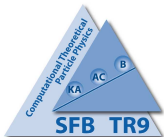


Towards thermodynamics with $N_f = 2 + 1 + 1$ twisted mass quarks



Florian Burger
Humboldt University Berlin



for the tmFT Collaboration:

G. Hotzel, M. Müller-Preussker (HU Berlin),
E. M. Ilgenfritz (JINR Dubna)
M. P. Lombardo (INFN Frascati)

Lattice '13
July, 30 2013

- 1 Introduction
- 2 First estimate of T_c , some observables
- 3 Towards the thermodynamic equation of state
- 4 Conclusions & Outlook

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- Explore finite T crossover with dynamical second quark generation
- Study role of charm quark - mass thresholds in thermodynamics
- Dynamical charm little studied, so far only with staggered quarks ([Budapest-Wuppertal & MILC, 2012])
- Goal of project: EoS with (near to) physical masses
- In this talk: some first **preliminary** results

Fixed scale approach

- First used by [\[WHOT-QCD, 2009\]](#)
- Used to compare Wilson and staggered quark results [\[Budapest-Wuppertal, 2012\]](#)
- Fewer $T = 0$ lattices needed for EoS
- At small T and around T_c : close to continuum (here T_χ at $N_\tau \sim 12$)
- Large T : Need several lattice spacings to check for lattice artefacts
- Temperature resolution limited - possible improvement: odd N_τ

$N_f = 2 + 1 + 1$ twisted mass action

- $N_f = 2 + 1 + 1$ lattice QCD with Wilson fermions at maximal twist.

Degenerate light sector:

$$S_f^l[\chi_l, \bar{\chi}_l, U] = \sum_x \bar{\chi}_l(x) [D_W + i\mu\gamma_5\tau^3] \chi_l(x)$$

Heavy sector:

$$S_f^h[\chi_h, \bar{\chi}_h, U] = \sum_x \bar{\chi}_h(x) [D_W + i\mu_\sigma\gamma_5\tau^1 + \mu_\delta\tau^3] \chi_h(x)$$

- Iwasaki Gauge Action:

$$S_g[U] = \beta \left(c_0 \sum_P \left[1 - \frac{1}{3} \text{ReTr}(U_P) \right] + c_1 \sum_R \left[1 - \frac{1}{3} \text{ReTr}(U_R) \right] \right)$$

T = 0 ETMC results

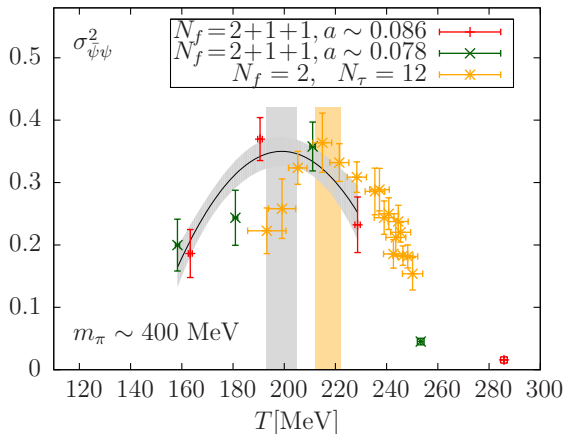
- $\mathcal{O}(30)$ $T = 0$ gauge field ensembles
- Pion masses in the range of 220 to 500 MeV
- Strange and charm tuned to reproduce physical values of K- and D-meson masses at the level of 10 %
- Three lattice spacings:
 $a \sim 0.086, 0.078$ and 0.061 fm (preliminary) [ETMC, 2011]
- $m_\pi L > 3.3$

T > 0:

- So far only $m_\pi \sim 400$ MeV
- Spatial sizes $N_\sigma = 24, 32$, $4 \leq N_\tau \leq 16$
- Aspect ratio ≤ 2

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Susceptibility of $\langle \bar{\psi}\psi \rangle$

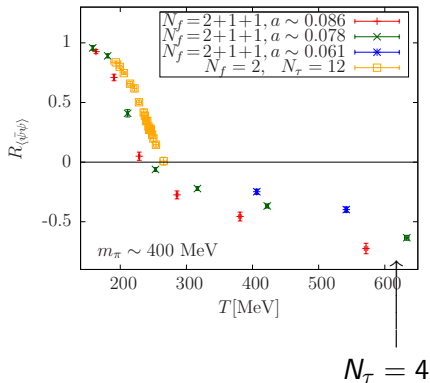


$T_\chi \sim 20$ MeV lower compared to $N_f = 2$

at same pion mass value [tmfT, 2012: FB et al. PRD 87,074508]

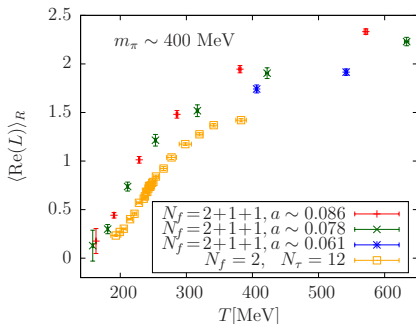
Renormalized $\text{Re}(L)$, $\langle \bar{\psi}\psi \rangle$

$$R_{\langle \bar{\psi}\psi \rangle} = \frac{\langle \bar{\psi}\psi \rangle(T, \mu) - \langle \bar{\psi}\psi \rangle(0, \mu) + \langle \bar{\psi}\psi \rangle(0, 0)}{\langle \bar{\psi}\psi \rangle(0, 0)}$$



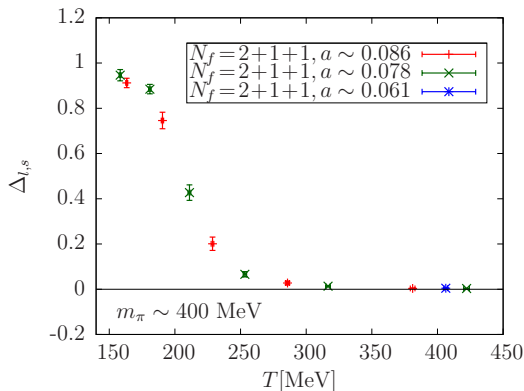
$N_f = 2$ cf. [tmfT, 2012]

$$\langle \text{Re}(L) \rangle_R = \text{Re}(L) \exp(V(r_0)/2T)$$



$$\Delta_{I,s} = \frac{\langle \bar{\psi}\psi \rangle_I - \frac{\mu_I}{\mu_s} \langle \bar{\psi}\psi \rangle_s}{\langle \bar{\psi}\psi \rangle_I^{T=0} - \frac{\mu_I}{\mu_s} \langle \bar{\psi}\psi \rangle_s^{T=0}}$$

[M. Cheng et al., 2008]

OS strange quark mass value
by courtesy of A. Ammon

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- Trace anomaly

$$I = \epsilon - 3p = -\frac{T}{V} \frac{d \ln Z}{d \ln a} .$$

- Starting point for $p(T)$ and $\epsilon(T)$ by integral method

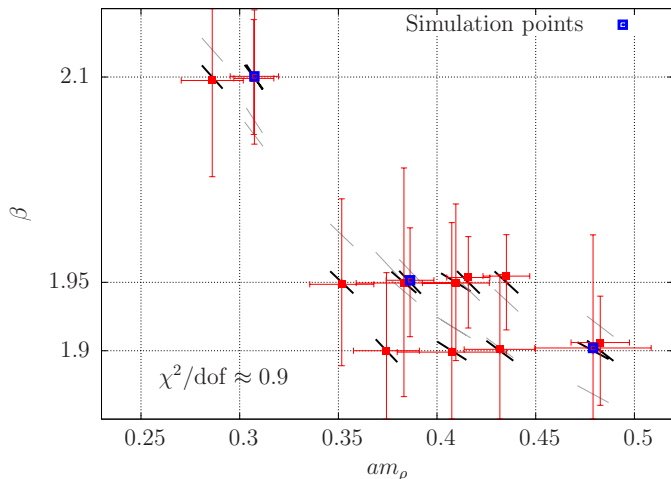
$$\frac{I}{T^4} = T \frac{\partial}{\partial T} \left(\frac{p}{T^4} \right) .$$

- Subtracted expectation values \leftrightarrow fixed scale advantageous
- Very preliminary results for gauge part:

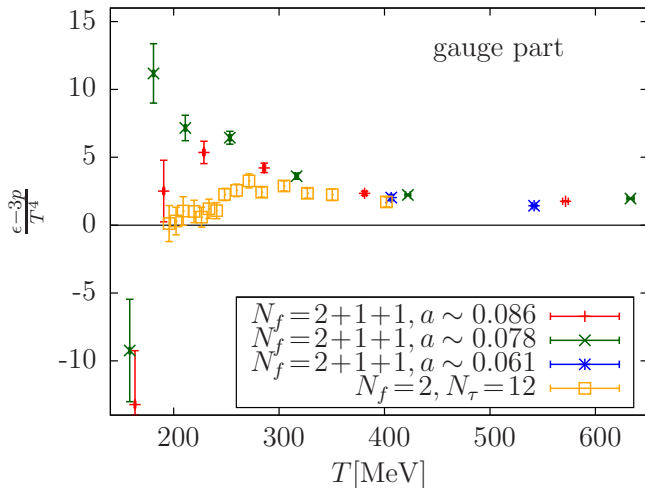
$$\frac{\epsilon - 3p}{T^4} = a \frac{d\beta}{da} \langle S_g \rangle_{\text{sub}} .$$

Beta function fit

$$\beta = c_0 \log(c_1(am_\rho)) + c_3(am_\pi) + c_4 \frac{(am_\rho)^2}{(am_\pi)^2}$$



A preliminary result



$N_f = 2$ cf. [tmfT, 2012]

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- Conclusions:

- First Wilson quark results for $N_f = 2 + 1 + 1$
- Crossover shifted to smaller T

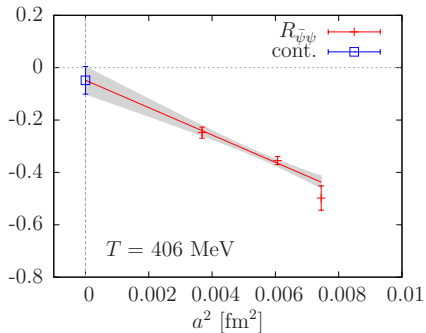
- Outlook:

- Improvement necessary on β -function and statistics
- In progress: light and heavy fermion contrib. to trace anomaly
- Extension of code to odd N_f
- Decrease light mass

Thank you

Continuum limit of $R_{\langle\bar{\psi}\psi\rangle}$

$T = 406$ MeV



$T = 541$ MeV

