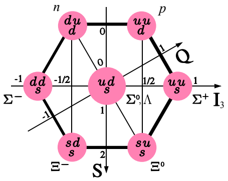

Isospin breaking effects from lattice QCD and QED

Antonin Portelli (University of Southampton)

29th of July 2013

This work is done within the **Budapest-Marseille-Wuppertal collaboration**.

Baryon octet mass splittings: `hep-lat/1306.2287`
Dashen's theorem and quark masses: in preparation

Plenary on isospin breaking effects: **N. Tantalo, Tue. at 11:30**

1. Motivation
2. Lattice methodology
3. Results
4. Conclusion

Motivation

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Isospin breaking effects

Sum of two little $\mathrm{O}(1\%)$ effects, possibly competing.

Nucleon mass splitting

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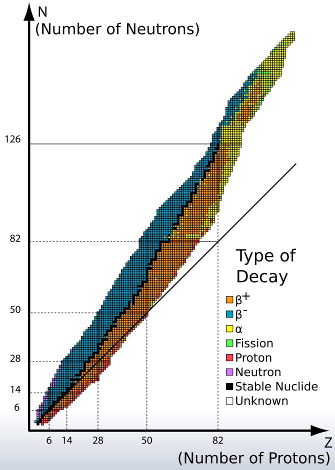

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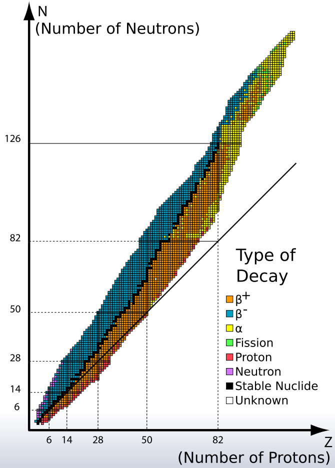

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Predicting $M_p - M_n$ is still an **open problem**.

Corrections to Dashen's theorem

In the $\mathrm{SU}(3)$ chiral limit [Dashen, 1969]:

$$\Delta_{\mathrm{QED}} M_K^2 = \Delta_{\mathrm{QED}} M_\pi^2 + \mathrm{O}(\alpha m_s)$$

with $\Delta_{\mathrm{QED}} M_X^2 = (M_{X^+}^2 - M_{X^0}^2)_{m_u = m_d}$.

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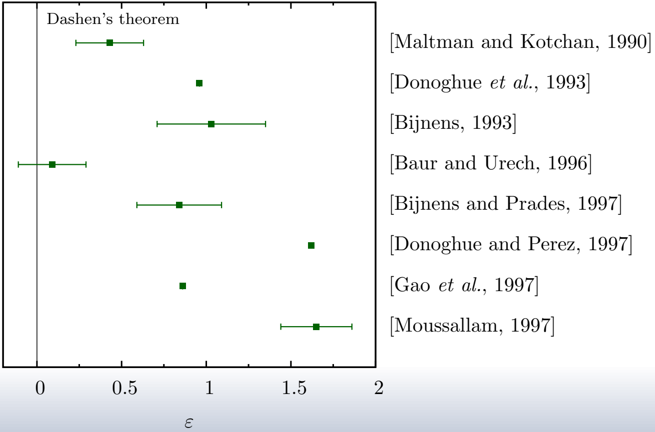
ε is important to determine **light quark mass ratios**.

Corrections to Dashen's theorem

Phenomenological results:



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- fermion action: tree-level $\mathrm{O}(a)$-improved Wilson fermions with 2 steps of HEX smearing;
- QCD is fully unquenched with $N_f = 2 + 1$, QED is quenched;
- QED quenching is suppressed by flavor $\mathrm{SU}(3)$ and $1/N_c$. Dimensionally it is an effect of $\mathrm{O}(10\%)$ on EM quantities.

QCD gauge configurations

39 independent gauge ensembles:
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$\mathrm{SU}(2)$ χPT paper in preparation: **A. Sastre talk, 9D, Fri. 14:00**.

Observables

Meson and baryon 2-point functions are computed in QCD+QED.
Mass splittings are extracted from the 2-point functions

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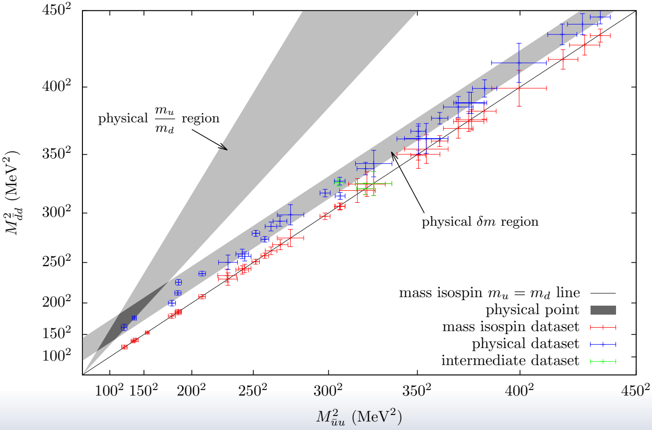
- 3 points with unphysical values of α.

Valence landscape



Physical point extrapolation

LO isospin expansion:

$$\Delta M_X = \alpha A_X + \Delta M^2 B_X$$

with $\Delta M^2 = M^2_{\bar{u}u} - M^2_{\bar{d}d} = 2B(m_u - m_d) + \dots$

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Physical point Taylor expansion of A_X and B_X:

$$A_X = a_0^X + a_1^X [M_\pi^2 - (M_\pi^\phi)^2] + a_2^X [M_{K^\chi}^2 - (M_{K^\chi}^\phi)^2] + a_3^X a + a_4^X \frac{1}{L}$$

$$B_X = b_0^X + b_1^X [M_\pi^2 - (M_\pi^\phi)^2] + b_2^X [M_{K^\chi}^2 - (M_{K^\chi}^\phi)^2] + b_3^X f(a)$$

with $M_{K^\chi}^2 = \frac{1}{2}(M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2)$ and $f(a) = a^2$ or $\alpha_s(a)a$.

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Unambiguous to LO at the physical masses.

(*cf.* [BMWc, `hep-lat/1306.2287`] and [A.P., `hep-lat/1307.6056`])

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- 2 types of QCD discretisation effects ($\mathrm{O}(a^2)$ and $\mathrm{O}(\alpha_s a)$);
- 2 different expansions for A_X and B_X (Taylor and Padé);
- 1 or 2 different choices of active fit parameters depending on the quantity.

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In total: between 64 and 256 fits for each quantity. Statistical sampling done through bootstrap resampling using 2000 samples.

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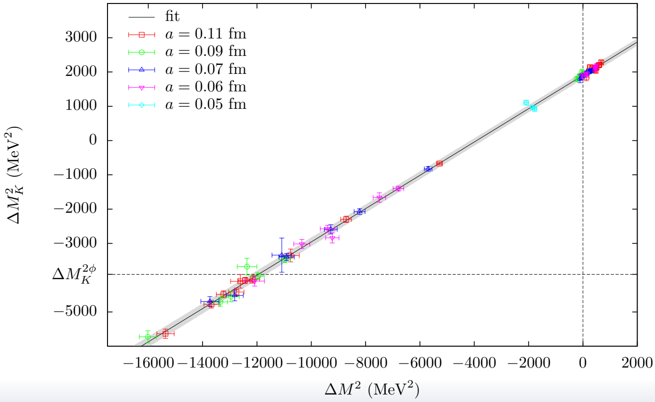
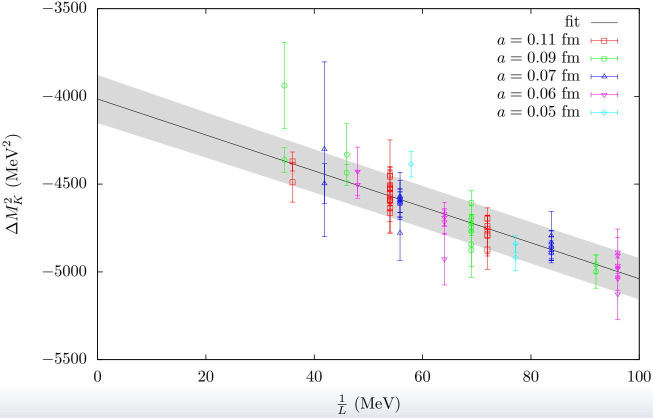
Results

Global fits


fit
a = 0.11 fm
a = 0.09 fm
a = 0.07 fm
a = 0.06 fm
a = 0.05 fm
3000
2000
1000
0
−1000
−2000
−3000
$\Delta M_K^{2\phi}$
−5000
ΔM_K^2 (MeV^2)
−16000 −14000 −12000 −10000 −8000 −6000 −4000 −2000 0 2000
ΔM^2 (MeV^2)

Global fits



−3500
−4000
−4500
−5000
−5500
0
20
40
60
80
100
ΔM_K^2 (MeV^2)
$\frac{1}{L}$ (MeV)
fit
$a = 0.11$ fm
$a = 0.09$ fm
$a = 0.07$ fm
$a = 0.06$ fm
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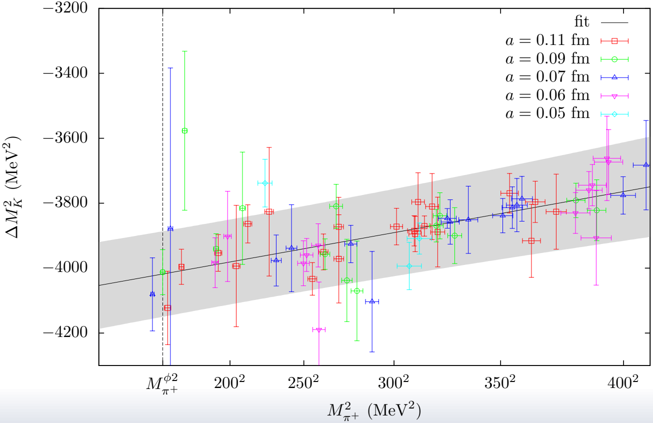
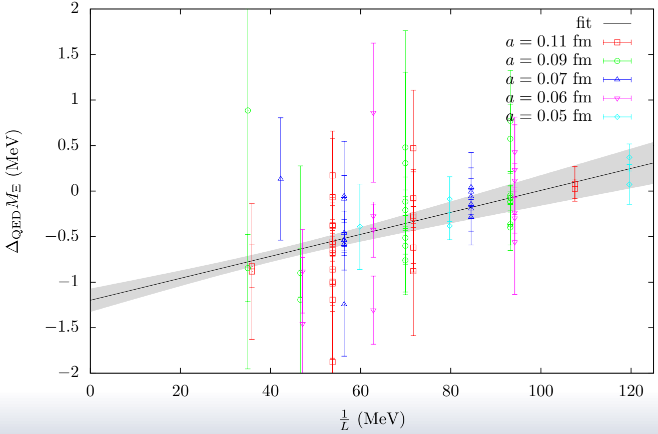
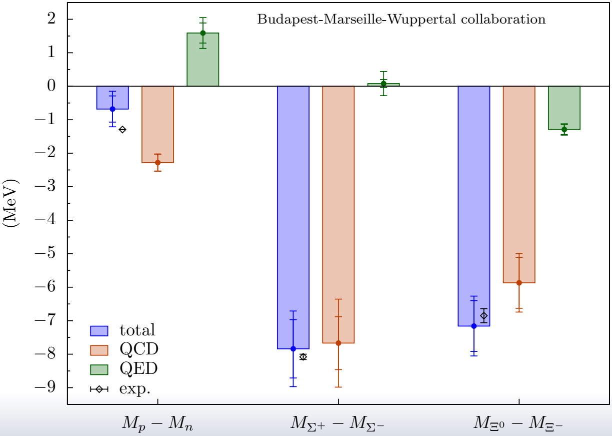

fit
$a = 0.11$ fm
$a = 0.09$ fm
$a = 0.07$ fm
$a = 0.06$ fm
$a = 0.05$ fm
-3200
-3400
-3600
-3800
-4000
-4200
ΔM_K^2 (MeV^2)
$M_{\pi^+}^{\phi 2}$
200^2
250^2
300^2
350^2
400^2
$M_{\pi^+}^2$ (MeV^2)

Global fits



fit
$a = 0.11$ fm
$a = 0.09$ fm
$a = 0.07$ fm
$a = 0.06$ fm
$a = 0.05$ fm
$\Delta_{\mathrm{QED}} M_{\Xi}$ (MeV)
$\frac{1}{L}$ (MeV)
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
0
20
40
60
80
100
120

Baryon octet splittings



Dashen's theorem

Preliminary results:

$$\varepsilon = \frac{\Delta_{\mathrm{QED}} M_K^2}{\Delta M_\pi^2} - (1 - \varepsilon_m) = 0.63(6)(7)(2)_{\varepsilon_m}$$

Using $\varepsilon_m = \Delta_{\mathrm{QCD}} M_\pi^2 / \Delta M_\pi^2 = 0.04(2)$ from FLAG.

Assuming 10% electro-quenching uncertainties:
25% quenching error on ε.

Light quark masses

Preliminary results ($\overline{\mathrm{MS}}$ scheme at $2\ \mathrm{GeV}$):

$$\delta m = m_u - m_d = \frac{\Delta M^2}{2B} = -2.36(8)(5)\ \mathrm{MeV}$$

$$\frac{m_u}{m_d} = \frac{m_{ud} + \delta m/2}{m_{ud} + \delta m/2} = 0.49(1)(1) \qquad (1)$$

B is taken from BMWc preliminary results (*cf.* A. Sastre talk).
m_{ud} is taken from [BMWc, `hep-lat/1011.2403`].

Assuming 10% electro-quenching uncertainties:
3% quenching error on δm and 2% on $\frac{m_u}{m_d}$.

Light quark masses

Preliminary results:

$$R = \frac{m_s - m_{ud}}{m_d - m_u} = \frac{m_s/m_{ud} - 1}{-\delta m/m_{ud}} = 39.0(1.5)(1.0)$$

$$Q = \sqrt{\frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}} = \sqrt{\frac{(m_s/m_{ud})^2 - 1}{-2\delta m/m_{ud}}} = 23.6(5)(3)$$

m_{ud} and m_s/m_{ud} are taken from [BMWc, `hep-lat/1011.2403`].

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- Significant progress on corrections to Dashen's theorem;
- Precise determination of the individual light quark masses;
- m_u is ~ 10 sigmas away from 0!
- Large power-like FV effects;
- For some important quantities, electro-quenching may already be the dominant source of uncertainty.

Thank you.

BMWc Collaboration

Budapest (Eötvös University)
S.D. Katz

Marseille (CPT)
J. Frison (now Univ. of Edinburgh), L. Lellouch, A. Portelli (now Univ. of Southampton), A. Ramos (now NIC DESY Zeuthen) and A. Sastre

Wuppertal (Bergische Universität)
Sz. Borsanyi, S. Dürr, Z. Fodor, C. Hölbling, S. Krieg, Th. Kurth, Th. Lippert and K. Szabo