The phase structure of a chirally-invariant Higgs-Yukawa model

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review article: Adv. High Energy Phys. 2013 (2013) 875612

Motivation

- With discovery of light Higgs boson $m_H \approx 125$ GeV at CERN

  [ATLAS/CMS 2012]

  $\rightarrow$ Standard Model completed

- Still questions open: dark matter, CP-violation, ...

- New physics if renormalized Yukawa coupling is large?
- Bare infinite Yukawa coupling corresponds to the O(4) model

  [Hasenfratz *et al.*1991]

- Investigation of phase transitions of a chirally-invariant Higgs-Yukawa model (non-perturbative in its nature):
  - Order?
  - Universality class?
  - Bound states of fermions?

The O(4)-model

- Lattice action with real four-vector Φ:

$$S_B[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\mu} + \Phi_{x-\mu}\right] + \sum_x \Phi_x^\dagger \Phi_x + \hat{\lambda} \sum_x \left[\Phi_x^\dagger \Phi_x - 1\right]^2$$

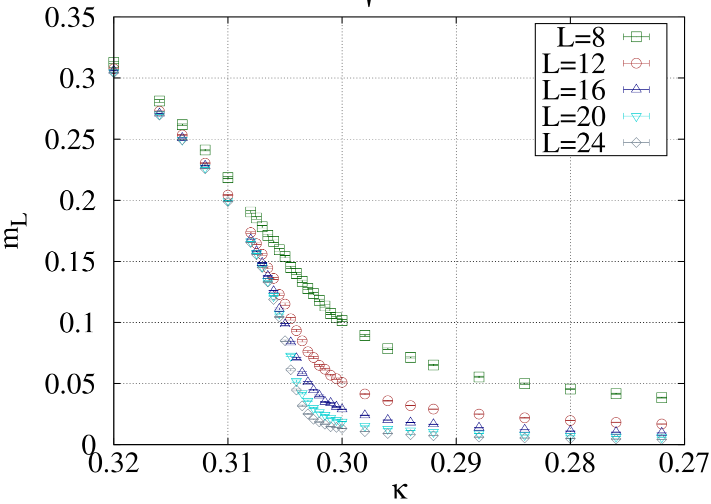
- Connection to continuum formulation:

$$\varphi = \sqrt{2\kappa} \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \quad \lambda_0 = \frac{\hat{\lambda}}{4\kappa^2}, \quad m_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}$$

- Simulations at upper Higgs boson mass bound ($\hat{\lambda} \to \infty$)
- Scan of phase transition in hopping parameter κ

Magnetization m_L of pure O(4)-model

$$m_L = V^{-1} \sqrt{\sum_{\alpha,x} |\Phi_x^\alpha|^2}$$



Investigation of phase structure with finite size scaling

- Use finite volume to compute critical exponents in infinite volume
- Critical exponents define universality class
- Investigation of susceptibility: $\chi_L = V\left[\langle m_L^2\rangle - \langle m_L\rangle^2\right]$
- Scales like:

$$\begin{aligned} \chi_L\left(|T - T_c^L| \gg 1\right) &\sim \left|T - T_c^L\right|^{-\gamma} \\ \chi_L\left(|T - T_c^L| \to 0\right) &\sim L^{1/\nu} \\ T_c^L - T_c^\infty &\sim L^{-1/\nu} \end{aligned}$$

with critical exponents $\nu = 1/2$ and $\gamma = 1$

- Focus on extraction of ν
- T represents either κ in O(4)-model or y in Higgs-Yukawa model
- log-corrections in case of triviality: $L^{1/\nu} \to L^2\left(\log L\right)^{1/2}$

Modelling of global fit function

- Modelling of **global** fit function to susceptibility: [Jansen, Seuferling 1990]

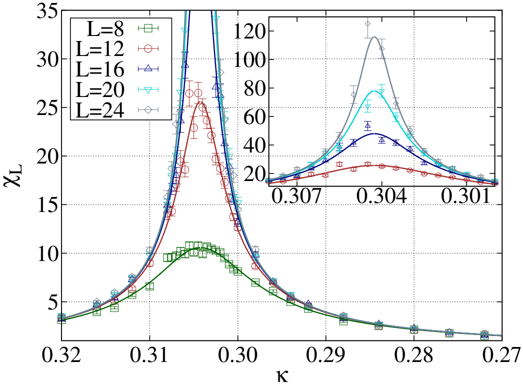
$$\chi_L(T;\xi) = A_1\left(\left[L^2(\log L)^{\xi}\right]^{-1/\nu} + A_{2,3}\cdot\tau^2\right)^{-\gamma/2}$$

$$\tau = (T - T_c^L) = \left[T - \left(T_c^{\infty} + C\cdot\left[L^{-1}\cdot(\log L)^{-\xi/2}\right]^{b}\right)\right]$$

- 8 free fit parameters: $A_1, A_2, A_3, C, \nu, b = 1/\nu, \gamma, T_c^{\infty}$
- $\log$-exponent ξ must be $1/2$ but can be changed
- Direct determination of critical exponents ν and γ

Susceptibility χ_L of pure O(4)-model

- Peak hight shows expected dependence on L
- Peak shift too small to see within given statistics

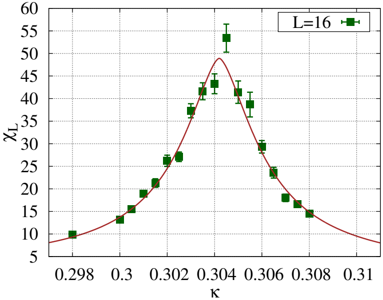


Individual volume fits

- Modelling of fit function for individual volumes: [Jansen *et al.*1986]

$$\chi_L(T) = a + c \cdot T + \frac{d}{1 + e \cdot |T - T_c^L|^g}$$

- 6 free fit parameters: a, c, d, e, g, T_c^L
- Extraction of T_c^L and χ_L^{max}

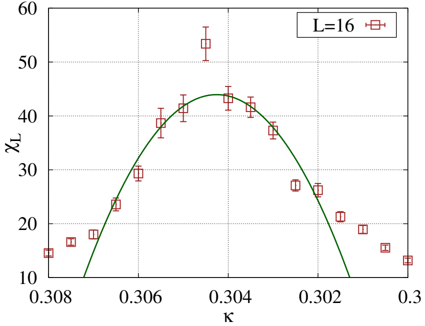


Naive fits to susceptibility

- χ_L^{max} computed from quadratic fit to χ_L close to maximum:

$$\chi_L(T) = m + p \cdot T + q \cdot T^2$$

- Three free fit parameters m, p, q
- Fits only very close to maximum
- Extraction of χ_L^{max}

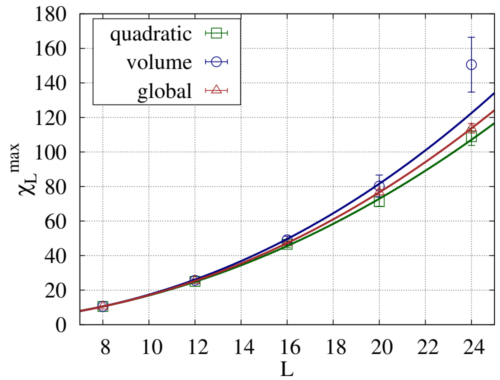


Fits to χ_L^{max}

- Fit function from finite size scaling:

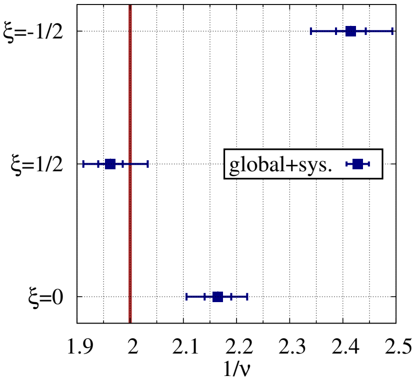
$$f_{\mathrm{max}}(L;\xi) = \left[A_1 \cdot \left(L[\log L]^{\xi}\right)^{1/\nu}\right]$$

- 2 free fit parameters: A_1, ν, parameter ξ must be $1/2$
- All methods agree within errors



Results of the O(4)-model study

- Comparison of $1/\nu$ for different $\log$ exponents ξ with its prediction of $1/\nu = 2$
- Systematics taken into account: variation of fit interval, volumes, and fit method
- Analysis in full agreement with expectation



Fermions on the lattice

- Fermion action:

$$S_\Psi = \sum_{x,x'} \bar{\Psi}_x \left[\mathcal{D}_{\text{ov}} + y P_+ \Phi^\alpha \theta_\alpha^\dagger \hat{P}_+ + y P_- \Phi^\alpha \theta_\alpha \hat{P}_- \right]_{x,x'} \Psi_{x'}$$

  with $\theta_{1,2,3} = -i\tau_{1,2,3}$ and $\theta_4 = 1_{2\times 2}$
  and the chiral projectors $P_\pm$ and $\hat{P}_\pm$

- Overlap operator $\mathcal{D}_{\text{ov}}$ usually numerically very expensive but here **no** gauge fields [Kaplan 1992; Neuberger, Lüscher, Hasenfratz 1998]

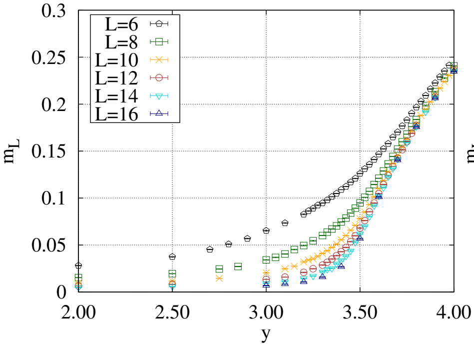
- Chirally-invariant lattice formulation

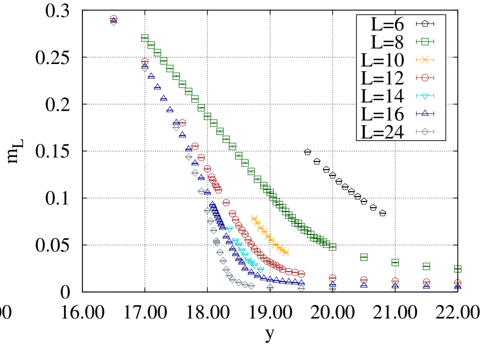
- Heavy mass degenerate quark doublet

- In the following: hopping parameter $\kappa = 0.06\,(0.00, 0.10)$ and quartic coupling $\hat{\lambda} \to \infty$ fixed and scan through Yukawa coupling y

Magnetization at small and large Yukawa couplings y

- Symmetric and broken phases easily distinguishable
- No jumps in magnetisation → phase transitions are of second order

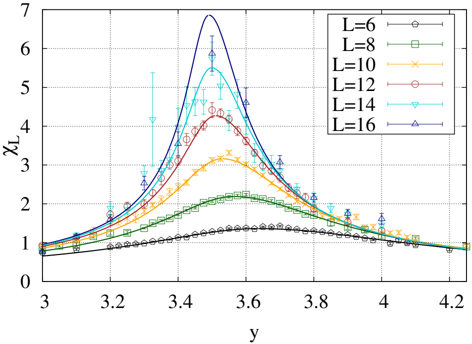


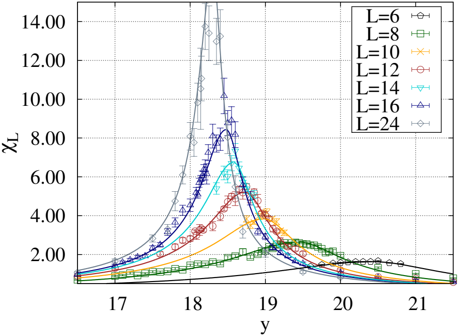


Susceptibility at at small and large Yukawa couplings y

- Peak shifts stronger at large y
- Global fits perform well

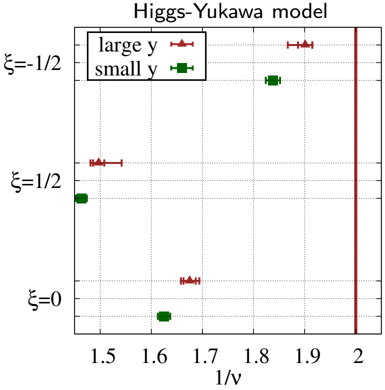
- At small y: more statistics and more points for L=14 and 16 needed
- At large y: more points for L=6, 10, and 14 far away from phase transition needed

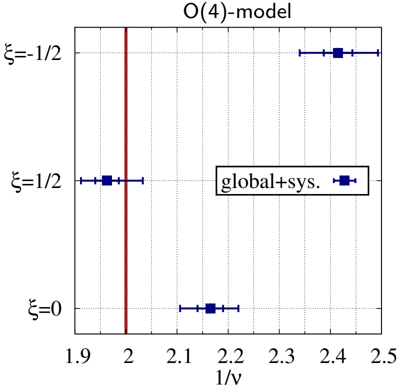




Comparison of Higgs-Yukawa model with O(4)-model

- ν agrees at small and large y $\rightarrow$ same universality class
- $\log$ exponent seems to be $-1/2$ if theory is trivial
  $\rightarrow$ different from O(4)-model
- Need of analytic computation of $\log$ exponent!





Spectrum observables

- Scale setting: $a = \frac{v_r}{246\ \text{GeV}}, \quad v_r = \frac{v}{\sqrt{Z_G}}, \quad v = \sqrt{2\kappa}\,\langle m_L \rangle$
- Higgs-/Goldstone boson masses from propagators:

$$0 = \Re\left(\left[G_{_{G/H}}(p^2)\right]^{-1}\right)\Bigg|_{p^2=-m^2_{G/H}}$$

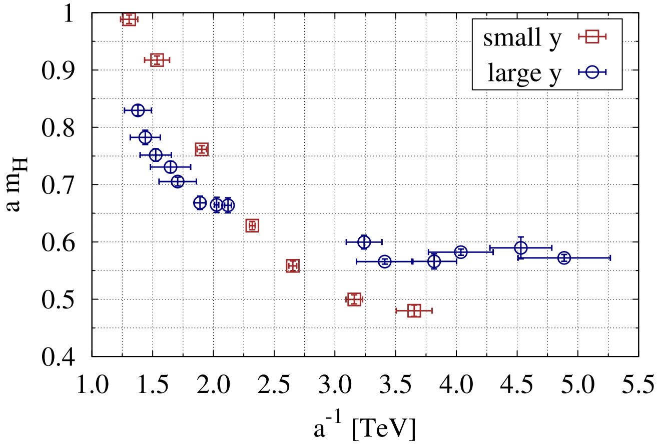
- Propagators are fitted according to a perturbative one-loop motivated expression
- Field renormalization constants:

$$Z^{-1}_{_{G/H}} = \frac{d}{d(p^2)}\Re\left(\left[G_{_{G/H}}(p^2)\right]^{-1}\right)\Bigg|_{p^2=-m^2_{G/H}}$$

- Fermion masses can be extracted from temporal correlation functions

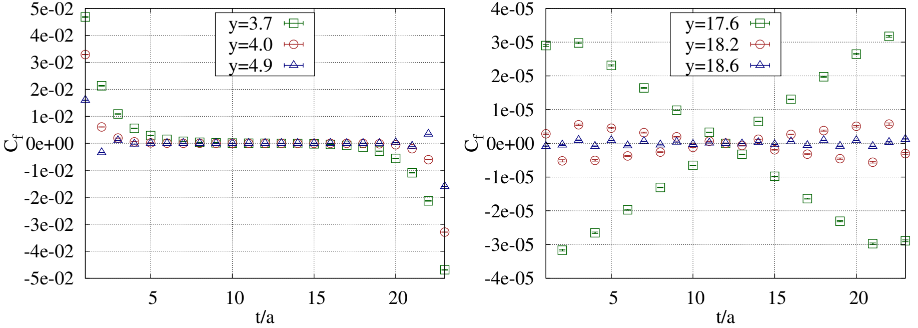
Higgs boson mass in dependence of cut-off (L=12)

- Very preliminary analysis - one volume, no systematic effects
- Higgs-boson mass dependence of cutoff similar for both regions



Fermion correlation functions (L=12)

- At small y close to phase transition expected $\sinh$ behaviour
- Deep in broken phase fermions become heavier than cutoff $\rightarrow$ doublers
- Happens also for L=16 and 24
- Further investigation necessary



Summary and outlook

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- $\log$ corrections might differ in Higgs-Yukawa model from O(4)-model
- Strong evidence that both phase transitions at $y \neq 0$ are in same universality class
- First spectrum calculations started

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Outlook:

- More statistics and data points are needed at small and large y
- Analytic calculation of $\log$ corrections in Higgs-Yukawa model
- Intensify spectrum calculations

Thank you!

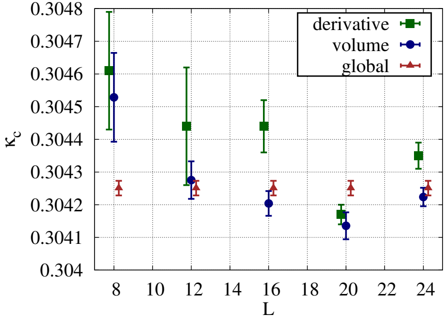
Backup

Fits to κ_c^L in O(4)-model

- Fit function from finite size scaling:

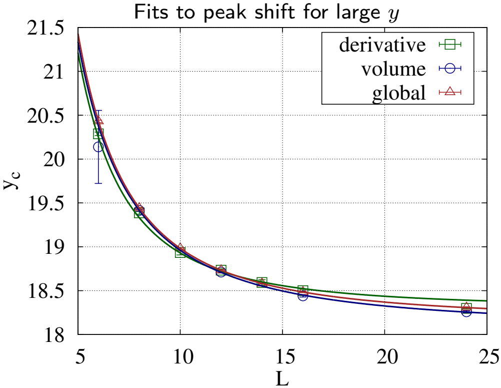
$$f_{\text{pos}}(L;\xi) = \left[C \cdot (L^{-1}[\log L]^{-\xi})^{1/\nu} + \kappa_c^\infty\right]$$

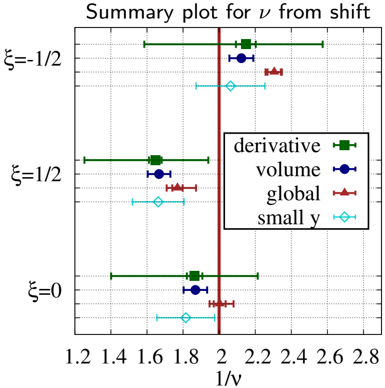
- 3 free fit parameters: C, ν, κ_c^∞, parameter ξ must be $1/2$
- Global fit procedure cannot resolve shift
- Errors are big and shift is very small $\rightarrow$ no reasonable fit possible



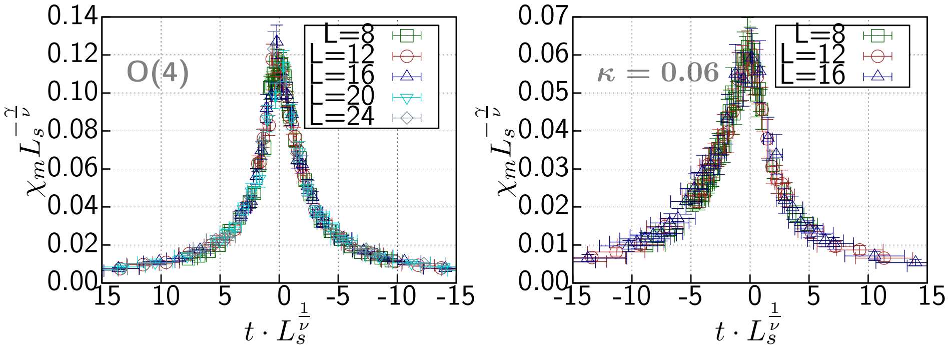
Critical exponent ν from peak shift at large y

- No systematic effects for volume method (only 4 volumes)
- Only global fit to small y without systematic study
- ν agrees at small and large y
- No information about $\log$ exponent





Rescaled susceptibility



Susceptibility at $\kappa = 0.00$ and $\kappa = 0.10$