Pion electromagnetic form factor from full Lattice QCD

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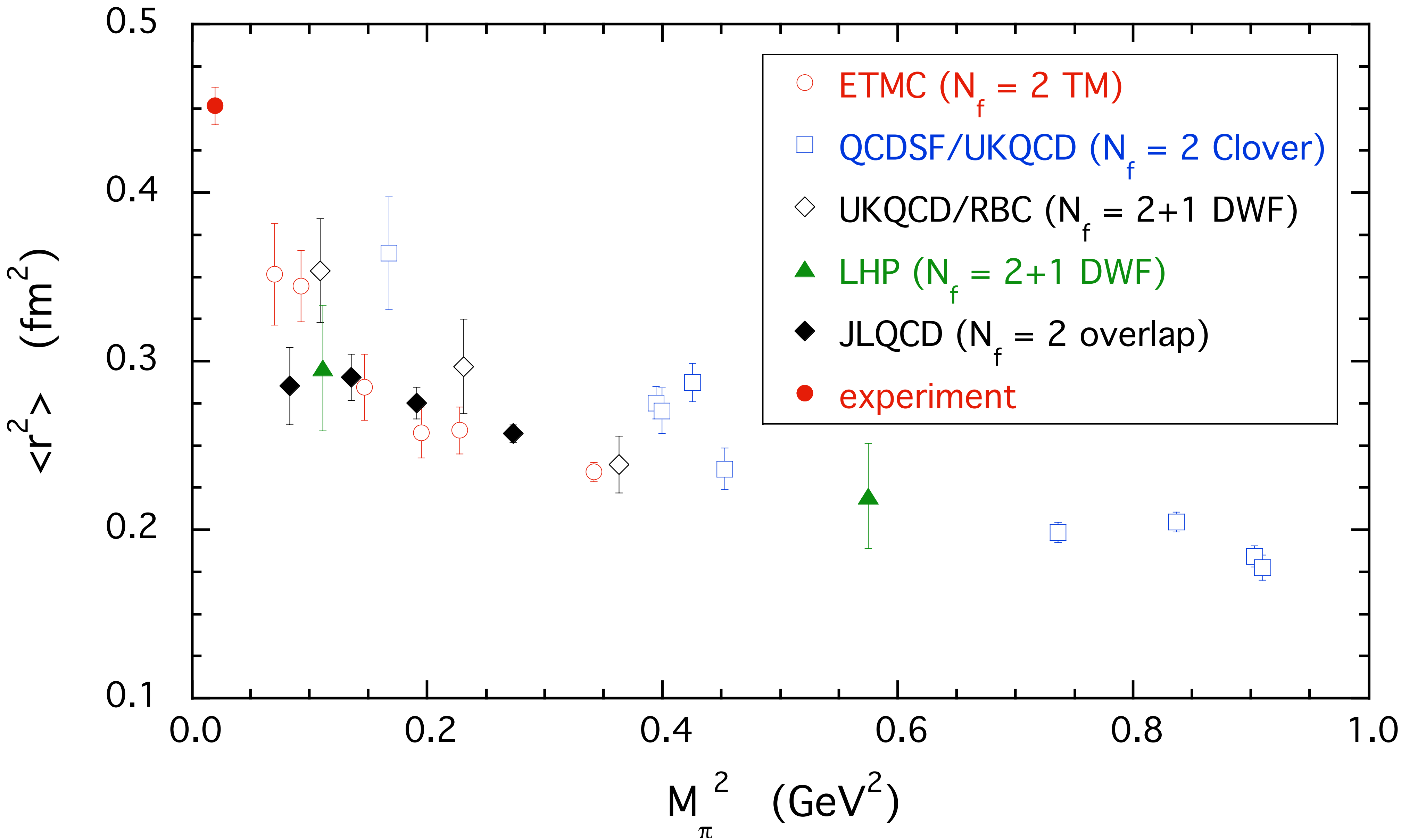
Motivation

- The electromagnetic form factor of the charged π meson parameterises the deviations from the behaviour of a point-like particle when struck by a photon
- These deviations arise from the internal structure of the π: constituent quarks and their strong interaction
- Can be calculated in QCD, but need fully nonperturbative treatment $\rightarrow$ use Lattice QCD
- Experimental determination from $\pi - e$ scattering
- Important to work at physical pion mass

Dependence on pion mass



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Lattice configurations

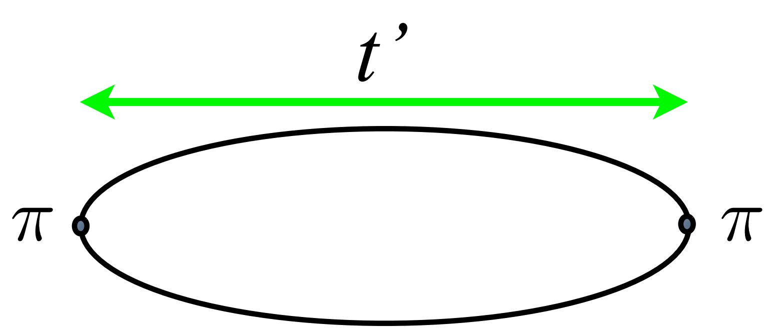
- MILC n_f=*2+1+1* HISQ lattice configurations
- HISQ action for valence quarks
- quark masses tuned to physical masses
- $Lm_\pi \approx 4$ for the coarse (a=*0.12* fm) and fine (a=*0.088* fm) lattices

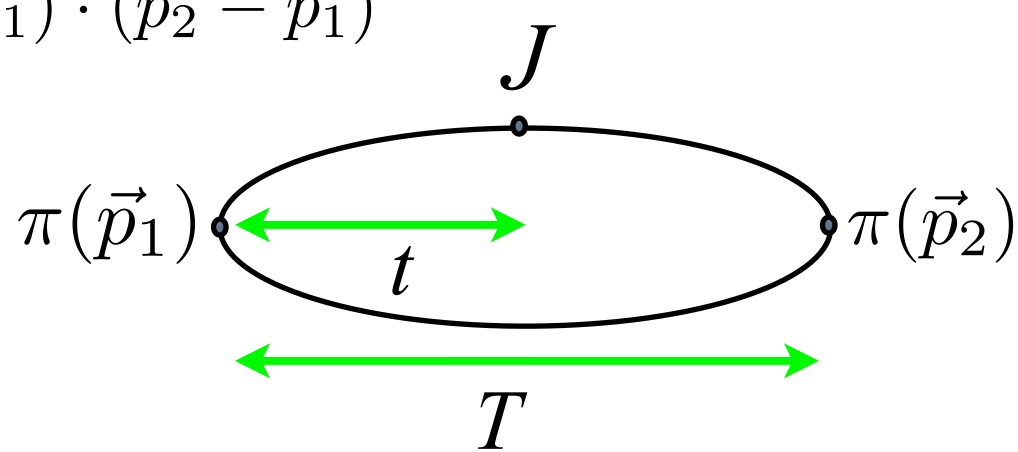
Set	a/fm	am_l	am_s	am_c	m_π/MeV	$L/a \times L_t$	$N_{\rm conf}$
1	0.15	0.00235	0.0647	0.831	133	32×48	1000
2	0.12	0.00184	0.0507	0.628	133	48×64	1000
3	0.088	0.00120	0.0363	0.432	128	64×96	223

Form factors = 3pt amplitudes

- Consider two currents, a 1-link spatial vector current and a scalar current
- Use a phase at the boundary to give a quark a momentum: $\Phi(x+\hat{e}_j L) = e^{i2\pi\theta_j}\Phi(x) \rightarrow p_j = \dfrac{2\pi\theta_j}{L}$
- Tune θ to get the desired q^2 and extract $f_+(q^2)$ in the space-like (negative) region of q^2 near zero

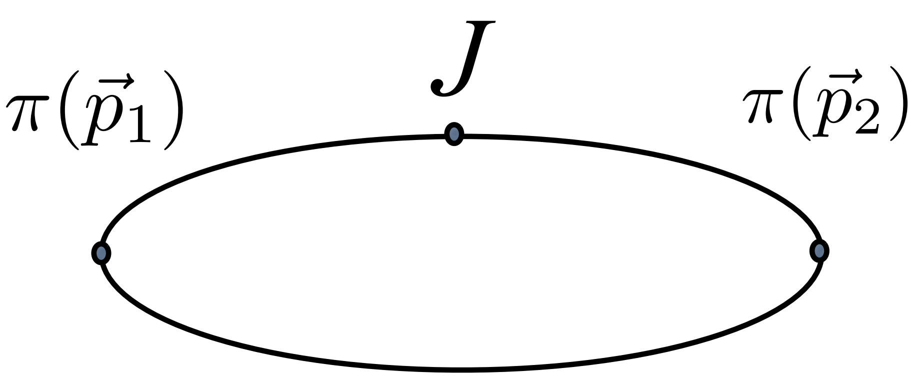
$$q^2 = (E(\vec{p}_2) - E(\vec{p}_1))^2 - (\vec{p}_2 - \vec{p}_1)\cdot(\vec{p}_2 - \vec{p}_1)$$

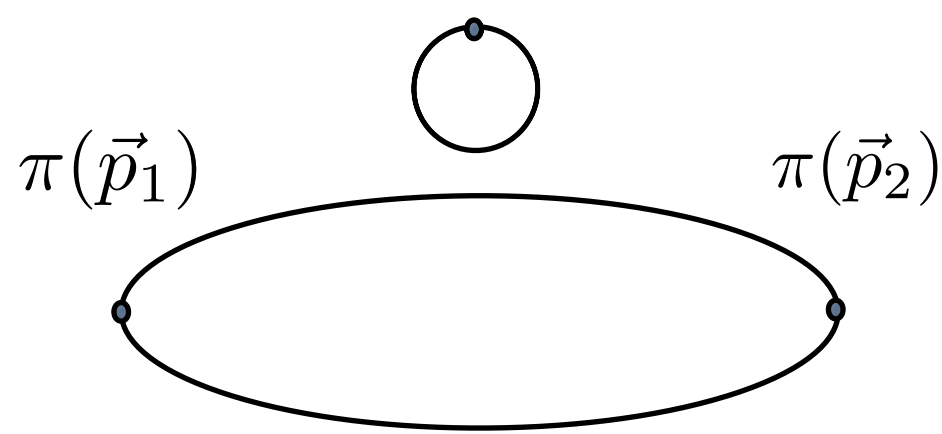




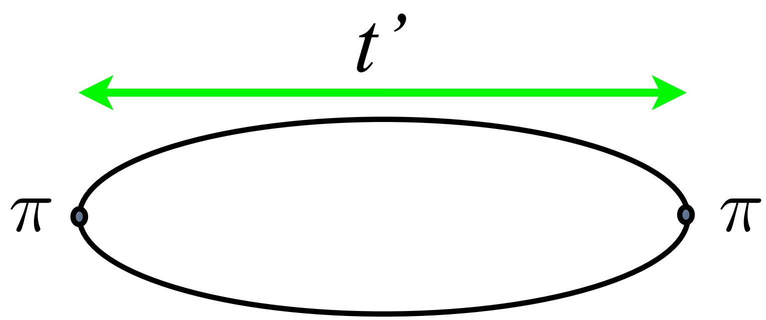
Connected and disconnected diagrams

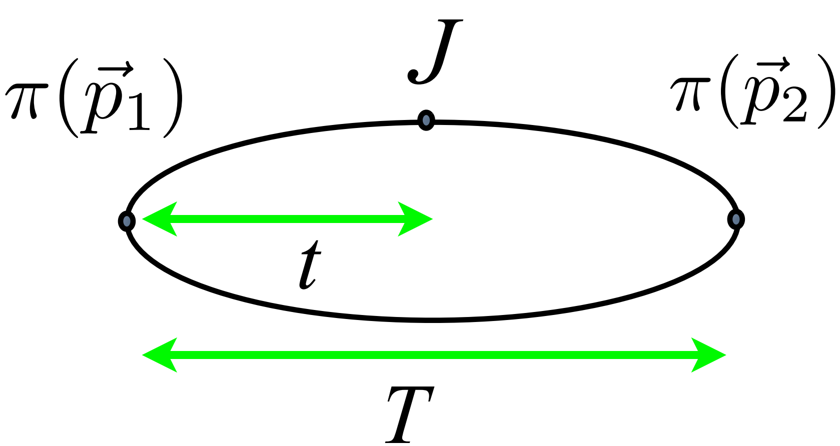
- Writing down 3-point matrix elements gives two types of terms, connected and disconnected
- Vector current: Disconnected diagrams cancel due to charge conjugation and isospin symmetries
- Scalar current: For a full calculation of the scalar form factor need both connected and disconnected diagrams, but here we only consider connected diagrams





Fitting the correlators





- Fit 2-point and 3-point correlators simultaneously
- Multi-exponential fits to reduce systematical errors from the excited states
- Use Bayesian priors to constrain fit parameters
- Fit all q^2 values simultaneously to take into account the correlations

Scalar and vector form factors

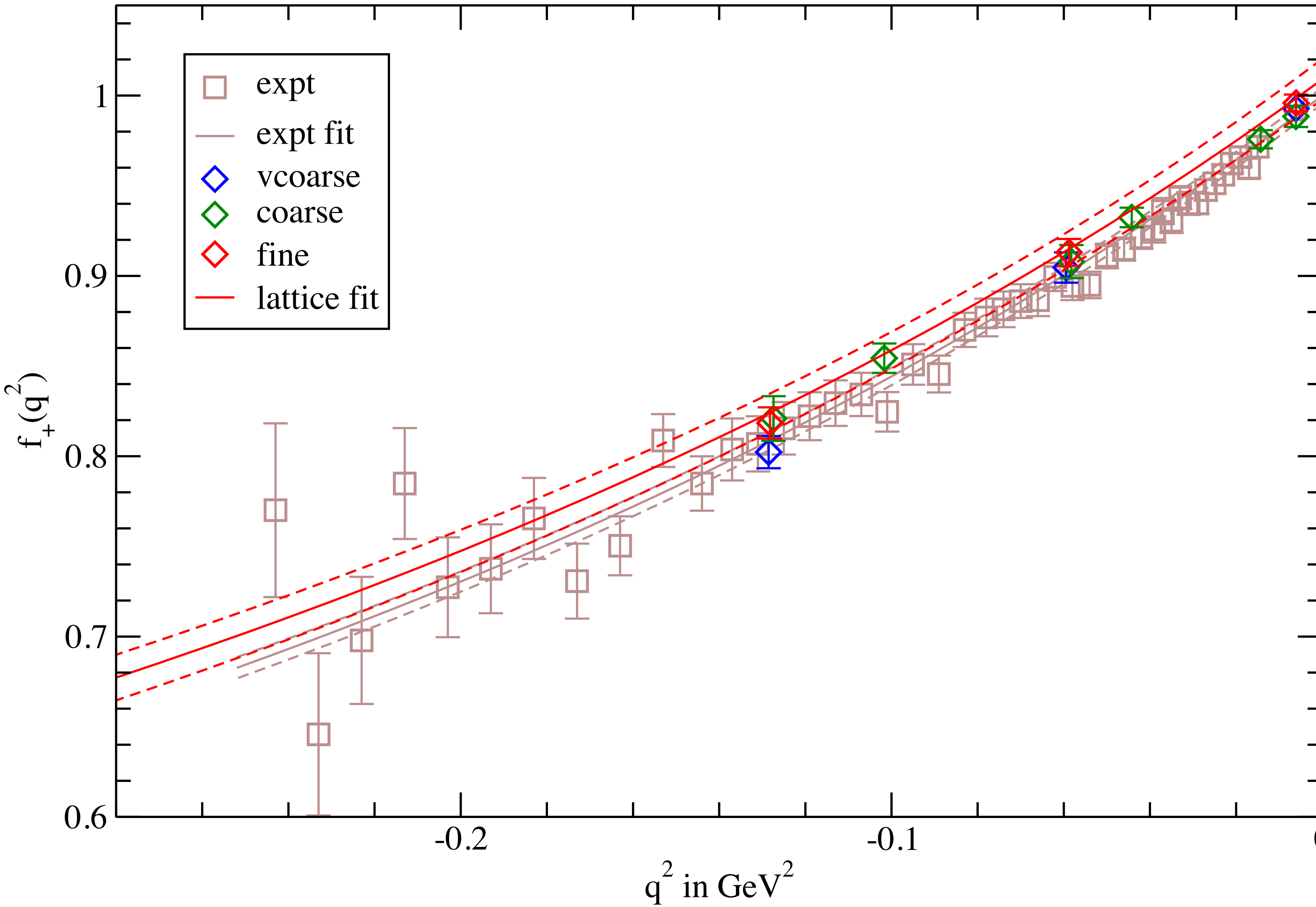
$$\langle\pi(\vec{p}_1)|J|\pi(\vec{p}_2)\rangle = Z\sqrt{4E_0(\vec{p}_1)E_0(\vec{p}_2)}J_{0,0}(\vec{p}_1,\vec{p}_2)$$

$$\langle\pi(\vec{p}_1)|V_i|\pi(\vec{p}_2)\rangle = f_+(q^2)(\vec{p}_1+\vec{p}_2)_i$$

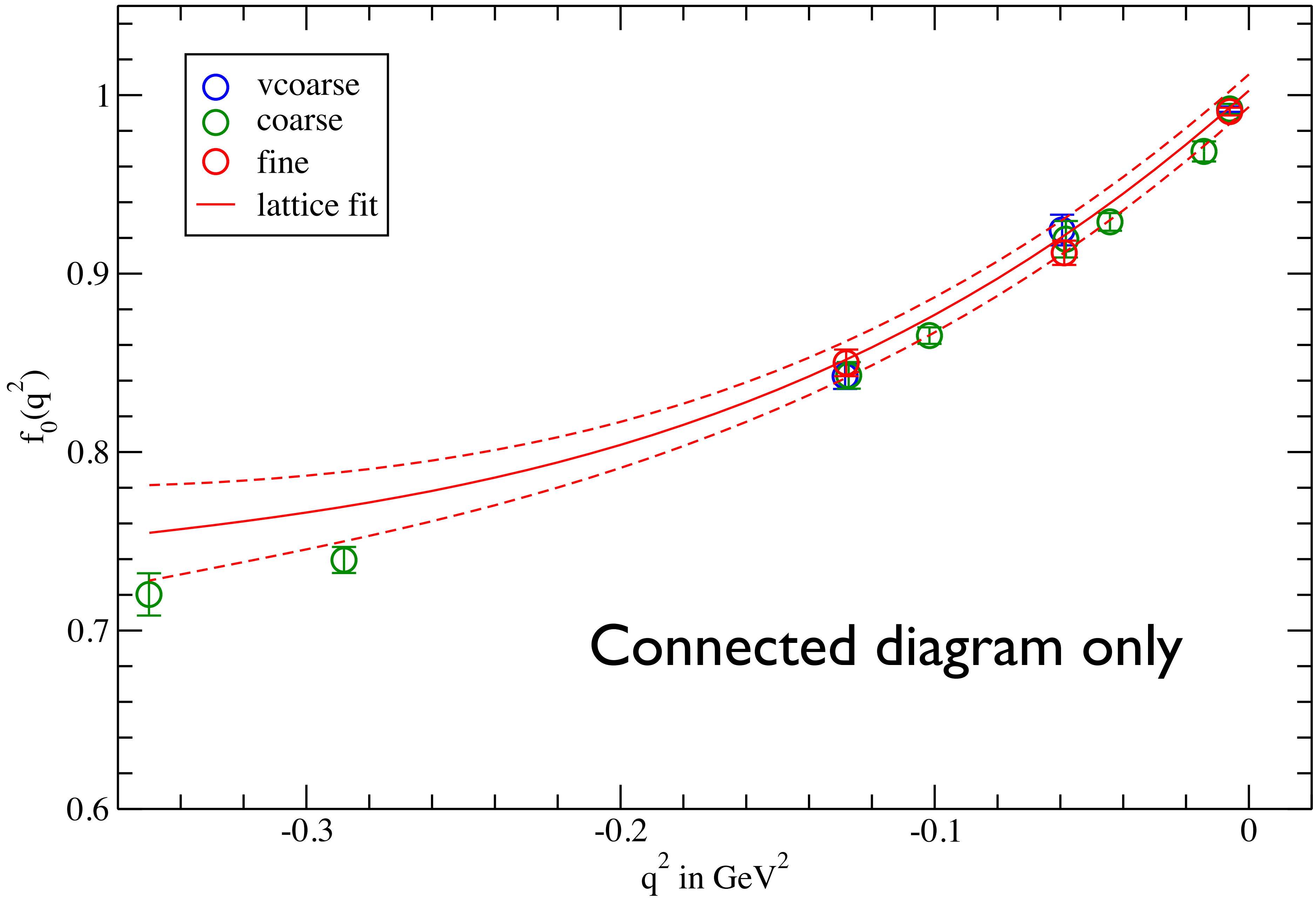
$$\langle\pi(\vec{p}_1)|S|\pi(\vec{p}_2)\rangle = f_0(q^2)\frac{\partial M_\pi^2}{\partial m_l}$$

- Need renormalisation constant Z for the vector current: demand that $f_+(0)=1$
- Scalar current is absolutely normalised, but we do not have complete calculation of the matrix element (only the connected 3pt correlator) - treat the scalar current as requiring a Z factor and set $f_0(0)=1$

Results: vector form factor



Results: scalar form factor



Continuum extrapolation

- Fit the form factors to the pole form

$$f(q^2) = \frac{1}{(1 + ba^2 + ca^4 + q^2\langle r^2\rangle/6)}$$

or as power series in q^2 allowing for a^2 and m_π dependence

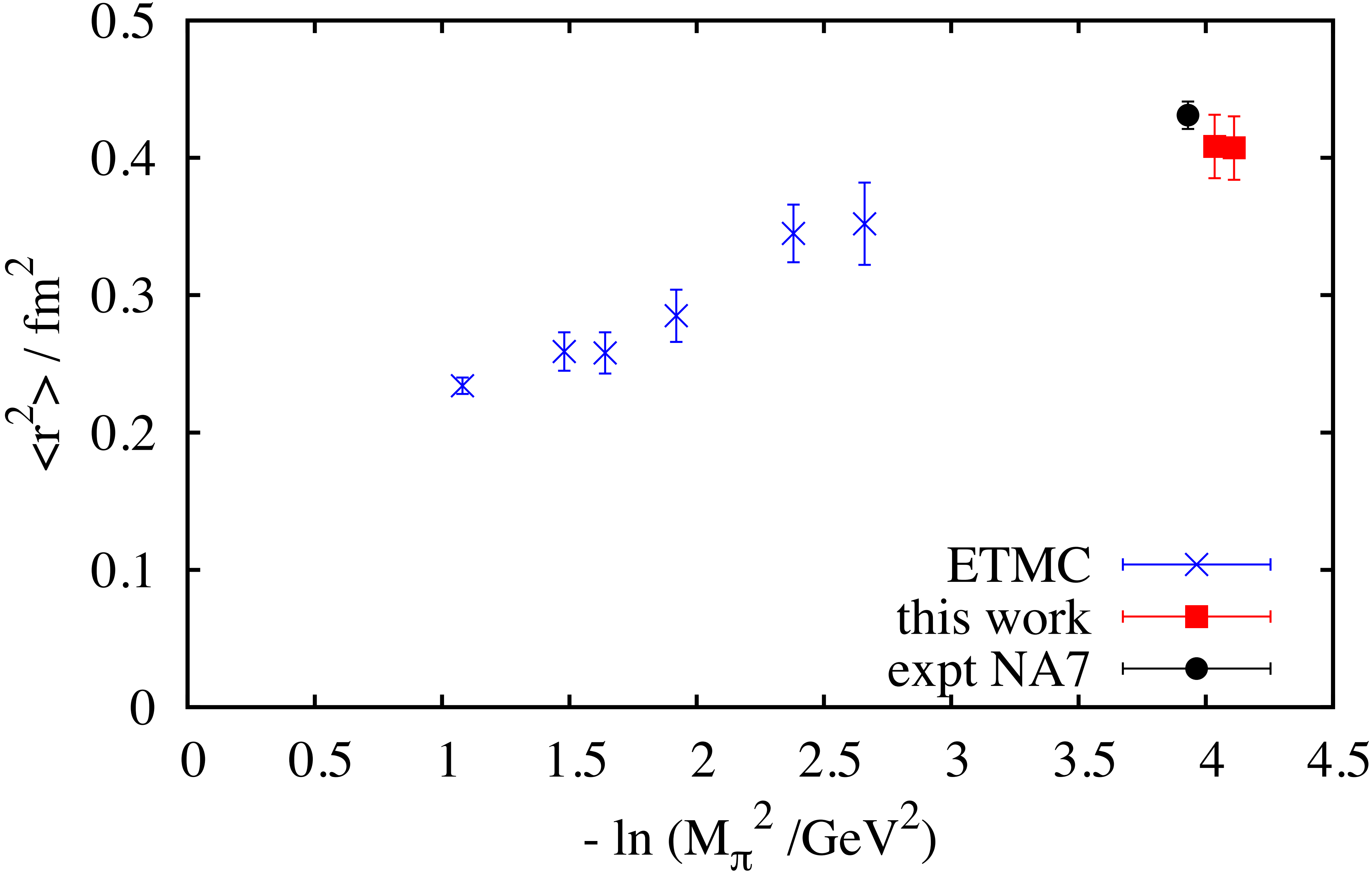
$$f(q^2) = A_0 + \frac{1}{6}\langle r^2\rangle q^2 + A_4 q^4 + A_6 q^6;\ A_i = d_i(1 + b_i a^2 + c_i a^4)$$

$$\langle r^2\rangle = A_2 + c_J \ln(m_\pi^2/\mu^2)$$

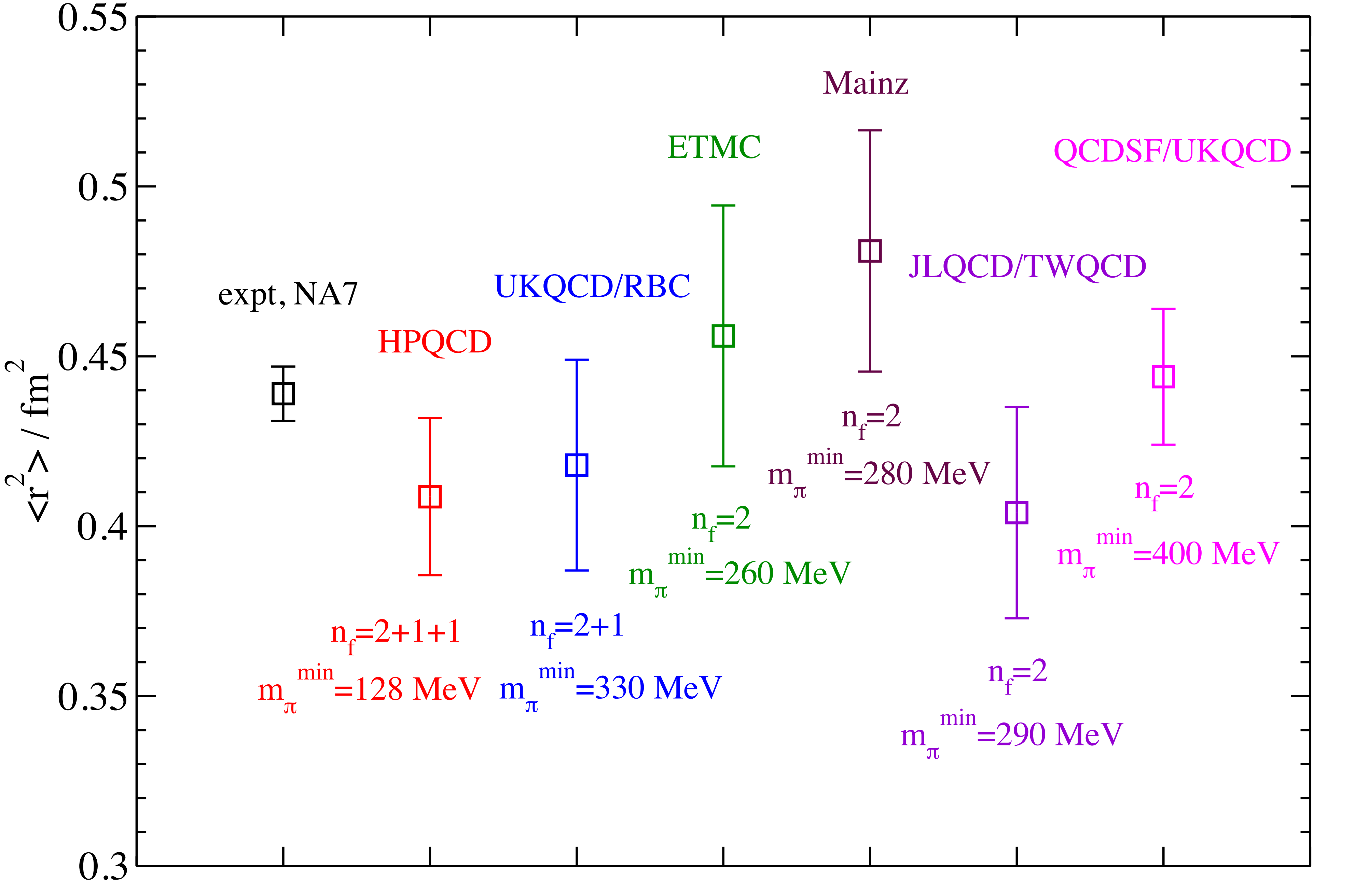
- The slope at $q^2=0$ gives the mean square of the charge radius:

$$\langle r_v^2\rangle = -6\frac{\mathrm{d}f_+(q^2)}{\mathrm{d}q^2}\bigg|_{q^2=0}$$

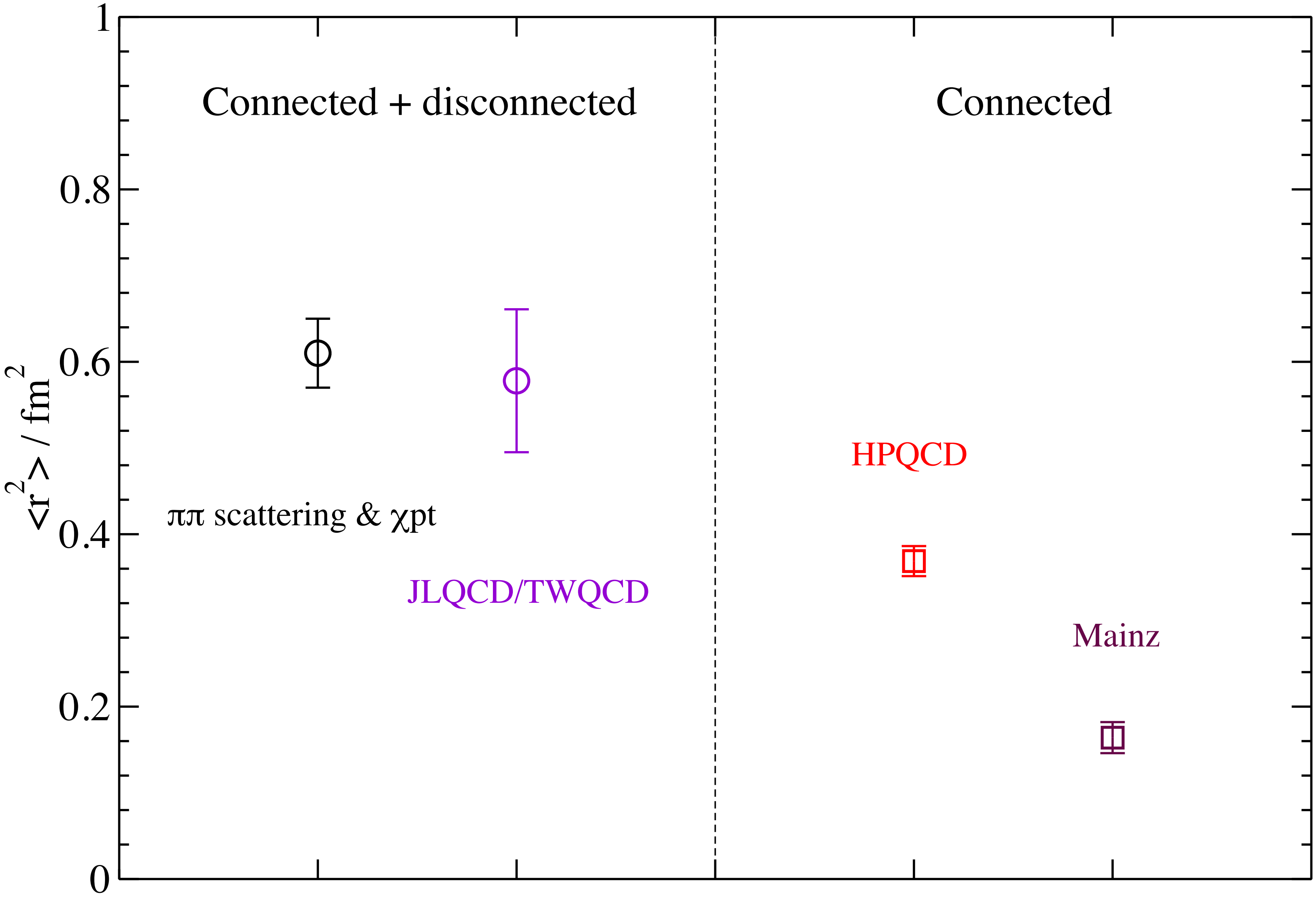
Dependence on pion mass



Vector mean square radius



Scalar mean square radius



Charge density

- In the non-relativistic limit, $q^2 \approx -(\vec{q})^2$, the form factor $f_+(q^2)$ can be viewed as the Fourier transform of the electric charge distribution

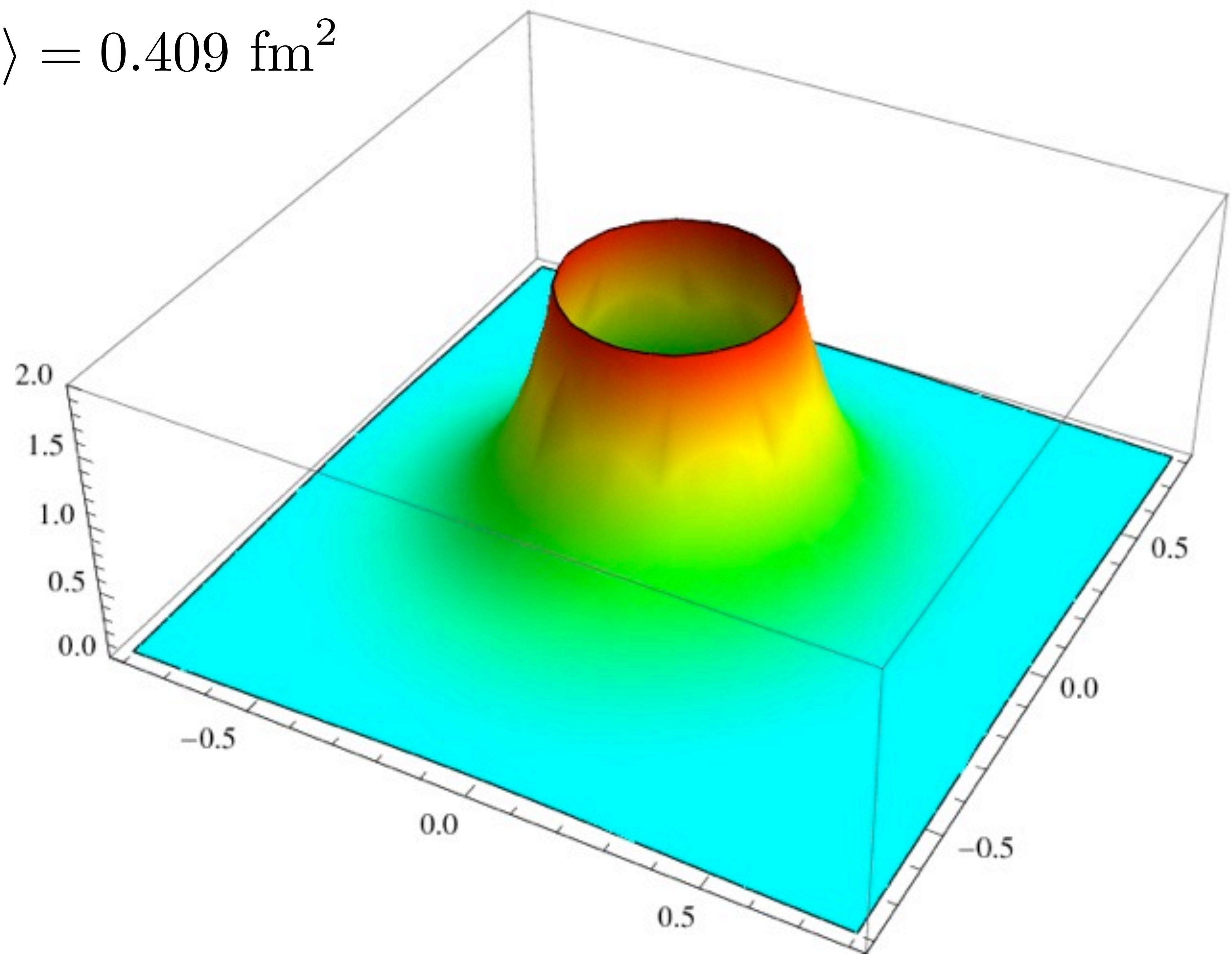
- The form factor is usually taken to be of pole form

$$f_+(q^2) = \frac{1}{(1 + q^2 \langle r_V^2 \rangle / 6)}$$

or a power series in q^2

Non-relativistic charge density

$\langle r_v^2 \rangle = 0.409\ \mathrm{fm}^2$



Summary

- Full Lattice QCD calculation of the pion vector electromagnetic form factor
  - physical pion mass
  - can choose the q^2 range
  - determine the charge radius:
  our preliminary result is $\langle r_v^2 \rangle = 0.409(23)\ \mathrm{fm}^2$
- Compare with experiment - get good agreement
- The scalar form factor needs much more work

Thank you!

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