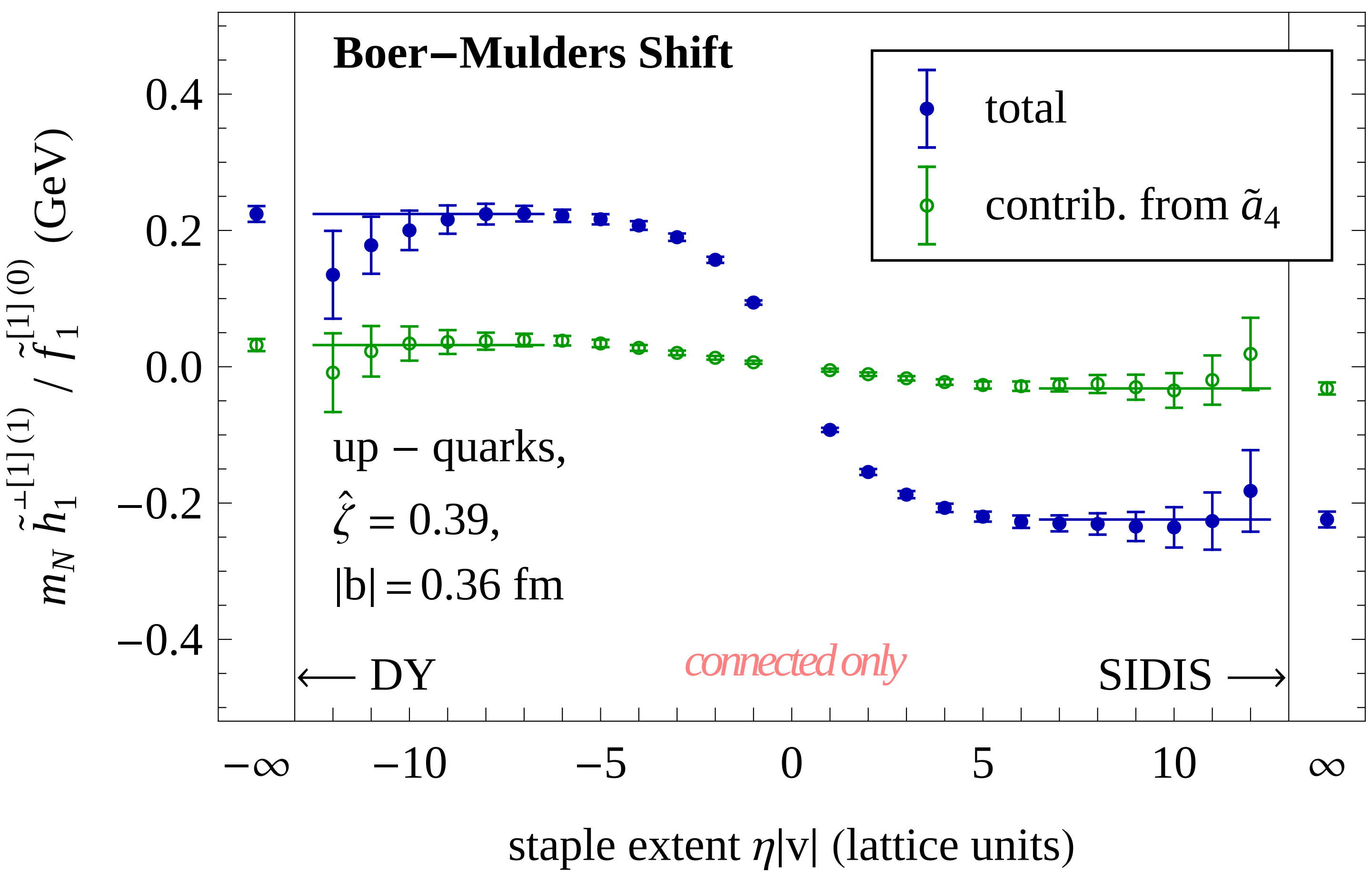
Nucleon

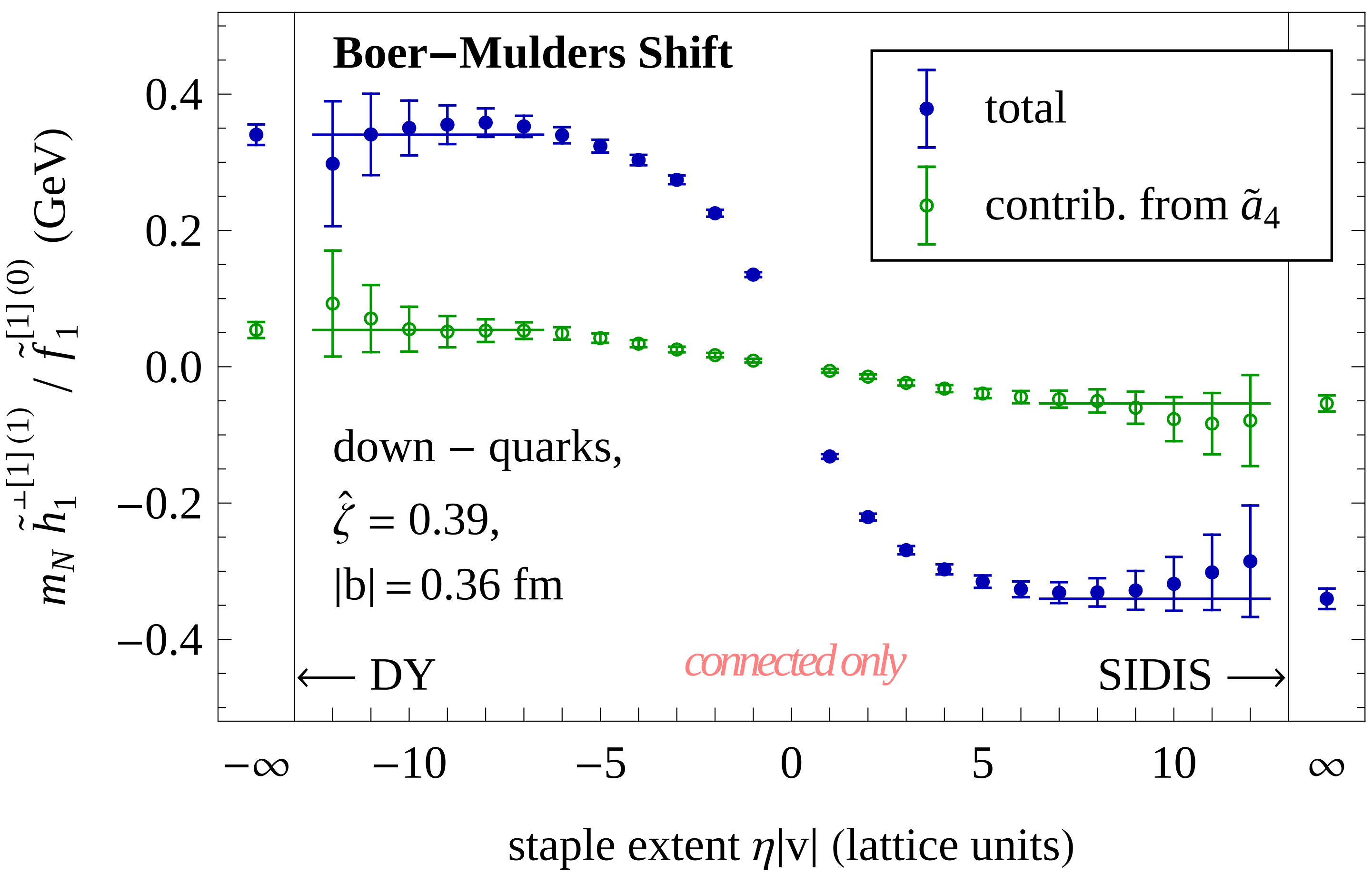


Pion

Results: Boer-Mulders shift

Nucleon; flavor separated





Conclusions

- Study of T-odd Boer-Mulders observable in the pion using staple-shaped gauge link structures.
- To avoid soft factors, multiplicative renormalization constants, constructed appropriate ratio of Fourier-transformed TMDs ("shift").
- v taken off light cone: Dependence on Collins-Soper parameter $\hat{\zeta}$. In addition to $\eta v \to \infty$, need to also consider $\hat{\zeta} \to \infty$.
- $\eta v \to \infty$ seems under good control; plateaux reached at moderate values.
- Significant progress concerning the $\hat{\zeta} \to \infty$ limit compared with earlier study in nucleon. Tentative statements concerning light cone limit possible. Perspective: Develop high momentum interpolating operators.
- Quantitative correspondence between u-quark Boer-Mulders ratios in proton, π^+ meson.