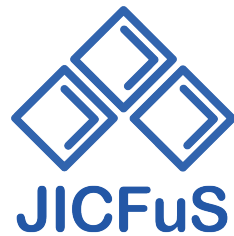


Fine lattice simulations with chirally symmetric fermions

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for JLQCD Collaboration



LATTICE2013, Mainz 2013 Jul. 29-- Aug. 3

JLQCD's new project

- Target: precise calculation for quark-flavor physics
 - eg. B -, D - form factors ← SuperKEKB/Belle-II @ KEK
- $N_f = 2+1$ simulation with good control of systematic errors
 - chiral symmetry: $m_{\text{res}} \ll m_{\text{ud}}$
 - continuum extrapolation: $a = 2.4, 3.6, 4.8$ GeV and more
 - light quarks: $m_\pi = 500, 400, 300$ MeV and lighter
 - large lattice volume satisfying $m_\pi L > 4$
- Domain-Wall (Möbius) fermion
 - $m_{\text{res}} < 0.5$ MeV with $L_s = 12$ or smaller

Basic studies / presentations

- Choice of the fermion action
 - T. Kaneko, study in HMC, Tue. 17:40--
 - S. Hashimoto, chiral symm. violation, Tue. Poster
- Code developments
 - G. Cossu, our code and its performance, Tue. Poster
- Tests for applications
 - Y. Cho, Brillouin improvement for heavy quark, Thu. 17:50--
 - H. Fukaya, reweighting to the overlap simulation, Fri. 14:20--
- First physics results : **this talk**

Plan of this talk

- Numerical simulation
 - Lattice action
 - Profile & status
- Measurements of observables
 - scale setting
 - hadron spectrum
- Inspective study of generated configs
 - thermalization / autocorrelation
- Summary & outlook



Numerical Simulation

Domain-Wall (Möbius) fermions

Kaplan 1992; Shamir 1994; Borici 1997; Chiu 1998; Brower et al. 2001

- 5D representation

$$D_{DW}^{(5)}(m) = 1 + b(4 + M)D_W - (1 - c(4 + M)D_W) \cdot \begin{bmatrix} 0 & P_- & & & & -mP_+ \\ P_+ & 0 & P_- & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & P_+ & 0 & P_- \\ -mP_- & & & & P_+ & 0 \end{bmatrix}$$

D_W : Wilson Dirac op with mass $-M$

- 4D effective operator

$$D_{DW}^{(4)}(m) = \left[\mathcal{P}^{-1} D_{DW}^{(5)}(m=1)^{-1} \cdot D_{DW}^{(5)}(m) \mathcal{P} \right]_{11}$$

$$= \frac{1+m}{2} + \frac{1-m}{2} \gamma_5 \tanh(L_s \tanh^{-1} H_M)$$

$$\mathcal{P} \equiv \begin{bmatrix} P_- & P_+ & & & \\ & P_- & \ddots & & \\ & & \ddots & \ddots & \\ & & & P_- & P_+ \\ P_+ & & & & P_- \end{bmatrix}$$

$L_s \rightarrow \infty$ → sign function approx.

– scaled Shamir kernel: $H_M = \gamma_5 \frac{bD_W}{2 + cD_W}$ we set $b = 2, c = 1$

See T. Kaneko's talk and S. Hashimoto's poster for more details.

Gauge generation : profile

- $N_f = 2+1$ QCD
 - action: Symanzik gauge + Domain-Wall (Möbius)
 - tree-level Symanzik action for topology changing
 - 3 levels of stout smearing
 - $m_{\text{res}} < 0.5$ MeV
 - $m_\pi L > 4$, $m_\pi = 500, 400, 300, \sim 220$ MeV
 - $\alpha^{-1} = 2.4, 3.0, 3.6, 4.8$ GeV
 - standard RHMC for the 5D representation with e/o preconditioning
 - Performance: 16 \rightarrow 30 Gflops/node
 - Gauge configs are stored in steps of 10 trajs.



IBM BG/Q, 1.2PFlops peak



see G. Cossu's poster for more details

Gauge generation: status

see T. Kaneko's talk for more details

- $\beta = 4.17, a^{-1} \sim 2.4 \text{ GeV}$

- $\beta = 4.35, a^{-1} \sim 3.6 \text{ GeV}$

$32^3 \times 64 \times 12$

m_{ud}	m_π [MeV]	# traj
$m_s = 0.030$		
0.007	310	3000
0.012	400	3000
0.019	500	3000
$m_s = 0.040$		
0.0035	240	3000
0.007	310	3000
0.012	400	3000
0.019	500	3000
$m_s = 0.040, 48^3 \times 96$		
0.0035	240	1500

$48^3 \times 96 \times 8$

m_{ud}	m_π [MeV]	#traj ($\tau = 1$)	#traj ($\tau = 2$)
$m_s = 0.018$			
0.0042	290	1850	280
0.0080	410	3280	260
0.0120	500	3360	—
$m_s = 0.025$			
0.0042	290	2470	235
0.0080	410	3580	330
0.0120	500	3540	430

so far, measurements are done on the $\tau=1$ configs.



Measuremnts of observables

Calculation of Wilson flow

- Wilson flow Lüscher 2010

- defined by $V_{x\mu}(0) = U_{x\mu}, \quad \left. \frac{dV_{x\mu}}{dt} \right|_t = -g_0^2 \partial_{x\mu} S_g[V] \cdot V_{x\mu}(t)$
(MD-force of Wilson gauge action)

- Runge-Kutta alg.: $t \rightarrow t + \varepsilon$ ➡ flow of gauge config.

- energy density $E \equiv \left. \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right|_t$

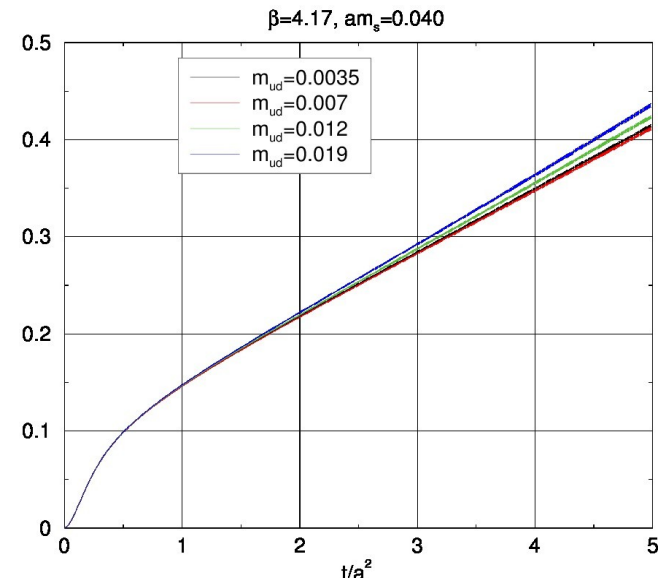
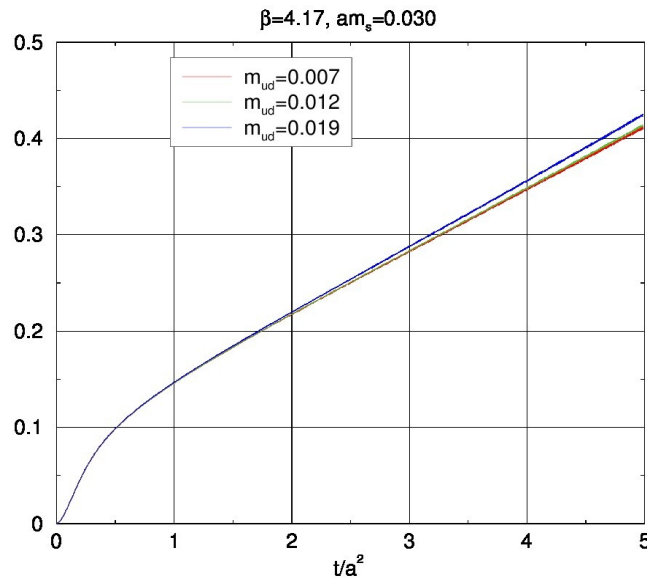
- reference values to determine lattice scale:

$$\left[t^2 \langle E \rangle \right]_{t=t_0} = 0.3 \quad \left\{ t \frac{d}{dt} [t^2 \langle E \rangle] \right\}_{t=w_0^2} = 0.3$$

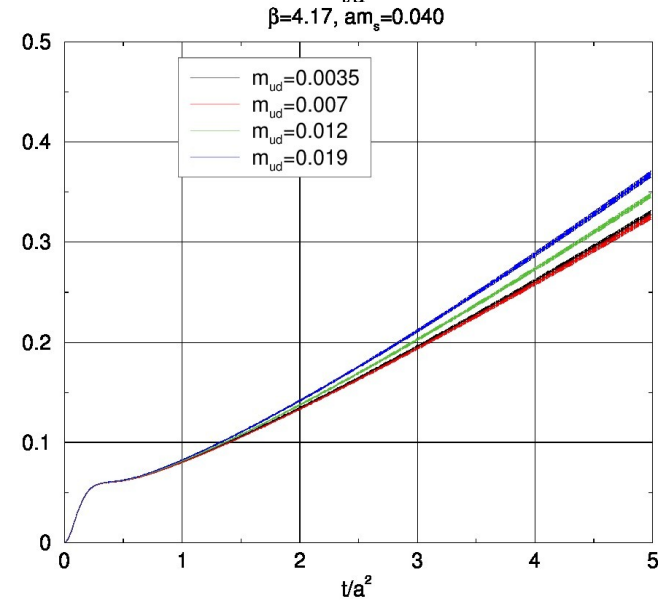
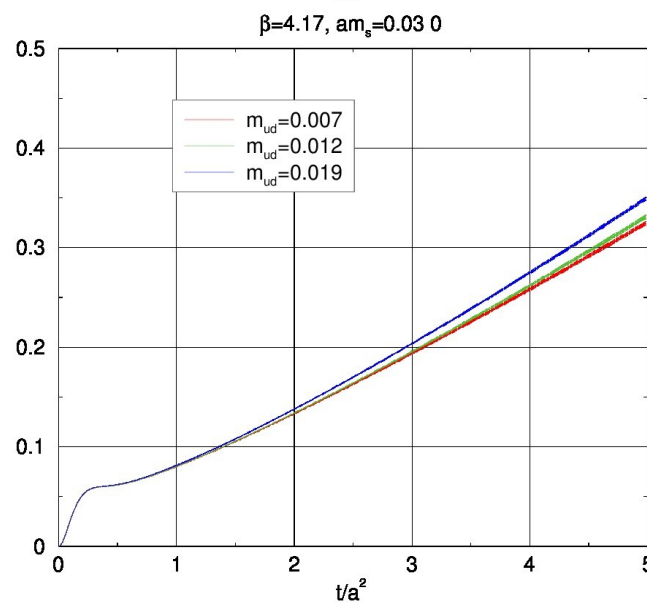
we use $t_0^{1/2} = 0.1465 \text{ fm}$, $w_0 = 0.1755 \text{ fm}$ BMW Collab. 2012

Wilson Flow result, $\beta = 4.17$ $32^3 \times 64$

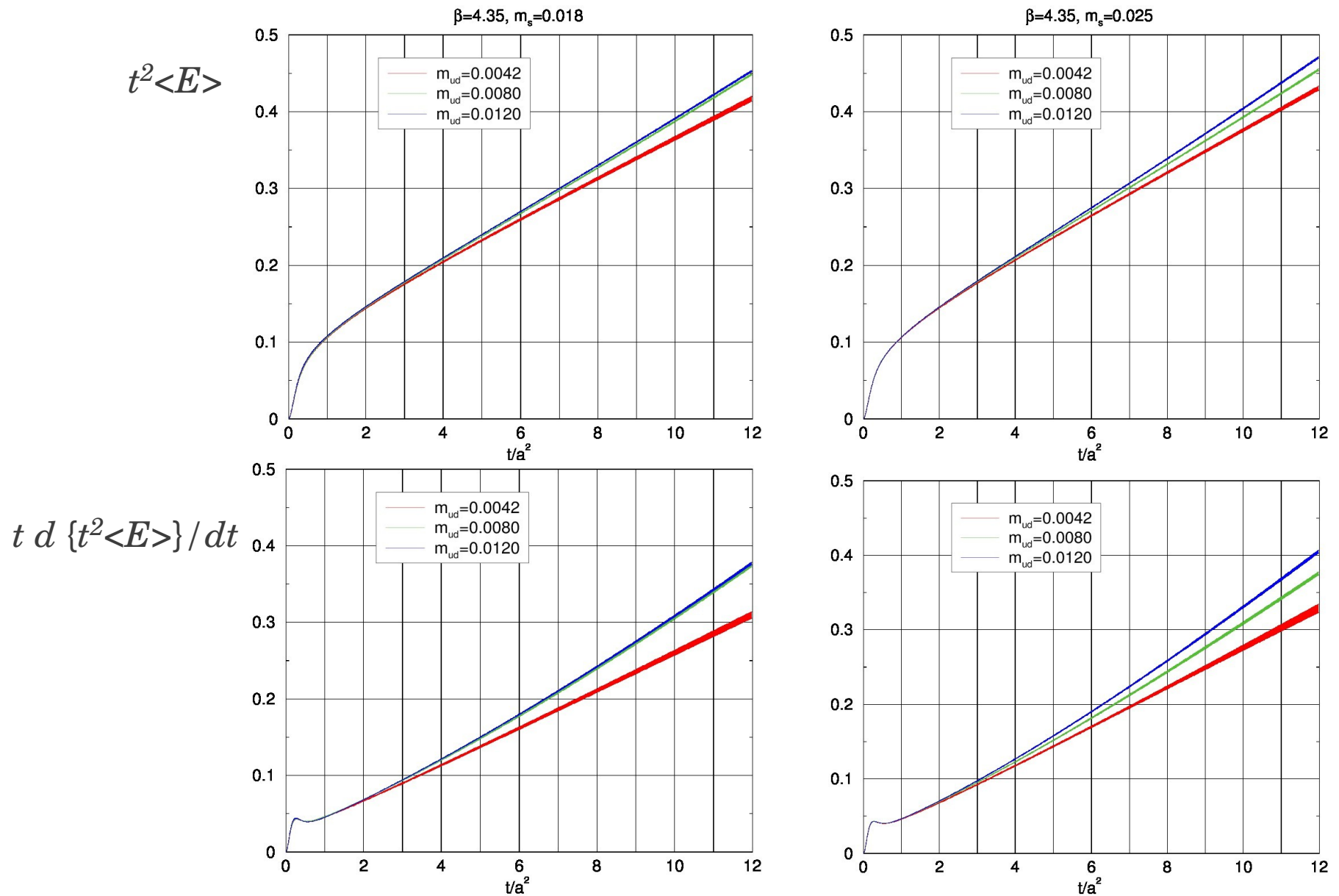
$t^2 \langle E \rangle$



$t d \{t^2 \langle E \rangle\} / dt$

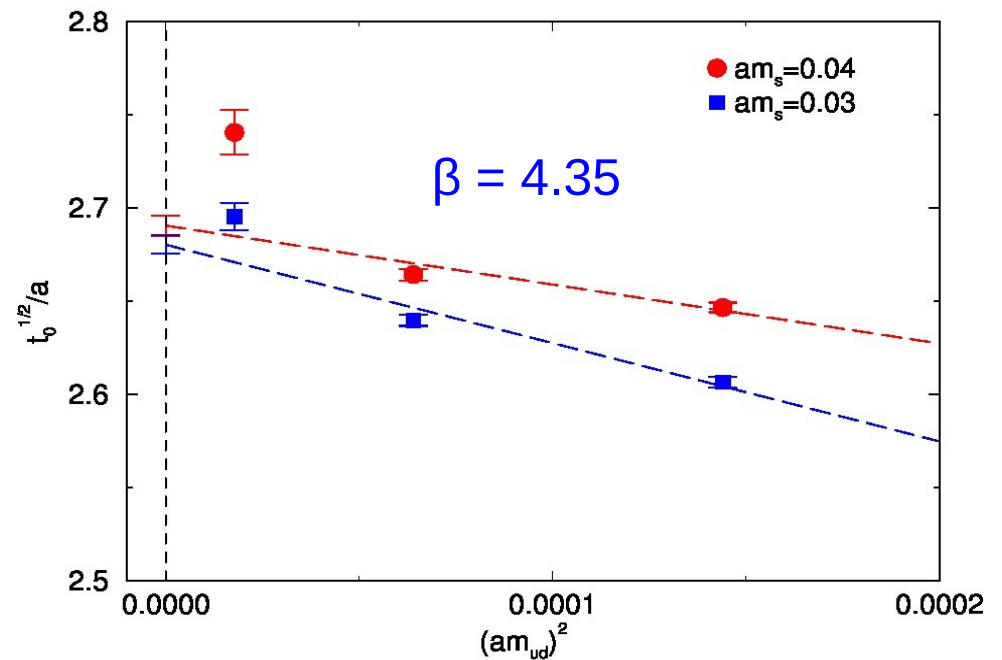
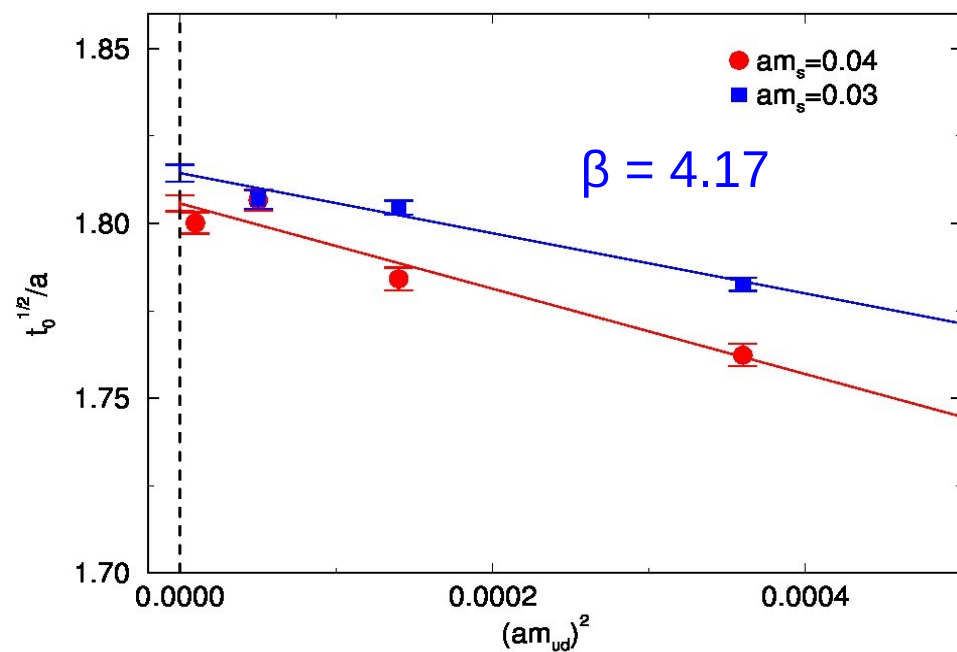


Wilson Flow result, $\beta = 4.35$, $48^3 \times 96$



Lattice scale

- **preliminary** determination by t_0
 - can be measured at smaller t than w_0



$\beta = 4.17$: $a^{-1} = 2.472(3) - 2.484(2)$ GeV ; $\beta = 4.35$: $a^{-1} \sim 3.68(1)$ GeV

systematic errors yet to be studied.

too small statistical error ?

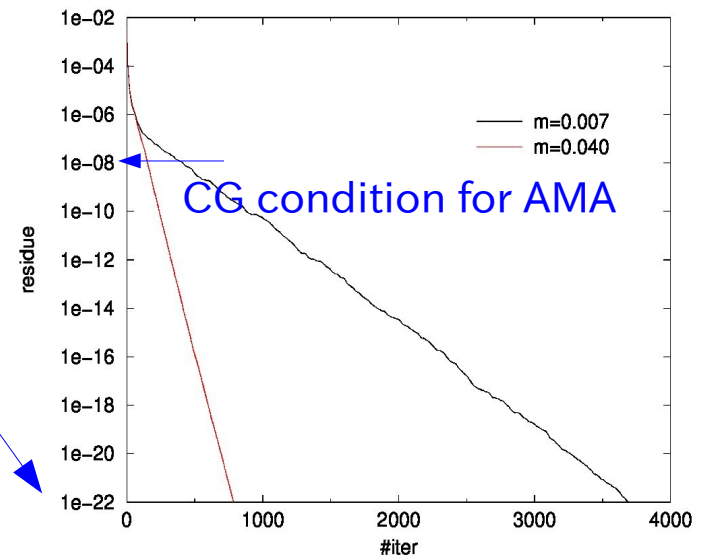
Light hadron mass spectrum

- 2-point correlators from $[D_{\text{DW}}^{(4)}]^{-1}$
 - source smearing: $e^{-\alpha r}$ with Coulomb gauge
 - **all-mode-averaging (AMA)** to improve signal Blum, Izubuchi and Shintani 2012
- compute correlators $C^{\text{bulk}}(t)$ with **relaxed stopping condition** as well

$$C(t) = (C(t) - C^{\text{bulk}}(t)) + C^{\text{bulk}}(t)$$

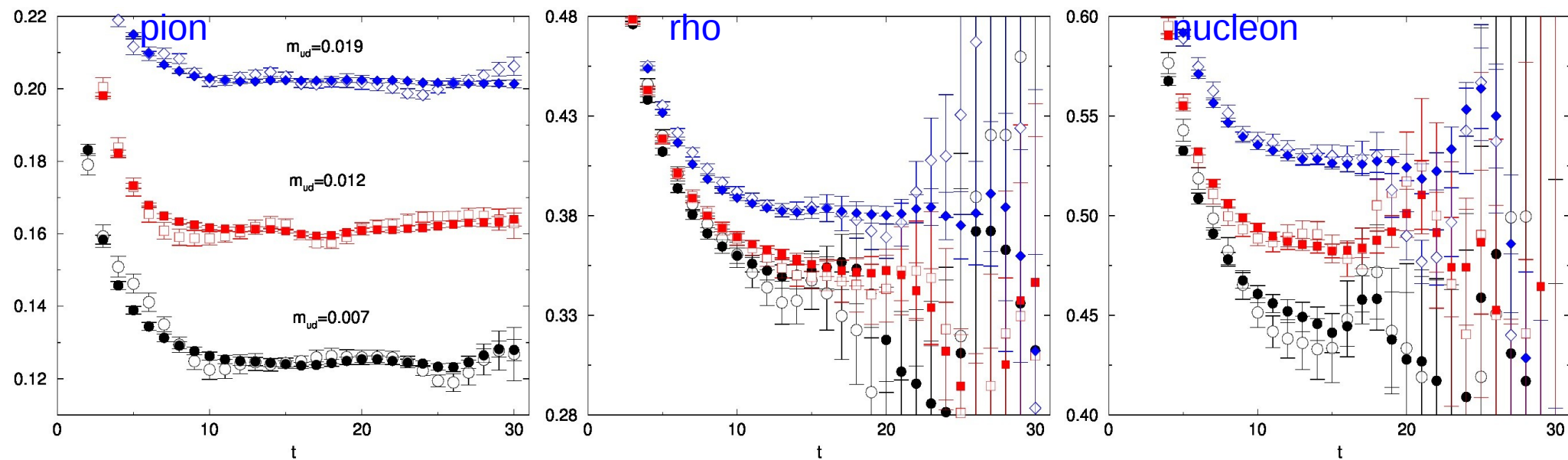
- **improve $C^{\text{bulk}}(t)$ by averaging over multiple time-slices**
 - ← much cheaper than usual case
- **how much gain?**

$\beta=4.17$: measurements at $t_{\text{src}} = 0, 2, 4, \dots, 62$
with $\sim 10\%$ iterations of the **regular precision**

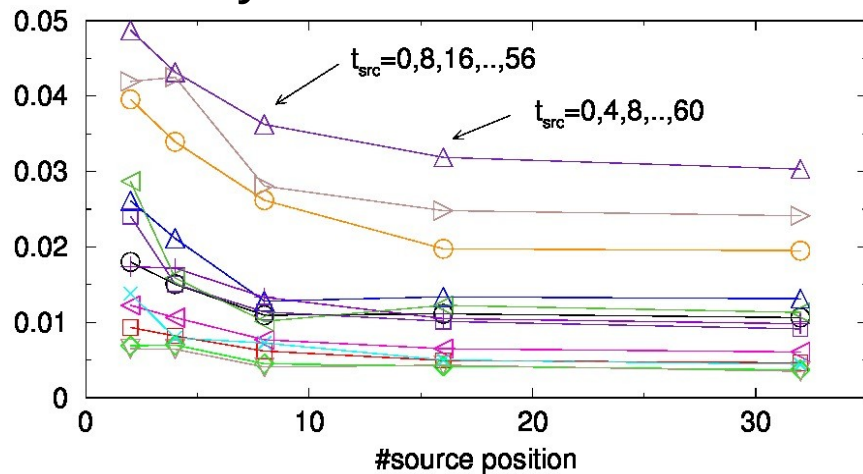


Performance of AMA

– at a glance : $\beta = 4.17$, $m_s = 0.030$ filled: AMA applied; open: conventional




how many sources for relaxed correlators?



error sizes of rho mass

→ start to saturate from $t_{step} = 8$

$t_{step} = 12$ for $48^3 \times 96$ lattice

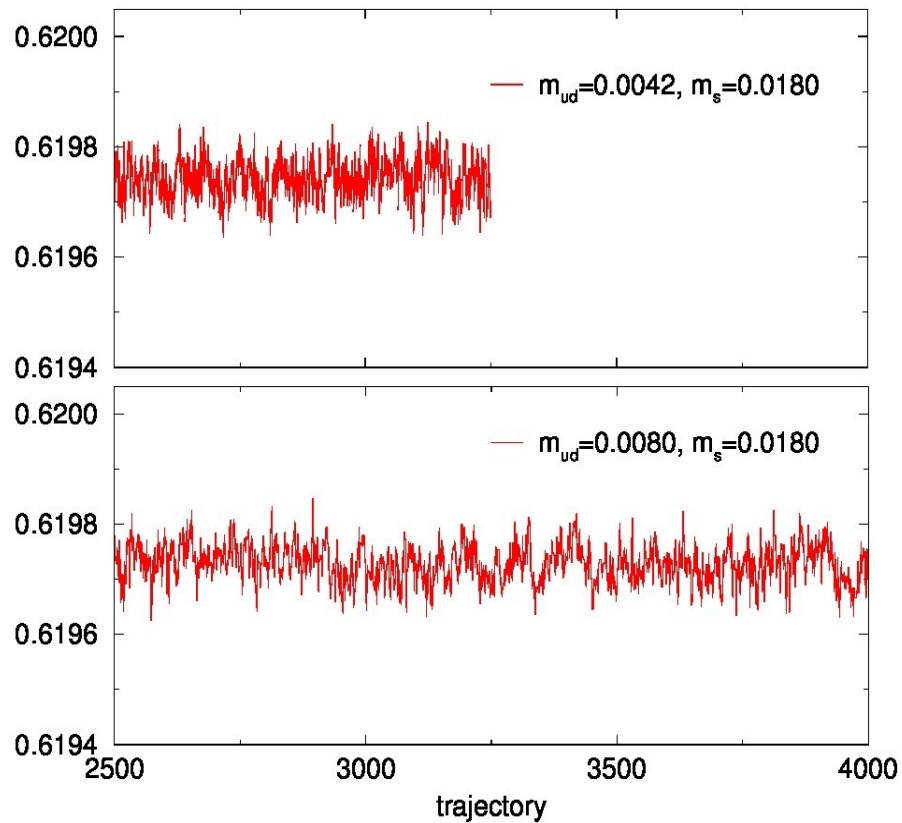


Inspective study of generated configurations ($\beta = 4.35$)

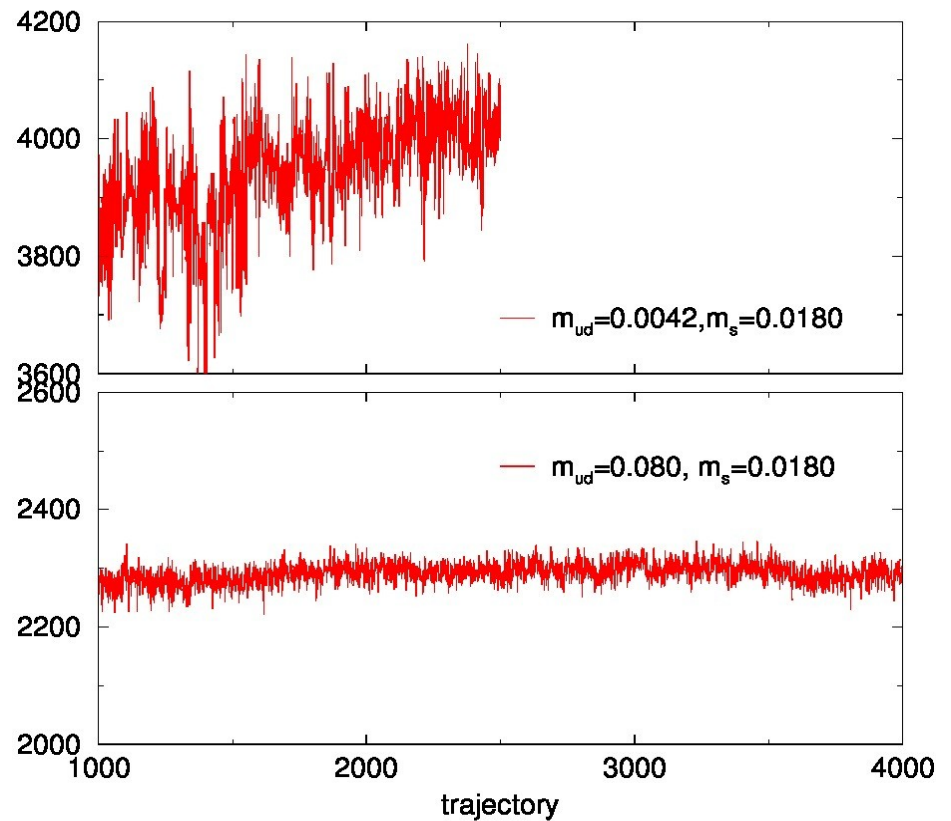
Thermalization: $\beta = 4.35$

- monitoring HMC from various angles

plaquette



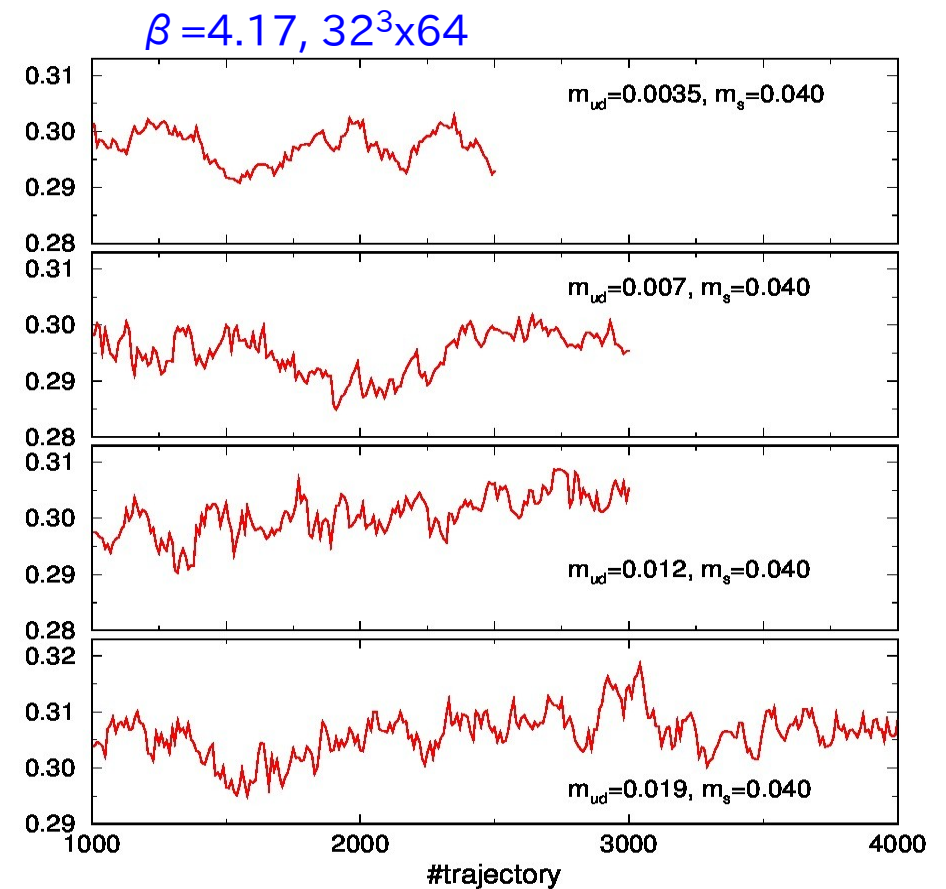
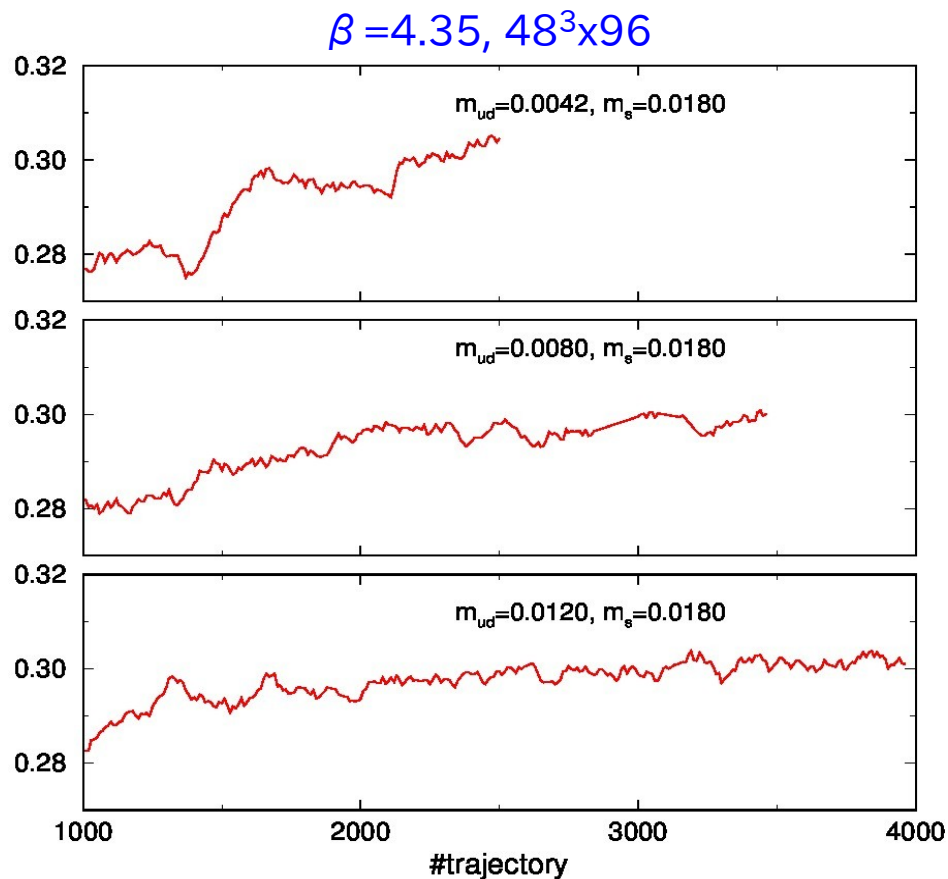
#iteration of CG



conventional plaquette monitoring ignores unthermalized configs!

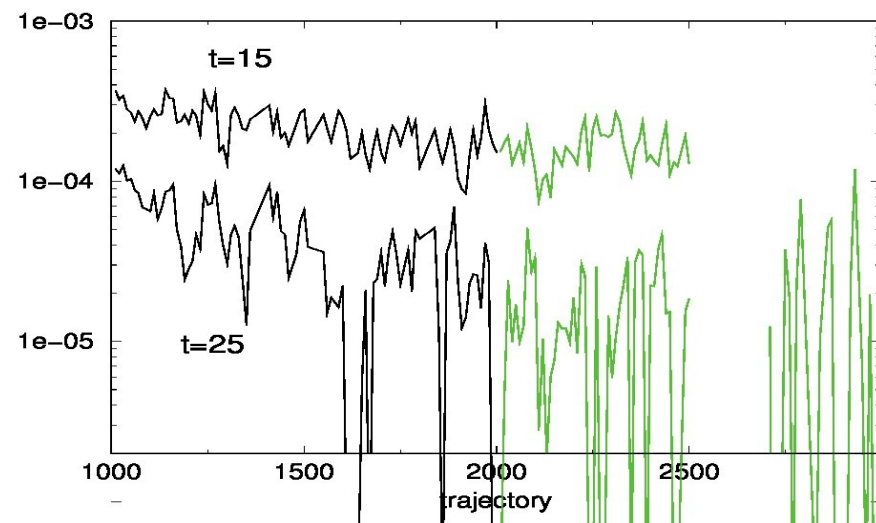
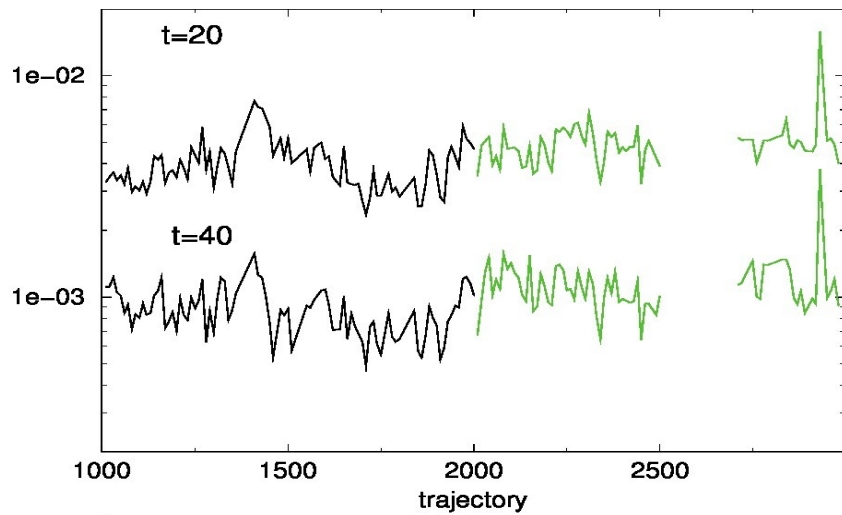
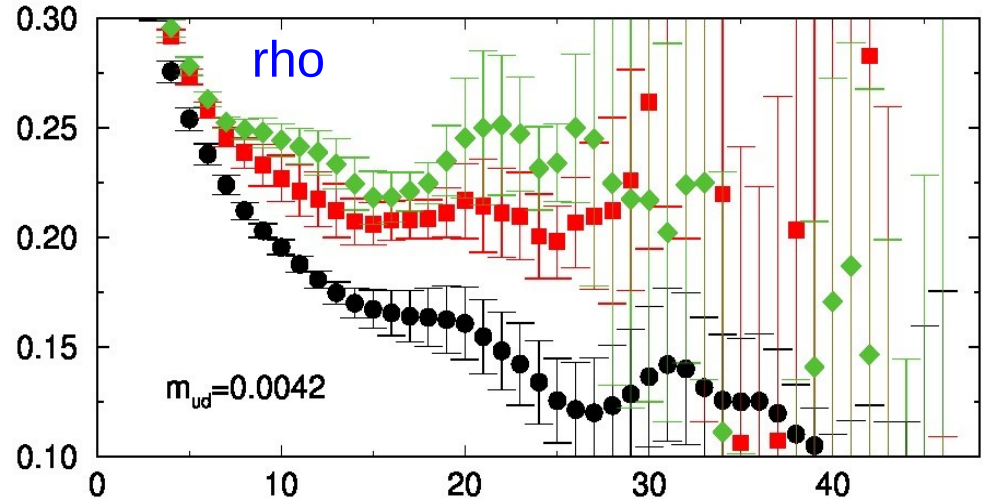
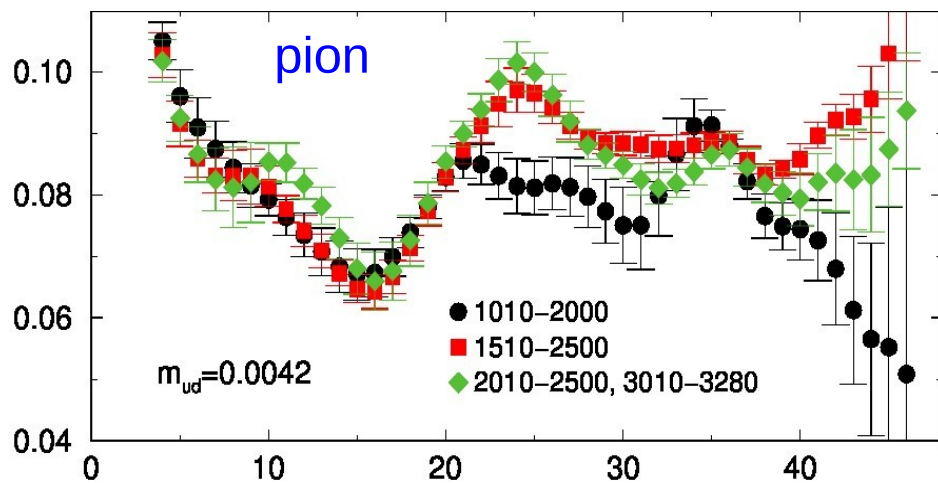
Thermalization in Wilson flow

- history of $t^2 \langle E \rangle$ (around t_0)



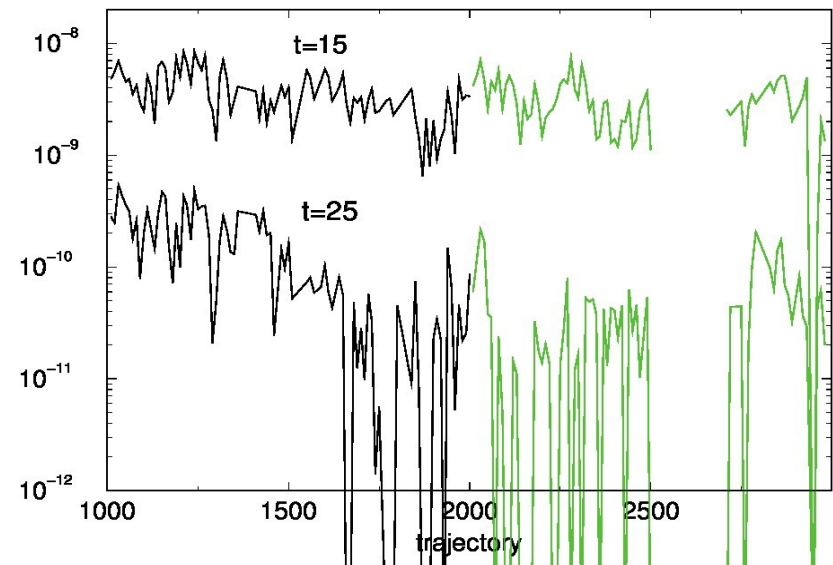
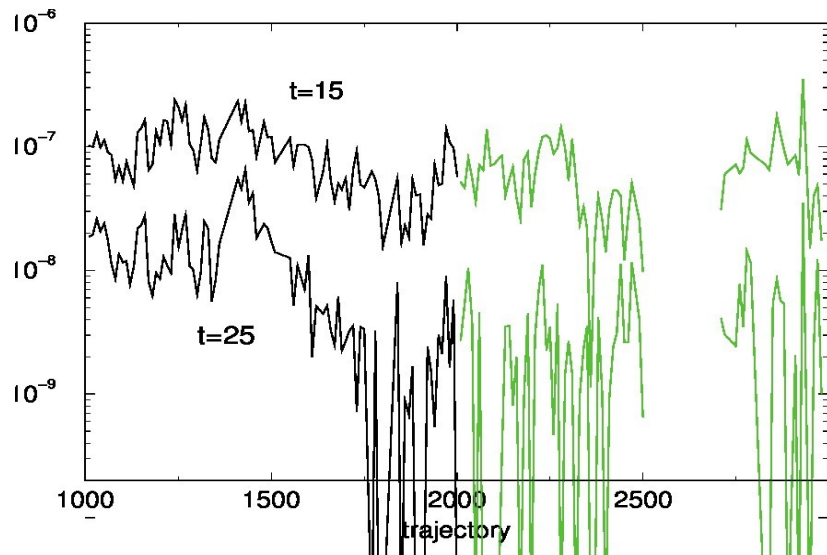
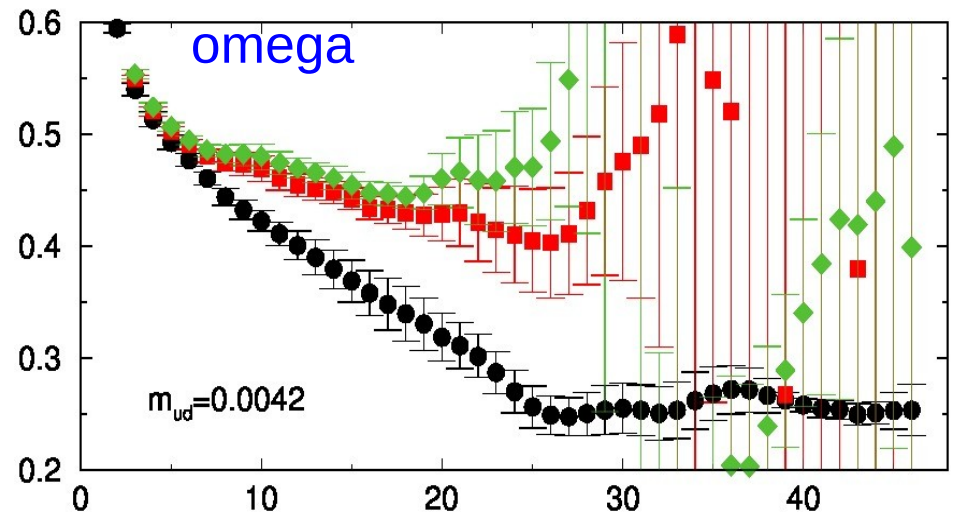
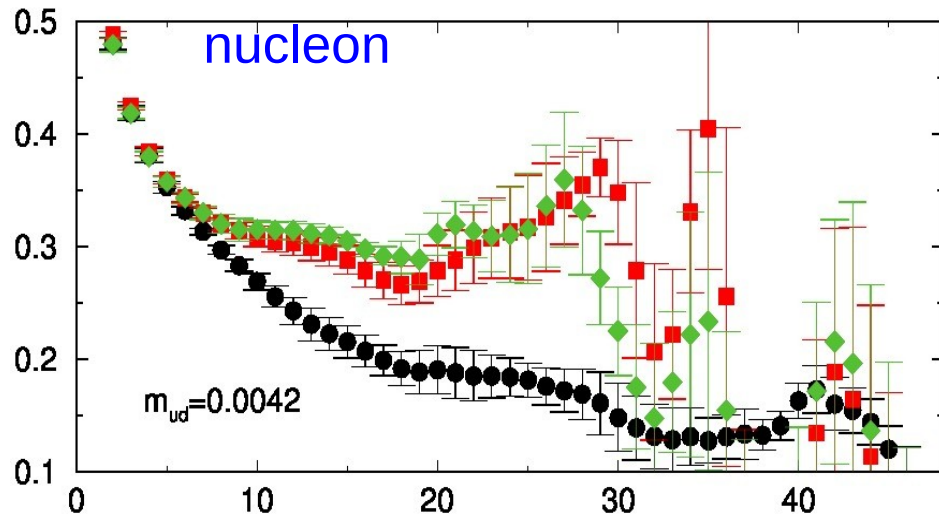
objects on WF is also sensitive to the thermalization

Thermalization in meson correlators



rho is more sensitive to the thermalization

Thermalization in baryon correlators

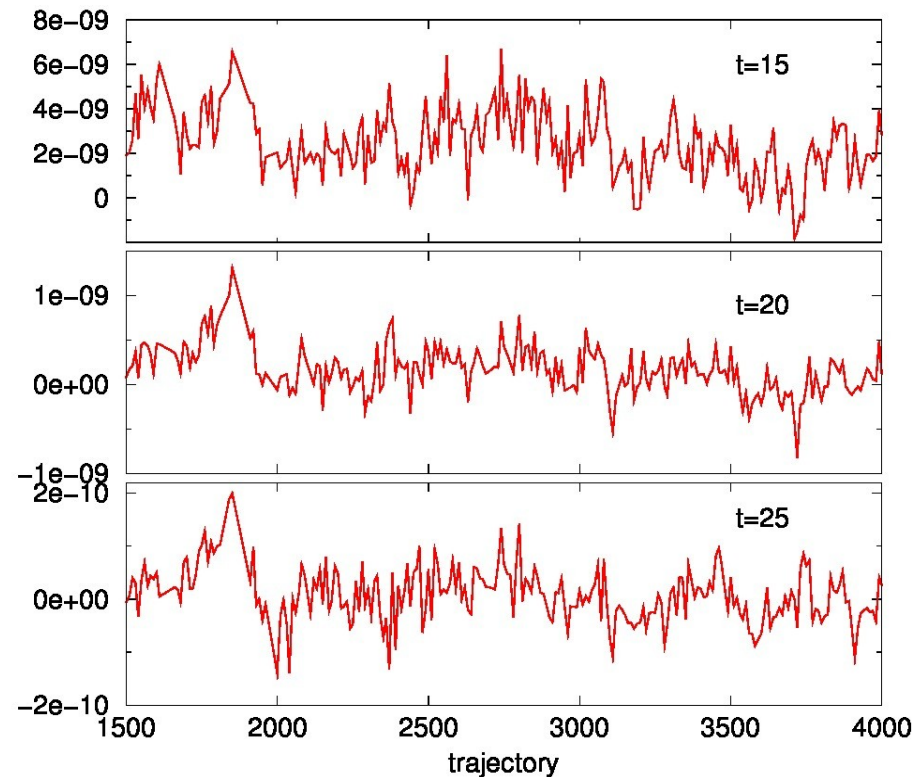


Autocorrelations

- history of omega-correlator is very sensitive to thermalization

→ useful to check **auto correlation**

$\beta = 4.35$, $mud = 0.008$, $ms = 0.018$:



no clear clustering observed 2000- 4000 traj.

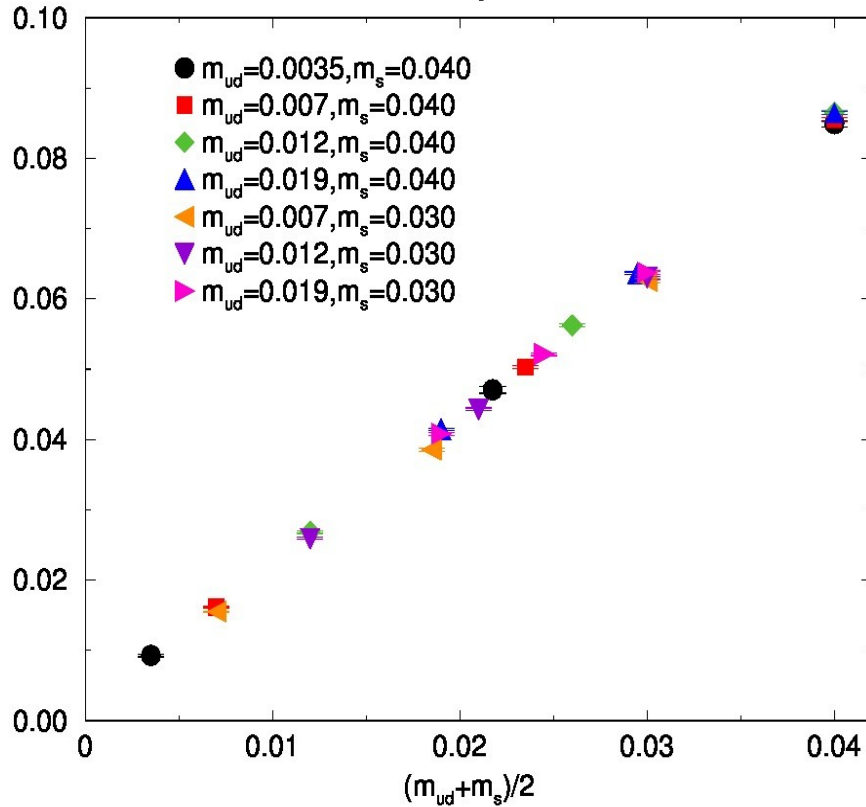


Summary & outlook

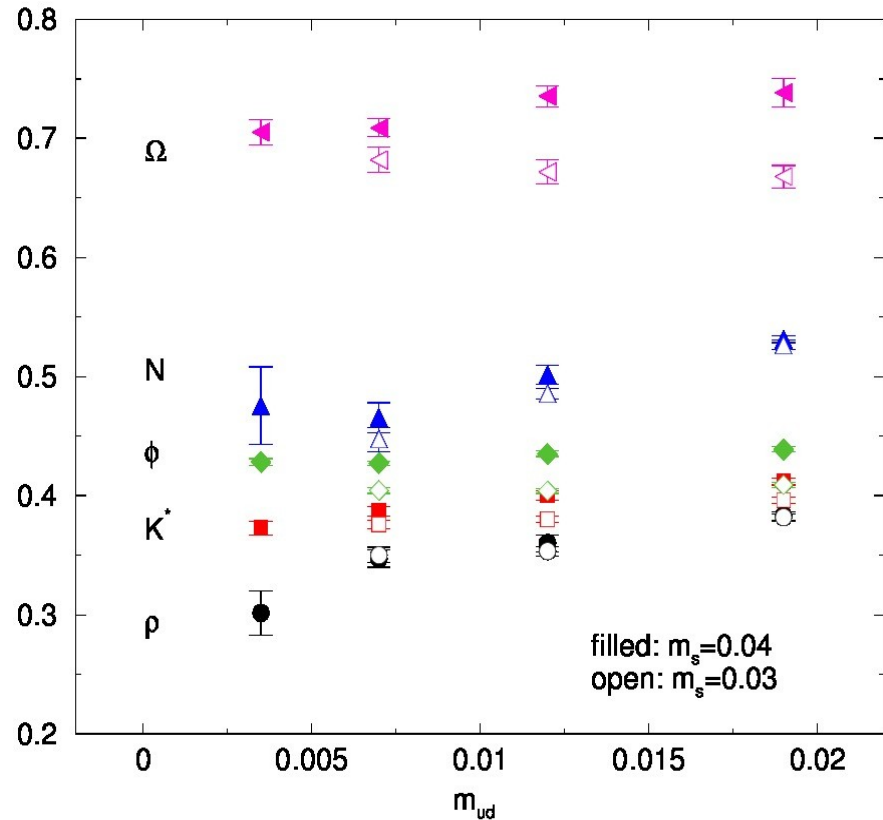
First look at the mass spectrum

$\beta = 4.17$

- PCAC relation in pseudo-scalar



- other hadrons



further improve the signal: increase the trajectories
and/or low-eigenmodes-averaging

General summary & outlook

- $N_f = 2+1$ simulation with Domain-Wall (Möbius) fermions
 - precise control of systematics with chiral symm. / discretization / finite volume
- Measurements of basic observables
 - Wilson flow & hadron mass spectrum
- Inspective study of the generated configurations
 - thermalization / autocorrelation **strongly** depends on objects to monitor
 - **new run with $\tau = 2$ is ongoing**
- Outlook, or future plan
 - further improvements of statistics
 - application to chiral symmetry dynamics, charm quark physics , etc.
see [H. Fukaya's talk](#) and [Y. Cho's talk](#)