Stabilizing the electroweak vacuum by higher dimensional operators in a Higgs-Yukawa model

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July 29th, 2013

Outline

Higgs boson and vacuum stability

- The discovery of *the/a* Higgs boson with a mass of ≈ 126 GeV may have impact on the stability of the electroweak vacuum
- Theoretical predictions suggest a lower bound for the stability of the vacuum in the range of around 129 GeV [Degrassi et.al. 2012]

  [Plenary talk by J. R. Espinosa]
- Results from evolutions of all couplings up to the Planck scale
- The meta stability occurs due to the quartic self coupling of the scalar field turning negative below the Planck scale
- Assumes nothing but Standard Model (SM)
- Any new physics might change this picture
- However, other results suggest stability even in the SM [Jegerlehner 2013]

Stabilizing the vacuum

- Higgs sector is a cutoff theory (triviality) $\rightarrow \lambda_6(\phi^\dagger\phi)^3$ term in the action is allowed
- with $\lambda_6 > 0$, the Higgs potential is stable even with negative λ
- Could emerge as a low energy effect of some higher scale physics
- Very easy extension of the SM
- We want to investigate the possibility to alter the Higgs boson mass bound in a Higgs-Yukawa model
- As a first step we want to map out the phase structure with a $\lambda_6(\phi^\dagger\phi)^3$ term included
    - Non-perturbatively via lattice simulations and perturbatively by the constrained effective potential (CEP)

Higgs-Yukawa model

$$S^{\text{cont}}[\bar{\psi}, \psi, \varphi] = \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi\right)^\dagger \left(\partial^\mu \varphi\right) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda \left(\varphi^\dagger \varphi\right)^2 + \lambda_6 \left(\varphi^\dagger \varphi\right)^3 \right\}$$

$$+ \sum_f \int d^4x \left\{ \bar{t} \not\partial t + \bar{b} \not\partial b + y_b \bar{\psi}_L \varphi \, b_R + y_t \bar{\psi}_L \tilde{\varphi} \, t_R + h.c. \right\}$$

Higgs-Yukawa model

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$$+ \sum_f \int d^4x \left\{ \bar{t}\partial\!\!\!/ t + \bar{b}\partial\!\!\!/ b + y_b\bar{\psi}_L\varphi\, b_R + y_t\bar{\psi}_L\tilde{\varphi}\, t_R + h.c. \right\}$$

$$S_B^{\text{lat}}[\Phi] = -\kappa\sum_{x,\mu}\Phi_x^\dagger\left[\Phi_{x+\mu}+\Phi_{x-\mu}\right] + \sum_x \Phi_x^\dagger\Phi_x +$$
$$\hat{\lambda}\sum_x\left[\Phi_x^\dagger\Phi_x - N_f\right]^2 + \hat{\lambda}_6\sum_x\left[\Phi_x^\dagger\Phi_x\right]^3$$

with:

$$\varphi = \sqrt{2\kappa}\begin{pmatrix}\Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3\end{pmatrix}, \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad \lambda_6 = \frac{\hat{\lambda}_6}{8\kappa^3}, \quad m_0^2 = \frac{1 - 2N_f\hat{\lambda} - 8\kappa}{\kappa}$$

Constrained effective potential in the broken phase

[O'Raifeartaigh, et al. 2007; Gerhold et.al. 2009]

- In the broken phase, the CEP only depends on the zero mode $\breve{v}$, determining the vev
- The scalar doublet can be decomposed in the Higgs and Goldstone modes

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- The scalar doublet can be decomposed in the Higgs and Goldstone modes

$$U(\breve{v}) = \frac{1}{2}m_0^2\breve{v}^2 + \lambda\breve{v}^4 + \lambda_6\breve{v}^6 + U_F(\breve{v}) + 6\lambda\breve{v}^2(P_H + P_G) + \lambda_6\breve{v}^4(15P_H + 9P_G) + \lambda_6\breve{v}^2(45P_H^2 + 54P_H P_G + 45P_G^2)$$

With the propagator sums $P_{G/H}$ given by:

$$P_G = \sum_{p\neq 0}\frac{1}{\hat{p}^2}, \qquad P_H = \sum_{p\neq 0}\frac{1}{\hat{p}^2 + m_H^2}$$

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$$+ \lambda_6\breve{v}^4(15P_H + 9P_G) + \lambda_6\breve{v}^2(45P_H^2 + 54P_H P_G + 45P_G^2)$$

The fermionic contribution is given by:

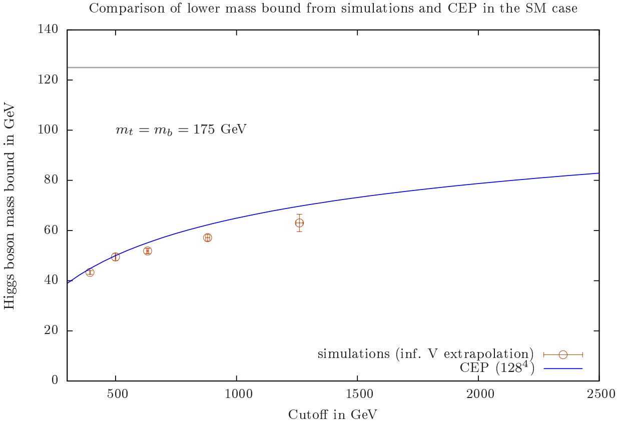
$$U_F(\breve{v}) = -\frac{2\,N_f}{V}\left[\sum_p \log\left|\nu(p) + y_t\cdot\breve{v}\cdot\left(1-\frac{1}{2\rho}\right)\nu(p)\right|^2 + \right.$$
$$\left.\sum_p \log\left|\nu(p) + y_b\cdot\breve{v}\cdot\left(1-\frac{1}{2\rho}\right)\nu(p)\right|^2\right]$$

The cutoff, vev and Higgs boson mass are given by:

$$\Lambda = \frac{246\ \text{GeV}}{vev}, \qquad U'(vev) = 0, \qquad U''(vev) = m_H^2$$

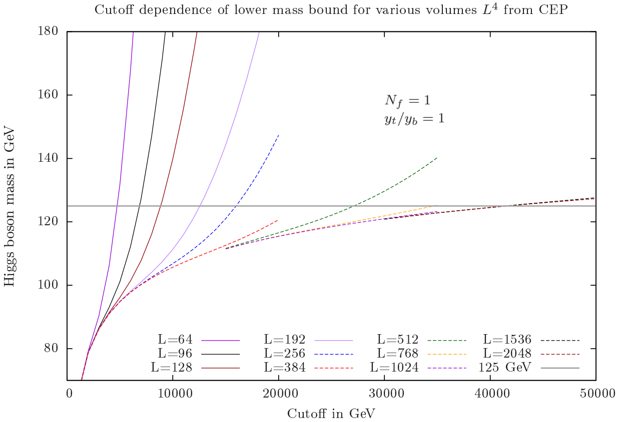
Mass bound

In the SM case ($\lambda_6 = 0$) the lowest accessible Higgs boson mass in a stable potential is obtained for $\lambda = 0$ [Gerhold et.al. 2009]



Cutoff dependence in the CEP $(N_f = 1, y_t = y_b)$

When is a 125 GeV Higgs boson incompatible with the Higgs-Yukawa-Model?



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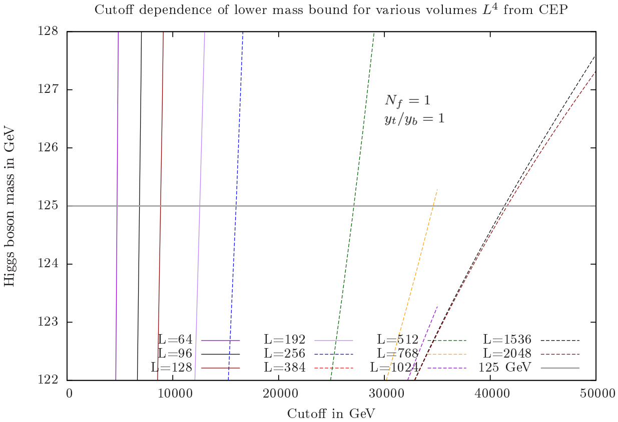
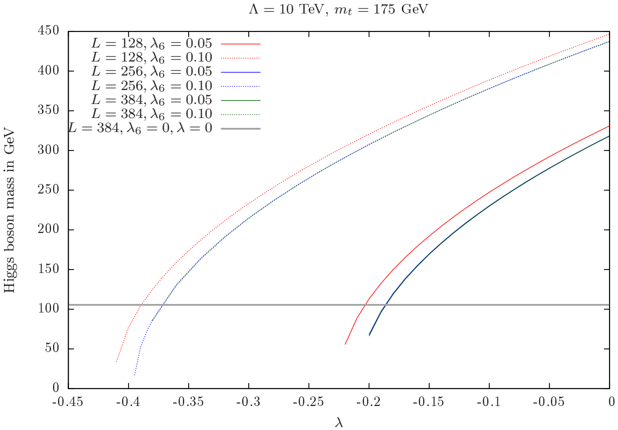


Mass bound for $\lambda_6 \neq 0$

- For $\lambda_6 > 0$ the potental is eventually stable, even with $\lambda < 0$
- How to define lowest accessible Higgs boson mass?
- Possibility: Demanding positivity of $U''(\breve{v})$ in the scaling regime ($\breve{v} < \breve{v}_{max} \approx 0.5$)
- Problem: determining that limit will end up in a potential that has negative curvature directly after $\breve{v}_{max}$

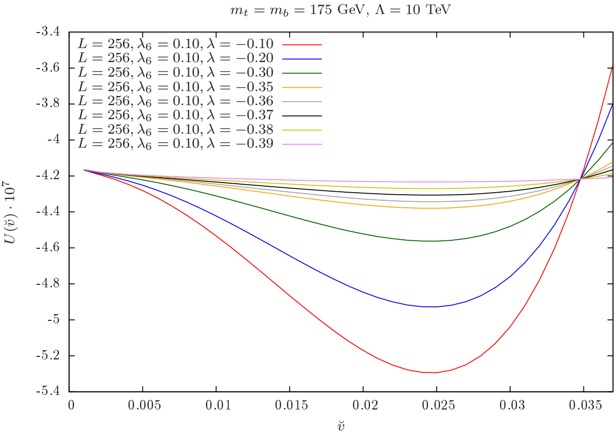
Higgs boson mass with non-zero λ_6

Naively (close your eyes and compute m_h), the Higgs boson mass can be reduced down to zero:



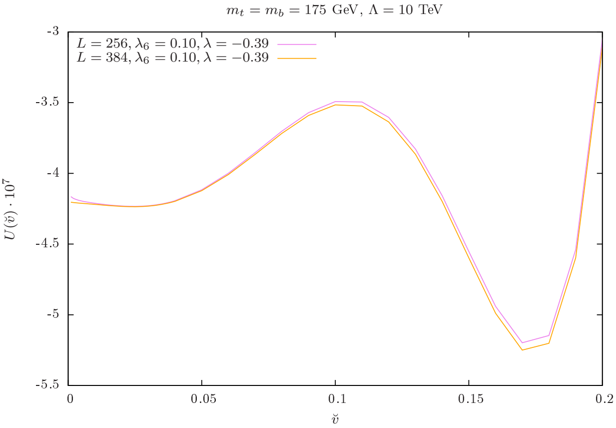
Looking at the potential

Clear from looking at the potential:



Looking at the potential

But...



HowTo: Lower bound with λ_6

- Second minimum might spoils further decrease of the Higgs boson mass
- Developement of second minimum corresponds to 1st order phase transition
- Not SM-like $\rightarrow$ avoid for mass determination
- Scan the phase structure of the model with finite λ_6
- determine Higgs boson mass in the regime of 2nd order phase transition but as close to the 1st order transition as possible (future)

CEP for determination of phase strucure

For determining the phase structure in the CEP, also the staggered mode ($p_s = (\pi, \pi, \pi, \pi)$) has to be taken into account

$$U(m,s) = U^{\mathsf{t}}(m,s) + U^{\mathsf{f}}(m,s) + U^{\mathsf{d}}(m,s) + U^{1}(m,s)$$

$$U^{\mathsf{t}} = -8\kappa\left(m^2 - s^2\right) + \left(m^2 + s^2\right) + \hat{\lambda}\left(m^4 + s^4 + 6m^2s^2 - 2N_f\left(m^2 + s^2\right)\right)$$
$$+ \hat{\lambda}_6\left(m^6 + s^6 + 15\left(m^4s^2 + m^2s^4\right)\right)$$

$$U^{\mathsf{d}} = -\frac{1}{2V}\sum_{0\neq p\neq p_s}\log\Big(2 - 4\hat{\lambda}N_f - 4\kappa\sum_{\mu}\cos(p_\mu)$$
$$+ 8\hat{\lambda}\left(m^2 + s^2\right) + 18\,\hat{\lambda}_6\left(m^4 + s^4 + 6\,m^2s^2\right)\Big)$$

$$U^{1} = 32\,\left(\hat{\lambda} + \hat{\lambda}_6\left(9\left(m^2 + s^2\right)\right)\right)\tilde{P}_B^2 + 384\,\hat{\lambda}_6\tilde{P}_B^3$$

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&\quad + \hat{\lambda}_6\left(m^6 + s^6 + 15\left(m^4 s^2 + m^2 s^4\right)\right) \\
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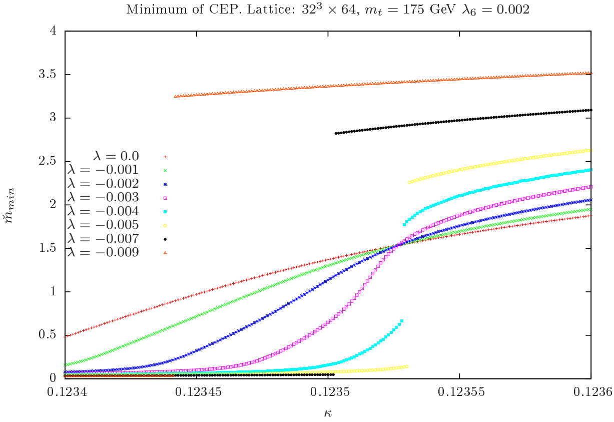
\end{aligned}
$$

$$
\tilde{P}_B = \frac{1}{V}\sum_{0\neq p\neq p_s}\frac{1}{2 - 4\hat{\lambda}N_f - 4\kappa\sum_{\mu}\cos(p_\mu) + 8\hat{\lambda}\left(m^2 + s^2\right) + 18\,\hat{\lambda}_6\left(m^4 + s^4 + 6\,m^2 s^2\right)}
$$

Minimum of U w.r.t. m and s determines the ground state.

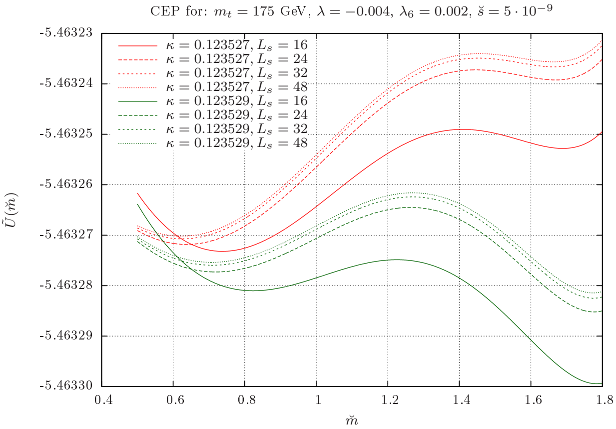
Phase structure in CEP

Qualitatively the CEP gives the expected picture



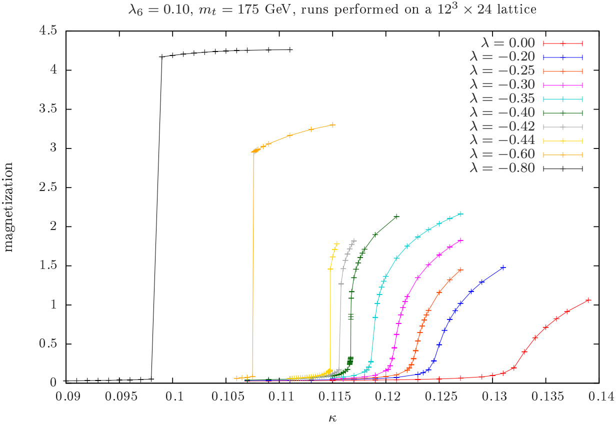
Potential around the phase transition

The potential shows the expected two minima



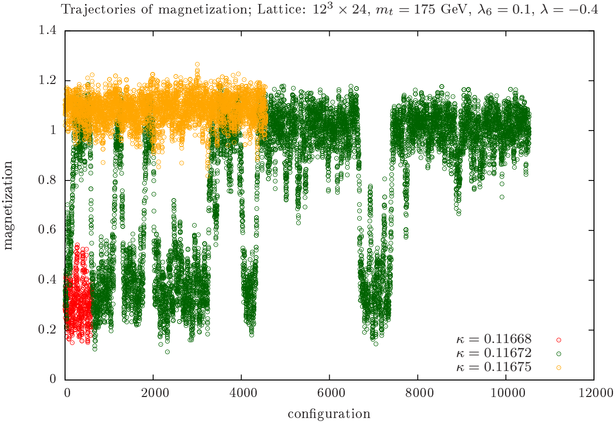
Phases from the simulation

However, the simulations look a little bit different...



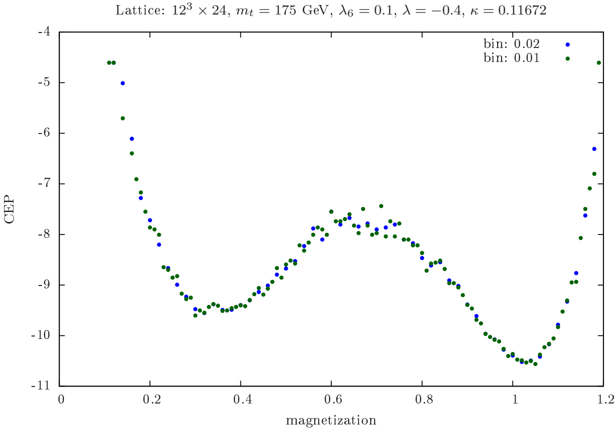
Signs for first order phase transition

Meta stabilities in one ensemble:



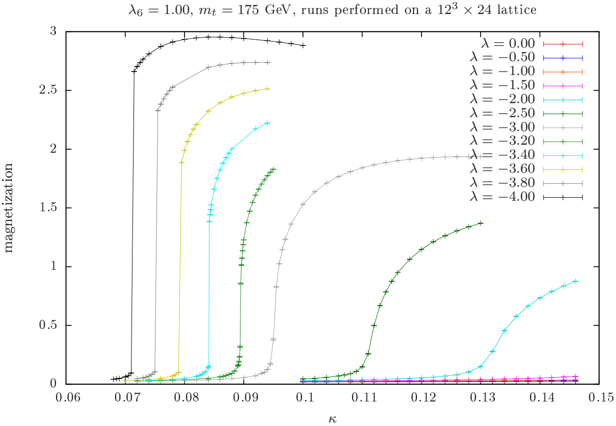
CEP from simulations

$$U(m) = -\log\left[\text{Frequency of occurence of a magnetization}\right]$$



Phases from the simulation

A non-perturbative λ_6



Summary

Conclusion:

- Showed perturbatively, that the lower Higgs boson mass bound can be further decreased by adding a ϕ^6 term
- Demonstrated how in principle a lower mass bound could be obtained non-perturbatively
- Results suggest the existence of a tri-critical point

Summary

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Outlook:

- Establish the nature of the observed phase transitions (FSSA)
- Determine the Higgs boson mass in the vicinity of the tri-critical point
- Find a better theoretical prescription for the CEP

BACKUP

Constrained effective potential

Ground state

$$\Phi^g_x = m_\Phi \cdot \hat{\Phi}_1 + s_\Phi \cdot e^{ip_s \cdot x} \cdot \hat{\Phi}_2 \qquad p_s = (\pi, \pi, \pi, \pi),$$

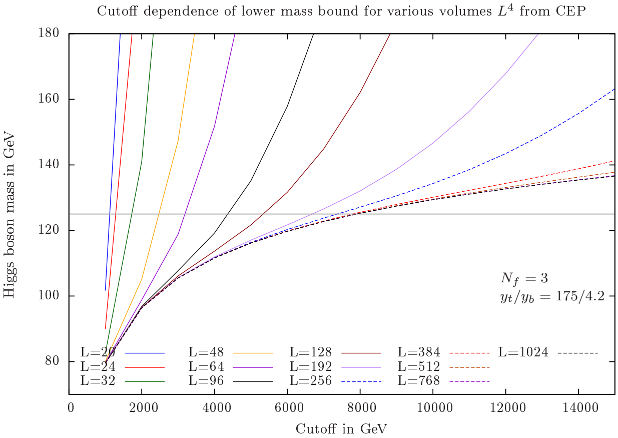
Definition of the potential:

$$V \cdot U(m,s) = -\log \left(\int D\Psi D\bar{\Psi} \left[\prod_{0 \neq p \neq p_s} d\tilde{\Phi}_p \right] e^{-S[\Psi,\bar{\Psi},\Phi]} \Big|_{\substack{\tilde{\Phi}_0 = \sqrt{V} m_\Phi \\ \tilde{\Phi}_{p_s} = \sqrt{V} s_\Phi}} \right)$$

$$\langle \mathcal{O}(m,s) \rangle = \frac{1}{Z} \int \mathrm{d}\Phi_0 \, \mathrm{d}\Phi_s \, \mathcal{O} \, e^{-V \cdot U(m,s)}$$

Cutoff dependence in the CEP (more physical setup)

Doing the same for $N_f = 3$ and the physical mass splitting



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