The Pion scalar radius from two-flavor Wilson Lattice QCD

Vera Gülpers

G. von Hippel H. Wittig

Helmholtz-Institut Mainz and Institut für Kernphysik
Johannes Gutenberg-Universität Mainz

July 29, 2013

HELMHOLTZ GEMEINSCHAFT
Helmholtz-Institut Mainz

LATTICE 13
JULY 29 – AUGUST 03 2013
MAINZ, GERMANY

JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

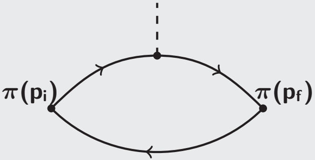
Outline

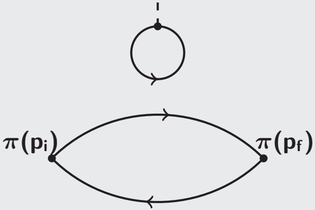
Introduction - The scalar Form Factor of the Pion

- describes the coupling of a charged pion to a scalar particle

$$\mathbf{F}_{\mathrm{S}}^{\pi}\left(\mathbf{Q}^2\right) \equiv \left\langle \pi^+(\mathbf{p_f}) \right| \mathbf{m_d \bar{d} d + m_u \bar{u} u} \left| \pi^+(\mathbf{p_i}) \right\rangle$$

with $\mathbf{Q^2 = -(p_f - p_i)^2}$





- disconnected loop $\sum_{\vec{\mathbf{x}}} \mathrm{Tr}\left(\mathbf{D}^{-1}(\mathbf{x},\mathbf{x})\right)$ requires all-to-all propagator
- stochastic sources and generalized hopping parameter expansion

Introduction - The scalar Radius of the Pion

- scalar radius

$$\langle r^2 \rangle_s^\pi = -\frac{6}{F_s^\pi(0)} \frac{\partial F_s^\pi(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$$

- depends only on $\bar{\ell}_4$ at NLO χPT
  [Gasser and Leutwyler, Phys. Lett. **B125**,325 (1983)]

$$\langle r^2 \rangle_s^\pi = \frac{1}{(4\pi F)^2}\left(-\frac{13}{2}\right) + \frac{6}{(4\pi F)^2}\left[\bar{\ell}_4 + \ln\left(\frac{m^2_{\pi,\mathrm{phys}}}{m^2_\pi}\right)\right]$$

$\rightarrow$ estimation of $\bar{\ell}_4$ alternative to the determination using f_K/f_π

- partially quenched χPT [Jüttner JHEP **1201**, 007 (2012)]
  $\rightarrow$ disconnected contribution to scalar radius not negligible

Outline

Calculation of disconnected loops

cf. [Bali et al. arXiv:0910.3970]

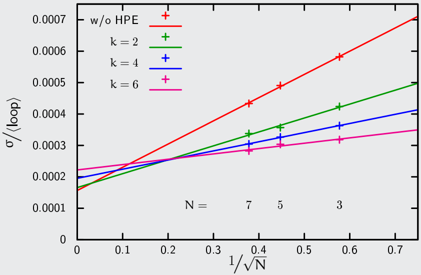
- $\mathcal{O}(\mathbf{a})$-improved Wilson-Dirac operator

$$\mathbf{D_{sw}} = \frac{1}{2\kappa}\mathbb{1} + \mathbf{c_{sw}B} - \frac{1}{2}\mathbf{H} \quad = \mathbf{A} - \frac{1}{2}\mathbf{H} \quad = \mathbf{A}\left(\mathbb{1} - \frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right)$$

- generalized hopping parameter expansion

$$\mathbf{D_{sw}^{-1}} = \sum_{i=0}^{k-1}\left(\frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right)^{i}\mathbf{A}^{-1} + \left(\frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right)^{k}\mathbf{D_{sw}^{-1}}$$

- $\mathbf{D_{sw}^{-1}}$ on the right hand side estimated using stochastic sources
- $\langle \mathrm{loop}\rangle = \left\langle \sum_{\vec{x}} \mathrm{Tr}\left(\mathbf{D}^{-1}(\mathbf{x},\mathbf{x})\right)\right\rangle_{\mathbf{G}}$
- choose $\mathbf{N = 3}$ sources with order $\mathbf{k = 6}$ of the generalized HPE

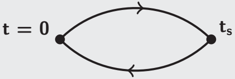


Extracting the form factor – 2pt and 3pt functions

- 2pt-function:

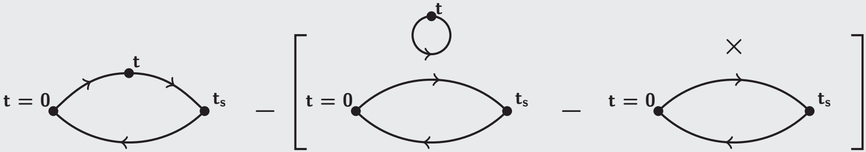
$$\mathbf{C_{2pt}(t_s, p) \sim \frac{Z_p^2}{2E_p}\left[e^{-t_sE_p} + e^{-(T-t_s)E_p}\right]}$$

with $\mathbf{Z_p^2} = |\langle \pi(\mathbf{p})| \, \phi(\mathbf{0}) \, |\mathbf{0}\rangle|^2$



- 3pt-function with subtracted vacuum $(\mathbf{0} < \mathbf{t} < \mathbf{t_s})$

$$\mathbf{C_{3pt}(t, t_s, p_i, p_f) \sim \frac{Z_{p_i}Z_{p_f}}{4E_{p_i}E_{p_f}} \langle \pi(p_f)| \, \mathcal{O}_S \, |\pi(p_i)\rangle \, e^{-(t_s-t)E_{p_f}} e^{-tE_{p_i}}}$$

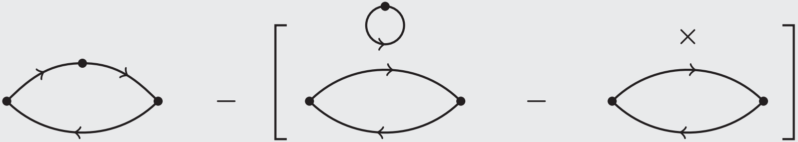


A few words on Renormalization

- chiral symmetry explicitly broken by Wilson fermions
- multiplicative and additive renormalization for the scalar operator

$$\left\langle \mathcal{O}^{\mathbf{R}} \right\rangle = \mathbf{Z_s} \left\langle \mathcal{O} - \mathbf{b_0} \right\rangle$$

- additive renormalization is canceled when subtracting the vacuum



- for all form factor data shown in this talk the multiplicative renormalization is not taken into account
- scalar radius independent of $\mathbf{Z_s}$

Extracting the form factor – Ratios I

- appropriate ratios of three- and two-point functions
  cf. [Boyle et al. JHEP **0705**, 016]

$$R_1(t, t_s, p_i, p_f) = \sqrt{\frac{C_{3pt}(t, t_s, p_i, p_f) C_{3pt}(t, t_s, p_f, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t_s, p_f)}}$$

$$\sim \frac{\langle \pi(p_f) | \mathcal{O}_S | \pi(p_i) \rangle}{2\sqrt{E_{p_i} E_{p_f}}} \sqrt{\frac{e^{-E_{p_i} t_s} e^{-E_{p_f} t_s}}{(e^{-E_{p_i} t_s} + e^{-E_{p_i}(T - t_s)}) \cdot (e^{-E_{p_f} t_s} + e^{-E_{p_f}(T - t_s)})}}$$

- all factors of Z_p cancel
- t-dependence is canceled
- remaining t_s-dependence parameter-free since E_p are known from two-point functions

Extracting the form factor – Ratios II

- appropriate ratios of three- and two-point functions
  cf. [Boyle et al. JHEP **0705**, 016]

$$R_3(t, t_s, p_i, p_f) = \frac{C_{3pt}(t, t_s, p_i, p_f)}{C_{2pt}(t_s, p_f)} \sqrt{\frac{C_{2pt}(t_s, p_f) C_{2pt}(t, p_f) C_{2pt}(t_s - t, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t, p_i) C_{2pt}(t_s - t, p_f)}}$$

$$\sim \frac{\langle \pi(p_f) | \mathcal{O}_S | \pi(p_i) \rangle}{2\sqrt{E_{p_i} E_{p_f}}} f(t, t_s)$$

- remaining t- and t_s-dependence $f(t, t_s)$ parameter-free
- $f(t, t_s) \to 1$ for $0 \ll t \ll t_s \ll T/2$
- the factors Z_p only cancel if the same type of source is used at pion source and sink
  $\to R_3$ can not be used for smeared-local correlators

Ensembles

- $\mathcal{O}(\mathbf{a})$-improved Wilson fermions with $\mathbf{N_f = 2}$ dynamical quarks
- overview over the CLS ensembles used

β	$\mathbf{a}$[fm]	lattice	$\mathbf{m_\pi}$[MeV]	$\mathbf{m_\pi L}$	κ	Label	Statistics
5.3	**0.063**	$\mathbf{64 \times 32^3}$	**650**	**6.6**	**0.13605**	E3	**156**
5.3	**0.063**	$\mathbf{64 \times 32^3}$	**605**	**6.2**	**0.13610**	E4	**162**
5.3	**0.063**	$\mathbf{64 \times 32^3}$	**455**	**4.7**	**0.13625**	E5	**1000**
5.3	**0.063**	$\mathbf{96 \times 48^3}$	**325**	**5.0**	**0.13635**	F6	**300**
5.3	**0.063**	$\mathbf{96 \times 48^3}$	**280**	**4.3**	**0.13638**	F7	**351**

- one lattice spacing $\mathbf{a = 0.063}$ fm
- all ensembles fulfill $\mathbf{m_\pi L \geq 4}$

Ensembles

- $\mathcal{O}(a)$-improved Wilson fermions with $\mathbf{N_f = 2}$ dynamical quarks
- overview over the CLS ensembles used

β	a[fm]	lattice	m_π[MeV]	$m_\pi L$	κ	Label	Statistics
5.3	0.063	64×32^3	650	6.6	0.13605	E3	156
5.3	0.063	64×32^3	605	6.2	0.13610	E4	162
5.3	0.063	64×32^3	455	4.7	0.13625	E5	1000
5.3	0.063	96×48^3	325	5.0	0.13635	F6	300
5.3	0.063	96×48^3	280	4.3	0.13638	F7	351

- one lattice spacing $\mathbf{a = 0.063}$ fm
- all ensembles fulfill $\mathbf{m_\pi L \geq 4}$

Outline

vanishing momentum transfer $Q^2 = 0$

$$R \equiv R_1(t, t_s, 0, 0) = R_3(t, t_s, 0, 0) = \frac{C_{3pt}(t, t_s, 0, 0)}{C_{2pt}(t_s, 0)}$$

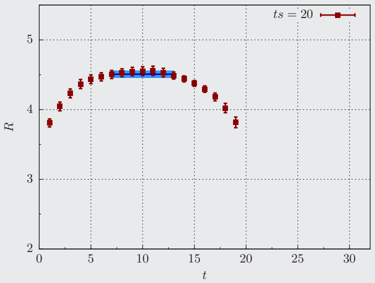
$$\sim \frac{\langle \pi(0)| \mathcal{O}_S |\pi(0)\rangle}{2m_\pi} \underbrace{\frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}}}_{= f(t_s)}$$

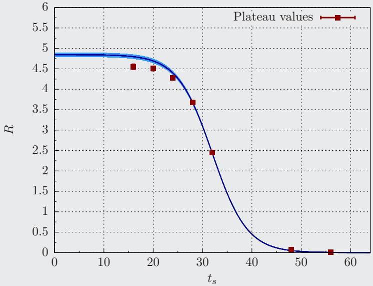
vanishing momentum transfer $Q^2 = 0$

$$R \equiv R_1(t, t_s, 0, 0) = R_3(t, t_s, 0, 0) = \frac{C_{3pt}(t, t_s, 0, 0)}{C_{2pt}(t_s, 0)}$$

$$\sim \frac{\langle \pi(0) | \mathcal{O}_S | \pi(0) \rangle}{2m_\pi} \underbrace{\frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}}}_{=f(t_s)}$$

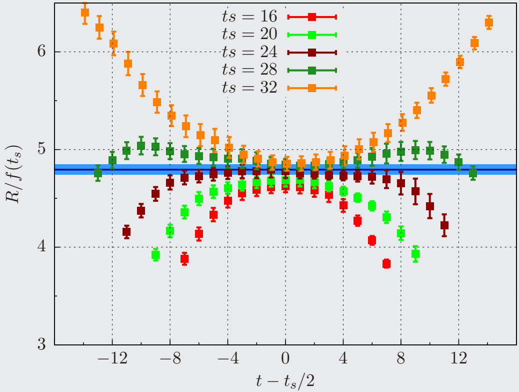
connected $Q^2 = 0$





connected $\mathbf{Q^2 = 0}$

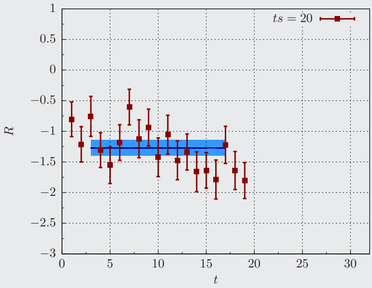
- divide out known $\mathbf{t_s}$-dependence
- excited state contributions for small $\mathbf{t_s}$
- global fit to $\mathbf{t_s \geq 24}$

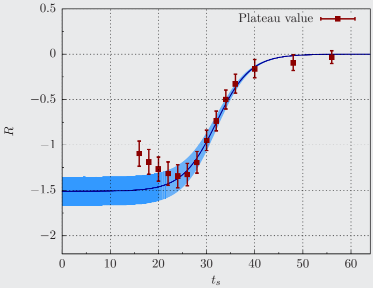


vanishing momentum transfer $Q^2 = 0$

$$R = \frac{C_{3pt}(t, t_s, 0, 0)}{C_{2pt}(t_s, 0)} \sim \frac{\langle \pi(0) | \mathcal{O}_S | \pi(0) \rangle}{2m_\pi} \underbrace{\frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}}}_{= f(t_s)}$$

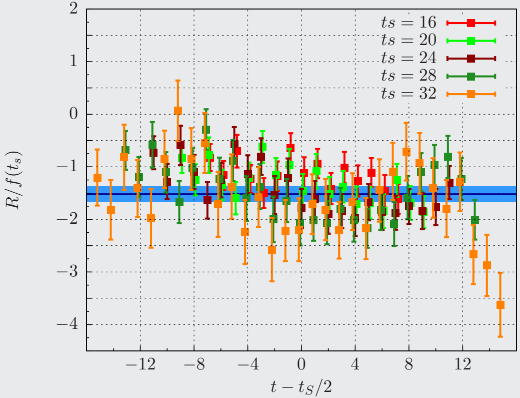
disconnected $Q^2 = 0$





disconnected $Q^2 = 0$

- divide out known t_s-dependence
- excited state contributions for small t_s
- global fit to $t_s \geq 24$



non-vanishing momentum transfer - connected contribution

- only smeared-local data available so far
  $\rightarrow$ use $\mathbf{R_1}$

$$R_1(t, t_s, p_i, p_f) = \sqrt{\frac{C_{3pt}(t, t_s, p_i, p_f) C_{3pt}(t, t_s, p_f, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t_s, p_f)}}$$

$$\sim \frac{\langle \pi(p_f) | \mathcal{O}_S | \pi(p_i) \rangle}{2\sqrt{E_{p_i} E_{p_f}}} \sqrt{\frac{e^{-E_{p_i} t_s} e^{-E_{p_f} t_s}}{(e^{-E_{p_i} t_s} + e^{-E_{p_i}(T-t_s)}) \cdot (e^{-E_{p_f} t_s} + e^{-E_{p_f}(T-t_s)})}}$$

non-vanishing momentum transfer - connected contribution

- only smeared-local data available so far
  $\rightarrow$ use $\mathbf{R_1}$

$$\mathbf{R_1(t, t_s, p_i, p_f) = \sqrt{\frac{C_{3pt}(t, t_s, p_i, p_f) C_{3pt}(t, t_s, p_f, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t_s, p_f)}}}$$

$$\sim \frac{\langle \pi(\mathbf{p_f}) | \mathcal{O}_S | \pi(\mathbf{p_i}) \rangle}{2\sqrt{E_{p_i} E_{p_f}}} \sqrt{\frac{e^{-E_{p_i} t_s} e^{-E_{p_f} t_s}}{(e^{-E_{p_i} t_s} + e^{-E_{p_i}(T - t_s)}) \cdot (e^{-E_{p_f} t_s} + e^{-E_{p_f}(T - t_s)})}}$$

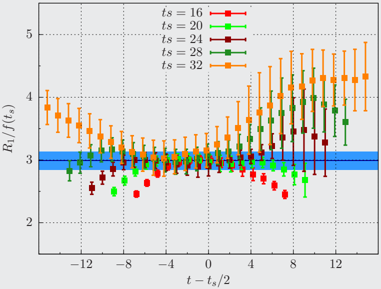
- momentum insertion via Fourier transformation
- momentum insertion at the operator:

$$\vec{\mathbf{q}} = (0, 0, 1) \frac{2\pi}{\mathbf{L}}$$

- momentum transfer:

$$\mathbf{Q^2 = -q^2 = 0.278}\,\mathrm{GeV}^2$$

- global fit to $\mathbf{t_s \geq 24}$



non-vanishing momentum transfer - disconnected contribution

- use smeared-smeared pion two-point functions
  $\rightarrow$ $\mathbf{R_1}$ or $\mathbf{R_3}$ can be used
- $\mathbf{R_3}$ gives a much cleaner signal

$$R_3(t, t_s, p_i, p_f) = \frac{C_{3pt}(t, t_s, p_i, p_f)}{C_{2pt}(t_s, p_f)} \sqrt{\frac{C_{2pt}(t_s, p_f) C_{2pt}(t, p_f) C_{2pt}(t_s - t, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t, p_i) C_{2pt}(t_s - t, p_f)}}$$

$$\sim \frac{\langle \pi(p_f) | \mathcal{O}_S | \pi(p_i) \rangle}{2\sqrt{E_\pi(p_i) E_\pi(p_f)}} f(t, t_s)$$

non-vanishing momentum transfer - disconnected contribution

- use smeared-smeared pion two-point functions
  $\rightarrow$ $\mathbf{R_1}$ or $\mathbf{R_3}$ can be used
- $\mathbf{R_3}$ gives a much cleaner signal

$$R_3(t, t_s, p_i, p_f) = \frac{C_{3pt}(t, t_s, p_i, p_f)}{C_{2pt}(t_s, p_f)} \sqrt{\frac{C_{2pt}(t_s, p_f) C_{2pt}(t, p_f) C_{2pt}(t_s - t, p_i)}{C_{2pt}(t_s, p_i) C_{2pt}(t, p_i) C_{2pt}(t_s - t, p_f)}}$$

$$\sim \frac{\langle \pi(p_f) | \mathcal{O}_S | \pi(p_i) \rangle}{2\sqrt{E_\pi(p_i) E_\pi(p_f)}} f(t, t_s)$$

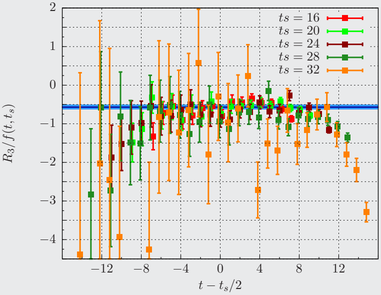
- momentum insertion via Fourier transformation
- momentum insertion at the operator:

$$\vec{q} = (0, 0, 1) \frac{2\pi}{L}$$

- momentum transfer:

$$Q^2 = -q^2 = 0.278\,\text{GeV}^2$$

- global fit to $\mathbf{t_s} \geq \mathbf{24}$



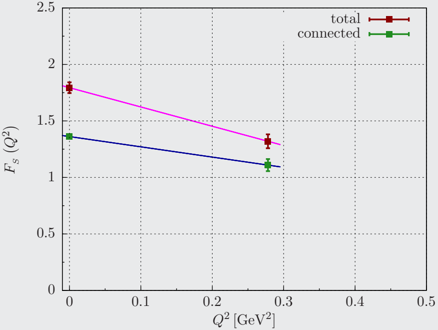
scalar radius

- $\mathbf{Q^2}$ - dependence of the form factor at $\mathbf{Q^2 = 0}$

$$\langle r^2 \rangle_s^\pi = -\frac{6}{F_s^\pi(0)} \frac{\partial F_s^\pi(Q^2)}{\partial Q^2}\Big|_{Q^2=0}$$

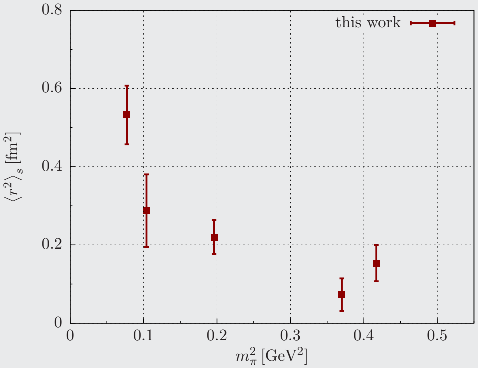
- two momentum transfers

$$F_s^\pi(Q^2) = F_s^\pi(0)\left(1 - \frac{1}{6}\langle r^2 \rangle_s^\pi Q^2 + \mathcal{O}(Q^4)\right)$$

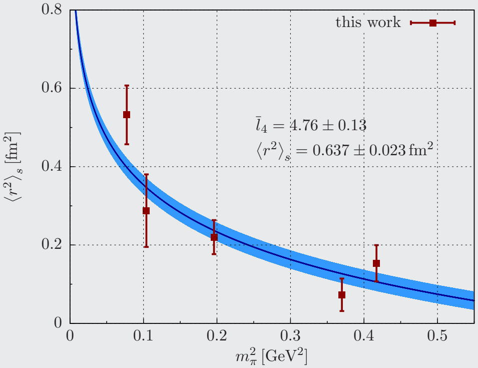


Outline

m_π^2-dependence

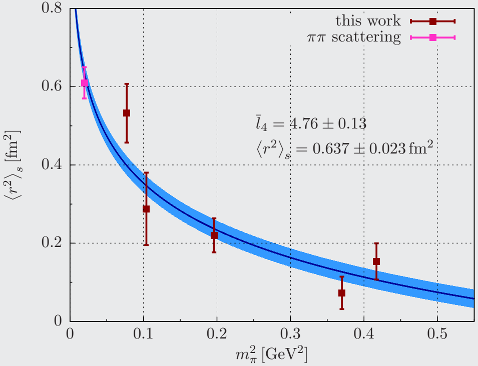


m_π^2-dependence



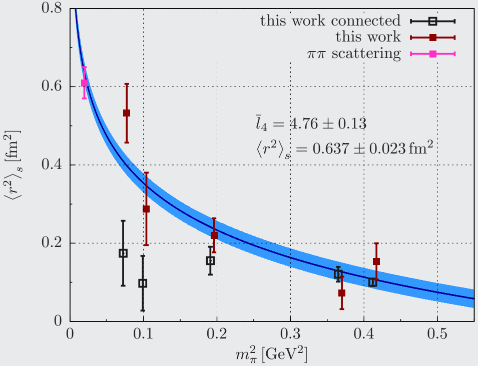
- NLO χPT: $\langle \mathbf{r^2} \rangle_s^\pi = \frac{1}{(4\pi F)^2}\left(-\frac{13}{2}\right) + \frac{6}{(4\pi F)^2}\left[\bar{\ell}_4 + \ln\left(\frac{m^2_{\pi,\mathrm{phys}}}{m^2_\pi}\right)\right]$

m_π^2-dependence



- NLO χPT: $\langle \mathbf{r}^2 \rangle_{\mathrm{s}}^{\pi} = \frac{1}{(4\pi F)^2}\left(-\frac{13}{2}\right) + \frac{6}{(4\pi F)^2}\left[\bar{\ell}_4 + \ln\left(\frac{m^2_{\pi,\mathrm{phys}}}{m^2_\pi}\right)\right]$
- $\pi\pi$-scattering: Colangelo et al. Nucl. Phys. **B603**, 125 (2001)

m_π^2-dependence



- NLO χPT: $\langle \mathbf{r^2} \rangle_s^\pi = \frac{1}{(4\pi F)^2}\left(-\frac{13}{2}\right) + \frac{6}{(4\pi F)^2}\left[\bar{\ell}_4 + \ln\left(\frac{m_{\pi,\mathrm{phys}}^2}{m_\pi^2}\right)\right]$
- $\pi\pi$-scattering: Colangelo et al. Nucl. Phys. **B603**, 125 (2001)
- disconnected not negligible (χPT: Jüttner JHEP **1201**, 007 (2012))
- disconnected required for expected behaviour

Outline

Conclusion

- disconnected contribution can be calculated precisely using the generalised hopping parameter expansion
- disconnected contribution to the scalar radius not negligible
  $\rightarrow$ required to obtain behaviour from NLO χPT
- extract low-energy constant $\bar{\ell}_4$
- scalar radius at physical pion mass in agreement with value from $\pi\pi$-scattering

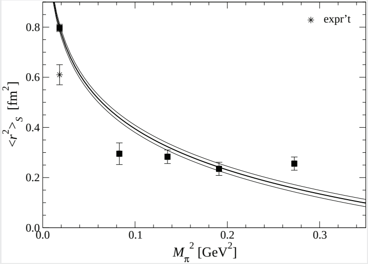
Outlook

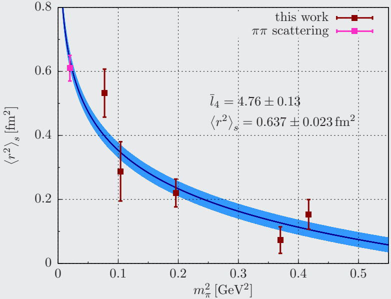
- smaller pion masses
- other lattice spacings
- study of systematic errors
- NNLO χPT (combined with vector form factor)

Outline

comparison with JLQCD/TWQCD

[Aoki et al. Phys. Rev. **D80**, 034508 (2009)]





- overlap fermions
- $\mathbf{a = 0.1184}$ fm
- $\mathbf{T \times L^3 = 32 \times 16^3}$
- two lightest ensembles $\mathbf{m_\pi L < 4}$

- $\mathcal{O}(\mathbf{a})$-improved Wilson fermions
- $\mathbf{a = 0.063}$ fm
- $\mathbf{T \times L^3 = 64 \times 32^3}$ and $\mathbf{96 \times 48^3}$
- all ensembles $\mathbf{m_\pi L \geq 4}$