Preliminary results from maximally twisted mass lattice QCD at the physical point


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DESY, Zeuthen Site
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Lattice 2013, Mainz – 2\textsuperscript{nd} August 2013
Overview

1. Introduction

2. Tuning and Stability

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4. Path to $N_f = 2 + 1 + 1$

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<table>
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<th>Requirements</th>
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<tr>
<td>• Physical point with $a \sim 0.1\text{fm} \leftrightarrow$ reasonable computing resources</td>
</tr>
<tr>
<td>• Stable simulation and controlled $O(a^2)$ cutoff effects $\leftrightarrow$ pion splitting</td>
</tr>
<tr>
<td>• Maintain all nice properties of tmLQCD</td>
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</tbody>
</table>

Twisted mass action: [Frezzotti, Grassi, Sint, Weisz, 2000; Frezzotti, Rossi; 2004]

$$ S = \beta \sum_{x;P} \left[ b_0 \left\{ 1 - \frac{1}{3} \text{ReTr} P^{1 \times 1}(x) \right\} + b_1 \left\{ 1 - \frac{1}{3} \text{ReTr} P^{1 \times 2}(x) \right\} \right] $$

$$ + \sum_x \bar{\chi}(x) \left[ D_W(U) + m_0 + i \mu \gamma^5 \tau^3 + \frac{i}{4} C_{SW} \sigma^{\mu \nu} F^{\mu \nu}(U) \right] \chi(x) $$

- $N_f = 2$
- $b_0 = 1 - 8b_1$, $b_1 = -0.331$ [Iwasaki; 1983]
- $C_{SW} = 1.57551$ from Padé fit of CP-PACS data [Aoki et al.; Phys.Rev. D73 (2006) 034501]
Introduction
Run Details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/a</td>
<td>48</td>
</tr>
<tr>
<td>T/a</td>
<td>96</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.10</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-0.331</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.13729</td>
</tr>
<tr>
<td>$a\mu_l$</td>
<td>0.0009</td>
</tr>
<tr>
<td>$C_{SW}$</td>
<td>1.57551</td>
</tr>
<tr>
<td>$N_{traj}$</td>
<td>$&gt; 1500$</td>
</tr>
<tr>
<td>$P$</td>
<td>0.603531(6)</td>
</tr>
<tr>
<td>$\tau_{int}(P)$</td>
<td>10.0(3.5)</td>
</tr>
<tr>
<td>$am_{PCAC}$</td>
<td>0.00004(2)</td>
</tr>
<tr>
<td>$m_{\pi}L$</td>
<td>3.00(2)</td>
</tr>
<tr>
<td>$a$</td>
<td>0.91(5) fm $^a$</td>
</tr>
</tbody>
</table>

$^a$very preliminary: large uncertainty to accomodate possible FS / discretization effects

- Substantial updates to tmLQCD software suite:
  - BG/Q optimizations
  - OpenMP
  - Clover term with EO pre-conditioning and twisted mass
  - RHMC implementation

- Details: Carsten Urbach, Parallels 9G, Friday 14:40

- O(10) exploratory runs on $24^3 \times 48$
- 2 (short) tuning runs on target volume
- Production runs on BG/Q in Juelich, replica on SuperMUC

B. Kostrzewa (bartosz.kostrzewa@desy.de) Preliminary mtmLQCD @ the phys. point 2nd August 2013 4 / 17
Tuning and Stability

Tuning to maximal twist at small quark mass and coarse lattice spacing

- $N_f = 2 + 1 + 1$
  
  tlSym action $\rightarrow$ remnant signs of 1$^{\text{st}}$ order phase-trans.

- Iwasaki gauge action, situation much improved but trouble with $N_f = 4$

- Clover term $+$ Iwasaki $\rightarrow$ very fine tuning possible, linear behaviour in $1/2\kappa$, no metastabilities
Tuning and Stability

Monte Carlo histories

- 2nd order minimal norm integrator on four timescales
- four Hasenbusch mass shifts, lightest two on same timescale
- \( \tau = 1.0 \) trajectory length, 75% acceptance for efficiency
First results
Neutral connected pion splitting

- Indications that $W'_8$ is reduced markedly $\rightarrow$ stable simulations, particular $O(a^2)$ effects under control
- Measurement of full pion splitting in progress $\rightarrow$ obtain estimate of $c_2$
First results
Pion decay constant

- $r_0f_{\pi}$
  - data consistent with old $N_f = 2$ runs
  - no FS corrections applied

\[
(r_0m_\pi)^2
\]

\[
r_0f_\pi
\]

- $\beta = 3.80$
- $\beta = 3.90$
- $\beta = 4.05$
- $\beta = 4.20$
- this work

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First results

Heavy-light meson sector then and now

\[
\frac{f_K}{f_\pi}
\]

- old \( N_f = 2 \)
- new \( N_f = 2 \) with cloak term
- consistent with previous values
- some extrapolation necessary

![Graph showing the ratio of \( f_K \) to \( f_\pi \) for different values of \( a \) and \( M_{\pi}^{(\text{lat})} / f_\pi^{(\text{PDG})} \).](image)
First results

Heavy-light meson sector then and now

- \( \frac{f_D}{f_\pi} \)
  - old \( N_f = 2 \)
  - new \( N_f = 2 \) with clover term
  - consistent over previous values
  - consistent with experimental point

![Graph showing the relationship between \( \frac{f_D}{f_\pi} \) and \( (M_\pi^{(lat)}/f_\pi^{(PDG)})^2 \) for different values of \( a \).](image)

- \( a = 0.054 \text{ fm} \)
- \( a = 0.067 \text{ fm} \)
- \( a = 0.085 \text{ fm} \)
- \( a = 0.098 \text{ fm} \)

This work

PDG
First results

Heavy-light meson sector then and now

\[ \frac{f_{D_s}}{f_\pi} \]

- old \( N_f = 2 \)
- new \( N_f = 2 \) with clover term
  - consistent with previous values
  - consistent with physical value

\[
\left( \frac{M_{\pi}^{(\text{lat})}}{f_\pi^{(\text{PDG})}} \right)^2
\]

- \( a = 0.098 \text{ fm} \)
- \( a = 0.085 \text{ fm} \)
- \( a = 0.067 \text{ fm} \)
- \( a = 0.054 \text{ fm} \)
- this work
- PDG

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First results

Heavy-light meson sector then and now

\[ \frac{f_{D_s}}{f_D} \]

- old \( N_f = 2 \)
- old \( N_f = 2 \) chiral extrapolation
- new \( N_f = 2 \) with clover term
- apparent improvement over previous values
- consistent with experimental point

\[ \left( \frac{M_{\pi}^{(\text{lat})}}{f_{\pi}^{(\text{PDG})}} \right)^2 \]

ETMC C.L. [2011]

PDG

this work

\( a = 0.098 \text{ fm} \)
\( a = 0.085 \text{ fm} \)
\( a = 0.067 \text{ fm} \)
\( a = 0.054 \text{ fm} \)

B. Kostrzewa (bartosz.kostrzewa@desy.de)

Preliminary mtmLQCD @ the phys. point

2\textsuperscript{nd} August 2013
First results
RI-MOM Renormalization constants

<table>
<thead>
<tr>
<th>RC</th>
<th>new $N_f = 2$ $\beta = 2.10 (C_{SW})$</th>
<th>$N_f = 2$ $\beta = 3.90$</th>
<th>$N_f = 2 + 1 + 1$ $\beta = 1.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_A$</td>
<td>0.805(05)</td>
<td>0.730(03)</td>
<td>0.746(05)</td>
</tr>
<tr>
<td>$Z_V$</td>
<td>0.762(04)</td>
<td>0.634(03)</td>
<td>0.614(03)</td>
</tr>
<tr>
<td>$Z_P/Z_S$</td>
<td>0.805(34)</td>
<td>0.669(08)</td>
<td>0.700(08)</td>
</tr>
</tbody>
</table>

- Renormalization constants closer to 1
- Goldstone boson pole subtraction in $Z_P$ effective
- $Z$-factors also available from momentum sources
  - M. Constantinou, Parallels 3B, Tuesday 15:40

\[
124^3 \times 48, \ a_\mu_{\text{sea}} = 0.0060, \ a_\mu_{\text{val}} = 0.0050 - 0.0090, \ no \ sea \ quark \ chiral \ limit \ yet!
\]
First results

Comparison to experimental values

- $Q_{\text{lat}} \div Q_{\text{phys}}$ for example: $Q_{\text{lat}} = \frac{m_{\pi}^{\text{lat}}}{f_{\pi}^{\text{lat}}} \quad Q_{\text{phys}} = \frac{m_{\pi}^{\text{phys}}}{f_{\pi}^{\text{phys}}}$
- Tuning strange and charm quark mass $\sim$ PDG quark mass ratios

\[\begin{array}{c|c|c|c|c}
\hline
& m_{\pi}/f_{\pi} & m_K/m_{\pi} & f_K/f_{\pi} & m_{D}/f_{\pi} \\
\hline
Q_{\text{lat}}/Q_{\text{phys}} & & & & \\
\hline
\end{array}\]

\[\begin{array}{c|c|c|c|c}
\hline
& m_{D}/f_{\pi} & m_{D_s}/f_{\pi} & f_{D}/f_{\pi} & f_{D_s}/f_{\pi} \\
\hline
Q_{\text{lat}}/Q_{\text{phys}} & & & & \\
\hline
\end{array}\]

B. Kostrzewa (bartosz.kostrzewa@desy.de)
Path to $N_f = 2 + 1 + 1$

Preliminaries

Tuning $C_{SW}$

- No existing NP study of $C_{SW}$ for $N_f = 2 + 1 + 1$
- For $N_f = 2$, discrepancy between Padé and direct approach $\sim 10\%$
  - Full determination of $C_{SW}$ has $O(a\Lambda_{QCD})$ systematic uncertainty
  - $10\%$ accuracy should be sufficient
  - Use simple algorithm to obtain tadpole improved value (next slide)

Idea: If simulation stable for $N_f = 2l + 2s$
- also stable for $N_f = 4$ and $N_f = 2 + 1 + 1$

\[ \begin{align*}
    N_f &= 2 + 2 \\
    a\mu_l^{(1)}, a\mu_l^{(2)}, a\mu_s \\
    am_{cr}(a\mu_l^{(phys)}, a\mu_s) \\
    \Downarrow K_C \\
    N_f &= 4 \\
    \mu_l \sim 30-70 \text{ MeV} \\
    \mu_{sea} \to 0, \mu_{val} \to 0 \end{align*} \]
Path to $N_f = 2 + 1 + 1$

Tuning $C_{SW}$

- Use simple approximate formula at some bare coupling $g_0$:

$$C_{SW} \sim 1 + 0.113(3) \frac{g_0^2}{\langle P \rangle}$$

[Sheikoleslami, Wohlert; 1985]

1. Start with $C_{SW} = N_f$-independent 1-loop value
2. Simulate at zero twisted mass, neg./pos. Wilson quark masses bracketing $m_{PCAC} = 0$
3. Linearly interpolate $\langle P \rangle$ at $m_{PCAC} = 0$
4. Use formula to get better estimate of $C_{SW}$
5. Repeat (1) with new estimate as starting value
6. Stop when change is less than 3%

$N_f = 4, \ a_\mu_l = 0.0$

$\bullet C^{(1)}_{SW}, \bullet C^{(2)}_{SW}$

$\Rightarrow C_{SW}$-dependence not too strong

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Conclusion and Outlook

- Shown feasibility of mtmLQCD simulations at physical point
- All preliminary measurements look promising
- Indications of better $O(a^2)$ behaviour from connected pion splitting and lack of metastabilities
- First $N_f = 4$ and $N_f = 2 + 2$ runs started, $m_{PCAC}$ quite linear in $\frac{1}{2\kappa}$
- Tuning $C_{SW}$ for $N_f = 2 + 1 + 1$ using tadpole improved formula

- Extension to $N_f = 2 + 1 + 1$ outlined and first steps taken
  - Results suggest no problems with plan
- Continuation of rich ETMC physics programme at the physical point
  - nucleon: C. Alexandrou, Parallels 3B, Tuesday 15:00
  - nucleon: M. Constantinou, Parallels 3B, Tuesday 15:40
  - muon g-2: G. Hotzel, Parallels 9B, Friday 14:20