Analysis of topological structure of the QCD vacuum with overlap-Dirac operator eigenmode

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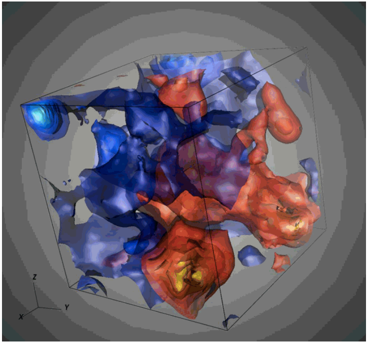
Coll. S. Hashimoto and G. Cossu

Vacuum Structure of QCD

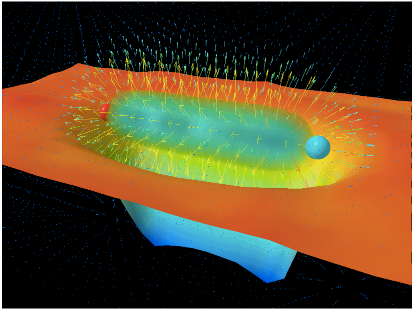
QCD vacuum is filled with interesting objects.

instanton/anti-instanton



⇒ chiral symmetry breaking

flux-tube



⇒ quark confinement

Figs (left) JLQCD Coll. '12 (right) www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/

Overlap-Dirac Operator

$$D_{\rm ov} = m_0 \left[1 + \gamma_5 {\rm sgn}\ H_W(-m_0)\right]$$

$H_W(-m_0)$: hermitian Wilson-Dirac operator with a mass $-m_0$ (Neuberger '98)

overlap-Dirac eigenmode is an ideal probe to study QCD vacuum

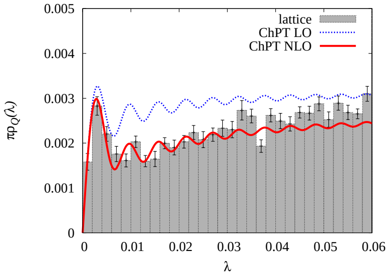
- exact chiral symmetry on the lattice
- Banks-Casher relation

  — chiral symmetry breaking

$$\langle \bar{q} q \rangle = -\pi \rho(0)$$

- index theorem — topology of QCD

$$\frac{1}{2} {\rm tr}\left[\gamma_5 D_{\rm ov}\right] = n_L - n_R$$



Dirac spectral density $\rho(\lambda)$
JLQCD Coll. '10

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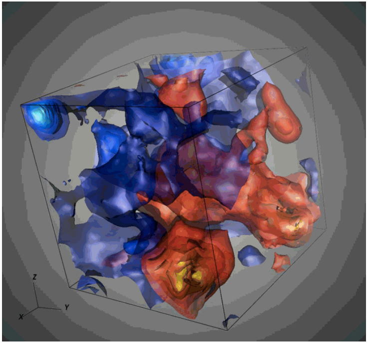
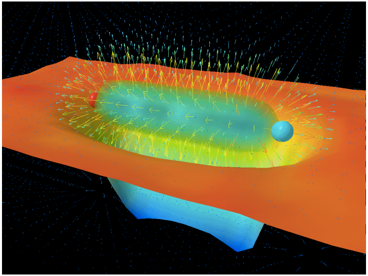
2. flux-tube formation from overlap-Dirac eigenmodes
3. chiral condensate in flux-tube





Dirac Eigenmode Decomposition of Field Strength

Dirac eigenmodes $\psi_\lambda(x)$ $\Longrightarrow$ Field strength tensor $F_{\mu\nu}$
Gattringer '02

$$[\not{D}(x)]^2 = \sum_\mu D_\mu^2(x) + \sum_{\mu<\nu} \gamma_\mu\gamma_\nu F_{\mu\nu}(x)$$

$$\therefore F_{\mu\nu}(x) = -\frac{1}{4}\mathrm{tr}\left[\gamma_\mu\gamma_\nu \not{D}^2(x)\right] \propto \sum_\lambda \lambda^2 f_{\mu\nu}(x)_\lambda$$

with Dirac-mode components $f_{\mu\nu}(x)_\lambda$

$$f_{\mu\nu}(x)_\lambda \equiv \psi_\lambda^\dagger(x)\gamma_\mu\gamma_\nu\psi_\lambda(x)$$

references Gattringer '02, Ilgenfritz *et al.* '07, '08.

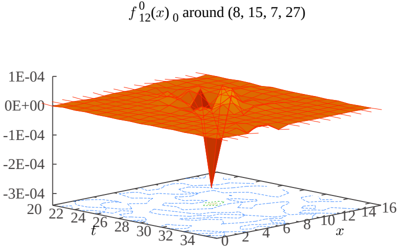
In this talk, we use
dynamical overlap-fermion configurations by JLQCD Coll.

$\beta = 2.3$ (Iwasaki action), $a \sim 0.11$ fm, Volume $16^3 \times 48$,
overlap-Dirac operator $N_f = 2+1$, $m_{ud} = 0.015$, $m_s = 0.080$

Duality of Field Strength Tensor by Overlap-eigenmode

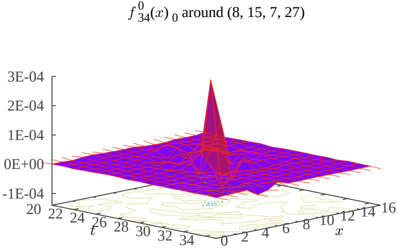
lowest eigenmode components of $f_{\mu\nu}$

$f_{12}(x)_0$



$\Rightarrow$ negative peak

$f_{34}(x)_0$



$\Rightarrow$ positive peak

at this point, an anti-self-dual lump exists

$$f_{12}(x)_0 \simeq -f_{34}(x)_0$$

Action and Topological Charge from Dirac Eigenmodes

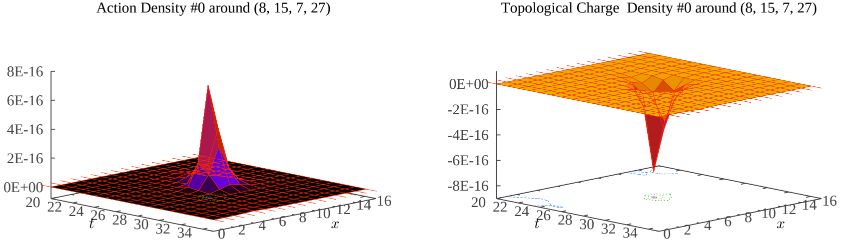
action density

$$\rho^{(N)}(x) = \sum_{i,j}^{N} \frac{\lambda_i^2 \lambda_j^2}{2} f^a_{\mu\nu}(x)_i f^a_{\mu\nu}(x)_j$$

topological charge density

$$q^{(N)}(x) = \sum_{i,j}^{N} \frac{\lambda_i^2 \lambda_j^2}{2} f^a_{\mu\nu}(x)_i \tilde{f}^a_{\mu\nu}(x)_j$$

Figure: action and topological charge densities of lowest eigenstate



there is an "anti-instanton" at this point

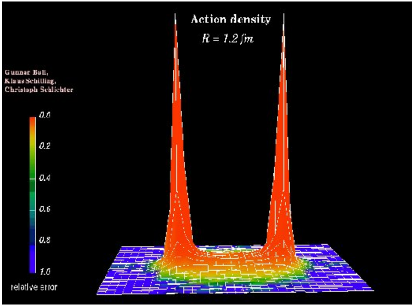
Flux-tube Formation Between Quarks

action density is “expelled” between quarks

$\Longrightarrow$ formation of flux-tube $\Longrightarrow$ confinement potential

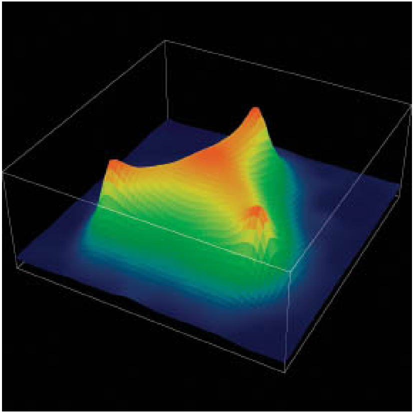
we discuss flux-tube structure from low-lying eigenmodes

flux-tube between
quark and antiquark



Bali-Schilling-Schlichter ’95

Y-type flux-tube for 3Q-system



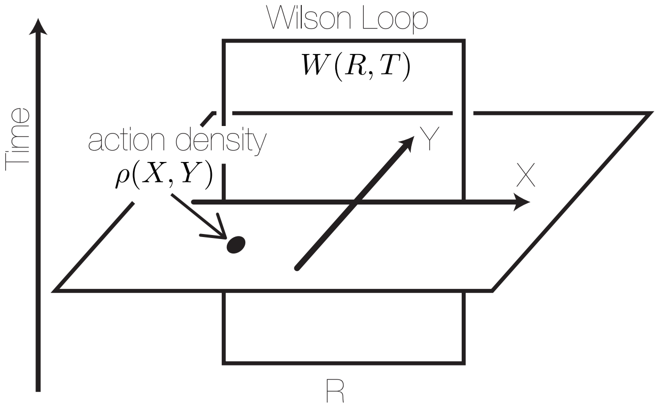
Ichie et al. ’03

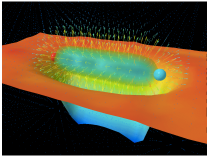
Measurement of Flux-tube Structure

difference of action density $\rho(x)$ with or without Wilson loop $W(R,T)$

$$\langle \rho(x) \rangle_W \equiv \frac{\langle \rho(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle < 0$$

N.B. action density is lowered in the flux-tube



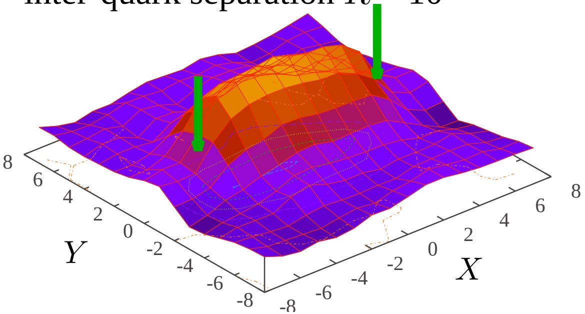


Flux-tube Formation by Low-lying Eigenmodes

$\rho^{(N)}(x)$: low-mode truncated action density

$$\langle \rho(x) \rangle_W^{(N)} \equiv \frac{\langle \rho^{(N)}(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle^{(N)}$$

inter-quark separation $R = 10$



$(-)$ $\langle \rho(x) \rangle_W^{(N)}$ $N = 100$ eigenmodes

- low-modes *really* contribute to flux-tube

  $\frac{100}{2,359,296} \sim 0.00004$

- $\lambda_{\max} \sim 300$ MeV

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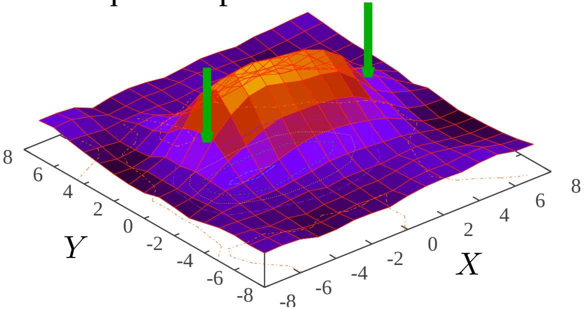
inter-quark separation $R = 9$



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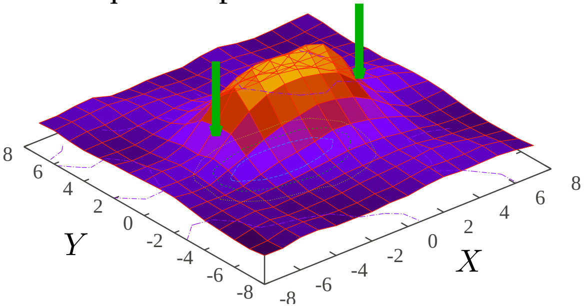
inter-quark separation $R = 8$



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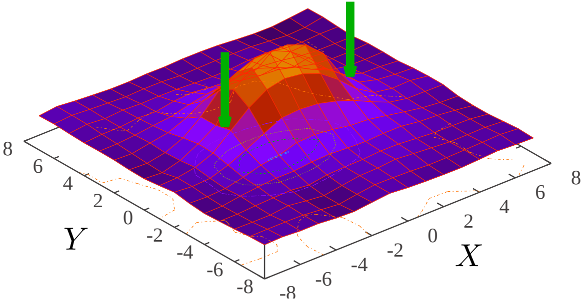
inter-quark separation $R = 7$



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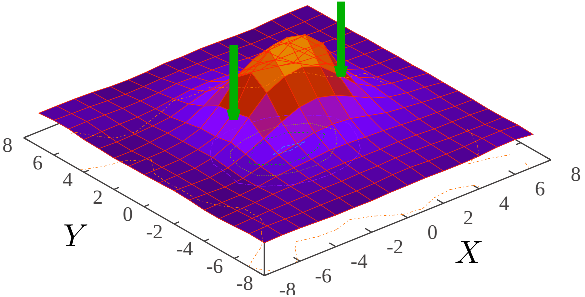
inter-quark separation $R = 6$



$(-)$ $\langle \rho(x) \rangle_W^{(N)}$ $N = 100$ eigenmodes

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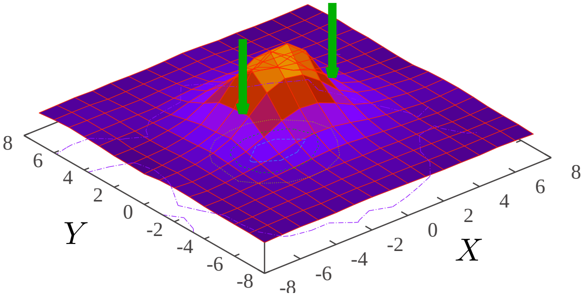
Flux-tube Formation by Low-lying Eigenmodes

$\rho^{(N)}(x)$: low-mode truncated action density

$$\langle \rho(x) \rangle_W^{(N)} \equiv \frac{\langle \rho^{(N)}(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \rho \rangle^{(N)}$$

inter-quark separation $R = 5$



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- low-modes *really* contribute to flux-tube

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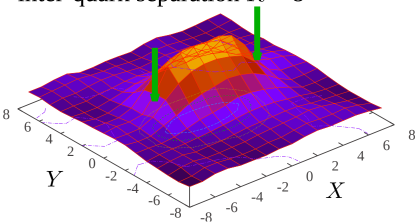
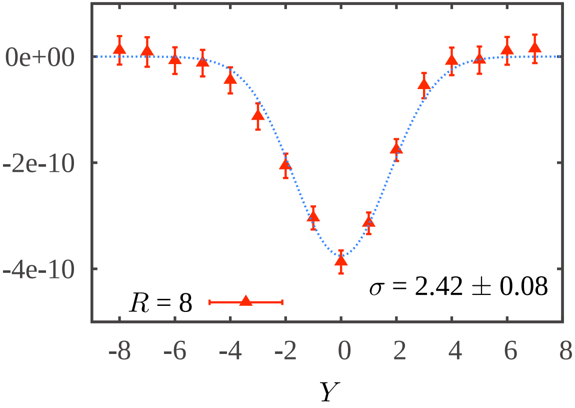
Cross Section of Flux-tube





- gaussian form $\rho(y) \propto e^{-y^2/\sigma^2}$
- thickness of flux ~ 0.6 fm

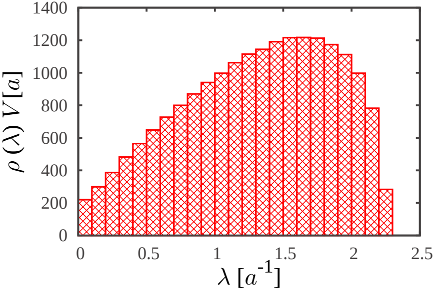
About Dirac Eigenmodes and Confinement

confinement remains without *low* or *intermediate* or *higher* eigenmodes

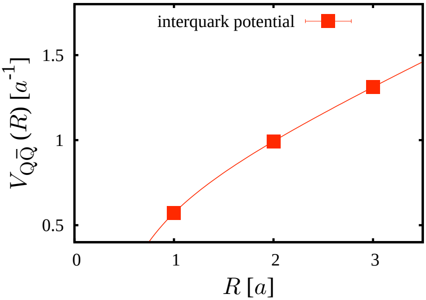
Refs. Gongyo-TI-Suganuma '12, TI-Suganuma '13

$\Longrightarrow$ "seeds" of confinement seem to be widely distributed in eigenmodes

removing Dirac eigenmodes



interquark potential



Figs. Gongyo-TI-Suganuma '12

flux-tube formation by low-lying modes

$\Longrightarrow$ direct evidence of the seed of confinement in the low-lying modes

$\neq$ low-lying modes are essential for confinement

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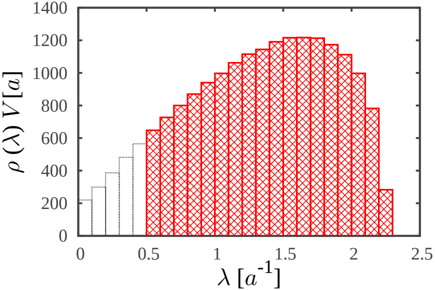

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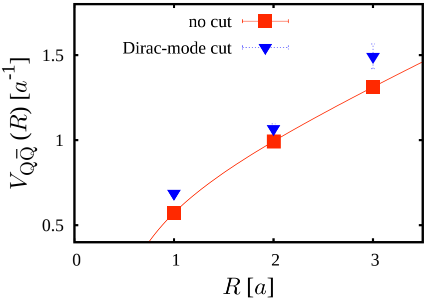

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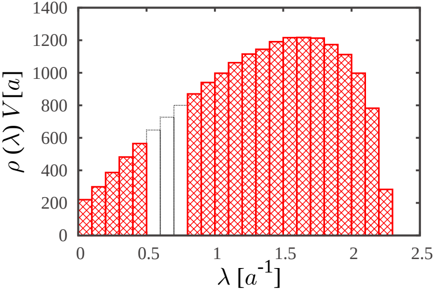

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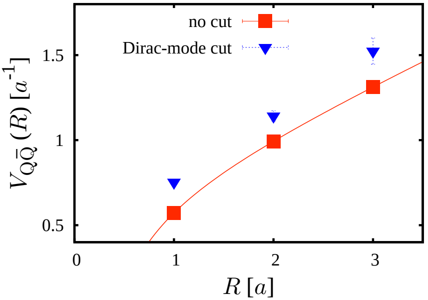

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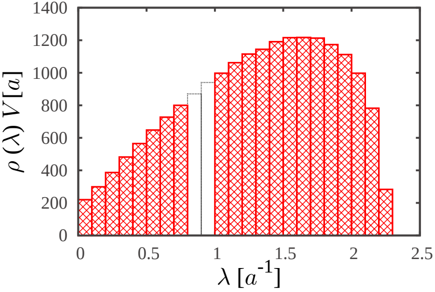

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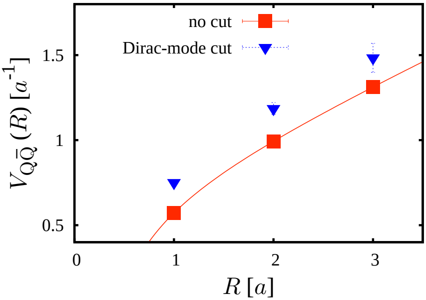

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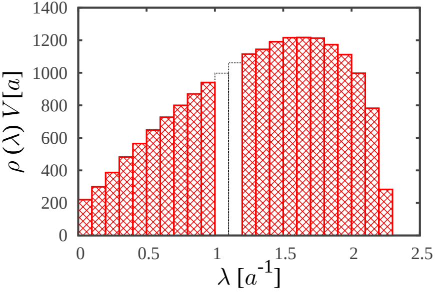

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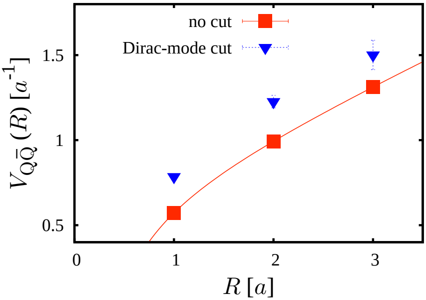

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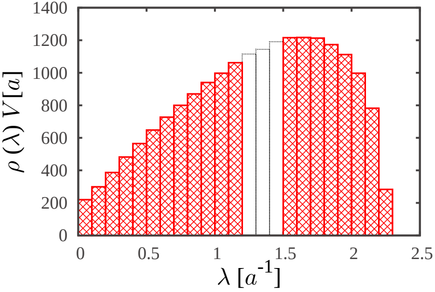

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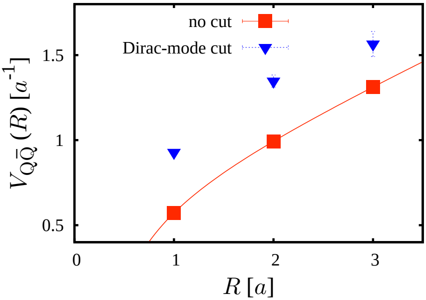

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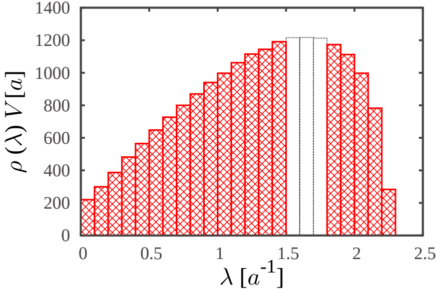

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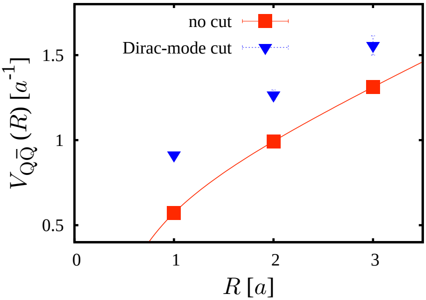

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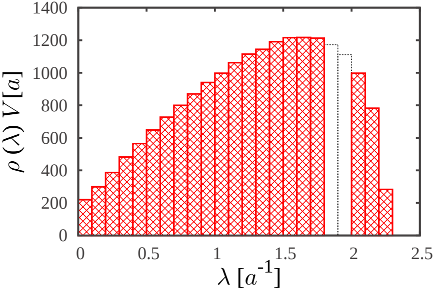

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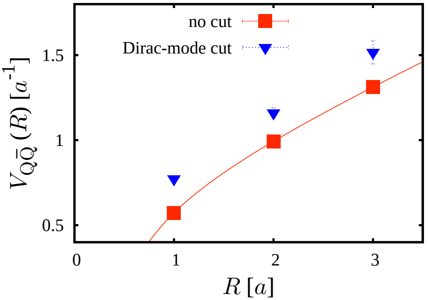

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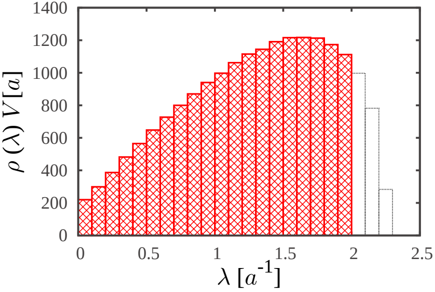

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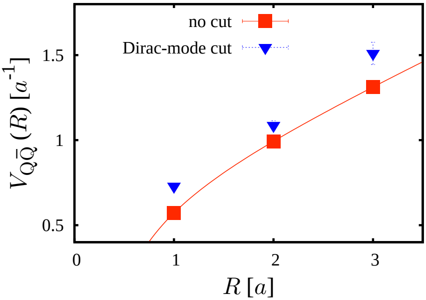

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Dirac Eigenmodes and Chiral Condensate

chiral condensate $\langle \bar{q}q \rangle$

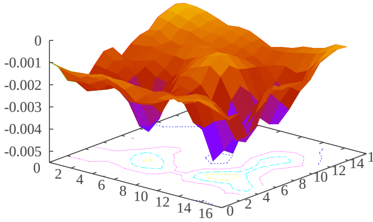
$$\langle \bar{q}q \rangle = -\mathrm{Tr}\frac{1}{\not{D} + m_q} = -\frac{1}{V}\sum_{\lambda}\frac{1}{\lambda + m_q}$$

cf. Banks-Casher relation $\langle \bar{q}q \rangle = -\pi\rho(0)$

"local chiral condensate" $\bar{q}q(x)$ is given by

$$\bar{q}q(x) = -\sum_{\lambda}\frac{\psi_\lambda^\dagger(x)\psi_\lambda(x)}{\lambda + m_q}$$

with Dirac eigenmodes $\psi_\lambda(x)$

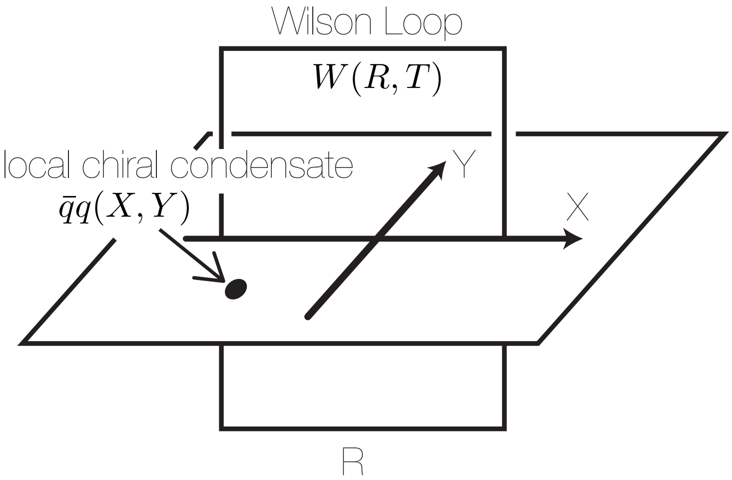


sample of a local condensate

Chiral Condensate in Flux-tube

difference of chiral condensate $\bar{q}q(x)$ with or without Wilson loop $W(R,T)$

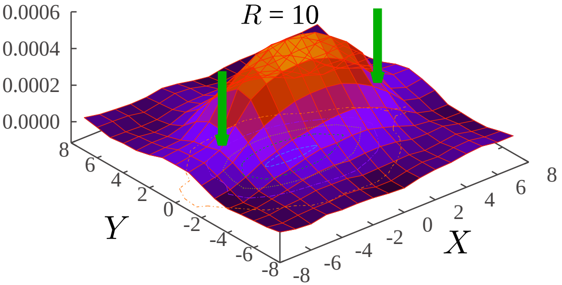
$$\langle \bar{q}q(x) \rangle_W \equiv \frac{\langle \bar{q}q(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle$$



Reduction of Chiral Condensate in Flux

difference of "truncated local chiral condensate"

$$\langle \bar{q}q(x) \rangle_W^{(N)} \equiv \frac{\langle \bar{q}q^{(N)}(x) W(R,T) \rangle}{\langle W(R,T) \rangle} - \langle \bar{q}q \rangle^{(N)}$$



- $\langle \bar{q}q(x) \rangle_W^{(N)} > 0$
- partially restored $|\langle \bar{q}q \rangle|_{\text{in flux}} < |\langle \bar{q}q \rangle|$
- "Bag-model" like

$m_q = 0.015$ using $N = 100$ eigenmodes

cf. chiral condensate in color-electro/magnetic fields (Suganuma-Tatsumi '93)

Reduction of Chiral Condensate in Flux

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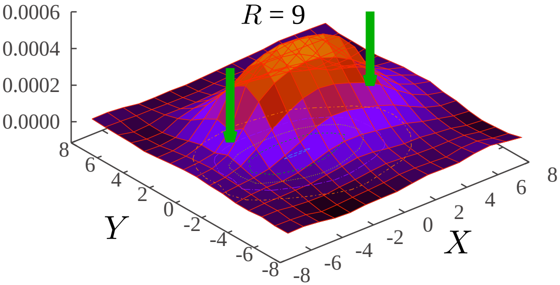

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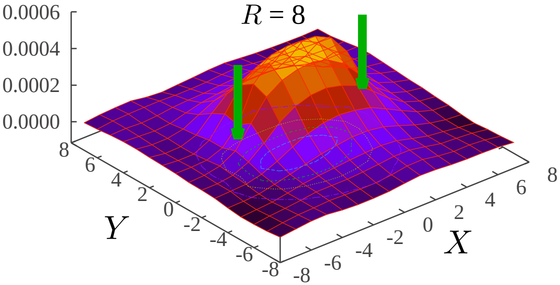

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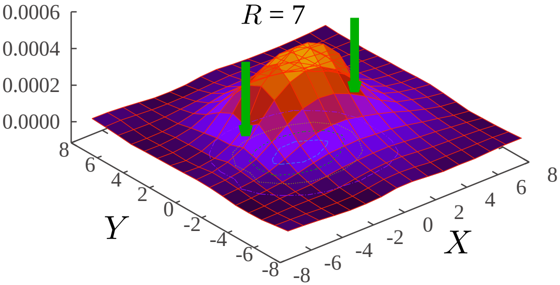

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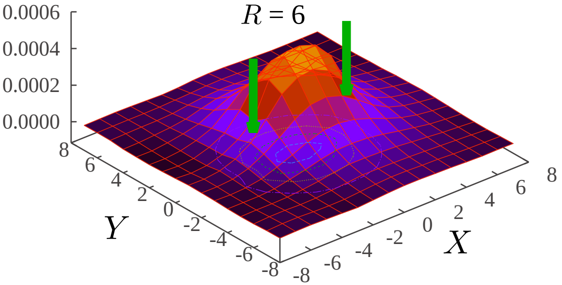

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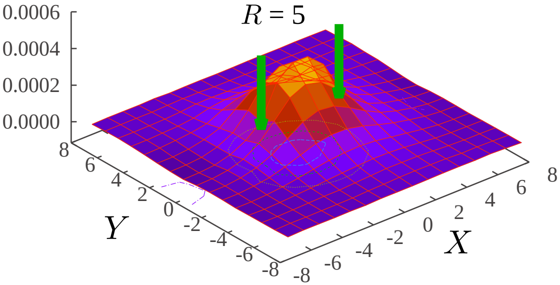

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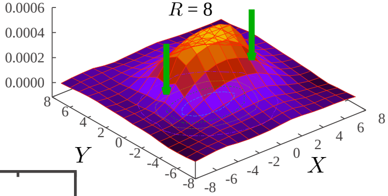
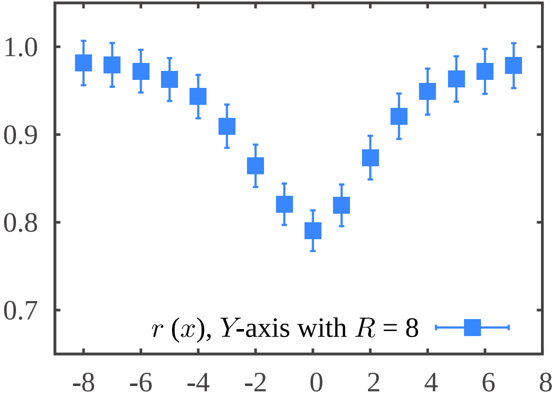
cf. chiral condensate in color-electro/magnetic fields (Suganuma-Tatsumi '93)

Reduction Ratio of Chiral Condensate in Flux-tube

$$r(x) \equiv \frac{\langle \bar{q}q^{(\mathrm{subt})}(x) W(R,T)\rangle}{\langle \bar{q}q^{(\mathrm{subt})}\rangle\langle W(R,T)\rangle} < 1$$





- at center of flux
- reduction of $|\langle \bar{q}q\rangle|$
  $\Longrightarrow$ about 20 %

cf. subtracted condensate $\langle \bar{q}q\rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})}\rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3$

Summary

we discuss vacuum structure of QCD using overlap-Dirac eigenmodes

- low-lying overlap-Dirac eigenmodes
  $\Longrightarrow$ show "instanton"-like behavior
  - (anti-)self-dual field strength $f_{\mu\nu} \simeq (-)\tilde{f}_{\mu\nu}$
  - (anti-)self-dual lump of action density / topological charge density
- flux-tube formation by low-lying Dirac eigenmodes
  $\Longrightarrow$ low-lying eigenmodes contribute to the formation of flux-tube
  i.e., confinement
- chiral condensation in the flux-tube
  $\Longrightarrow$ chiral symmetry is partially restored in the flux-tube
  reduction of $|\langle \bar{q}q \rangle|$ is about 20% at the center of flux

outlooks

- chiral condensate in 3Q-system, quark-number densities in the flux-tube, light-quark content in quarkonia, topological susceptibility from eigenmodes, *and so on*

6 Appendix

Appendix

Lattice Setup

- gauge configurations
  - JLQCD dynamical overlap simulation $N_f = 2 + 1$
  - $16^3 \times 48$, $\beta = 2.3$, $m_{ud} = 0.015$, $m_s = 0.080$, $Q = 0$
  - $a^{-1} = 1.759(10)$ GeV ($m_\pi \sim 300$ MeV)
  - 100 low-lying Dirac eigenmodes ($\lambda_{\rm UV} \sim 400$ MeV)
  - 50 configurations
- Wilson loop $W(R,T)$ measurement
  - APE smearing for spatial link-variable U_i — 16 sweeps
  - $T = 4$ — ground-state component is dominant
  - measurement of action density/local chiral condensate at $T = 2$

Subtracted Chiral Condensate

$$\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\mathrm{subt})} \rangle + c_1^{(N)} \frac{m_q}{a^2} + c_2^{(N)} m_q^3$$

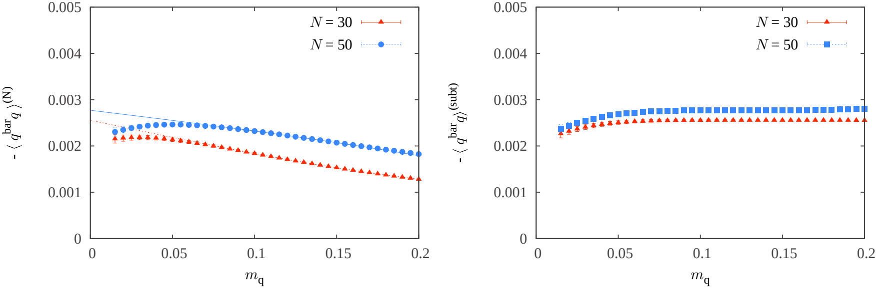


Figure: (a) $\langle \bar{q}q \rangle^{(N)}$ (b) $\langle \bar{q}q \rangle^{(\mathrm{subt})}$

ref. J. Noaki et al., Phys. Rev. D**81**, 034502 (2010).