The thermodynamic and the continuum limit of meson screening masses in quenched lattice QCD

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Motivation

Situation

- Two fermion actions with $O(a^2)$ discretization errors:
  - Non-pert. clover-improved Wilson action
  - Standard staggered action
- Continuum physics should be independent of the action
- However, results on finite lattice differ

A systematic study of both actions

- Employing meson screening masses as a precise observable
- At two temperatures $1.5\,T_c$ and $3.0\,T_c$ in the deconfined phase
- Taking the thermodynamic $N_\sigma \to \infty$ and continuum $a \to 0$ limit
- In the quenched approximation, so $128^3 \times 16$ lattices are possible

Setup

- 4 spacings $a = 1/(N_\tau \cdot T_{\text{fixed}})$: $N_\tau = 8, 10, 12, 16$
- 5 aspect ratios: $N_\sigma / N_\tau = 2, 3, 4, 6, 8$
- Quark mass tuned to < 10MeV (no dependence up to 20MeV)
- Between 100 and 300 independent configurations per ensemble

Channels

Ch.	Wilson Γ_{Wilson}	staggered Phasefactor	Γ non-osc.	Γ osc.	non-osc.	osc.
S	1	-1^{x+y+t}	$\gamma_3\gamma_5$	1	π	$\underline{a_0}$
PS	γ_5	1	γ_5	γ_3	$\underline{\pi}$	-
AV	$\gamma_1\gamma_5$	$-1^{x+t}, -1^{y+t}$	$\gamma_1\gamma_3, \gamma_2\gamma_3$	$\gamma_1\gamma_5, \gamma_2\gamma_5$	ρ_2	$\underline{a_1}$
V	γ_1	$-1^{x}, -1^{y}$	γ_1, γ_2	$\gamma_2\gamma_4, \gamma_1\gamma_3$	$\underline{\rho_1}$	b_1

Meson screening masses

Correlator (for Wilson fermions)

$$G(n_z) = \langle O(n_z), \bar{O}(0) \rangle = \sum_k \langle 0|\hat{O}|k\rangle\langle k|\hat{O}^\dagger|0\rangle e^{-n_z E_k}$$
$$= A_0 e^{-n_z E_0} + A_1 e^{-n_z E_1} + \ldots$$

symmetric on the lattice:

$$G(n_z) = \underbrace{2A_0 e^{-N_\sigma E_0/2}}_{A_0'} \cosh((N_\sigma/2 - n_z) \cdot E_0) + \ldots$$

Extracting the screening mass

- As **effective masses**: $m_{\text{eff.}}(n_z) = \log(G(n_z)/G(n_z+1))$
- By **fitting** $A_0', E_0, \ldots$ to lattice results

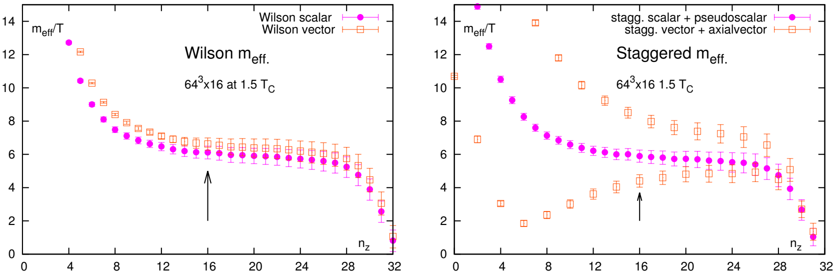
Staggered fermions

Correlator (for staggered fermions)

$$G(n_z) = (-1)^{n_z} \cdot \underbrace{2A_0^{\text{osc}} e^{-N_\sigma E_0^{\text{osc}}/2}}_{A_0^{\prime,\text{osc}}} \cosh((N_\sigma/2 - n_z) \cdot E_0^{\text{osc}})$$
$$+ \underbrace{2A_0^{\text{n.o.}} e^{-N_\sigma E_0^{\text{n.o.}}/2}}_{A_0^{\prime,\text{n.o.}}} \cosh((N_\sigma/2 - n_z) \cdot E_0^{\text{n.o.}})$$
$$+ \dots$$

- Each staggered channel may hold an oscillating and a non-oscillating state
- For $A^{\text{n.o}} \ll A^{\text{osc.}}$ or $A^{\text{n.o}} \gg A^{\text{osc.}}$ one state clearly dominates
  - The same analysis as for Wilson correlators can be used
- If $A^{\text{n.o}} \approx A^{\text{osc.}}$ both contributions have to be considered
  - The analysis has to take both states into account

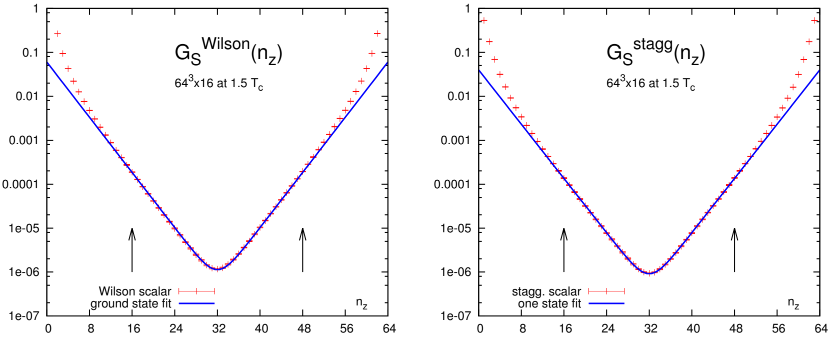
Effective masses



$$m_{\text{eff.}}(n_z) = \log\left(\frac{G(n_z)}{G(n_z+1)}\right)$$

- Plateau starting at $N_\sigma/4$ motivates to define $m_{\text{eff}}(N_\sigma/4)$ as (near) ground state.
- Does not work for staggered channels V, AV.

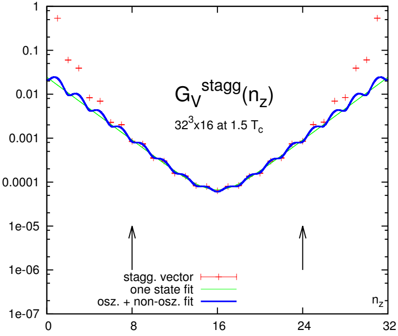
Fitting one state (PS and S)

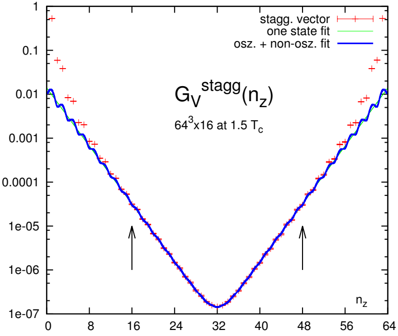


$$G(n_z) = A'_0 \ \cosh((N_\sigma/2 - n_z) \cdot E_0)$$

- Mimics the effective mass definition: Fit range $\frac{1}{4}N_\sigma$ to $\frac{3}{4}N_\sigma$.
- Works for staggered channels S, PS, where $A^{\text{osc}} \ll A^{\text{n.o.}}$.

Fitting both contribution for the staggered case (V and AV)

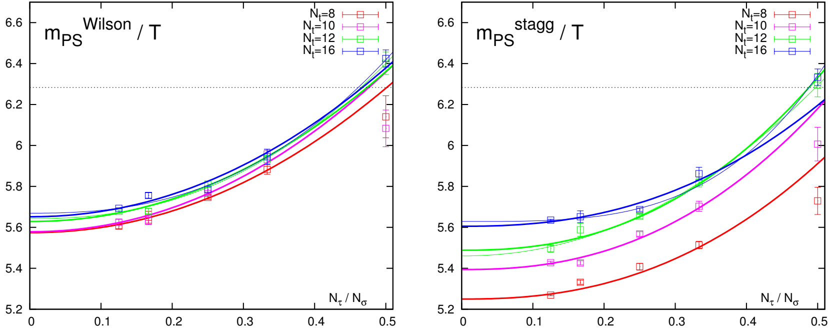




$$G(n_z) = A'^{0,\text{n.o.}} \cosh((N_\sigma/2 - n_z)\cdot E_0^{\text{n.o.}}) + (-1)^{n_z}\cdot A'^{0,\text{osc}} \cosh((N_\sigma/2 - n_z)\cdot E_0^{\text{osc}})$$

Allows to fit when $A^{\text{n.o}} \approx A^{\text{osc.}}$: Vector (V) and axial-vector (AV) channels

1.5 T_c The thermodynamic limit

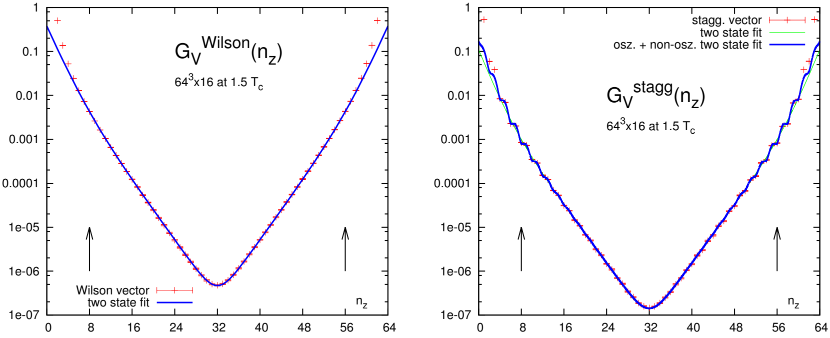


Ansatz

$$m_{a_{\text{fixed}}, V\to\infty} \cdot (1 + b_a \cdot (N_\tau / N_\sigma)^c) = m_{\text{lat.}} \quad \text{with} \quad b_a = b(a, T)\ ,\ c = c(T)$$

- Linear behavior $m \cdot (1 + b \cdot (N_\tau/N_\sigma)^1) \sim m_{\text{lat}}$ for free theory
- Cubic dependence $m \cdot (1 + b \cdot (N_\tau/N_\sigma)^3) \sim m_{\text{lat}}$ at zero temperature

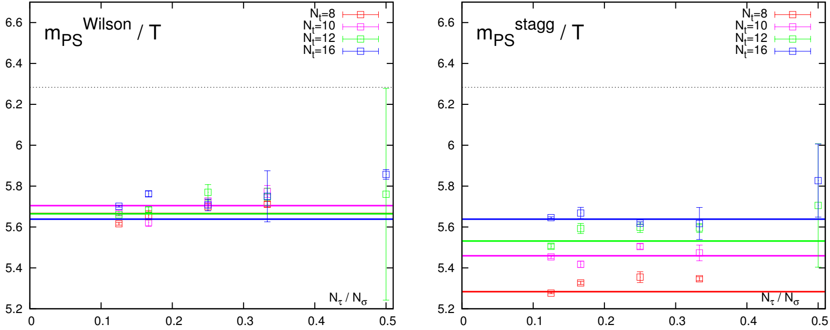
$1.5\,T_c$ Alternative thermodynamic limit: Fitting two states



$$G(n_z) = A'_0 \cosh((N_\sigma/2 - n_z)\cdot E_0) + A'_1 \cosh((N_\sigma/2 - n_z)\cdot E_1)$$
$$+ (-1)^{n_z} \cdot A''_0 \cosh((N_\sigma/2 - n_z)\cdot E''_0)$$

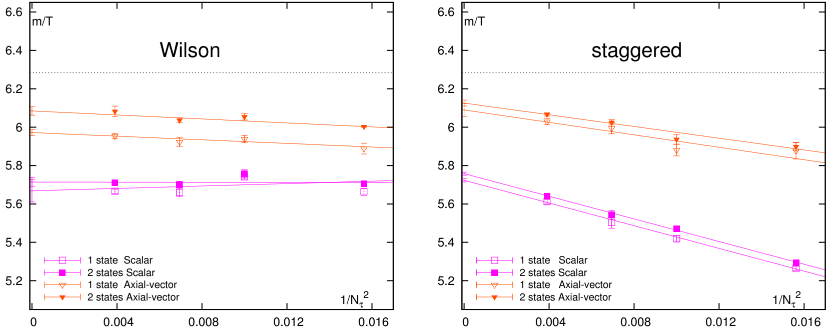
- Fit absorbs higher exited states, extracts ground state.
- Fit range is tuned to constant distance to source, e.g. $\frac{1}{2}N_\tau$.

$1.5\,T_c$ Two state fits volume dependence



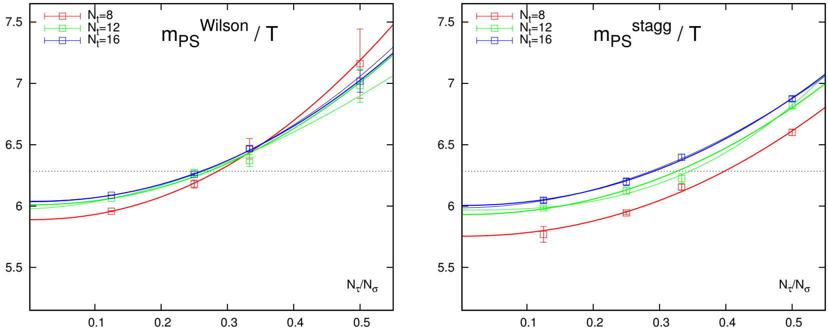
- Needs a minimal N_σ/N_τ of $4\ldots6$ depending on N_τ (for Wilson action).
- Results are consistent with the one-state fits at $N_\sigma/N_\tau = 8$.

$1.5\,T_c$ The continuum limit



- Extrapolation in $a^2 \sim 1/N_\tau^2$, for 1-state and 2-state fits
- Staggered results are more effected by lattice spacing.
- In the full continuum limit, **both actions yield compatible masses**.
- Thermodynamic and continuum limit can be interchanged with compatible results

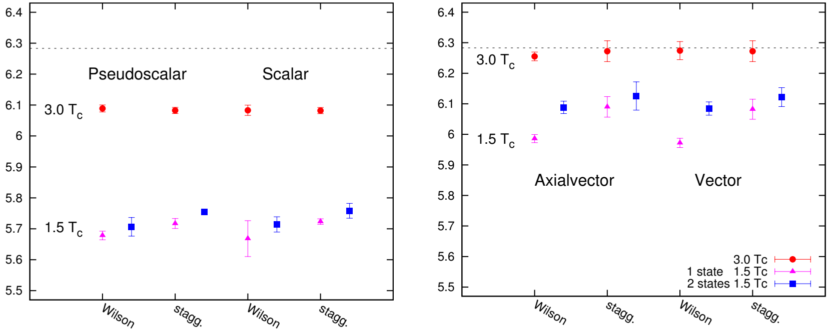
$3.0\,T_c$ dataset



Very robust fit results at $3.0\,T_c$

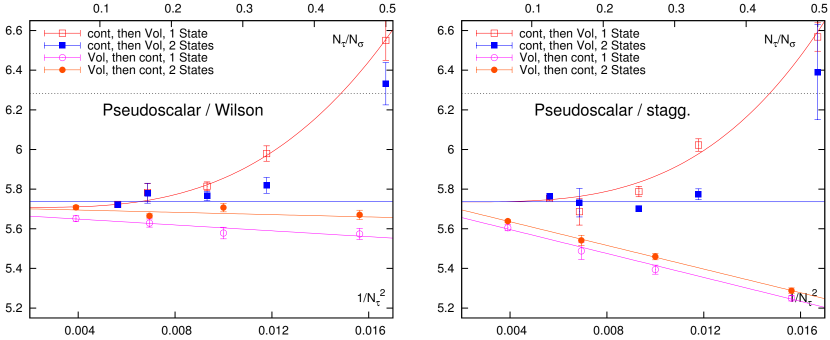
- Reduced dataset $N_\tau = 8, 12, 16$ with $N_\sigma / N_\tau = 2, 3, 4, 8$
- Only 1-state fits are performed, the continuum limit is taken after thermodynamic limit

Conclusion

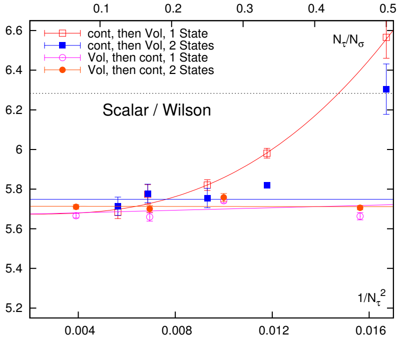


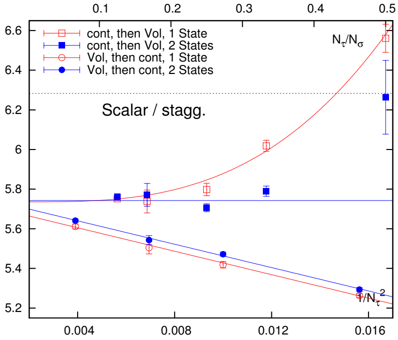
- Both actions yield compatible continuum results at $1.5\,T_c$ and $3.0\,T_c$
- Pseudo-scalar/scalar and vector/axial-vector degenerate at both temperatures
- Vector / axial vector at $3.0\,T_c$ agrees with 2π
- $1.5\,T_c$ correlators need a much more involved analysis than $3.0\,T_c$

Backup: All PS extrapolations

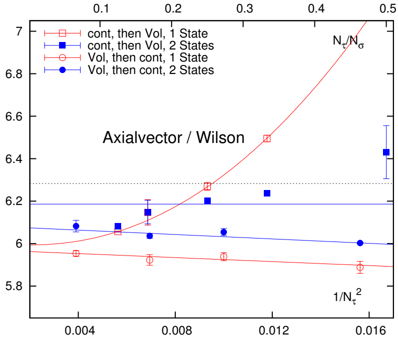


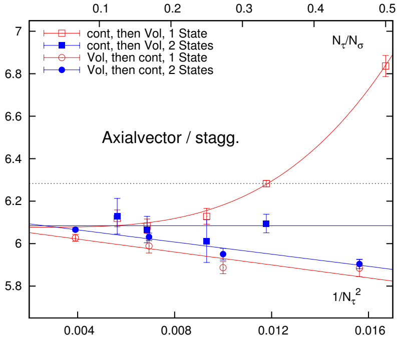
Backup: All S extrapolations



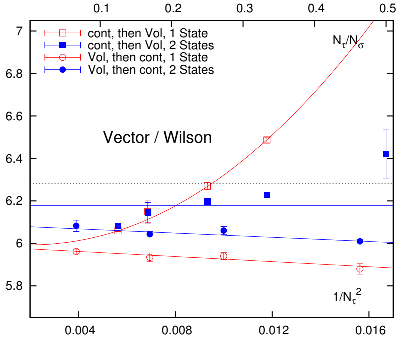


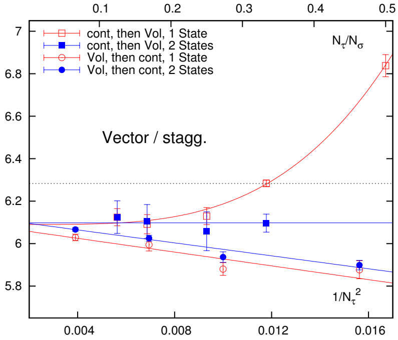
Backup: All AV extrapolations





Backup: All V extrapolations





Backup: Simulation data overview

N_τ	N_σ	β	κ_{Wilson}	C_{SW}	m_q^{stagg}
8	$16, 24, 32, 48, 64$	6.338	0.13572	1.548725	0.003864
10	$20, 30, 40, 60, 80$	6.503	0.13554	1.493023	0.002190
12	$24, 36, 48, 72, 96$	6.640	0.13536	1.457898	0.001707
16	$32, 48, 64, 96, 128$	6.872	0.13495	1.412488	0.001028