2+1 flavour thermal studies on an anisotropic lattice

Chris Allton (Swansea University)

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Gert Aarts, CRA, Alessandro Amato, Wynne Evans, Pietro Giudice, Simon Hands, Aoife Kelly, Seyong Kim, Maria-Paola Lombardo, Dhagash Mehta, Bugra Oktay, Sinead Ryan, Jon-Ivar Skullerud, Don Sinclair, Tim Harris

FASTSUM Collaboration

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Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

Lattice Parameters

	1st Generation	2nd Generation *(HSC parameters)*
Flavours	2	2+1
Volume(s)	(2fm)^3	(3fm)^3 & (4fm)^3
a_s [fm]	0.167	0.123
a_t [fm]	0.028	0.035
anisotropy	6	3.5
M_π/M_ρ	~ 0.55	~ 0.45
Action	Gauge: Symanzik Improved Fermion: fine-Wilson, coarse-Hamber-Wu stout-link	Gauge: Symanzik Improved Fermion: Clover, Tadpole Improved

Lattice Temperatures

1st Generation

2 flavours

smaller volume: (2fm)^3

coarser lattices: $a_s = 0.167$fm

quark mass: $M_\pi / M_\rho =\sim 0.55$

N_s	N_τ	T(MeV)	T/T_c
12	16	460	2.10
12	18	409	1.86
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

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2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3$ – $(4\text{fm})^3$

finer lattices: $a_s = 0.123$fm

quark mass: $M_\pi/M_\rho =\sim 0.45$

N_s	N_τ	T(MeV)	T/T_c
24, 32	16	350	1.89
24	20	280	1.52
24, 32	24	235	1.26
24, 32	28	200	1.08
24, 32	32	175	0.95
24	36	155	0.84
24	40	140	0.76
32	48	115	0.63
16	128	45	0.24

Outline

- Polyakov Loop & its Susceptibility
- Light Mesons: Pseudoscalar vs Scalar
- Electric Charge Susceptibility, χ
- Electrical Conductivity, σ
- Charmonium Potential, $V(r)$
- NRQCD (Bottomonium) Spectral Functions, $\rho(\omega)$

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BlueGene Q (DiRAC/Edinburgh) 200M core-hours = 1.5 rack-years

BlueGene Q (PRACE/Cineca) 34M core-hours

Renormalising the Polyakov Loop

Polyakov Loop, L, related to free energy, F, via:

$$L(T) = e^{-F(T)/T}$$

However, F only defined up to addivitive renormatlisation constant $\Delta F = f(\beta, \kappa)$.
Imposing renormalisation condition:

$$L_R(T_R) \equiv \text{some number}$$

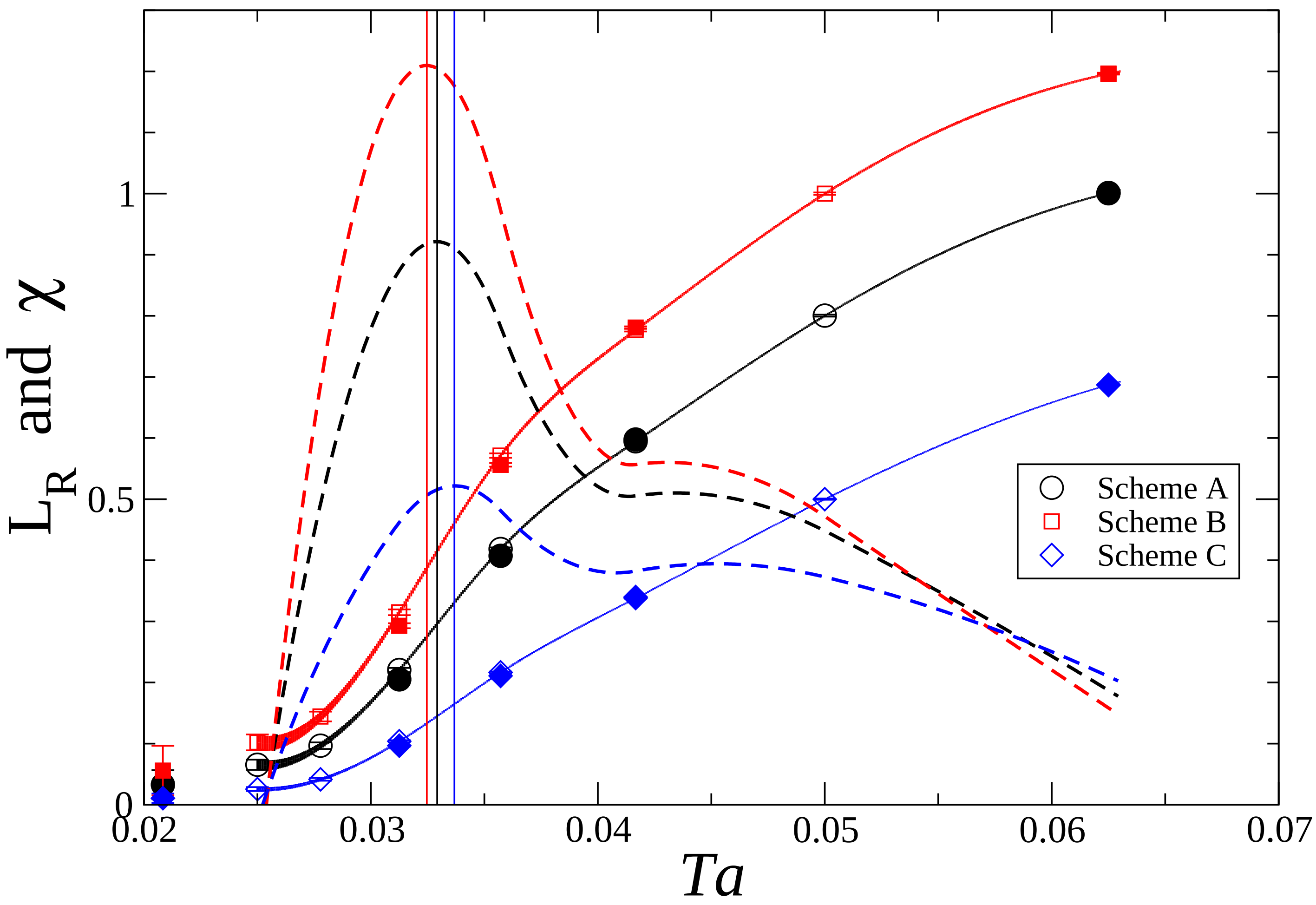
gives us

$$L_R(T) = e^{-F_R(T)/T} = e^{-(F_0(T)+\Delta F)/T} = L_0(T)e^{-\Delta F/T} = L_0(T){Z_L}^{N_\tau}$$

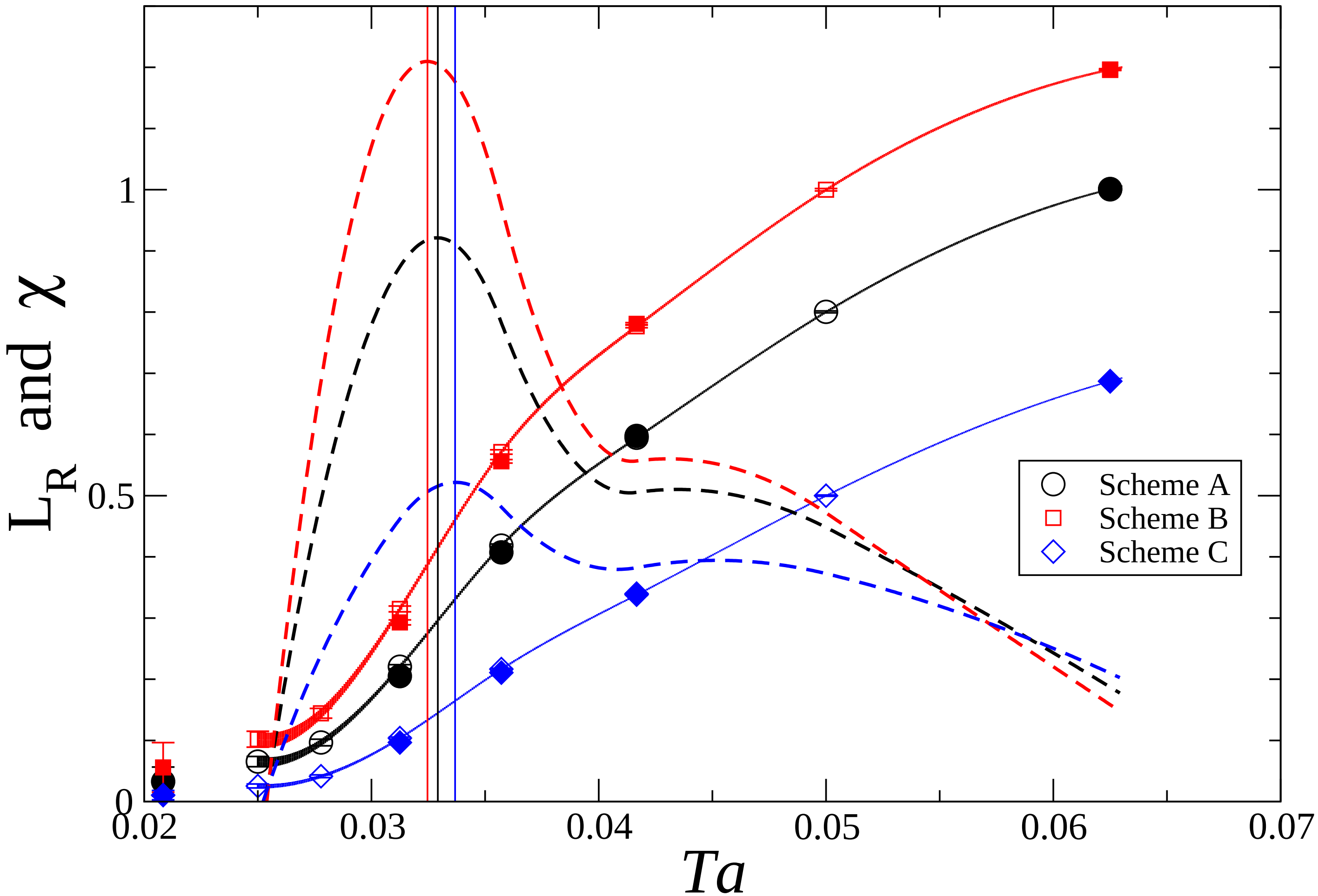
and Z_L defined from renormalisation condition.

Wuppertal-Budapest, PLB713(2012)342 [1204.4089]

Polyakov Loop



Polyakov Loop

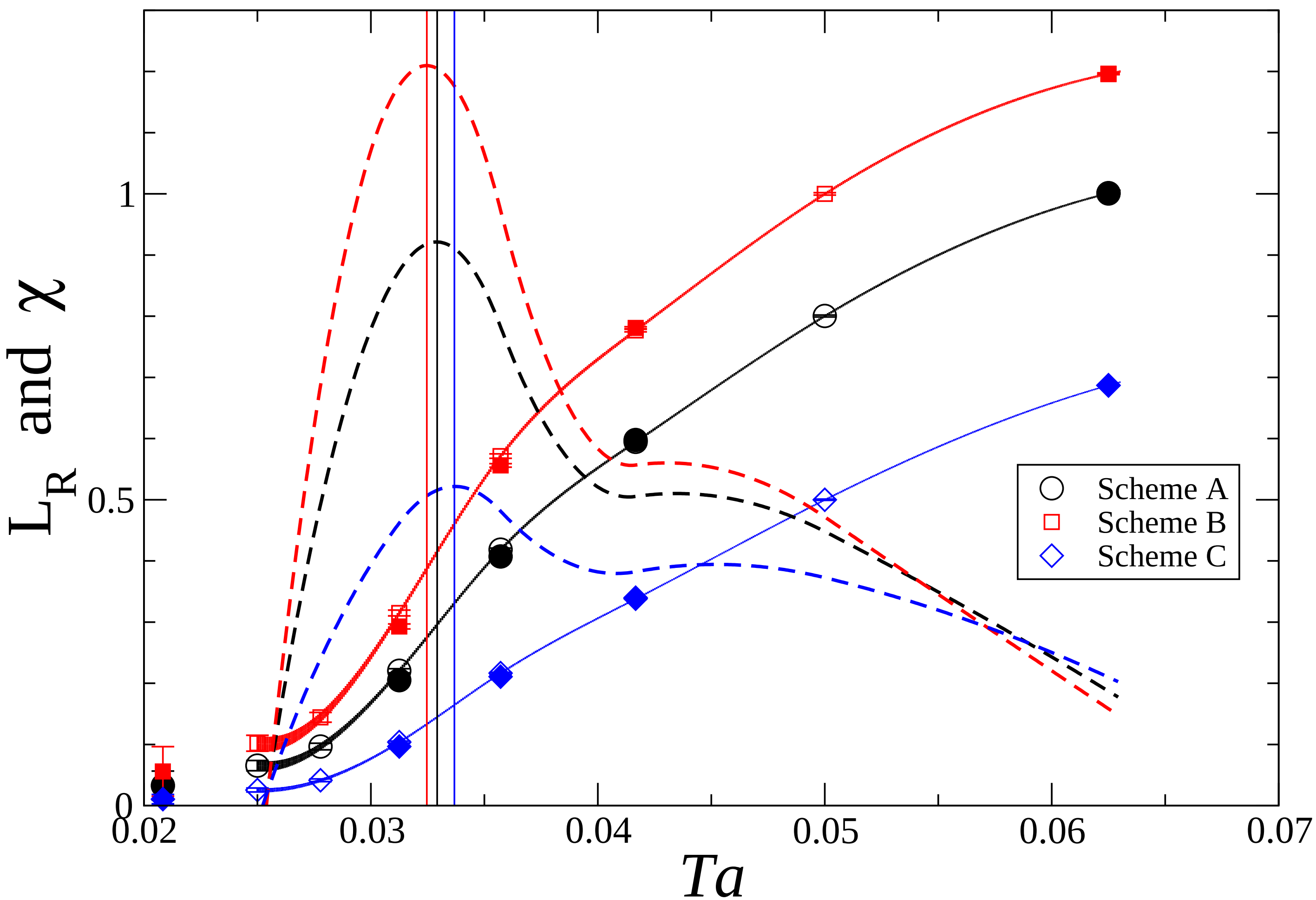


Scheme A: $L_R(Nt = 16) = 1.0$

Scheme B: $L_R(Nt = 20) = 1.0$

Scheme C: $L_R(Nt = 20) = 0.5$

Polyakov Loop



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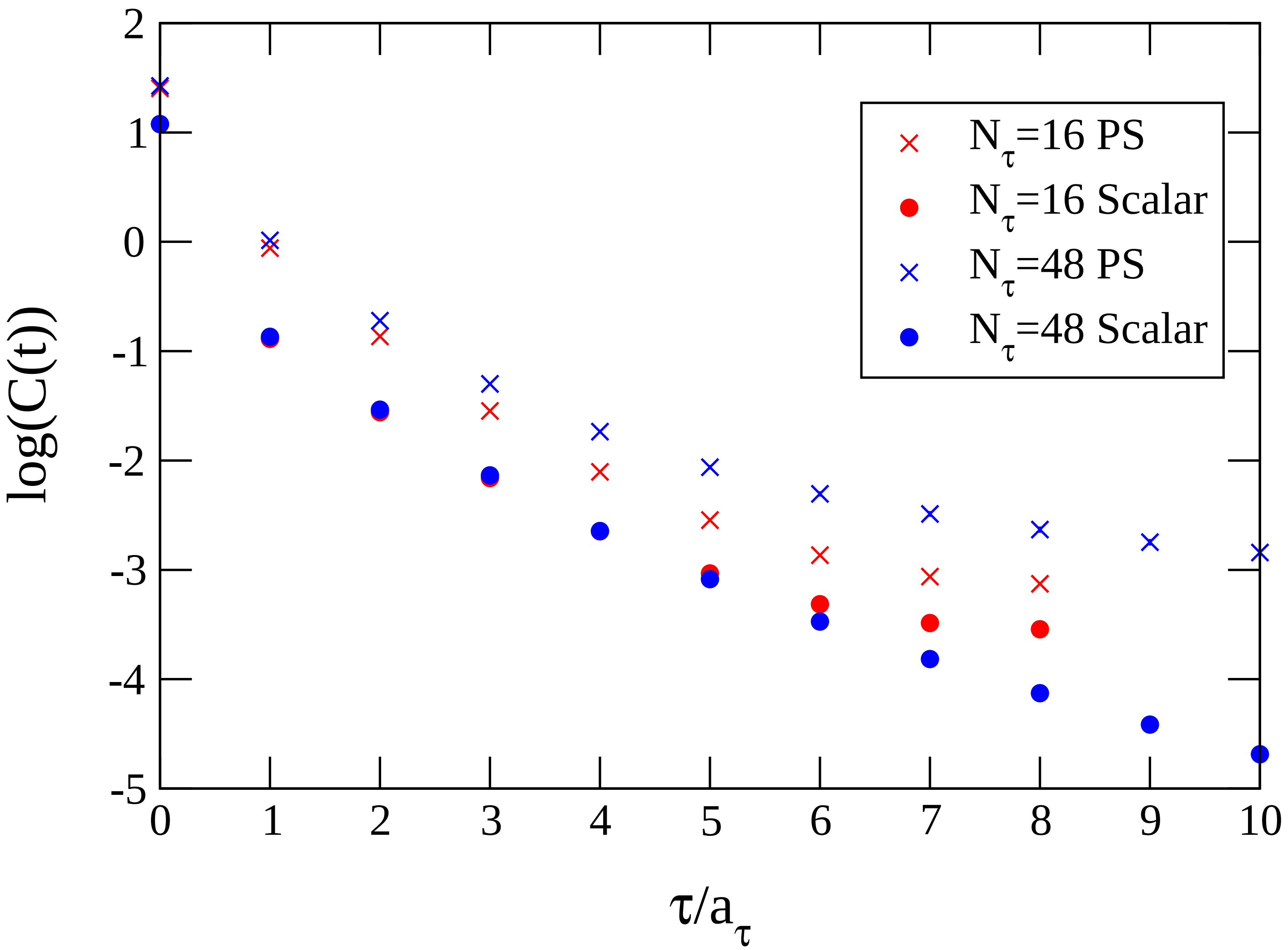
Cubic spline, solid = 32^3, open = 24^3

$\longrightarrow a_\tau T_c = 0.0329(7)$

i.e. $N_\tau^{\text{crit}} = 30.4(7)$ or $T_c = 171(4)$ MeV

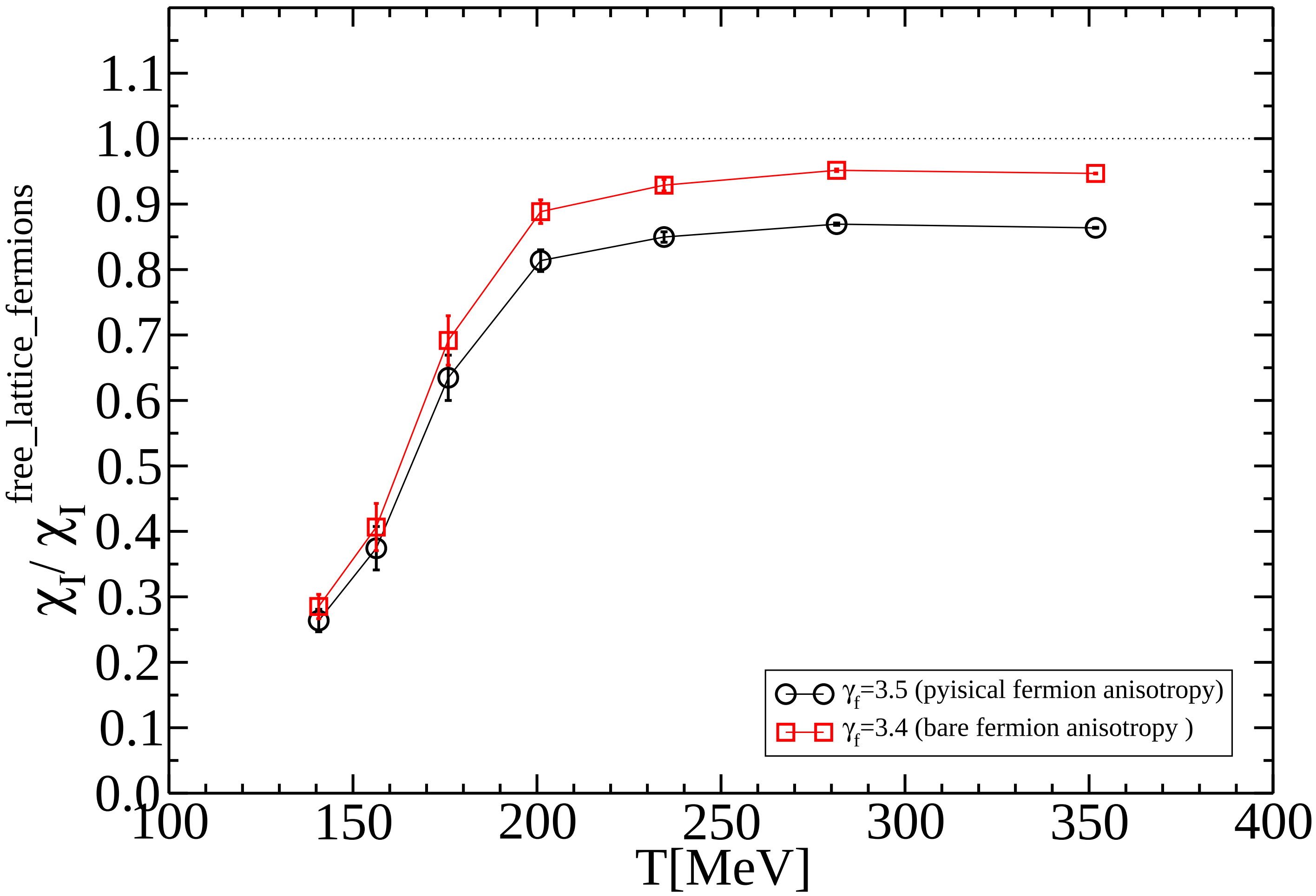
Light mesons & Chiral Symmetry



$\longrightarrow$ (partial) restoration of chiral symmetry at high T

Electric Charge Susceptibility, χ

Poster: Pietro Giudice, Tuesday 6-8pm



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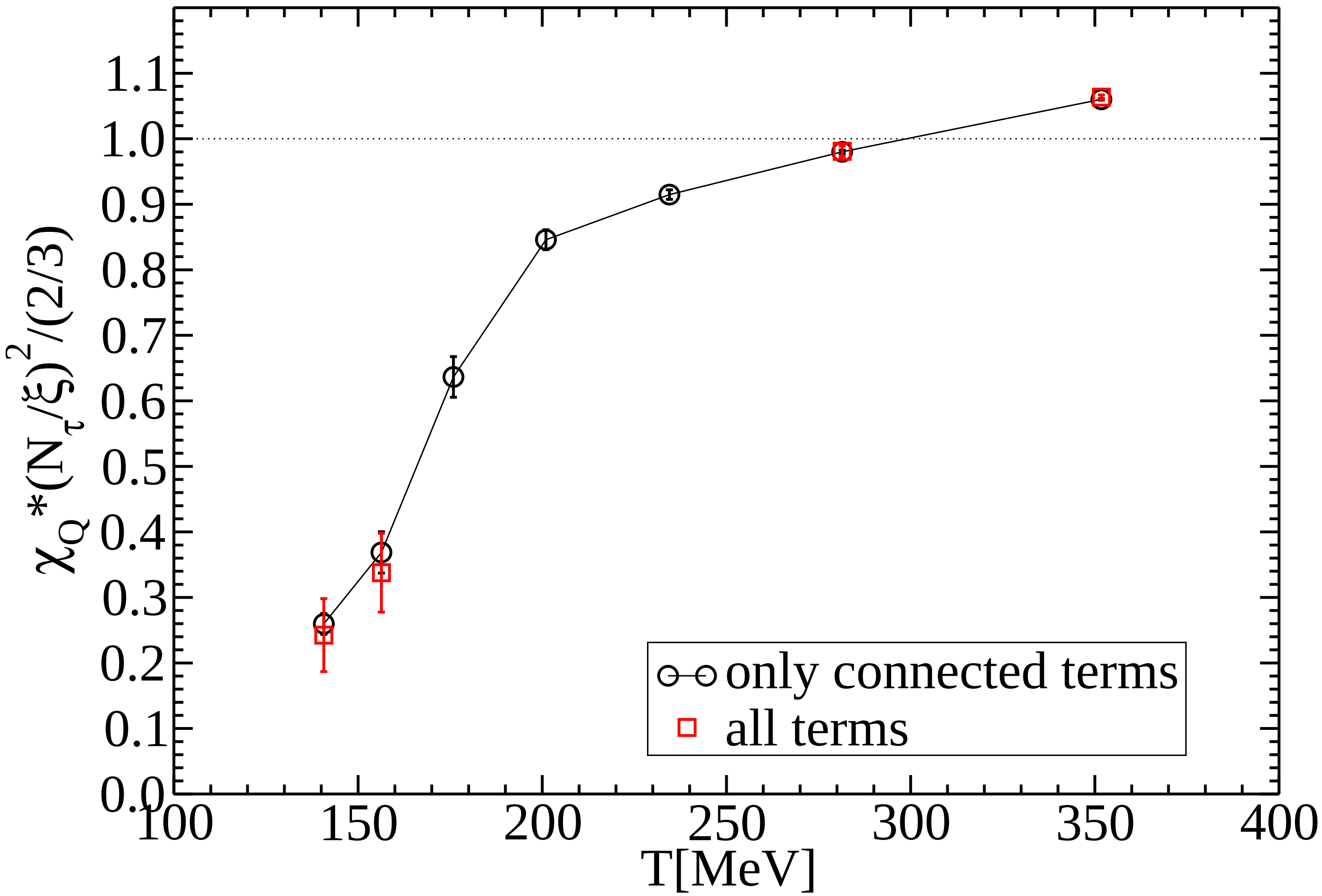

Conductivity (Theory)

Electromagnetic current:

$$j_\mu^{\text{em}}(x) = e \sum_f q_f j_\mu^f(x)$$

Correlation F'ns: $$G_{\mu\nu}(\tau, \mathbf{p}) = \int d^3x\, e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle\, j_\mu^{\text{em}}(0, \mathbf{x}) j_\nu^{\text{em}}(\tau, \mathbf{y})^\dagger \,\rangle$$

$$= \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega)\, \rho_{\mu\nu}(\omega, \mathbf{p})$$

with kernel: $$K(\tau, \omega) = \frac{\cosh[\omega(\tau - /2T)]}{\sinh[\omega/2T]}$$

Conductivity: $$\frac{\sigma}{T} = \frac{1}{6T} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}, \qquad \rho(\omega) = \sum_{i=1}^{3} \rho_{ii}(\omega)$$

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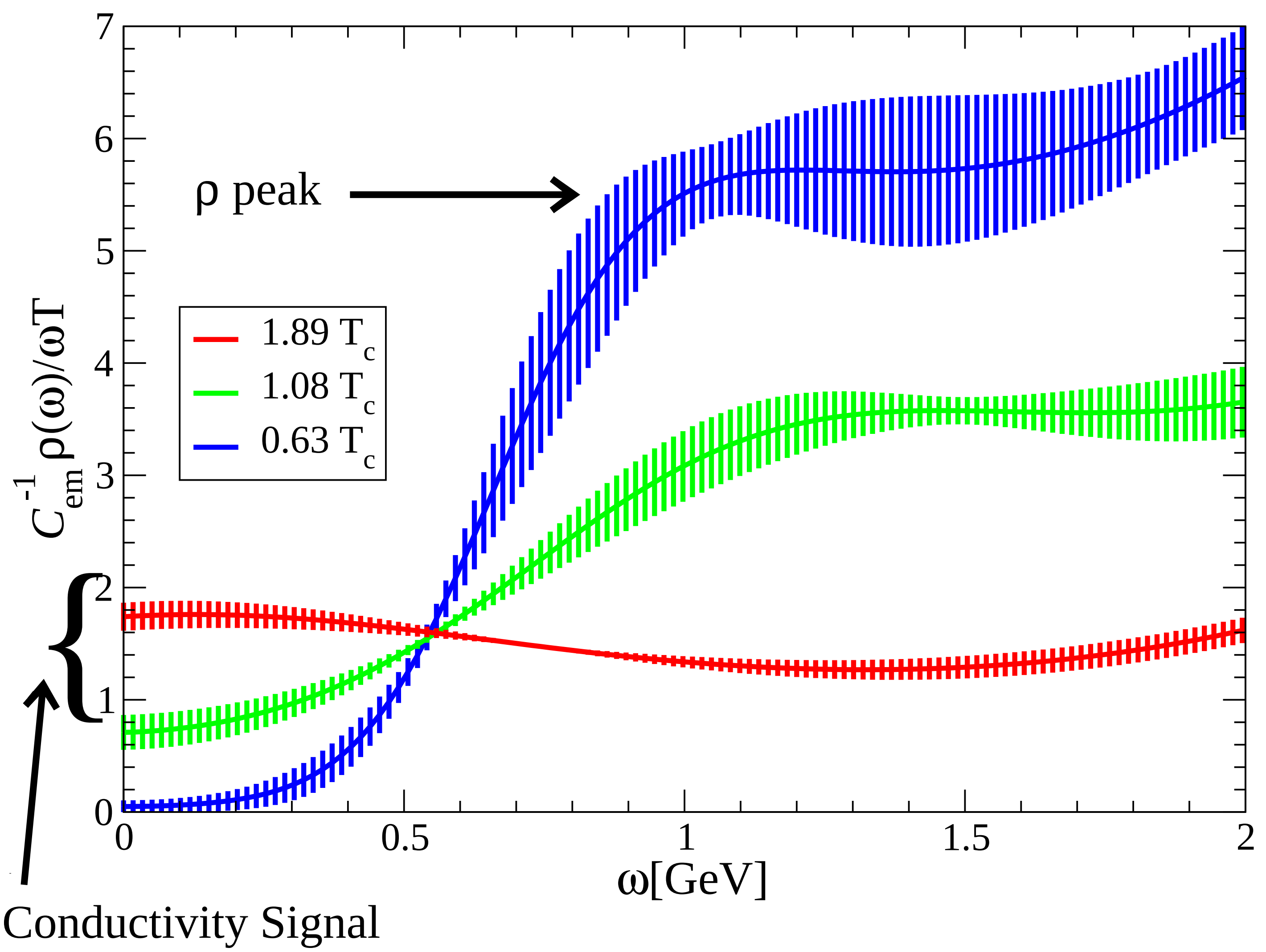
MEM approach

M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 46, 459(2001)

Electrical Conductivity, σ

Alessandro Amato, *(Mon 17:30, Room A)*

$$C_{\rm em} = e^2 \sum_f q_f^2 = 5/9 e^2$$



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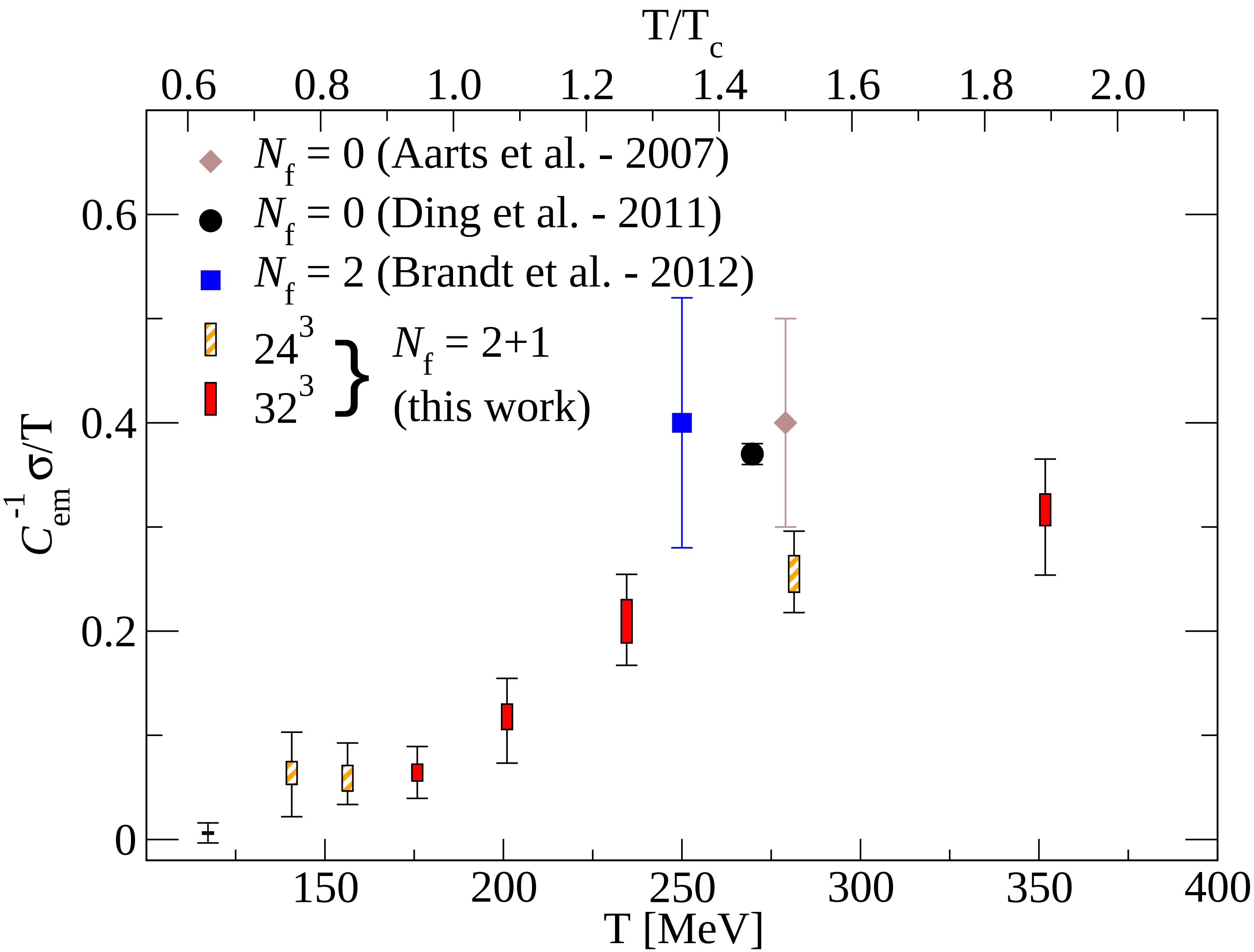
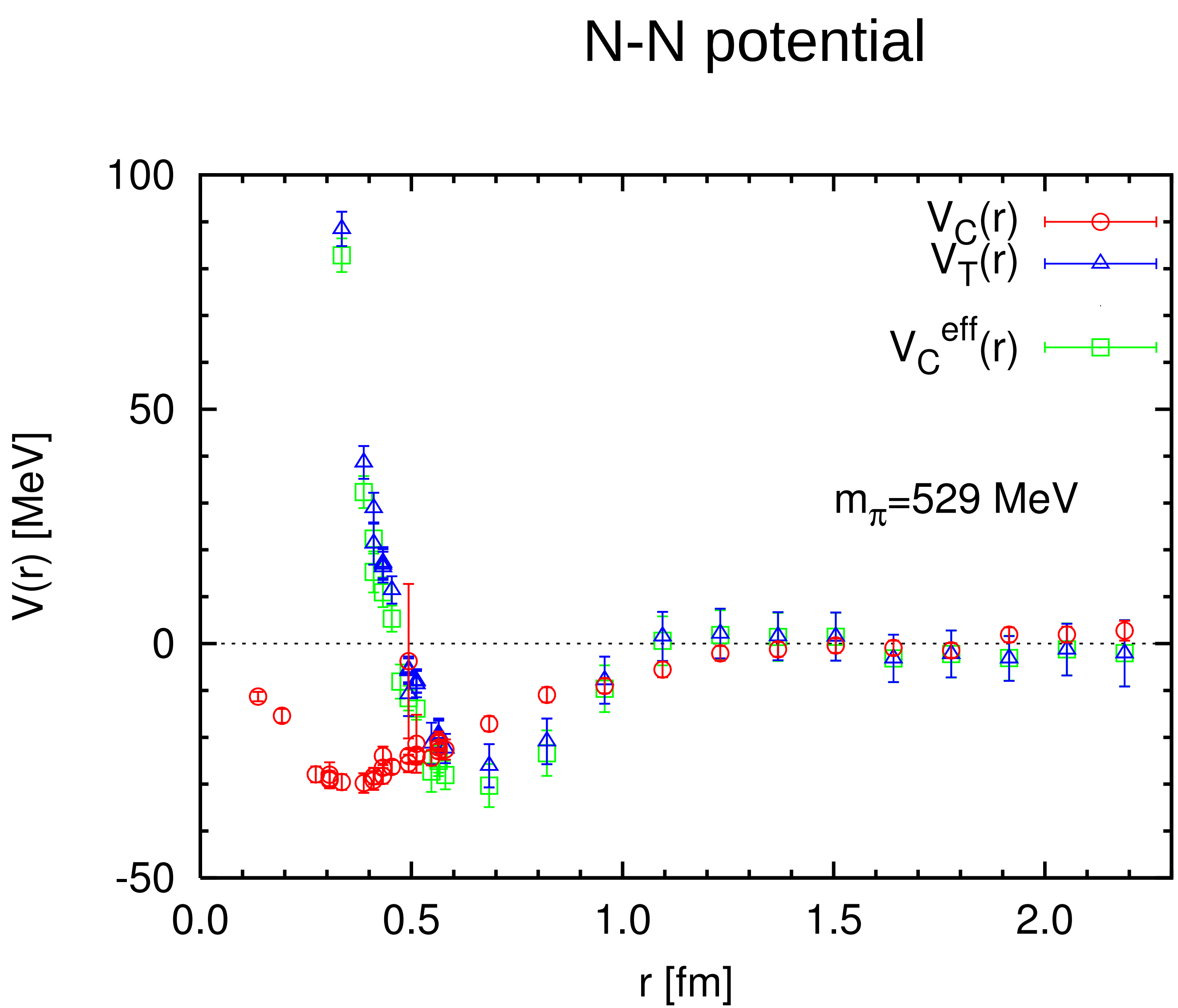


Lattice goes Nuclear



HAL QCD Collaboration, Aoki, Doi, Hatsuda, Ikeda, Inoue, Ishii, Murano, Nemura, Sasaki

Schrödinger Equation Approach

Hatsuda, PoS CD09 (2009) 068 use the Schrödinger equation to "reverse engineer" the potential, $V(r)$, given the Nambu-Bethe-Salpeter wavefunction, $\psi(r)$:

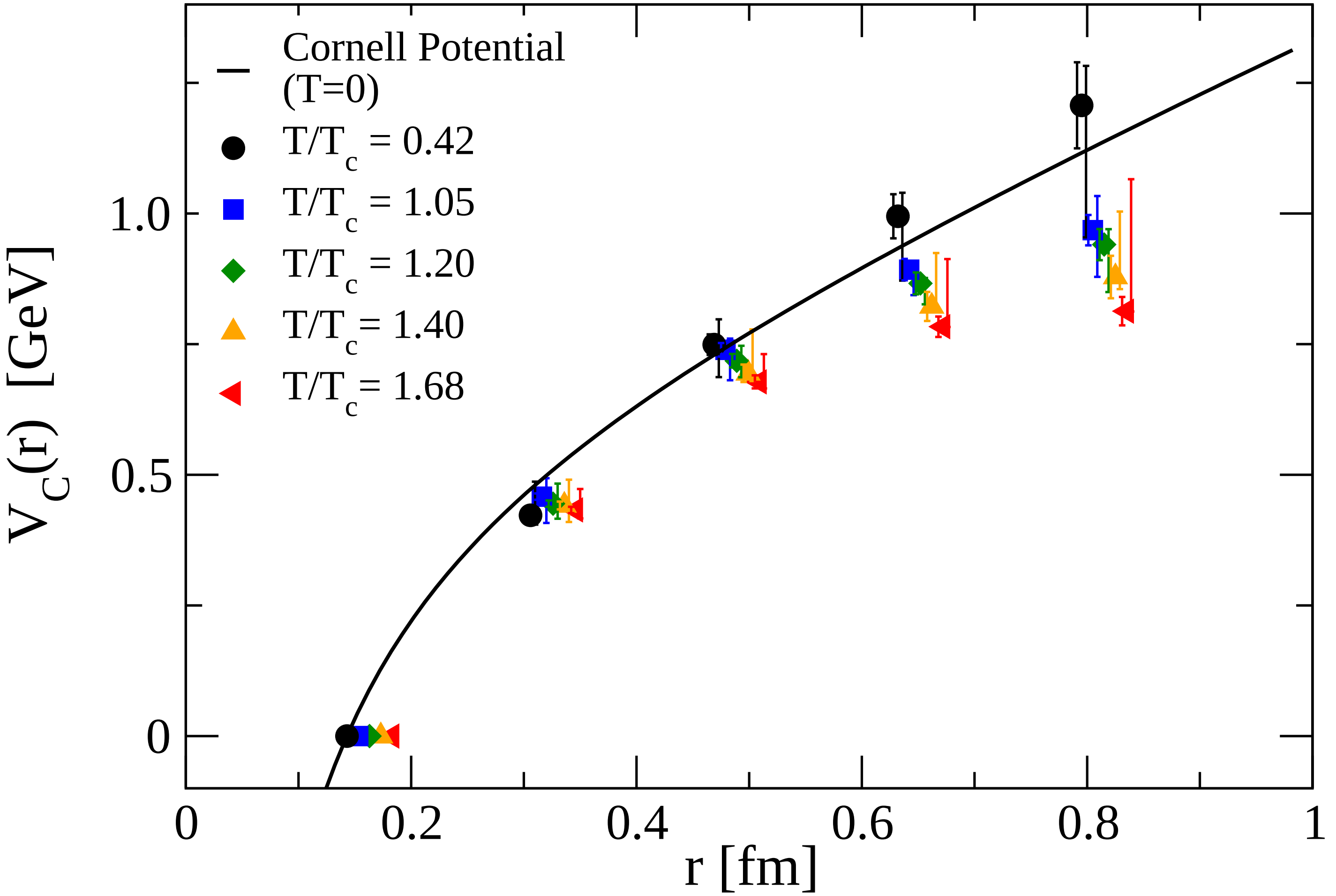
$$\left(\frac{p^2}{2M} + \underset{\substack{\downarrow \\ \text{output}}}{V(r)}\right) \overset{\substack{\text{input} \\ \downarrow}}{\psi(r)} = \overset{\substack{\text{input} \\ \downarrow}}{E} \ \overset{\downarrow}{\psi(r)}$$

$\psi(r)$ is determined from a lattice simulation from correlators of *non-local* (point-split) operators, $J(x;\vec{r}) = q(x)\,\Gamma\, U(x, x+\vec{r})\overline{q}(x+\vec{r})$

$$\begin{aligned} C(\vec{r}, t) &= \sum_{\vec{x}} < J(0;\vec{r})\ J(x;\vec{r}) > \\ &\longrightarrow \ |\psi(r)|^2\ e^{-Et} \end{aligned}$$

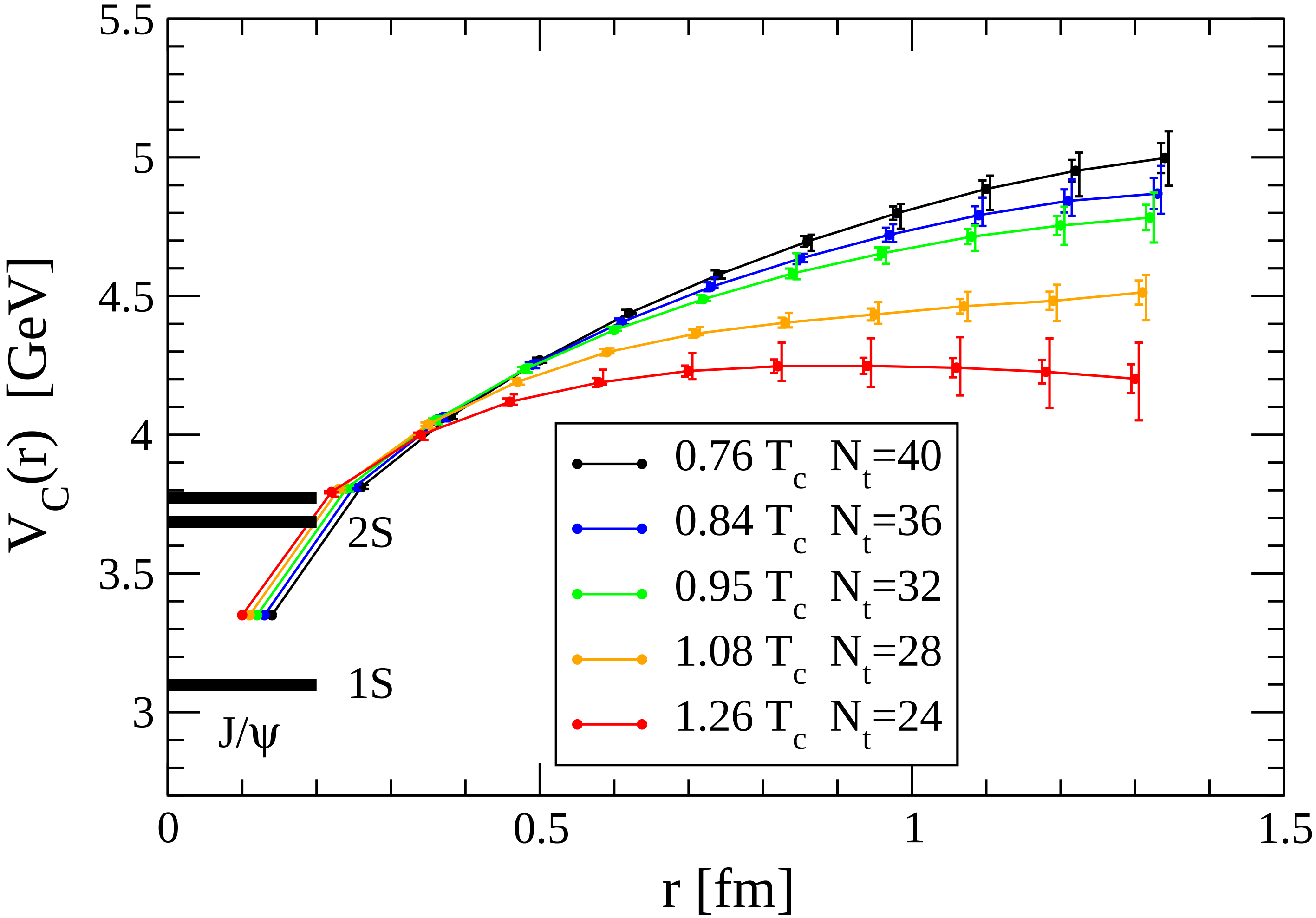
$V_C(r)$ (1st Generation)

P.W.M. Evans, CRA and J.-I. Skullerud, arXiv:1303.5331



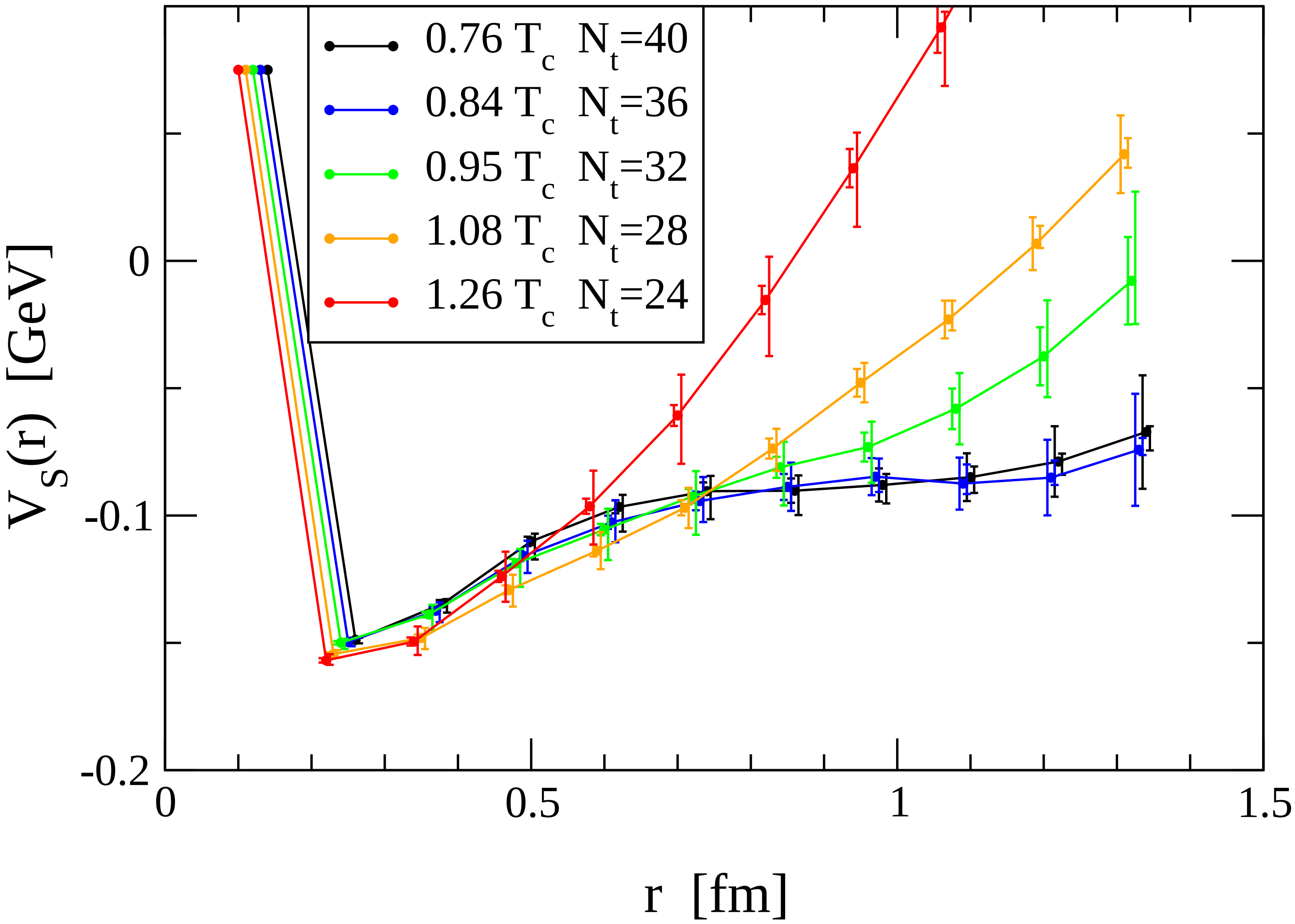
Charmonium Central Potential, $V(r)$

Wynne Evans, Fri 17:10, Room A



Charmonium Spin Dept Potential, $V(r)$

Wynne Evans, Fri 17:10, Room A

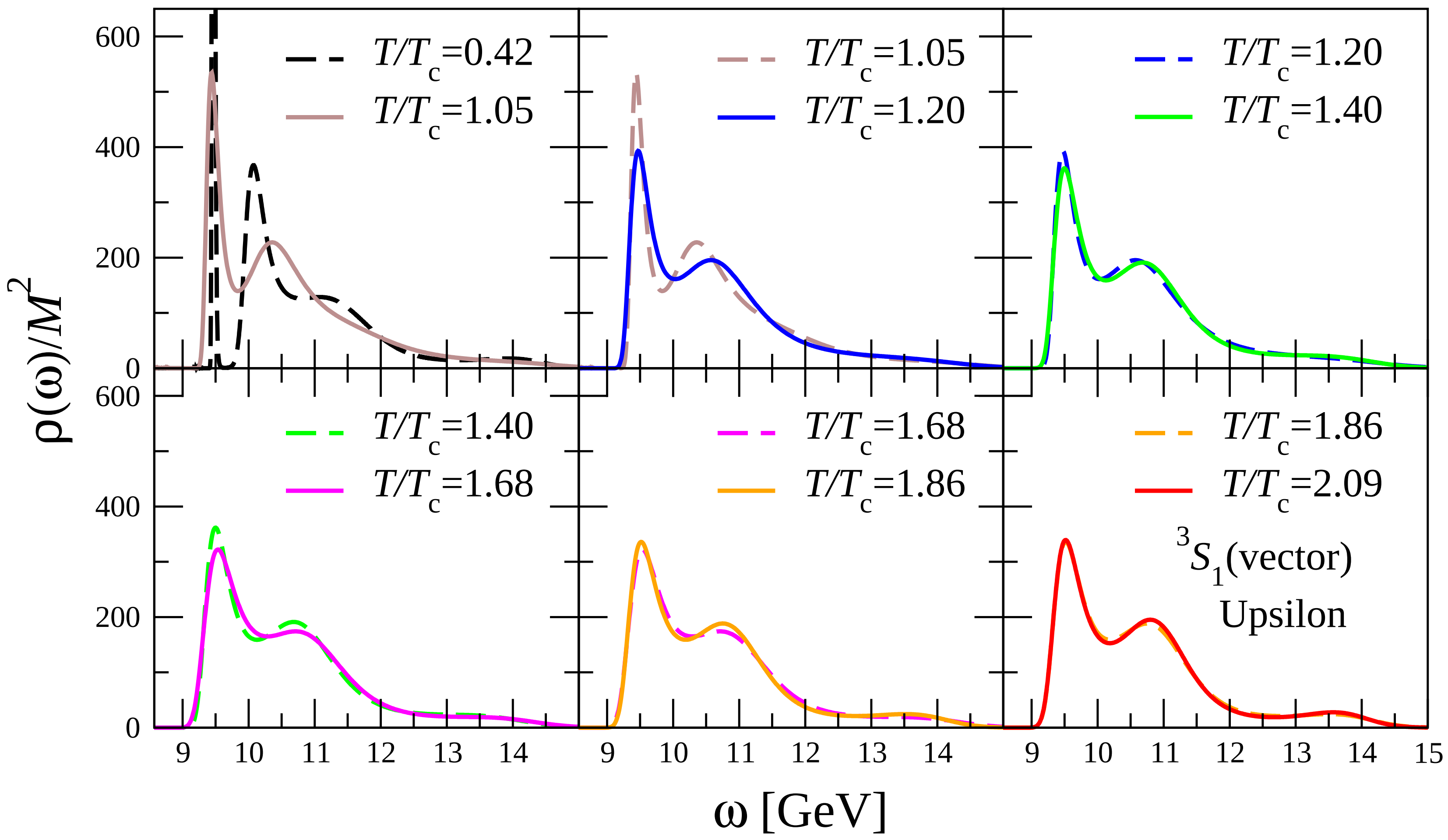


NRQCD

- An expansion in v/c valid as quark mass $M \to \infty$
  - applicable for b-quarks
- Heavy quark mass, $M > T$
- M factored out of energy scale: $\omega \to \omega - M$
- no periodicity in time
  - bottom quark is a **probe** of thermal media
  - simpler numerically to deal with correlation f'ns
- NRQCD formulism we use is correct to $\mathcal{O}(v^4)$

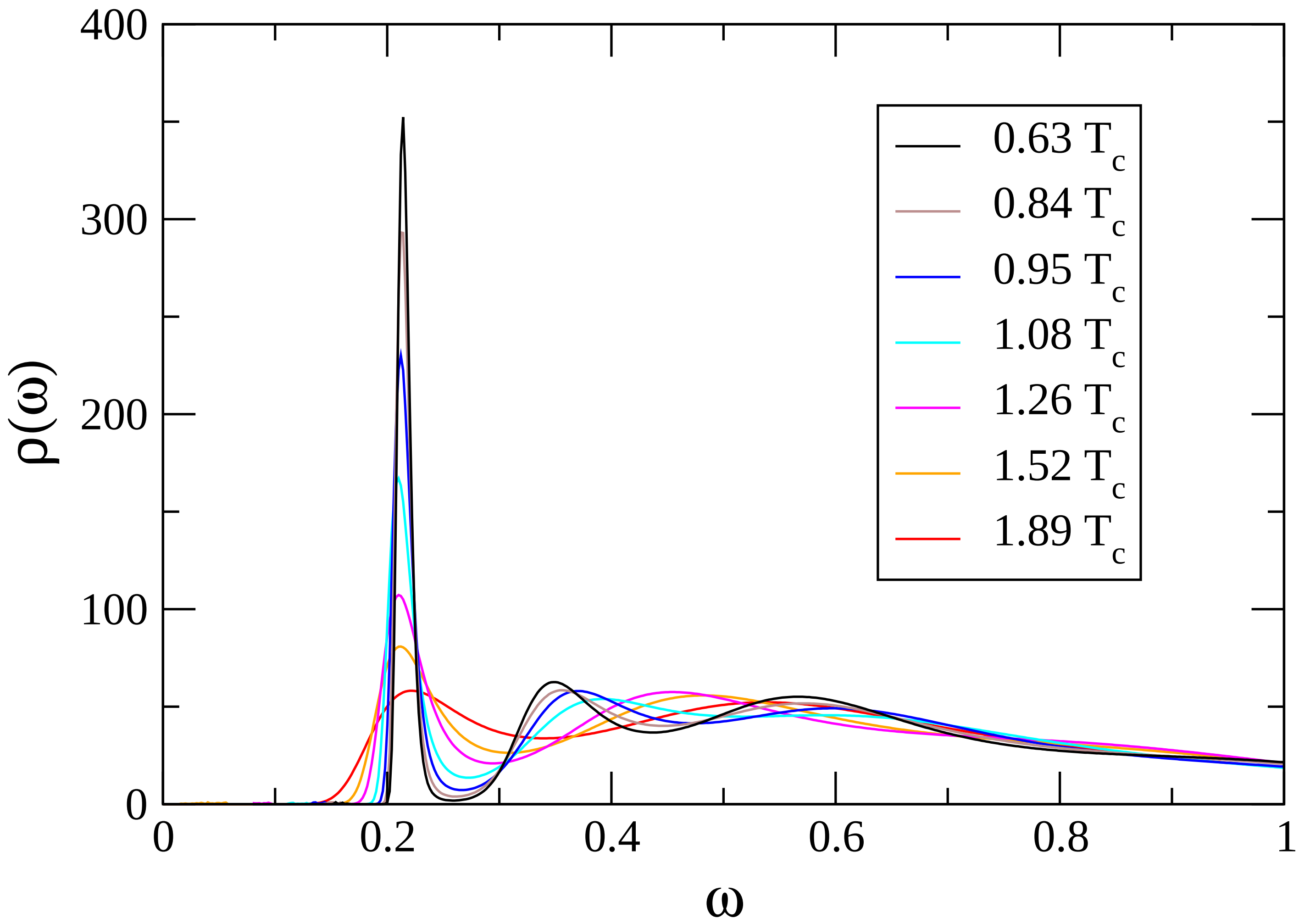
$T = 0$ spectral functions (1st Generation)

Aarts et al, JHEP **1111** (2011) 103 [arXiv:1109.4496]



NRQCD spectral functions, s-wave

Tim Harris, Fri 18:10, Room A



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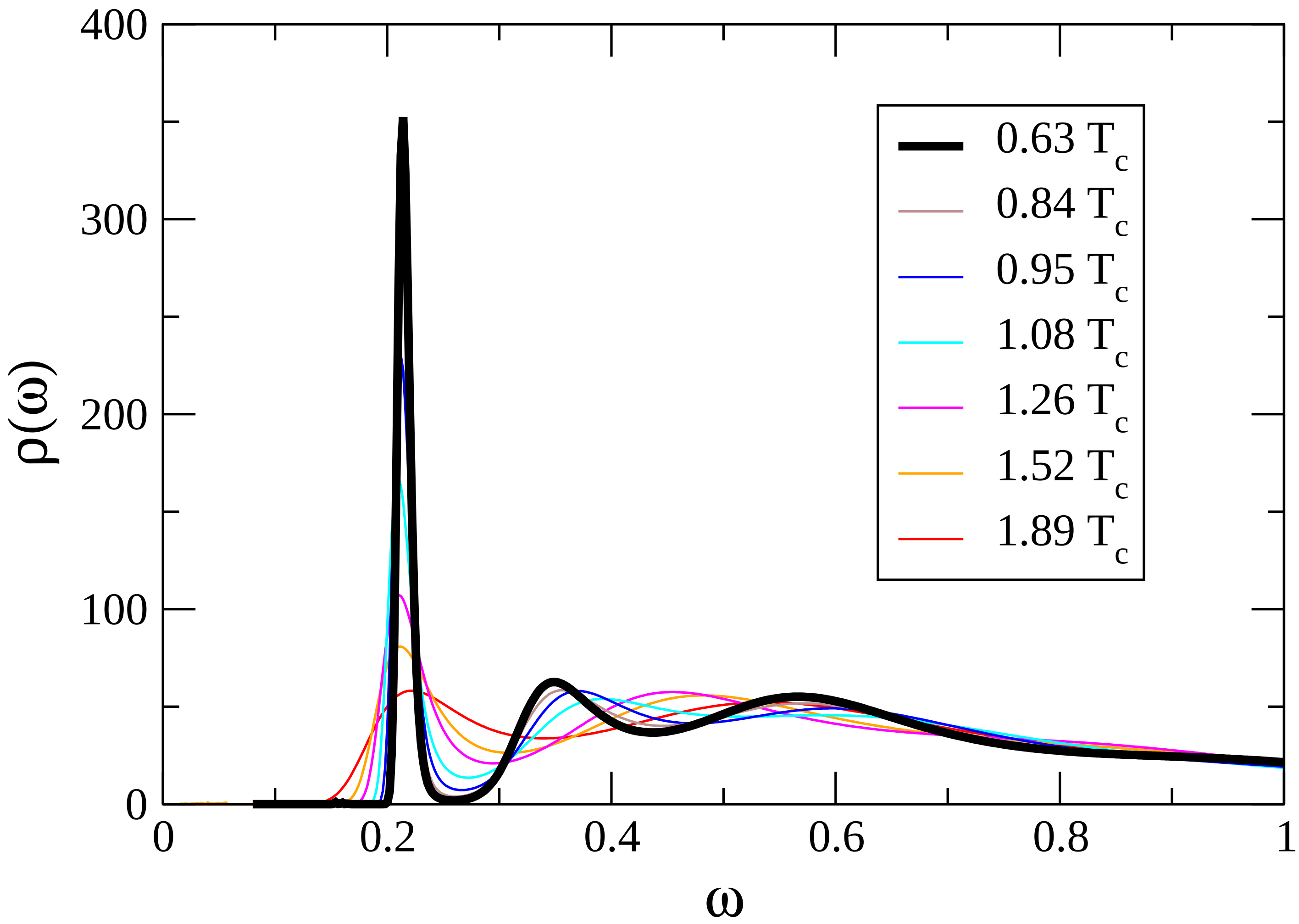

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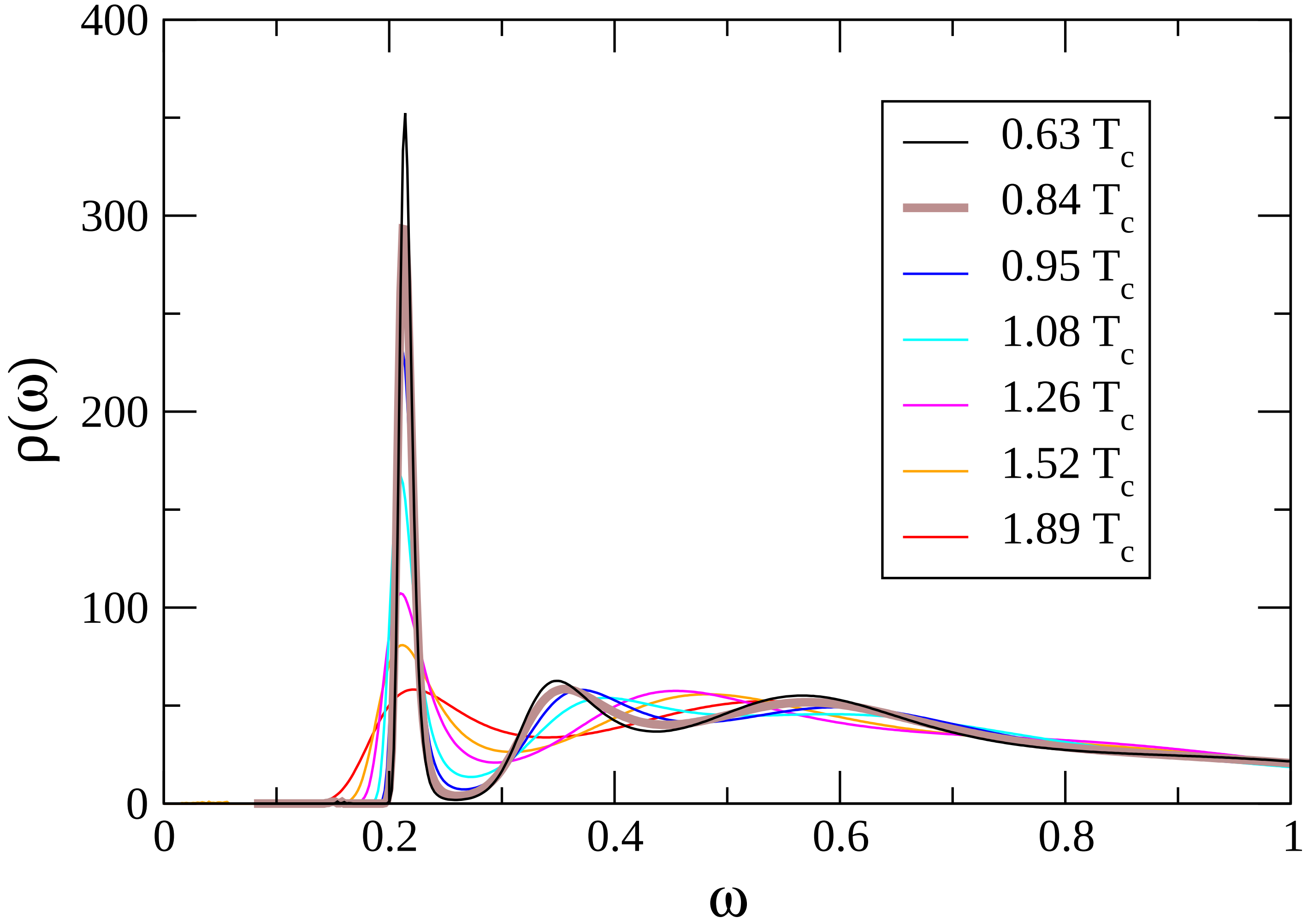

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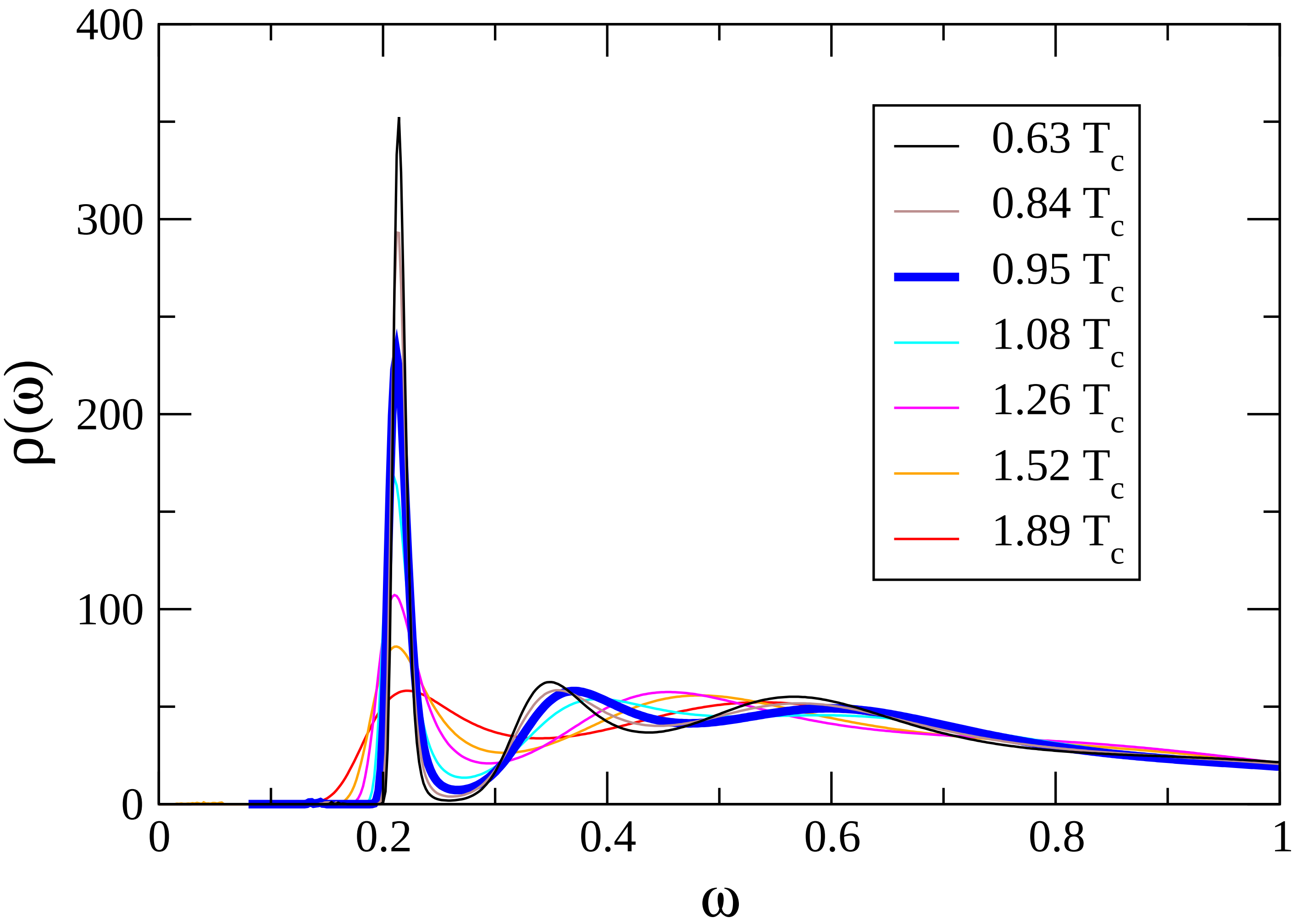

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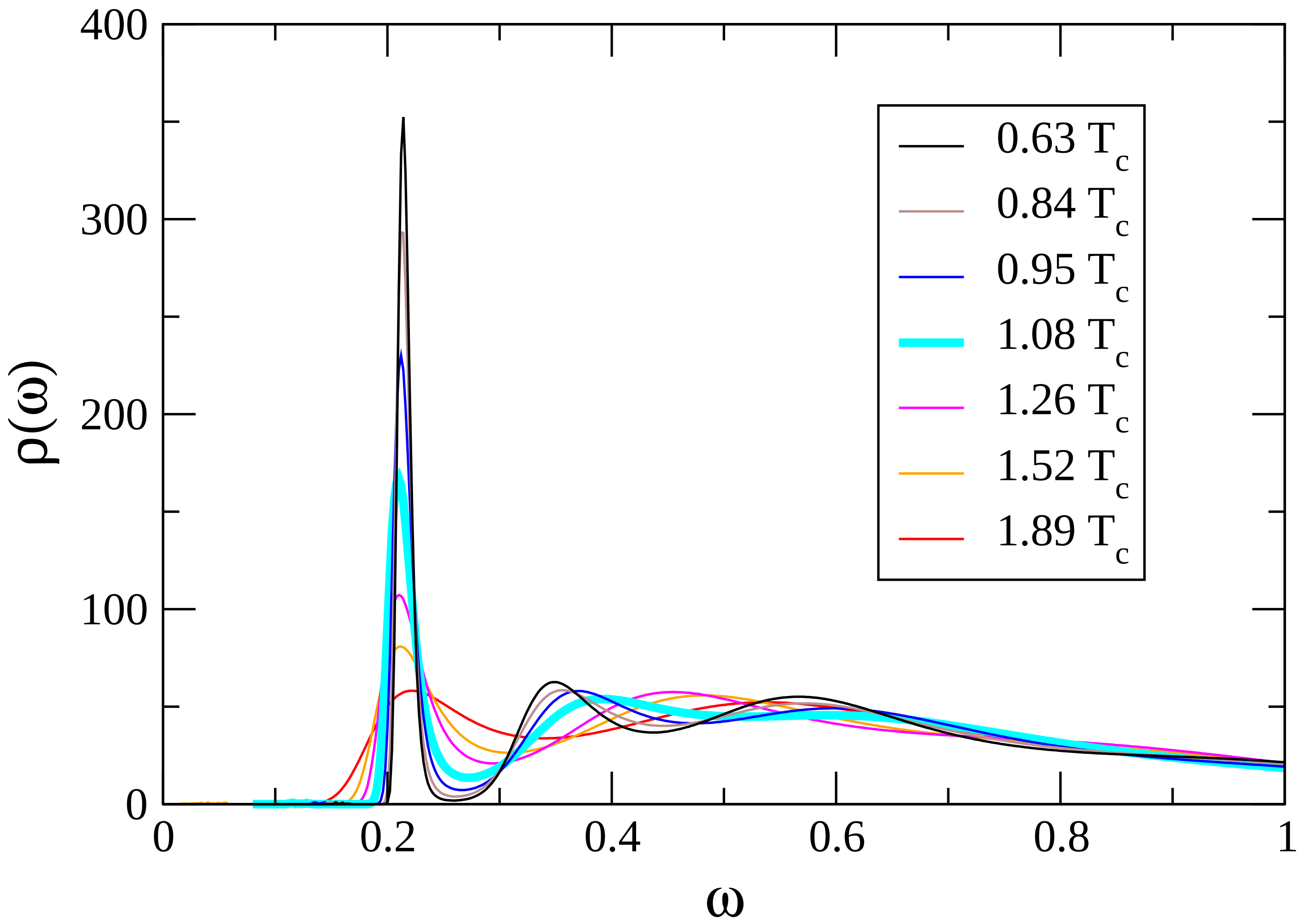

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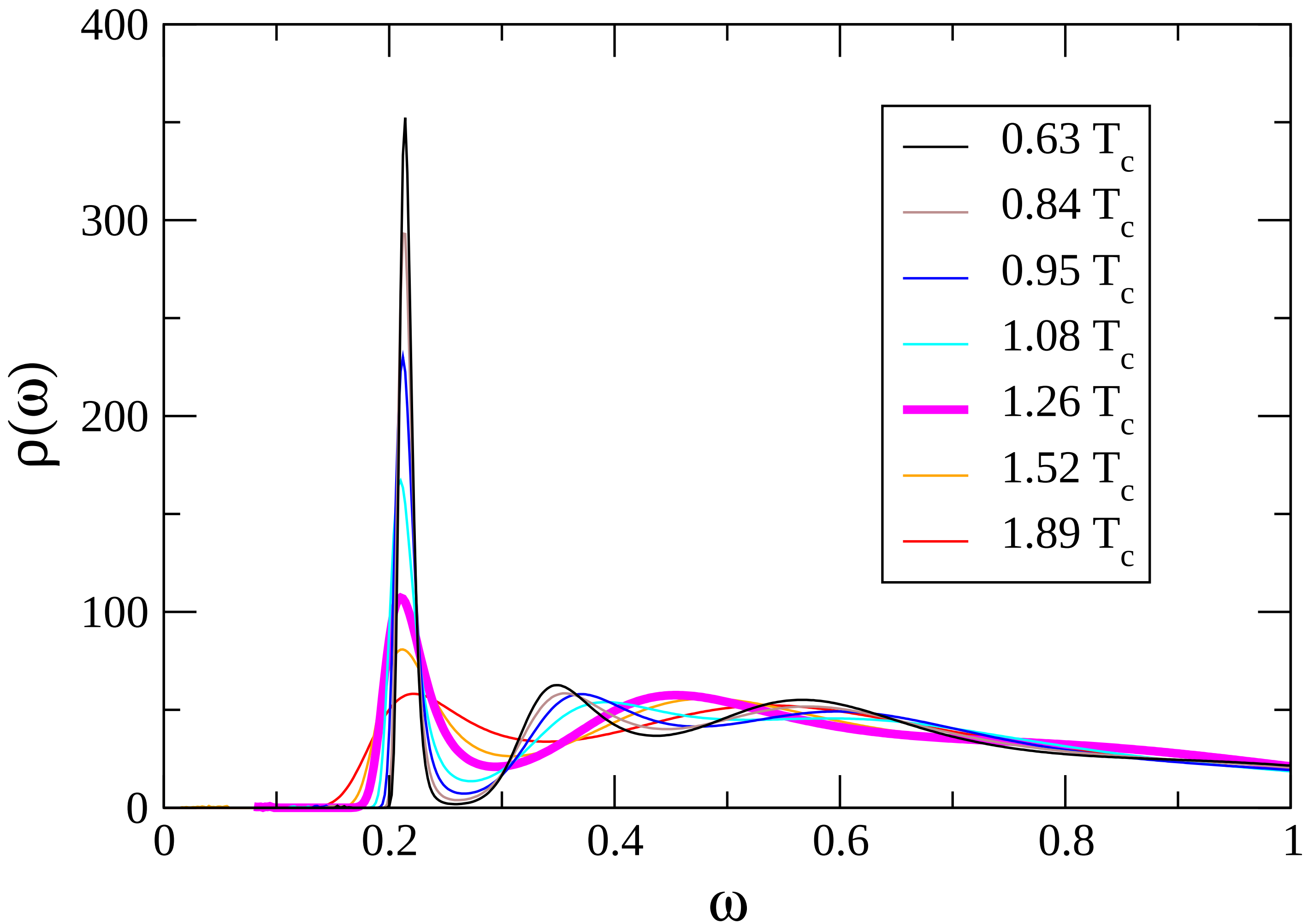

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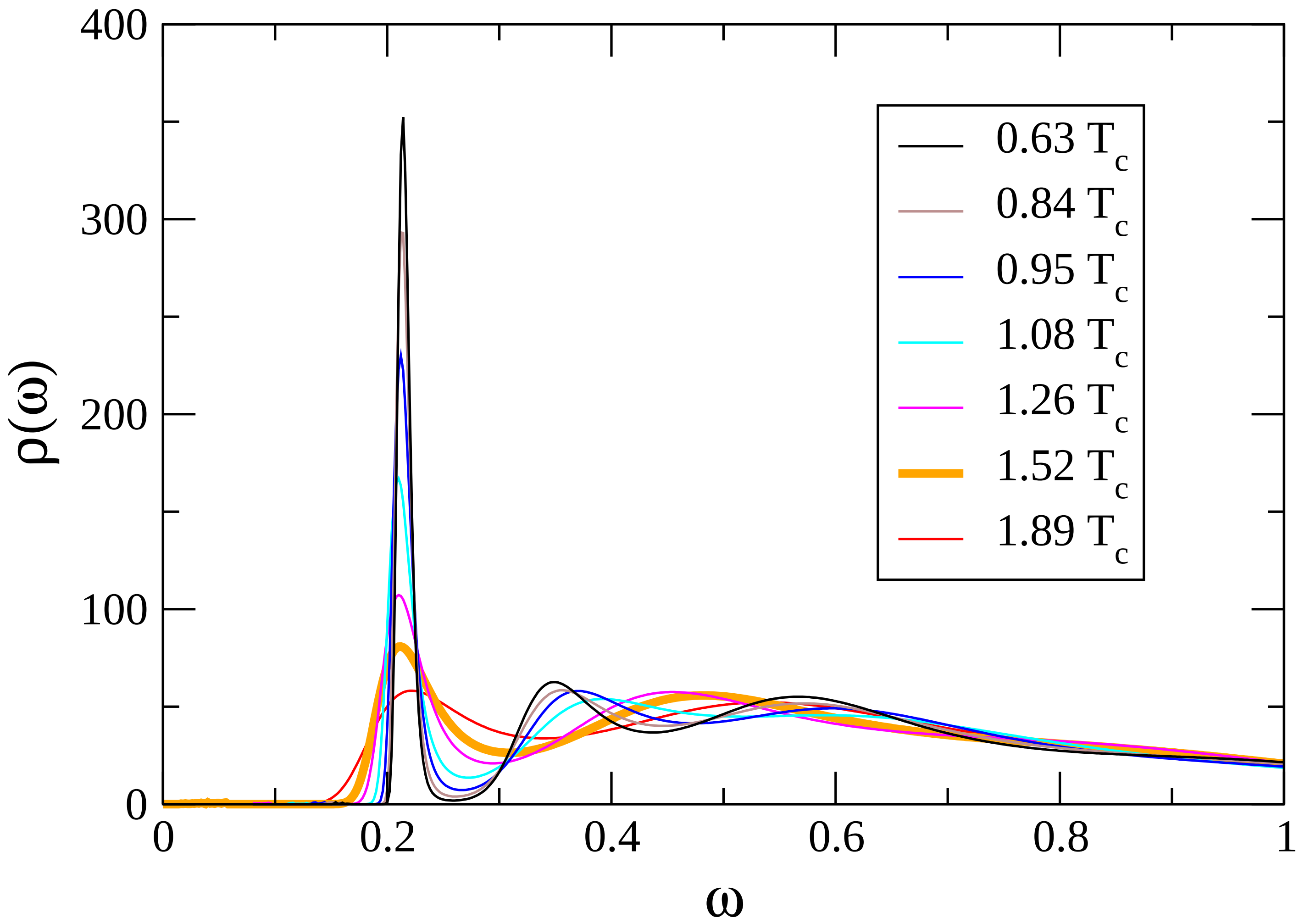

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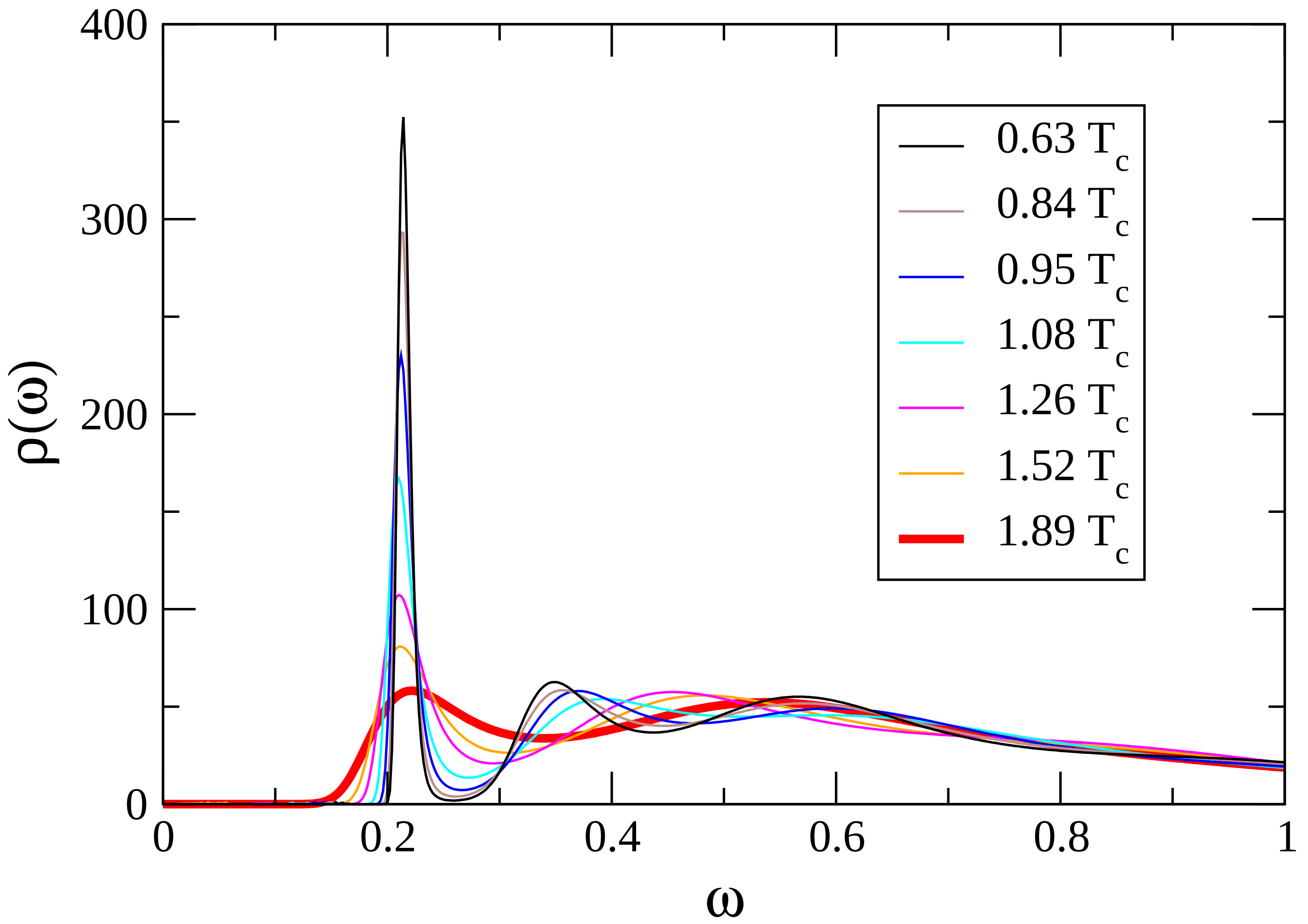
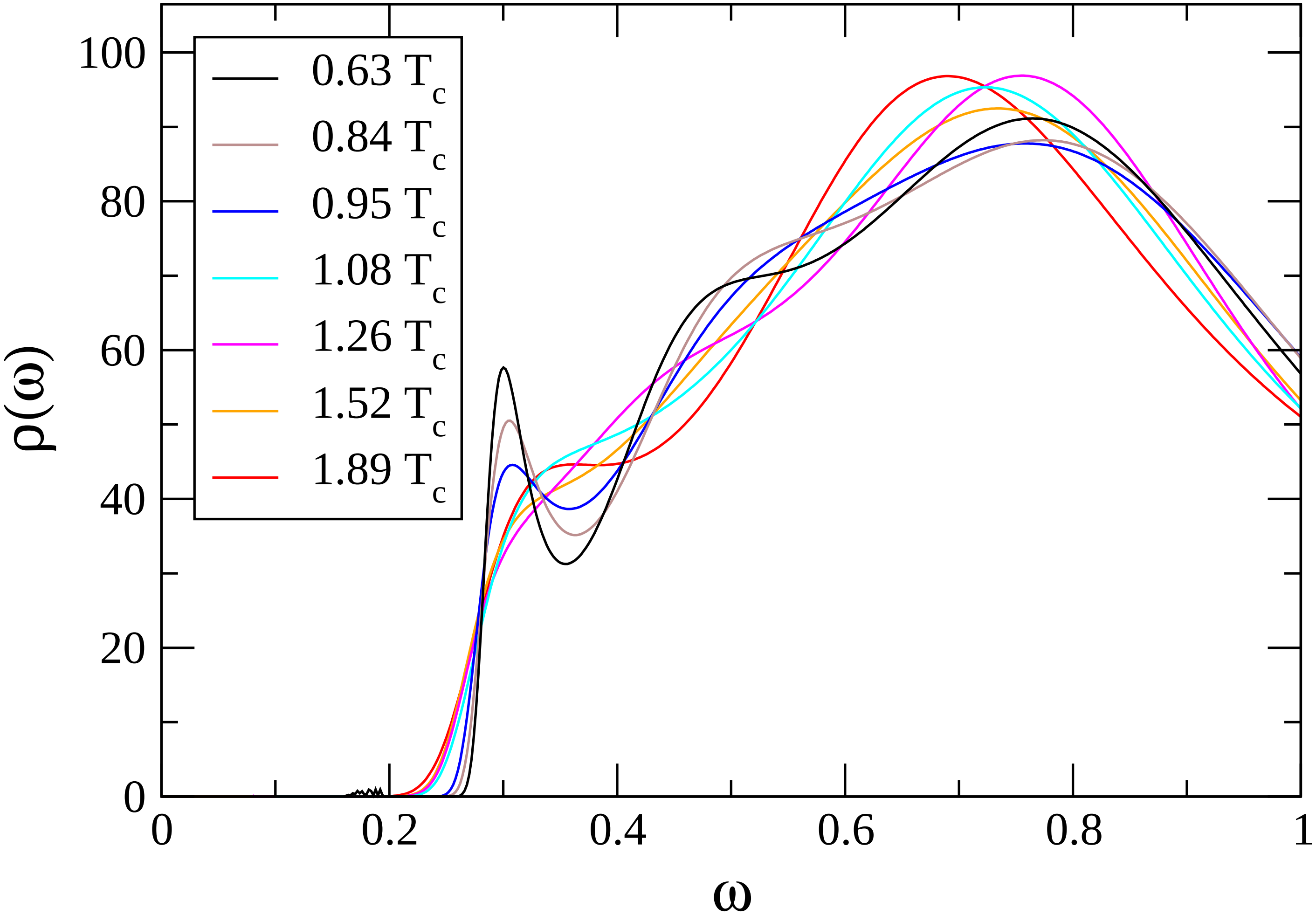


NRQCD spectral functions, p-wave

Tim Harris, Fri 18:10, Room A



Summary

Electrical Conductivity

- First time the temperature dependency has been uncovered on lattice
- Results compatible with previous determinations

Inter-quark potential in charmonium at finite temperature

First time this was done with:

- relativisitc quarks rather than static quarks
  - No issue with Free Energy and the Entropy Term...
- finite temperature rather than $T = 0$

Bottomonium spectral functions at finite temperature

- s-wave (J/ψ and η_b) survive to large T
- p-wave (χ_{b1}) melts at $T \sim T_c$

Future Plans

3rd Generation $a_t \to 0$ *Currently being tuned*

4th Generation $a_s \to 0$

Future Plans

3rd Generation $a_t \to 0$ *Currently being tuned*

4th Generation $a_s \to 0$

Conductivity

- Other transport coefficients
- Continuum Limit

Inter-quark potential in charmonium at finite temperature

- Study P-wave states
- Understand excited states
- Larger volumes
- Continuum Limit

Bottomonium spectral functions at finite temperature

- Momenta
- Take continuum limit

Other FASTSUM Lat13 Presentations

- Transport Coefficients of the QGP
  - Alessandro Amato *Mon, 17:30, Seminar Room A – Parallels 2A*
- Electric charge susceptibility
  - Pietro Giudice Tuesday evening – Poster Session
- P wave bottomonium spectral functions
  - Gert Aarts Fri, 16:50, Seminar Room A – Parallels 10A
- Charmonium Potentials at Non-Zero Temperature
  - Wynne Evans Fri, 17:10, Seminar Room A – Parallels 10A
- Spectral functions of charmonium
  - Aoife Kelly Fri, 17:50, Seminar Room A – Parallels 10A
- Bottomonium spectrum
  - Tim Harris Fri, 18:10, Seminar Room A – Parallels 10A