Progress in Gauge-Higgs Unification on the Lattice (I)

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Introduction

Gauge Higgs Unification Model

- 5-dimensional gauge theory
- The Higgs fields is identified with some of the extra dimensional components of the gauge fields.
- Higgs mass and potential are finite.(Hierarchy problem is solved) [Gersdorff, Irges, Quiros 2002] [Antoniadis, Benakli and Quiros 2001] [Cheng, Matchev and Schmaltz 20002][Irges and Knechtli 2006, 2007]
- Higgs potential can break gauge symmetry.(Origin of Spontaneous Symmetry Breaking SSB) [Hosotani 1983]

Lattice Gauge theory

- You can study non-perturbative region by Monte Carlo and Mean-Field.
- You can take gauge-invariant ultra-violet cut off $(\Lambda = 1/a)$.

Is there SSB without fermion in non-perturbative region ?

- Torus boundary condition ; No, there is no SSB [Irges and Knechtli 2009]
- Orbifold boundary conditions ; Yes! there is SSB [Irges, Knechtli and Yoneyama 2012]

$\longrightarrow$ I will talk about dimensional reduction and continuum limit

The Orbifold Boundary Conditions

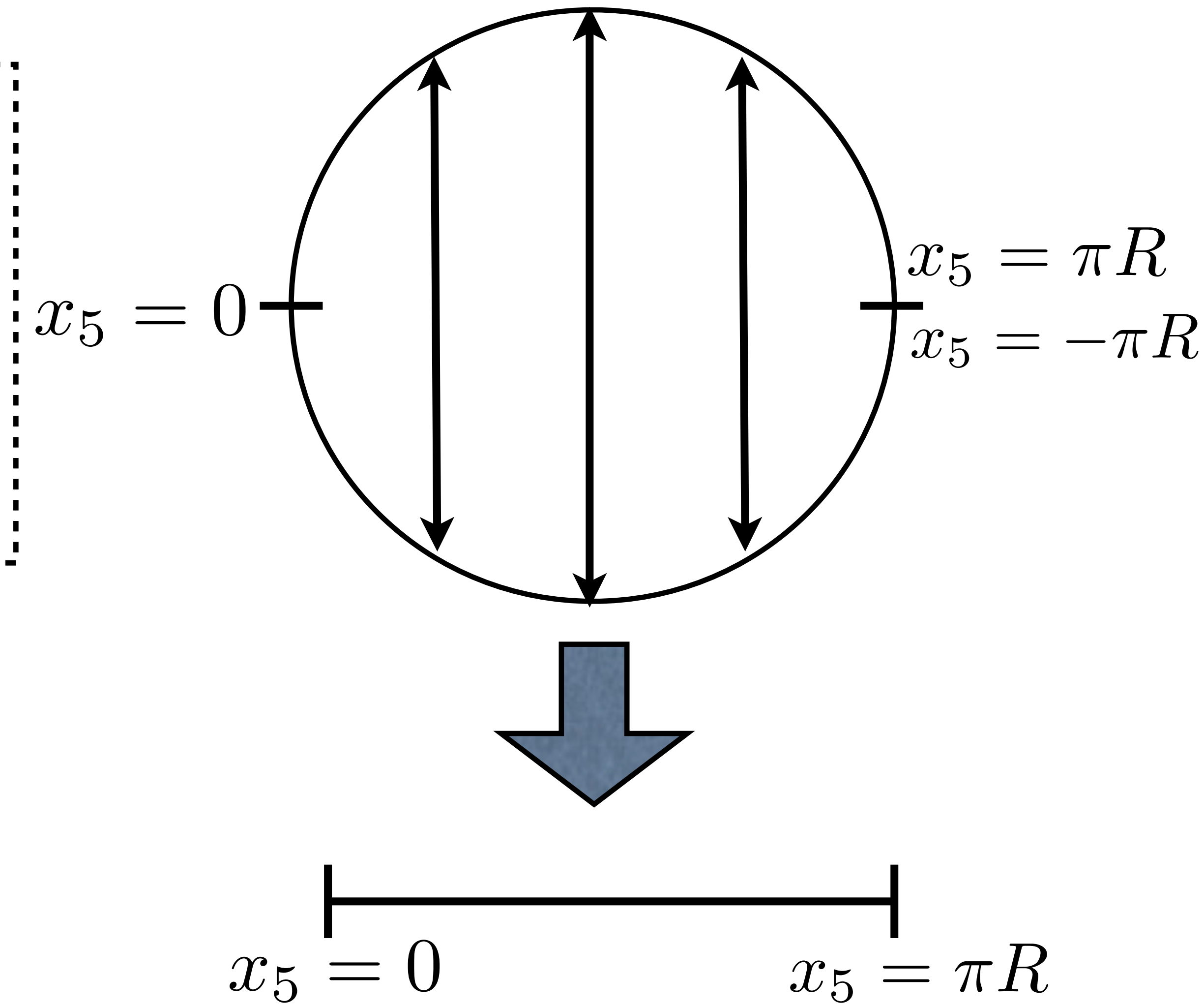
Reflection

$$\mathcal{R} \;:\quad z = (x_\mu, x_5) \;\rightarrow\; \bar{z} = (x_\mu, -x_5)$$

$$A_M(z) \;\rightarrow\; \alpha_M A_M(\bar{z}),\;\; \alpha_\mu = 1,\;\; \alpha_5 = -1$$

Group conjugation

$$\mathcal{C}:\quad A_M(z) \rightarrow g A_M(z) g^{-1} \qquad ,\; g^2 \in SU(N)$$



$$\mathcal{T} = \mathcal{RC}:\quad A_M(z) \rightarrow \alpha_M g A_M(\bar{z}) g^{-1}$$

Orbifold projection

$$A_M(z) = \alpha_M \; g A_M(\bar{z}) g^{-1} = \mathcal{T} A_M(z)$$

On the boundary $(x_5 = 0, \pi R)$, $z = \bar{z}$

$$A_M(z) = \alpha_M g A_M(z) g^{-1}$$

→ only even components A_μ^a and $A_5^{\hat{a}}$ are $\neq 0$

generator

$$g T^a g^{-1} = T^a$$

$$g T^{\hat{a}} g^{-1} = -T^{\hat{a}}$$

$SU(2)$ case, we choose $g = -i\sigma^3$.

Then, only A_5^1, A_5^2 and A_μ^3 are $\neq 0$ on the boundaries

The symmetry break explicitly by boundary conditions ; $SU(2) \rightarrow U(1)$

Mean-Field Expansion

[Drouffe and Zuber, 1983]

The partition function of SU(N) gauge theory on lattice

$$Z = \int \mathrm{D}U \; e^{S_G[U]}$$

$SU(N)$ gauge links U are replaced by $N \times N$ complex matrices V and Lagrange multipliers H

$$Z = \int \mathrm{D}V \int \mathrm{D}H e^{-S_{eff}[V,H]}$$

$$S_{eff} = S_G[V] + u(H) + (1/N)\mathrm{Re}\,\mathrm{tr}\{VH\}$$

$$e^{-u(H)} = \int \mathrm{D}U \; e^{-(1/N)\mathrm{Re}\,\mathrm{tr}\{UH\}}$$

Saddle point solution (background)

$$\bar{V}(n,M) = -\frac{\partial S_{eff}}{\partial H(n,M)}\Big|_{\bar{H}(n,M)}, \quad \bar{H}(n,M) = -\frac{\partial S_{eff}}{\partial V(n,M)}\Big|_{\bar{V}(n,M)}$$

$$\bar{V} \rightarrow \bar{v}_0 \mathbf{1} \qquad \bar{H} \rightarrow \bar{h}_0 \mathbf{1} \qquad S_{eff}[\bar{V},\bar{H}] = \mathrm{minimal}$$

Expansion in Gaussian fluctuations

$$H = \bar{H} + h \quad V = \bar{V} + v$$

covariant gauge fixing on v [Rühl, 1982]

Five dimensional SU(2) Lattice Gauge Theory for Orbifold

$T \times L^3 \times N_5$ lattice, $SU(2)$ gauge theory on orbifold boundary conditions

Wilson plaquette action

$$S_W[U_1, U_2] = S_{W1}[U_1] + S_{W2}[U_1, U_2]$$

$$\beta = \frac{2Na_4}{g_5^2}, \quad \gamma = \frac{a_4}{a_5} \quad \text{(tree level)}$$

Where

$$U_1 \in U(1), \; U_2 \in SU(2)$$

$$S_{W1}[U_1] = \frac{1}{4N}\frac{\beta}{\gamma}\sum_{n_\mu}\sum_{n_5=0,N_5-1}\sum_{\nu,\rho}\mathrm{tr}\{1 - U(n,\nu,\rho)\} \qquad \text{boundary}$$

$$S_{W2}[U_2, U_1] = \frac{1}{2N}\frac{\beta}{\gamma}\sum_{n_\mu}\sum_{n_5=1}^{N_5-1}\sum_{\nu,\rho}\mathrm{tr}\{1 - U(n,\nu,\rho)\} + \frac{\gamma \cdot \beta}{N}\sum_{n_\mu}\sum_{n_5=0}^{N_5-1}\sum_{\nu}\mathrm{tr}\{1 - U(n,\nu,5)\} \qquad \text{bulk}$$

gauge trans formation

on a boundary link

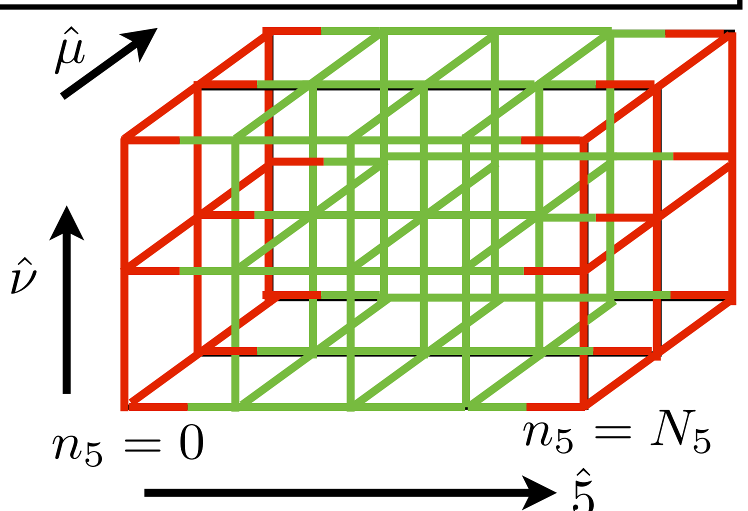
$$U(n, M) \longrightarrow \Omega^{(U(1))}(n)U(n,M)\Omega^{(U(1))\dagger}(n+\hat{M})$$

on the bulk link

$$U(n, M) \longrightarrow \Omega^{(SU(2))}(n)U(n,M)\Omega^{(SU(2))\dagger}(n+\hat{M})$$

on a link whose one end is in the bulk and the other touches the boundary

$$U(n, M) \longrightarrow \Omega^{(U(1))}(n)U(n,M)\Omega^{(SU(2))\dagger}(n+\hat{M})$$



The Mean-Field Background

The action is expressed with $N \times N$ complex matrices V and Lagrange multipliers H.

$$S_W[U_1, U_2] \rightarrow S_W(V) + u(H) + (1/N)\mathrm{Re}\ \mathrm{tr}\{VH\}$$

Then, we can get mean-field background from these minimization equations.

$$\bar{V}(n,M) = -\frac{\partial S_{eff}}{\partial H(n,M)}\Big|_{\bar{H}(n,M)}, \quad \bar{H}(n,M) = -\frac{\partial S_{eff}}{\partial V(n,M)}\Big|_{\bar{V}(n,M)}$$

parametrization of the mean-field background (saddle point solution)

for 4-dimensional links $(n_5 = 0, 1, ..., N_5)$

$$\bar{H}(n,\mu) = \bar{h}_0(n_5), \quad \bar{V}(n,\mu) = \bar{v}_0(n_5)$$

for extra-dimensional links $(n_5 = 0, 1, ..., N_5 - 1)$

$$\bar{H}(n,5) = \bar{h}_0(n_5 + 1/2), \quad \bar{V}(n,5) = \bar{v}_0(n_5 + 1/2)$$

The mean-field back ground depend on the position of 5th dimension.

The parametrization of the field

in the bulk

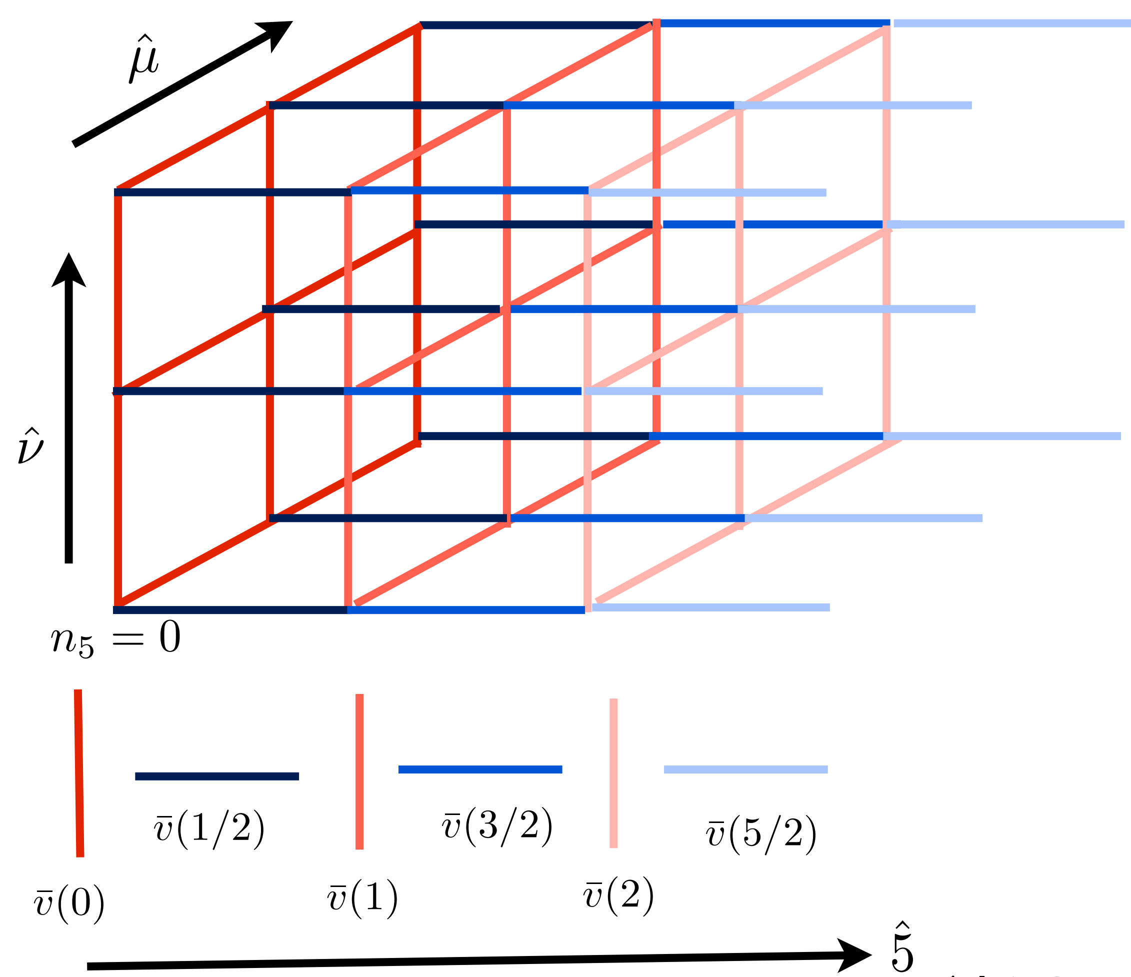
$$V(m,M) = v_0(n,M) + i\sum_{A=1}^{3} v_A(n,M)\sigma^A$$

$$H(m,M) = h_0(n,M) - i\sum_{A=1}^{3} h_A(n,M)\sigma^A$$

on the boundaries

$$V(n,M) = v_0(n,M) + iv_3(n,M)\sigma^3$$

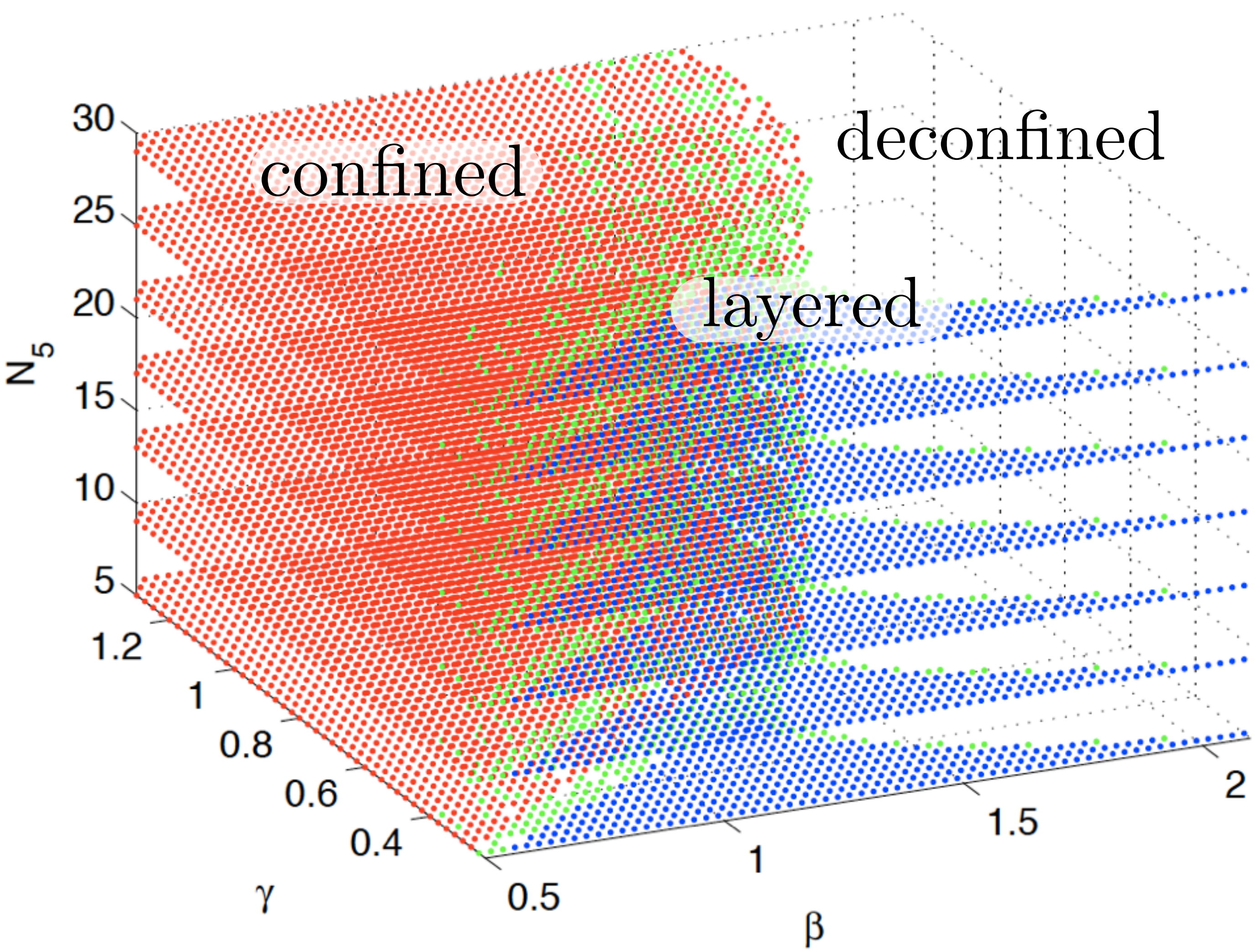
$$H(n,M) = h_0(n,M) - ih_3(n,M)\sigma^3$$



The Phase Diagram

The parameters are β, γ and N_5.

$$\beta = \frac{2Na_4}{g_5^2}, \quad \gamma = \frac{a_4}{a_5} \quad \text{(tree level)}$$



There are three phases.

deconfined phase: $\bar{v}_0(n_5) \neq 0$ for all n_5

confined phase: $\bar{v}_0(n_5) = 0$ for all n_5

layered phase:

$\bar{v}_0(n_5) \neq 0$ for $n_5 = 0, 1, \ldots N_5$ (along 4d hyperplanes)

$\bar{v}_0(n_5) = 0$ for $n_5 = \frac{1}{2}, \frac{3}{2}, \ldots, N_5 - \frac{1}{2}$ (along the 5th dimension)

$\gamma \gg 1$; compactification

$\gamma < 1$; localization

Mean-Field Expansion

Mean-field expansion of the expectation value of the observables

$$\mathcal{O}[V] = \mathcal{O}[\overline{V}] + \frac{\delta \mathcal{O}}{\delta V}\bigg|_{\bar{V}} v^{+} \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\bar{V}} v^2 + \dots$$

$$\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{1}{Z}\int Dv \int Dh \left(\mathcal{O}[\overline{V}] + \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\bar{V}} v^2 \right) e^{-S_{eff}[\bar{V},\bar{H}]+S^{(2)}[v,h]} \\
&= \mathcal{O}[\overline{V}] + \frac{1}{2}\frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\bar{V}} \frac{1}{Z}\int Dv \int Dh \; v^2 e^{-S_{eff}[\bar{V},\bar{H}]+S^{(2)}[v,h]} \\
&= \mathcal{O}[\overline{V}] + \frac{1}{2}\mathrm{tr}\left\{ \frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\bar{V}} K^{-1} \right\}
\end{aligned}$$

$$\frac{\delta^2 S_{eff}}{\delta V^2}\bigg|_{\bar{V},\bar{H}} v^2 = v_i K_{ij}^{(vv)} v_j = v^T K^{(vv)} v$$

$$\frac{\delta^2 S_{eff}}{\delta V \delta H}\bigg|_{\bar{V},\bar{H}} vh = v_i K_{ij}^{(vh)} h_j = v^T K^{(vh)} h$$

$$\frac{\delta^2 S_{eff}}{\delta V^2}\bigg|_{\bar{V},\bar{H}} v^2 = v_i K_{ij}^{(vv)} v_j = v^T K^{(hh)} v$$

where the lattice propagator

$$K = -K^{(vh)} K^{(hh)^{-1}} K^{(vh)} + K^{(vv)}$$

1st order of the expectation value of the observable

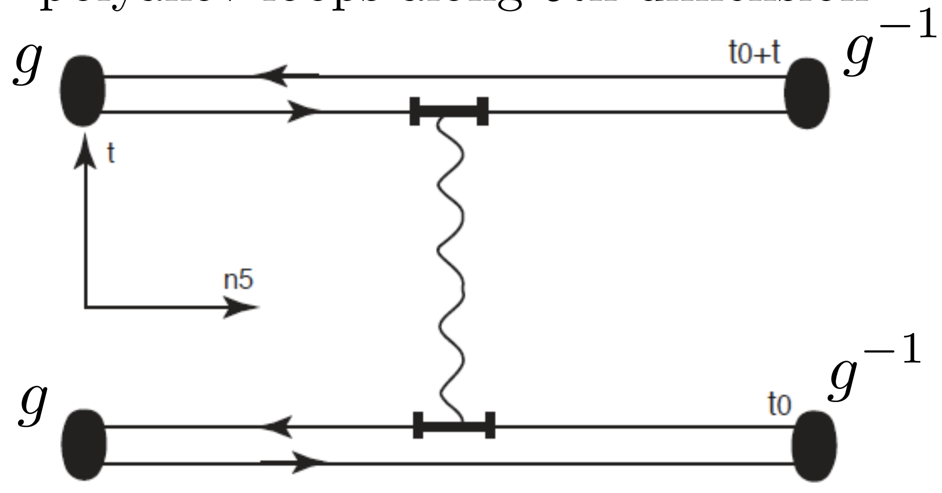
$$\boxed{\langle \mathcal{O} \rangle = \mathcal{O}[\overline{V}] + \frac{1}{2}\mathrm{tr}\left\{ \frac{\delta^2 \mathcal{O}}{\delta V^2}\bigg|_{\bar{V}} K^{-1} \right\}}$$

Observables

the expectation value of the observable

Higgs mass

polyakov loops along 5th dimension



$g = -i\sigma^3$

$$\mathcal{O}(t_0+t)\mathcal{O}(t_0) = \frac{1}{L^6 T}\sum_{t_0}\sum_{\vec{m}'\vec{m}''} \mathrm{tr}\left\{P^{(0)}(t_0,\vec{m}')\right\} \mathrm{tr}\left\{P^{(0)}(t_0+t,\vec{m}'')\right\}$$

$$C(t) =< \mathcal{O}(t_0+t)\mathcal{O}(t_0) > - < \mathcal{O}(t_0+t) >< \mathcal{O}(t_0) >$$

$$= C^{(0)}(t) + C^{(1)}(t) + \dots$$

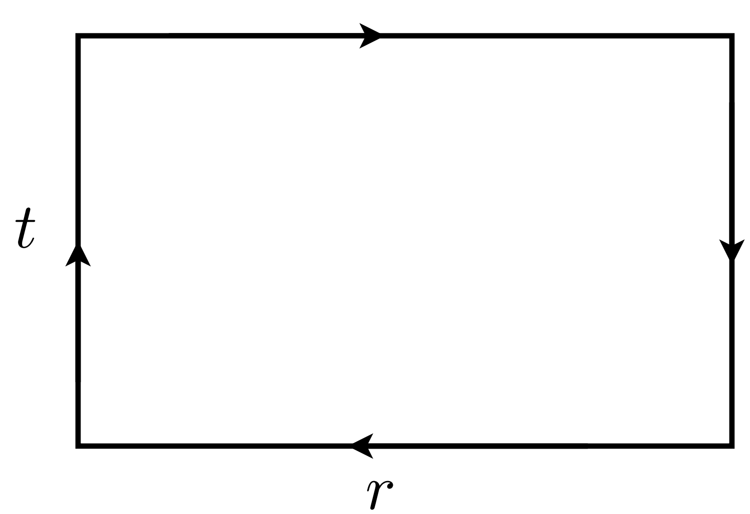
$$C^{(1)}(t) = \frac{1}{2}\mathrm{tr}\left\{\frac{\delta^{(1,1)}(\mathcal{O}(t_0+t)\mathcal{O}(t_0))}{\delta^2 v}K^{-1}\right\}$$

$$C^{(1)}(t) = \sum_{\lambda} c_\lambda e^{-E_\lambda t} \qquad E_0 = m_H$$

$$m_H = \lim_{t\to\infty} \ln\frac{C^{(1)}(t)}{C^{(1)}(t-1)}$$

Static potential

wilson loop



wilson loop : $< \mathcal{O}_W^{(t.r)} >$

$$t \to \infty : \quad e^{-V(r)t} \simeq < \mathcal{O}_W^{(t.r)} >$$

$V(r)$: static potential

Dimensional Reduction

Conditions for the dimensional reduction to 4-dimensions

1) The fit to $V(r) = const. + b\frac{e^{-m_z r}}{r}$ is possible with $m_Z \neq 0$.

This ensures that there is SSB, signaled by the presence of the massive $U(1)$ gauge boson. Otherwise the gauge boson is massless and only a Coulomb fit is possible.

2) The quantities $M_H = a_4 m_H$ and $M_Z = a_4 m_Z < 1$.

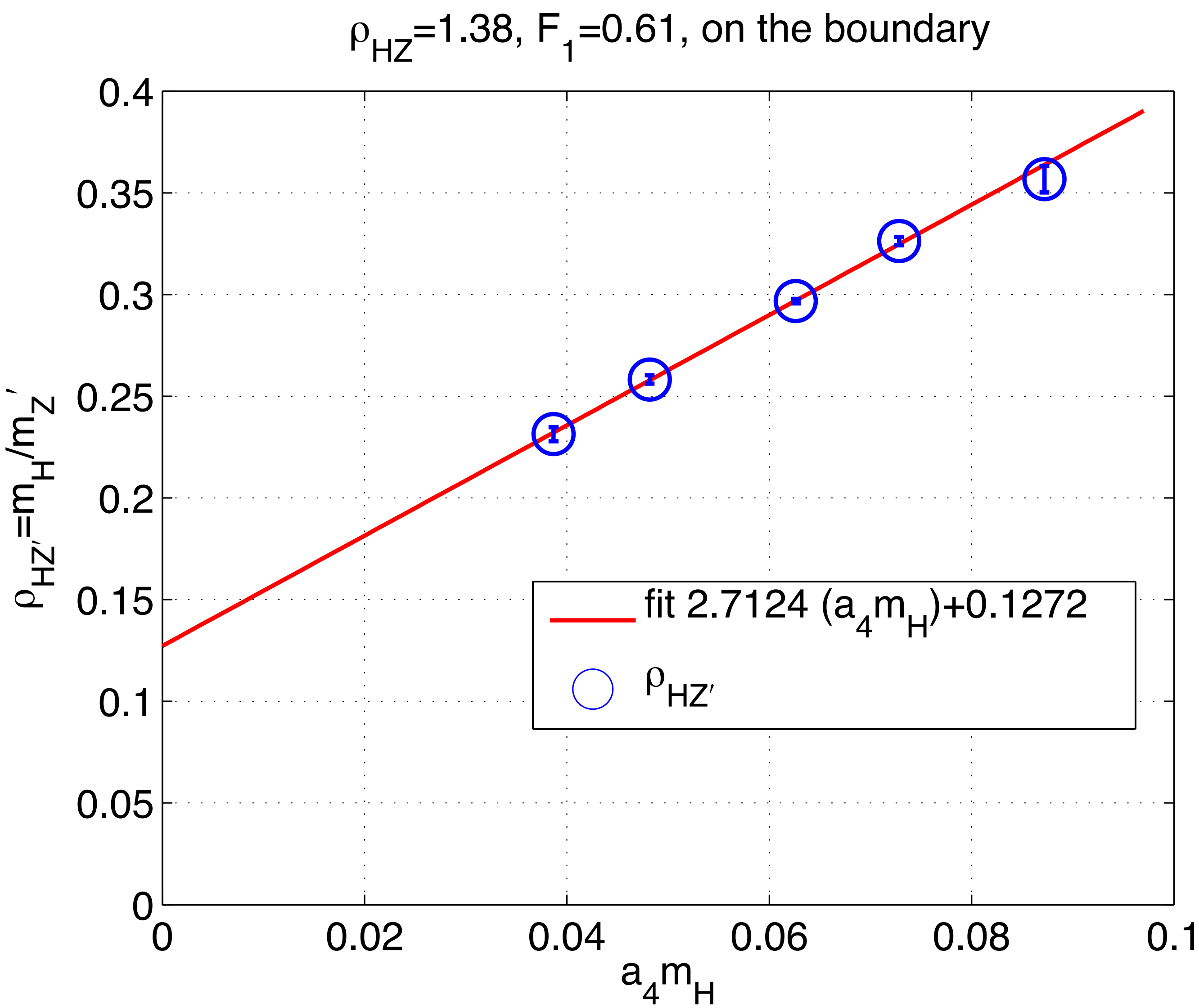
The observables are not dominated by the cut off.

3) $F_1 = m_H R < 1$ and $\rho_{MZ} = m_H/m_Z > 1$.

The Higgs and the Z are lighter than $1/R$ and the Higgs is heavier than the Z. We will target the value $\rho_{HZ} = 1.38$

The Line of Constant Physics

We have 3 observables: $M_H(\beta,\gamma,N_5)$, $M_Z(\beta,\gamma,N_5)$ and $M_{Z'}(\beta,\gamma,N_5)$



LCP; Change the Lattice spacing by keeping two dimensionless physical quantities fixed

$$\rho_{HZ} = \frac{m_H}{m_Z} = 126/91.19 = 1.38$$

$$F_1 = m_H \cdot R = 0.61$$

$N_5 \to \infty$ means $a_4 \to 0$ on the LCP

$$(\because N_5 = \frac{\pi R}{a_5} = \frac{\pi R}{a_4}\gamma)$$

Prediction of the Z' mass $N_5 \to \infty$ (i.e. $a_4 = 0$) on the LCP

$$\rho_{HZ'} = \frac{m_H}{m_{Z'}} = 0.1272$$

$$m_{Z'} = \frac{m_H}{\rho_{HZ'}} = 126/0.1272 = \underline{989\ [\mathrm{GeV}]}$$

Where χ^2 per degree of freedom is 0.025/3.

Summary

Setup

We studied 5-dimensional SU(2) pure gauge theory (Gauge Higgs Unification) on Euclidean lattice whose 5th dimension is orbifolded.

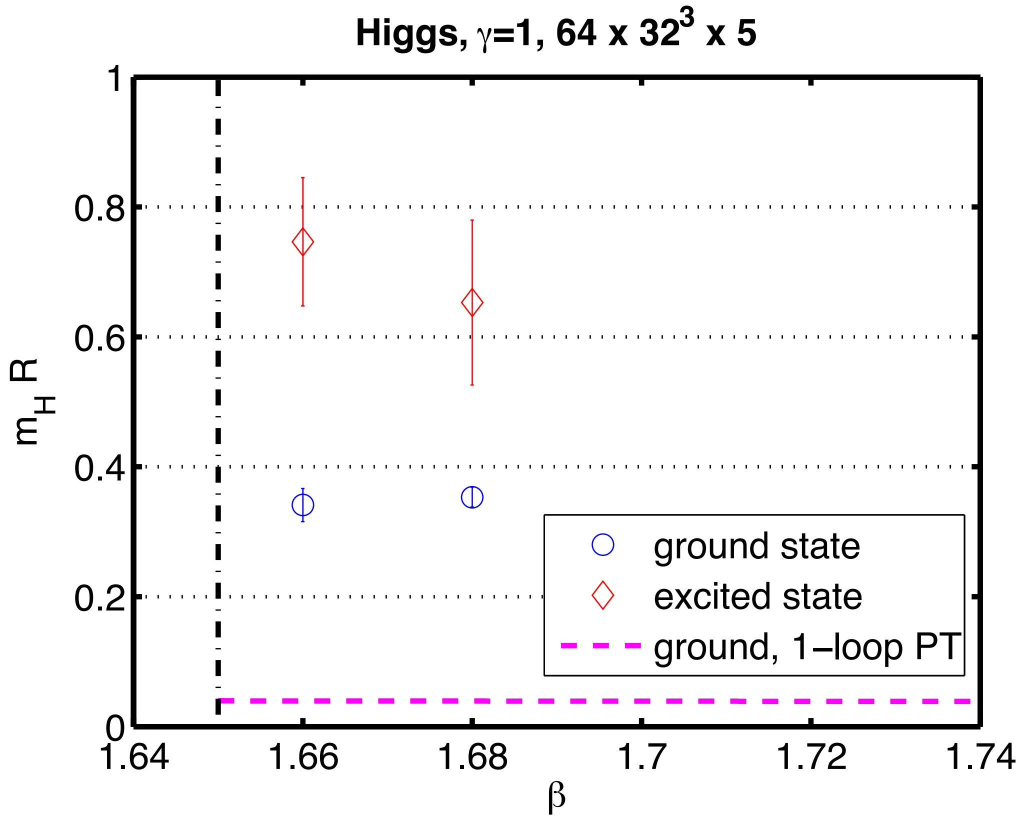
Result from mean-field expansion

We found there is SSB for orbifold boundary condition. This result is different from perturbative studies but support the Monte Carlo simulation [Irges and Knechtli 2007]. It means SSB occur even if there are no fermions and the Higgs mass can be large enough in the non-perturbative region. Also, it is possible to construct LCP's and predict the existence of a Z' state with a mass around 1 TeV by taking the continuum limit for small γ.

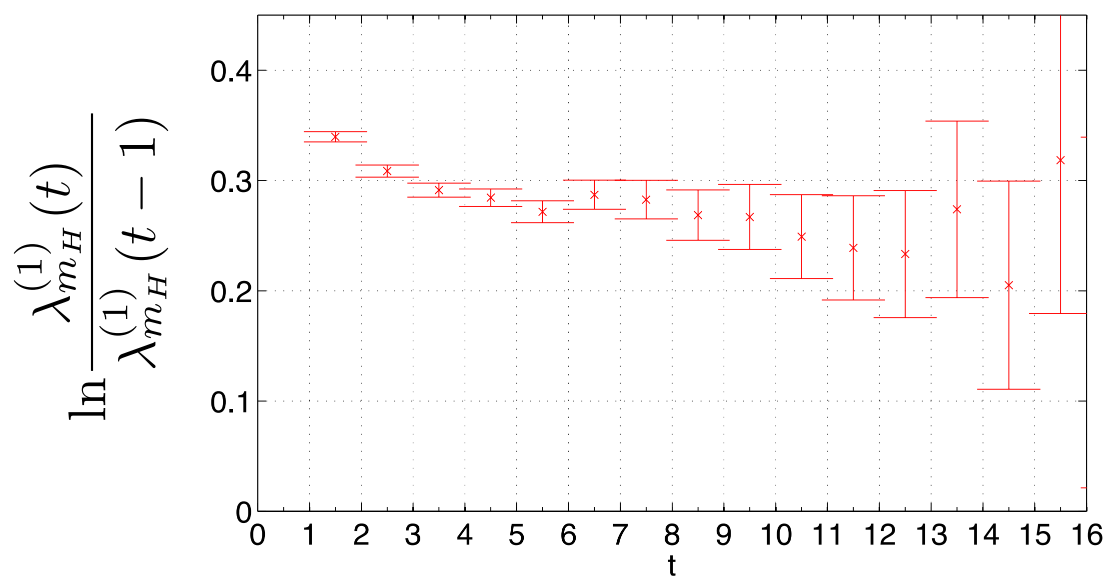
Monte Carlo Simulation

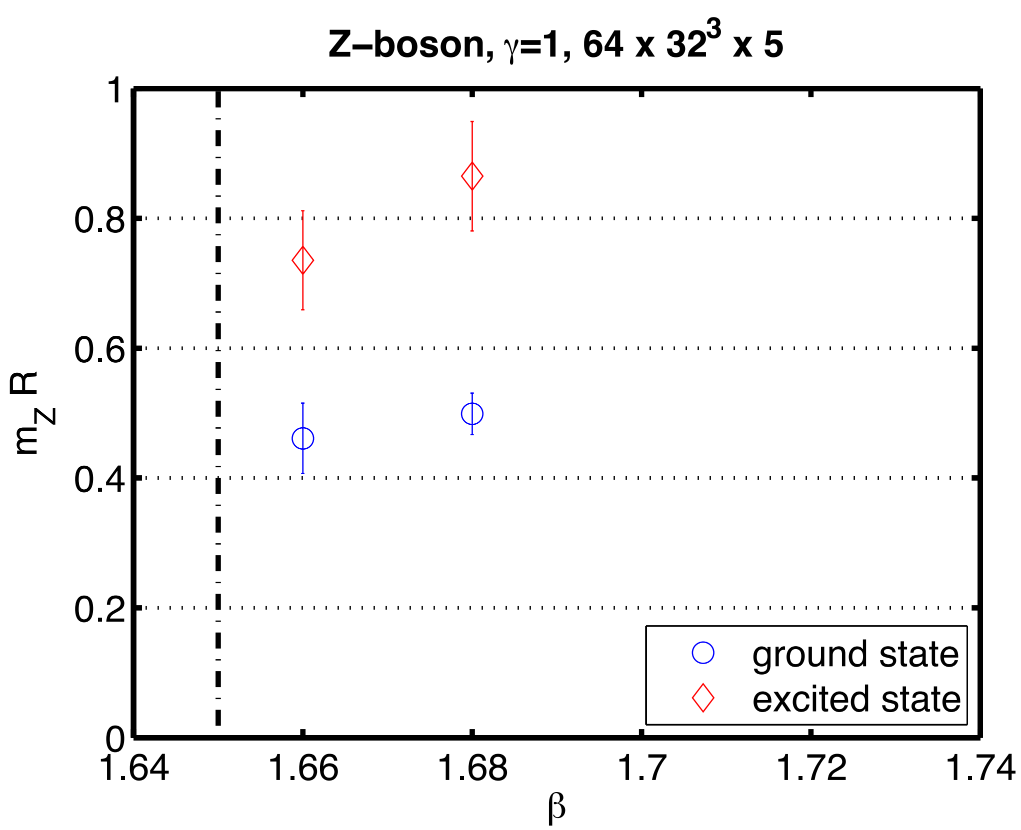
Although the mean-field expansion works better in higher dimensions, there is still uncertainty of the convergence of the mean-field expansion. Therefore, we have to confirm the mean-field result by Monte Carlo simulation. Now, we are working on Monte Carlo Simulation and already have some result.

Monte Carlo Simulations

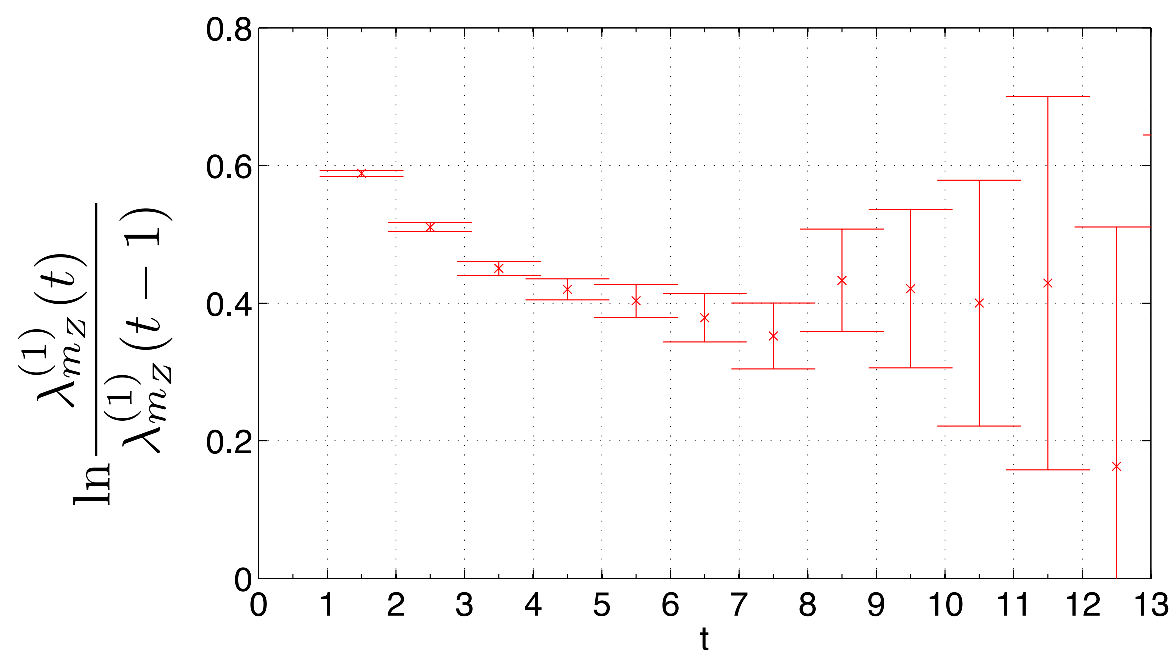


Higgs mass, $T = 64,\ L = 32,\ N_5 = 4,\ \gamma = 1,\ \beta = 1.68$





Z-boson mass, $T = 64,\ L = 32,\ N_5 = 4,\ \gamma = 1,\ \beta = 1.68$



$\longrightarrow$ The details will be in the next talk by Francesco Knechtli

The Mean-Field Background

parametrization of the mean-field background (saddle point solution)

for 4-dimensional links $(n_5 = 0, 1, ..., N_5)$

$$\bar{H}(n,\mu) = \bar{h}_0(n_5), \quad \bar{V}(n,\mu) = \bar{v}_0(n_5)$$

for extra-dimensional links $(n_5 = 0, 1, ..., N_5 - 1)$

$$\bar{H}(n,5) = \bar{h}_0(n_5 + 1/2), \quad \bar{V}(n,5) = \bar{v}_0(n_5 + 1/2)$$

minimization equations

$$\bar{V}(n,M) = -\frac{\partial S_{eff}}{\partial H(n,M)}\bigg|_{\bar{H}(n,M)}, \quad \bar{H}(n,M) = -\frac{\partial S_{eff}}{\partial V(n,M)}\bigg|_{\bar{V}(n,M)}$$

for $n_5 = 0$ (boundary)

$$\begin{aligned}\bar{v}_0(0) &= -u_1'(\bar{h}_0(0)) = \frac{I_1(\bar{h}_0(0))}{I_0(\bar{h}_0(0))} \\ \bar{h}_0(0) &= \beta_4\left[(d-2)(\bar{v}_0(0))^3 + \gamma^2(\bar{v}_0(1/2))^2\bar{v}_0(1)\right]\end{aligned}$$

for $n_5 = N_5$ (boundary)

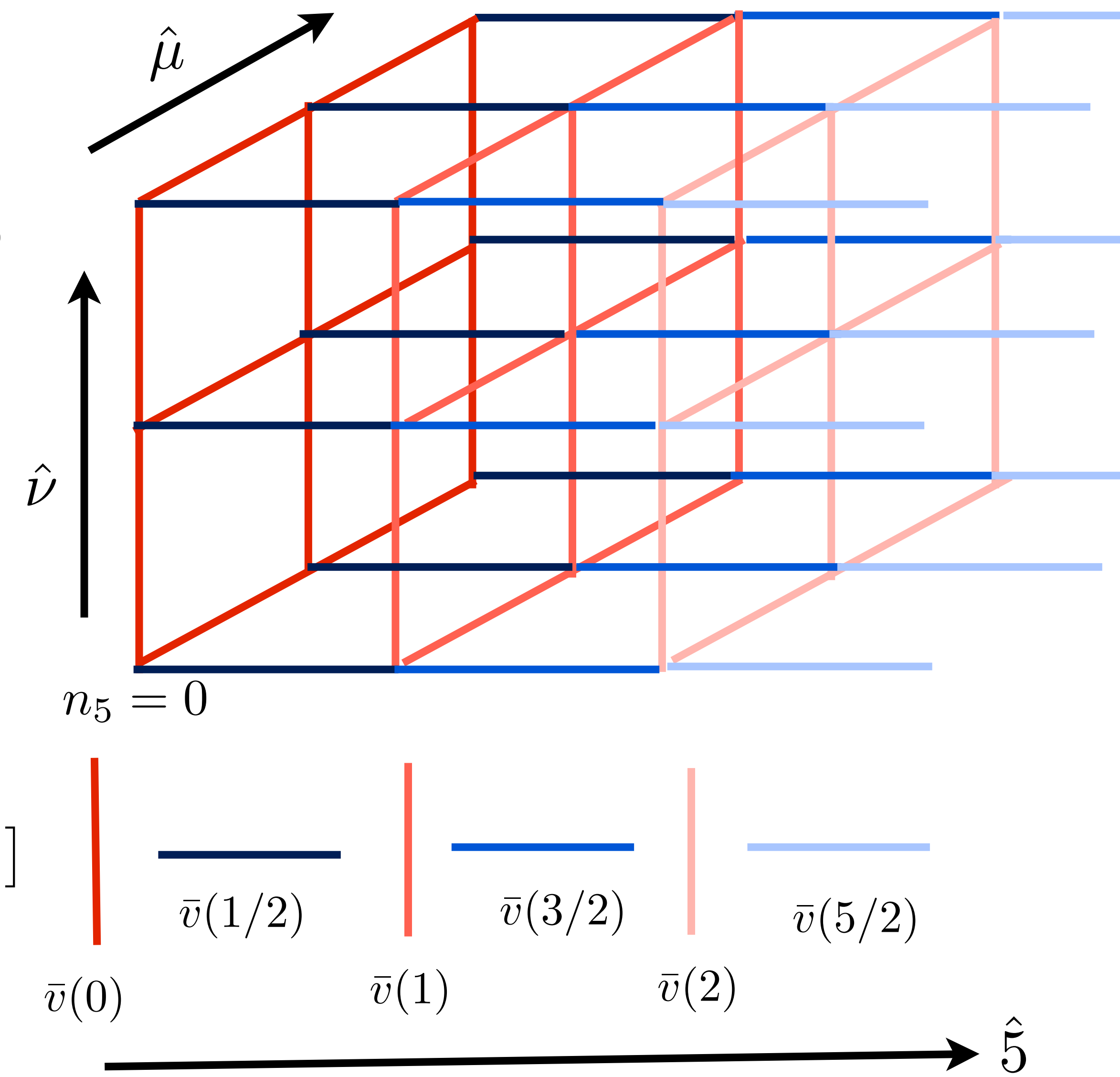
$$\begin{aligned}\bar{v}_0(N_5) &= -u_1'(\bar{h}_0(N_5)) = \frac{I_1(\bar{h}_0(N_5))}{I_0(\bar{h}_0(N_5))} \\ \bar{h}_0(N_5) &= \beta_4\left[(d-2)(\bar{v}_0(N_5))^3 + \gamma^2\bar{v}_0(N_5-1)(\bar{v}_0(N_5-1/2))^2\right]\end{aligned}$$

for $n_5 = 1, ..., N_5 - 1$ (4-dimensional links in the bulk)

$$\begin{aligned}\bar{v}_0(n_5) &= -u_2'(\bar{h}_0(n_5)) = \frac{I_2(\bar{h}_0(n_5))}{I_1(\bar{h}_0(n_5))} \\ \bar{h}_0(n_5) &= \beta_4\left[2(d-2)(\bar{v}_0(n_5))^3 + \gamma^2\Big((\bar{v}_0(n_5+1/2))^2\bar{v}_0(n_5+1) + \bar{v}_0(n_5-1)(\bar{v}_0(n_5-1/2))^2\Big)\right]\end{aligned}$$

for $n_5 = 0, ..., N_5 - 1$ (extra-dimensional links)

$$\begin{aligned}\bar{v}_0(n_5+1/2) &= -u_2'(\bar{h}_0(n_5+1/2)) = \frac{I_2(\bar{h}_0(n_5+1/2))}{I_1(\bar{h}_0(n_5+1/2))} \\ \bar{h}_0(n_5+1/2) &= 2\beta_5(d-1)\bar{v}_0(n_5)\bar{v}_0(n_5+1/2)\bar{v}_0(n_5+1)\end{aligned}$$



Five dimensional SU(2) Lattice Gauge Theory for Orbifold

$T \times L^3 \times N_5$ lattice, $SU(2)$ gauge theory on orbifold boundary condition

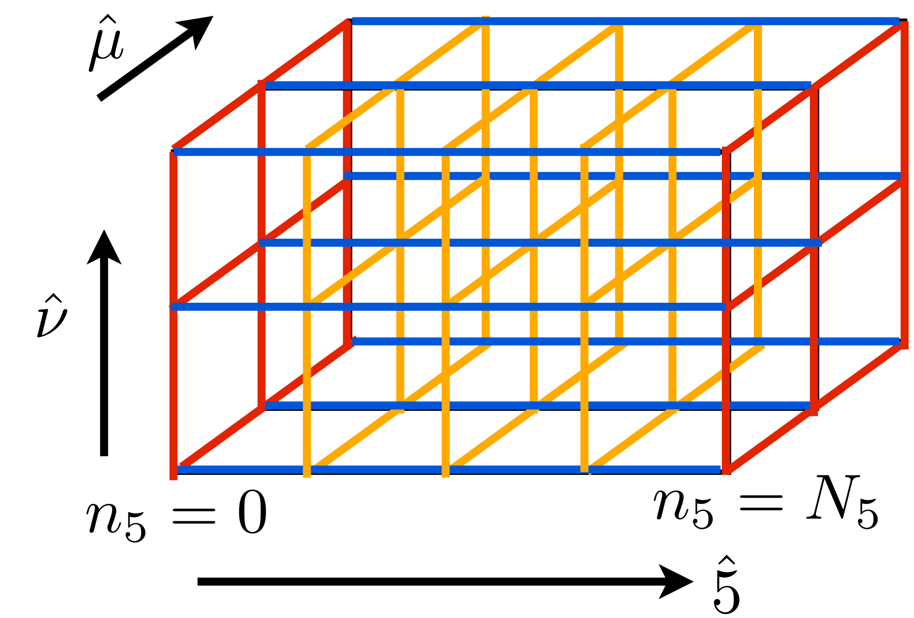
Wilson plaquette action

$$
\begin{aligned}
S_{\text{eff}} = & -\frac{\beta_4}{2}\sum_{n_\mu}\sum_{n_5=1}^{N_5-1}\left[\sum_{\mu<\nu}\text{Re tr } V_{p\notin\text{bound}}(n;\mu,\nu)\right] \qquad \text{bulk} \\
& -\frac{\beta_5}{2}\sum_{n_\mu}\sum_{n_5=0}^{N_5-1}\left[\sum_{\mu}\text{Re tr } V_{p\notin\text{bound}}(n;\mu,5)\right] \qquad \text{bulk} \\
& -\frac{\beta_4}{4}\sum_{n_\mu}\left[\sum_{\mu<\nu}\sum_{n_5=0,N_5}\text{Re tr } V_{p\in\text{bound}}(n;\mu,\nu)\right] \qquad \text{boundary} \\
& +\sum_{n_\mu}\sum_{n_5=1}^{N_5-1}\sum_{\mu}\left[u_2(\rho(n,\mu)) + \sum_{\alpha} h_\alpha(n,\mu)v_\alpha(n,\mu)\right] \\
& +\sum_{n_\mu}\sum_{n_5=0}^{N_5-1}\left[u_2(\rho(n,5)) + \sum_{\alpha} h_\alpha(n,5)v_\alpha(n,5)\right] \\
& +\sum_{n_\mu}\sum_{\mu}\sum_{n_5=0,N_5}\left[u_1(\rho(n,\mu)) + \sum_{\alpha} h_\alpha(n,\mu)v_\alpha(n,\mu)\right]
\end{aligned}
$$

where

$$u_1(\rho(n,M)) = -\ln(I_0(\rho)), \quad u_2(\rho(n,M)) = \frac{2}{\rho}I_1(\rho)$$

$$\rho = \sqrt{(\text{Re } h_0(n,M))^2 + (\text{Re } h_A(n,M))^2}$$



The parametrization of the field

in the bulk

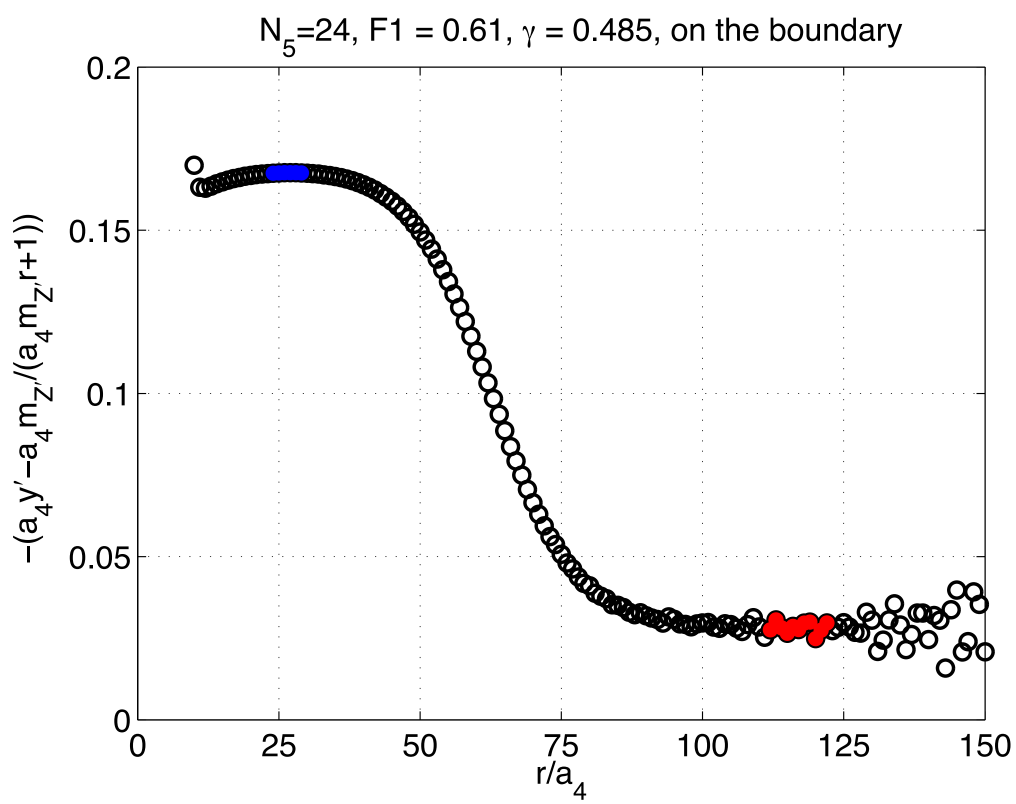
$$
\begin{aligned}
V(m,M) &= v_0(n,M) + i\sum_{A=1}^{3} v_A(n,M)\sigma^A \\
H(m,M) &= h_0(n,M) - i\sum_{A=1}^{3} h_A(n,M)\sigma^A
\end{aligned}
$$

on the boundaries

$$
\begin{aligned}
V(n,M) &= v_0(n,M) + iv_3(n,M)\sigma^3 \\
H(n,M) &= h_0(n,M) - ih_3(n,M)\sigma^3
\end{aligned}
$$

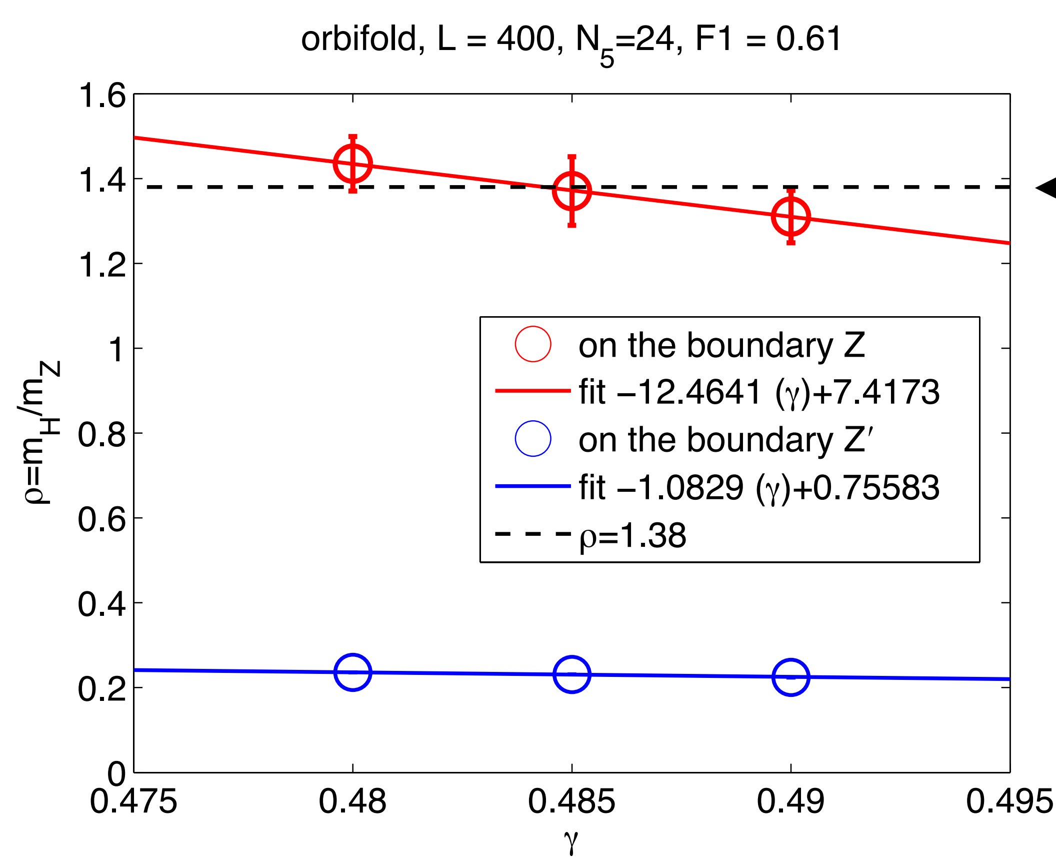
The Line of Constant Physics

We have 3 observables: $M_H(\beta, \gamma, N_5)$, $M_Z(\beta, \gamma, N_5)$ and $M_{Z'}(\beta, \gamma, N_5)$



4d Yukawa potential

$$V_4 = -\alpha \frac{e^{-mr}}{r} + C, \quad \alpha > 0$$
$$F_4 = V_4' = \alpha \frac{e^{-mr}}{r}(m + \frac{1}{r})$$
$$y = \log(r^2 F_4) = \log(\alpha) - mr + log(mr + 1)$$
$$y' = -m + \frac{m}{mr+1}$$



$\longleftarrow$ $\rho_{HZ} = 1.38$

LCP; Change the Lattice spacing by keeping two dimensionless physical quantities fixed

$$\rho_{HZ} = \frac{m_H}{m_Z} = 126/91.19 = 1.38$$
$$F_1 = m_H \cdot R = 0.61$$

$N_5 \to \infty$ means $a_4 \to 0$ on the LCP

$$(\because N_5 = \frac{\pi R}{a_5} = \frac{\pi R}{a_4}\gamma)$$