Nucleon transversity generalized form factors with twisted mass fermions

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In collaboration with:

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OUTLINE

A. Motivation

- ★ Introduced in 1979 by Ralston and Sober (Drell-Yan scattering)
- ★ Appeared again in 1990^s by Artru and Mekhfi / Jaffe and Ji
- ★ transversity distributions (TDs) are chirally-odd $\rightarrow$ fully inclusive DIS not useful
- ★ To measure TDs the chirality must be flipped twice:
  1. hadron hadron collisions (2 hadrons in initial state)
  2. semi-inclusive DIS (SIDIS) (1 hadron in initial state and 1 in final state)
- ★ Small contributions of tensor interactions in SM (10^{-3}): future experiments are planned

B. Nucleon Generalized Parton Distributions (GPDs)

- Parametrization of off-forward nucleon matrix of a bilocal quark operator

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\, \mathcal{O}\, \underbrace{\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)}}_{\text{gauge invariance}} \psi(\lambda n/2)|p\rangle$$

where $q = p' - p$, $\bar{P} = (p'+p)/2$, n: light-cone vector ($\bar{P}.n = 1$), $\xi = -n\cdot\Delta/2$

- Choices of operators within LQCD: towers of local twist-2 operators

- Operators $\&$ hadron structure observables related via moments in the momentum fraction x

$$f^n = \int_{-1}^{1} dx\, x^{n-1} f(x)$$

Twist-2 Parton Distributions:

complete set for describing the quark state inside the nucleon (leading-order hard processes)

A unpolarized $$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

B helicity (polarized) $$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{q}\,\gamma_5\gamma^{\{\mu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

Talk by C. Alexandrou

C **transversity** $$\mathcal{O}^{\mu_1\cdots\mu_{n-1}} = \bar{q}\,\sigma^{\mu\,\{\nu}\, iD^{\mu_1} \ldots iD^{\mu_{n-1}\}} q$$

→ net number of quarks with transverse polarization in a transversely polarized nucleon

- Transversity distribution (scheme and scale dependent):

$$\langle x^n \rangle_{\delta q} = \int_0^1 dx\, x^n \left[\delta q(x) + (-1)^{n+1} \delta\bar{q}(x)\right] \quad , \qquad \delta q = q_T + q_\perp$$

- Nucleon case: $H_T(x,\xi,q^2),\ E_T(x,\xi,q^2),\ \widetilde{H}_T(x,\xi,q^2),\ \widetilde{E}_T(x,\xi,q^2)$

Nucleon transversity generalized form factors

Decomposition of matrix elements into GFFs: contain FFs, PDFs

Special cases for the tensor operator:

$n = 0$: $A_{T10},\ B_{T10},\ \widetilde{A}_{T10}$ quark helicity flip form factors

$$\langle\langle \overline{q}(0) i\sigma^{\mu\nu} q(0) \rangle\rangle = \langle\langle i\sigma^{\mu\nu} \rangle\rangle A_{T10}(q^2) + \langle\langle \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N} \rangle\rangle B_{T10}(q^2) + \langle\langle \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2} \rangle\rangle \widetilde{A}_{T10}(q^2)$$

- $A_{T10}(0) \equiv \langle 1 \rangle_{\delta q(x)}$ (tensor charge g_T)

$n = 1$: $A_{T20},\ B_{T20},\ \widetilde{A}_{T20},\ \widetilde{B}_{T21}$ moments of parton distributions

$$\begin{aligned} \langle\langle \mathcal{O}_T^{\mu\nu\mu_1}(0) \rangle\rangle \ &= \ \mathcal{A}_{\mu\nu}\mathcal{S}_{\nu\mu_1} \Big\{ \langle\langle i\sigma^{\mu\nu}\overline{P}^{\mu_1} \rangle\rangle A_{T20}(q^2) + \langle\langle \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}\overline{P}^{\mu_1} \rangle\rangle B_{T20}(q^2) \\ &+ \ \langle\langle \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\overline{P}^{\mu_1} \rangle\rangle \widetilde{A}_{T20}(q^2) + \langle\langle \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\Delta^{\mu_1} \rangle\rangle \widetilde{B}_{T21}(q^2) \Big\} \end{aligned}$$

- $A_{T20}(0) \equiv \langle x \rangle_{\delta q(x)}$ (tensor moment)

FFs / GFFs and GPDs

FFs

$$
\begin{aligned}
A_{T10}(q^2) &= \int_{-1}^{1} dx H_T(x,\xi,q^2) \\
B_{T10}(q^2) &= \int_{-1}^{1} dx E_T(x,\xi,q^2) \\
\widetilde{A}_{T10}(q^2) &= \int_{-1}^{1} dx \widetilde{H}_T(x,\xi,q^2) \\
0 &= \int_{-1}^{1} dx \widetilde{E}_T(x,\xi,q^2)
\end{aligned}
$$

GFFs

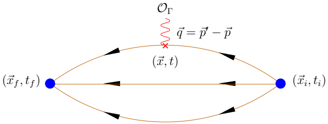
$$
\begin{aligned}
A_{T20}(q^2) &= \int_{-1}^{1} dx\, x H_T(x,\xi,q^2) \\
B_{T20}(q^2) &= \int_{-1}^{1} dx\, x E_T(x,\xi,q^2) \\
\widetilde{A}_{T20}(q^2) &= \int_{-1}^{1} dx\, x \widetilde{H}_T(x,\xi,q^2) \\
-2\xi \widetilde{B}_{T20}(q^2) &= \int_{-1}^{1} dx\, x \widetilde{E}_T(x,\xi,q^2)
\end{aligned}
$$

→ time reversal transformation properties

- $\overline{B}_{T10} \equiv B_{T10} + 2\widetilde{A}_{T10}$ $\qquad$ $\overline{B}_{T10}(0) \equiv \kappa_T$ (tensor magnetic moment)†

†[M. Burkardt, Phys. Rev. **D72** (2005) 094020]

C. Evaluation on the Lattice: Connected diagram



2pt : $$G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\, \Gamma^1_{\beta\alpha}\, \langle J_\alpha(\vec{x}_f, t_f)\overline{J}_\beta(0)\rangle$$

3pt : $$G^{T,\,DT}(\Gamma, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_f, t_f)\mathcal{O}^{T,\,DT}(\vec{x}, t)\overline{J}_\beta(0)\rangle$$

Operator Insertion:

$$\mathcal{O}^{\mathbf{T}} = \sigma^{\mu\,\nu}$$
$$\mathcal{O}^{\mathbf{DT}} = \sigma^{[\mu\,\{\nu]}\overleftrightarrow{D}^{\rho\}}$$

- **No mixing with lower dimension operators**
- **Isovector combinations: No disconnected diagrams**
- **DT: antisymm., symm. and subtraction of the traces**

- Sequential inversion “through the sink”: fix sink-source separation $t_f - t_i$
- Smearing techniques (Gausian/APE): improvement of ground state dominance in 3pt correlators

Types of projectors:

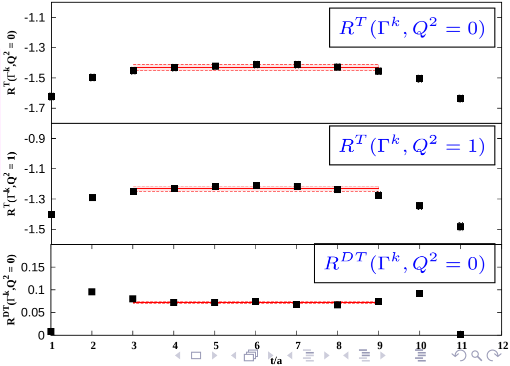
- $\Gamma^1 = (1+\gamma_0)$ (2pt & 3pt)
- $\Gamma^k = (1+\gamma_0)\, i\, \gamma_5\, \gamma_k$ (3pt) ($Q^2 = 0$ FFs and GFFs)

Optimized Ratios: Leading t dependence cancels

$$R(\Gamma,\vec{q},t) = \frac{G(\Gamma,\vec{q},t)}{G(\vec{0},t_f)} \times \sqrt{\frac{G(-\vec{q},t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(-\vec{q},t)G(-\vec{q},t_f)}}$$

$$\lim_{t_f-t\to\infty}\ \lim_{t-t_i\to\infty} R(\Gamma,\vec{q},t) \to \Pi(\Gamma,\vec{q})$$

$R(\Gamma,\vec{q},t)$ depends on operator indices



Continuum Decomposition (Euclidean)

Ultra-local Tensor

$$\Pi_T^{\mu\nu}(\Gamma^1) = \left(\frac{4\,i}{m}\,A_{T10} + 16\,i\,m\,B_{T10} + 16\,i\,\widetilde{A}_{T10}\,(E+m)\right)\left[\delta_{\nu\,0}\,p_\mu - \delta_{\mu\,0}\,p_\nu\right]$$

$$\Pi_T^{\mu\nu}(\Gamma^k) = 4\,A_{T10}\left[\epsilon_{\mu\,\nu\,k\,0} - \frac{i\,\epsilon_{\mu\,\nu\,k\,\rho}\,p_\rho}{m}\right]$$
$$+\,8\,B_{T10}\left[\left(i\,(-m\,\delta_{\mu\,0} - m\,\delta_{\nu\,0})\,\epsilon_{\mu\,\nu\,k\,\rho} + \epsilon_{\nu\,k\,0\,\rho}\,p_\mu - \epsilon_{\mu\,k\,0\,\rho}\,p_\nu\right)p_\rho\right]$$

1-D Tensor

$$\Pi_{DT}^{\mu\neq\nu\neq\rho\neq 0}(\Gamma^1) = \frac{-3}{m}\left(A_{T20}\,m + B_{T20}\,(E+m) + C_{T20}\,m\right)\left(\delta_{\mu(2),0}\,p_\mu - \delta_{\mu,0}\,p_\nu\right)\,p_\rho$$

$$\Pi_{DT}^{0\,0\neq\rho}(\Gamma^k) = A_{T20}\left[i\delta_{\nu k}\left(2E + \frac{E^2}{m} + m\right) + p_k\left(-\frac{1}{2} - \frac{E}{2m}\right)\right]$$
$$+\,C_{T20}\,i\left[p_k\left(\frac{-E^2}{2m^2} - \frac{E}{2m}\right) + i\left(\delta_{\nu k}\left(\frac{-E}{2} + \frac{E^3}{2m^2} + \frac{E^2}{2m} - \frac{m}{2}\right) + p_\nu p_k\left(\frac{-E}{2m^2} - \frac{1}{2m}\right)\right)\right]$$
$$+\,D_{T20}\,i\left[p_k\left(-\frac{E^2}{m^2} + \frac{E}{m}\right) + i\left(\delta_{\nu k}\left(-E + \frac{E^3}{m^2} - \frac{E^2}{m} + m\right) + p_\nu p_k\left(-\frac{E}{m^2} + \frac{1}{m}\right)\right)\right]$$

★ Combination of 2 projectors: FFs, GFFs disentanglement (via SVD)

Ensembles

- $N_f = 2+1+1$ twisted mass gauge configurations
- $N_f = 2$ twisted mass/Clover gauge configurations
- Iwasaki gluon action

N_f	β	a (fm)	$a\mu_0$	c_{sw}	m_π (MeV)	$L^3 \times T$	Stat.
2+1+1	1.95	0.082 ⋆	0.0055	0	373	$32^3 \times 64$	770
2+1+1	2.10	0.064 ⋆	0.0015	0	213	$48^3 \times 96$	425
2	2.10	0.097 ⋆	0.0009	1.57551	126	$48^3 \times 96$	420

⋆ Determination of lattice spacing from nucleon masses

We focus on:

- connected diagram: Isovector/Isoscalar nucleon transversity FFs/GFFs

since:

- disconnected diagram: Isoscalar computation ($\mathcal{O}^T$): negligible contribution

Talk by A. Vaquero

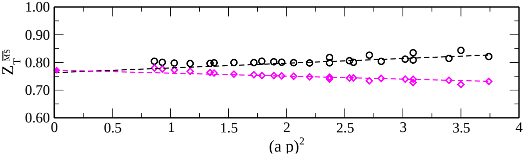
Thursday 14:00

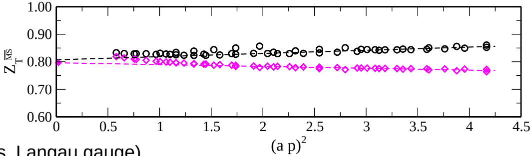
D1. Non-perturbative Renormalization

Ultra-local tensor operator

- Momentum source method† (high accuracy)
- RI'-MOM scheme
- Chiral extrapolation
- Continuum extrapolation
- Subtract perturbative $\mathcal{O}(a^2)$
- Conversion to $\overline{\rm MS}$ at 2GeV

†[M. Gockeler et al., Nucl. Phys. **B544** (1999) 699]





Perturbative $\mathcal{O}(a^2)$ terms: (Iwasaki gluons, Langau gauge)

(C. Alexandrou et al., Phys. Rev. **D86** (2012) 014505)

$$a^2\,\frac{g^2\,C_F}{16\,\pi^2}\Bigg[\;+\;\mu^2\,\left(0.2341-1.0950\,c_{\rm sw}-0.4297\,c_{\rm sw}^2\right)\;+\;\frac{\mu4}{\mu^2}\,\left(2.6676+0.1843\,c_{\rm sw}+0.1203\,c_{\rm sw}^2\right)\;+\;\log(a^2\,\mu^2)\,\left(\left(\frac{7271}{60000}+\frac{c_{\rm sw}}{2}+\frac{c_{\rm sw}^2}{4}\right)\mu^2-\frac{28891\,\mu4}{30000\,\mu^2}\right)\Bigg]$$

$$\mu4\equiv\sum_{i=1,4}\mu_i^4$$

D2. Renormalization at the physical point

- ★ Same ensemble as for the FFs/GFFs computation
- ★ m_π dependence expected insignificant [C. Alexandrou et al., Phys. Rev **D152** (1979) 109]
- ★ Democratic momenta in the spatial direction

$$\frac{1}{L^2} \ll \Lambda^2_{\rm QCD} \ll \mu^2 \ll \frac{1}{a^2}$$

2-4

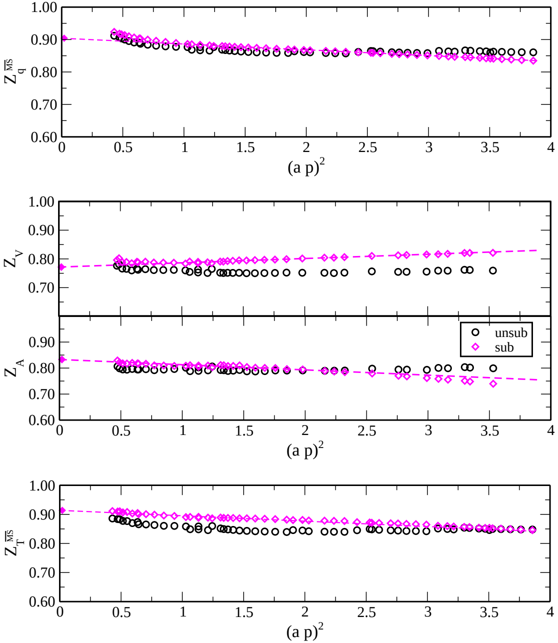
- Reliable perturbation theory
- Small $\mathcal{O}(a)$ lattice effects

Criterion for choosing the momenta

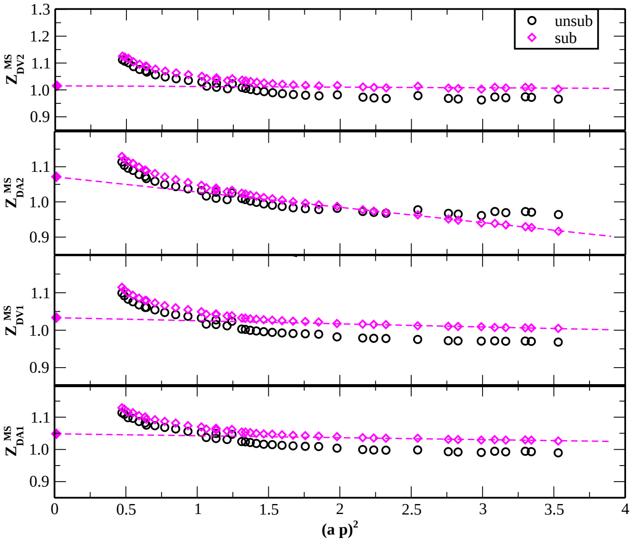
[M. Constantinou et al., JHEP 1008 (2010) 068]

$$\text{ultra}-\text{local}: \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.3 \qquad 1-\text{D}: \frac{\sum_\rho p_\rho^4}{\left(\sum_\rho p_\rho^2\right)^2} \leq 0.4$$

$\Rightarrow$ Non-Lorentz invariant contributions under control



$Z_q^{\overline{MS}}$
Z_V
Z_A
$Z_T^{\overline{MS}}$
$(a\,p)^2$
unsub
sub



unsub
sub
Z_{DV2}^{MS}
Z_{DA2}^{MS}
Z_{DV1}^{MS}
Z_{DA1}^{MS}
1.3
1.2
1.1
1.0
0.9
0
0.5
1
1.5
2
2.5
3
3.5
4
$(a\,p)^2$

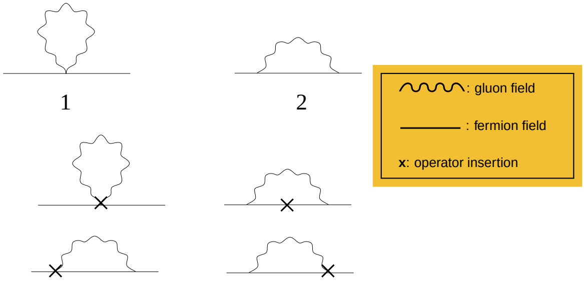
D3. Perturbative Renormalization

One-Derivative tensor operator

- 1-loop perturbation theory

Z_q

Z_{DT}



- RI'-MOM renormalization scheme

$$Z_q = \frac{1}{12}\mathrm{Tr}[(S^L(p))^{-1}\, S^{(0)}(p)]\Big|_{p^2=\bar{\mu}^2}$$

$$Z_q^{-1}\, Z_{DT}^{\mu\nu\rho}\, \frac{1}{12}\mathrm{Tr}[\Gamma^L_{\mu\nu\rho}(p)\,\Gamma^{(0)}_{\mu\nu\rho}(p)]\Big|_{p^2=\bar{\mu}^2} = \mathrm{Tr}[\Gamma^{(0)}_{\mu\nu\rho}(p)\,\Gamma^{(0)}_{\mu\nu\rho}(p)]\Big|_{p^2=\bar{\mu}^2}$$

- Conversion to $\overline{\mathrm{MS}}$ at 2GeV

Perturbative Results

(C. Alexandrou et al., Phys. Rev. **D83** (2011) 014503)

Iwasaki gluon action:

$$Z_{DT}(p=\bar{\mu}) = 1 + \frac{g^2\,C_F}{16\,\pi^2}\left(2.3285 - 2.2795\,c_{\rm sw} - 1.0117\,c_{\rm sw}^2 - 3\,\log(a^2\,\bar{\mu}^2)\right)$$

	RI′-MOM scheme:	$\overline{\rm MS}$ scheme:
$\beta = 1.95$:	$Z^{RI}_{DT}(p=\bar{\mu}) = 1.0891$	$Z^{\overline{\rm MS}}_{DT}(p=\bar{\mu}) = 0.9091$
$\beta = 2.1$:	$Z^{RI}_{DT}(p=\bar{\mu}) = 1.1177$	$Z^{\overline{\rm MS}}_{DT}(p=\bar{\mu}) = 0.9900$
$\beta = 2.1,\ c_{\rm sw}$:	$Z^{RI}_{DT}(p=\bar{\mu}) = 1.2059$	$Z^{\overline{\rm MS}}_{DT}(p=\bar{\mu}) = 1.1041$

Conversion factor:

$$C^{{\rm RI},\,\overline{\rm MS}}_{DT} = 1 - \frac{g^2\,C_F}{16\,\pi^2}(3\alpha + 7)$$

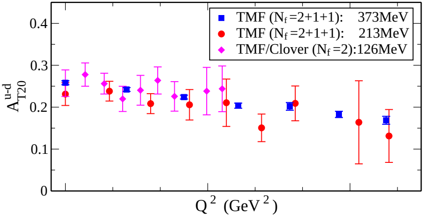
(J. Gracey, Nucl. Phys. **B667** (2003) 242)

★ Note: $Z^{\rm pert}_{DV/DA}$, $Z^{\rm nonpert}_{DV/DA}$: 10% difference

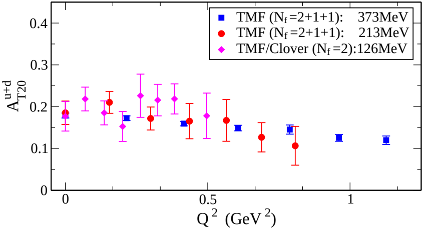
E. Results: Tensor GFFs

Preliminary

Isovector:



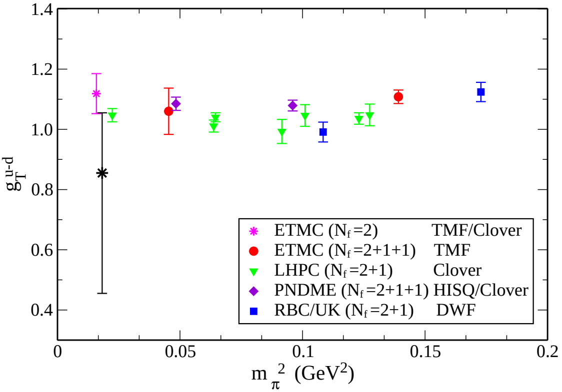
Isoscalar:



E2. Isovector Tensor FFs

Zero-momentum transfer Tensor charge

Fundamental parameter that characterize properties of the nucleon



[T. Bhattacharya et al., Phys. Rev. **D85** 5 (2012)]

LQCD. points:

Agreement

Mild m_π dependence

exp. point:

$A_{T10}^{exp}(0.8\mathrm{GeV}^2) = 0.77^{+0.18}_{-0.36}$ †

(at $\mu^2 = 110\mathrm{GeV}^2$)

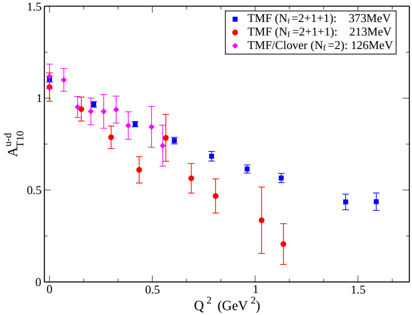
Global fit of HERMES, COMPASS, Belle $e^+ e^-$ data (9 parameters)

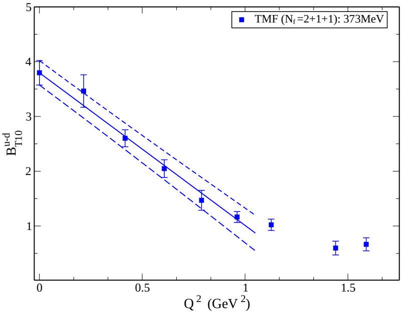
Running scale (3-loops):

$R(110, 4) = 1.11034523$

† (M. Anselmino et al., Nucl. Phys. Proc. Suppl. 191 (2009) 98)

Momentum dependence



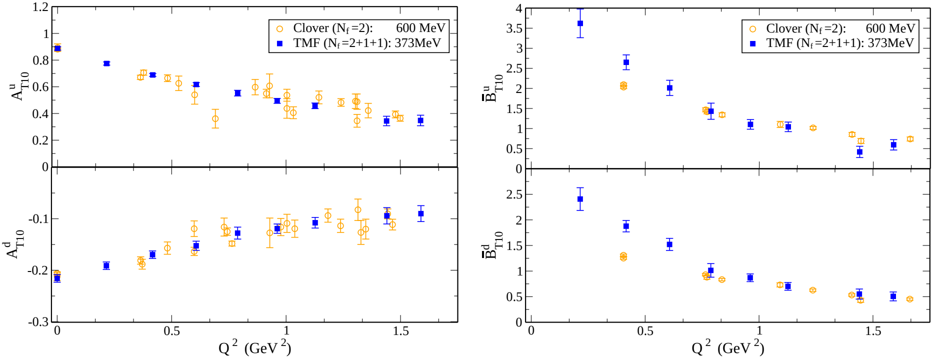


★ A_{T10} : linearly decreasing in Q^2

★ $B_{T10}(0)$ from fitting

★ $B_{T10}(0)$ Model dependent

Comparison for A_{T10}, $\overline{B}_{T10}$



★ Mild pion mass dependence

★ $A^u_{T10} > 0$ (decreasing) $A^d_{T10} < 0$ (increasing)

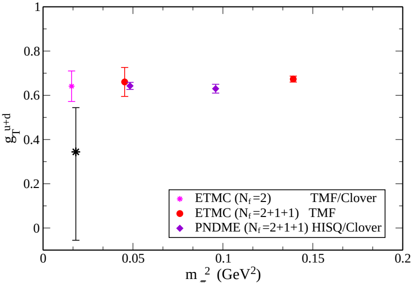
★ $|A^u_{T10}| > |A^d_{T10}|$

★ $\overline{B}^{u,d}_{T10} > 0$ (decreasing)

[QCDSF/UKQCD: M. Göckeler et al., Phys. Lett. B627 (2005) 113; Phys. Rev. Lett. 98 (2007) 222001]

E3. Isoscalar Tensor FFs

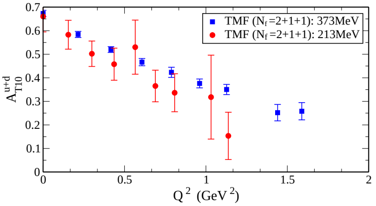
Tensor charge

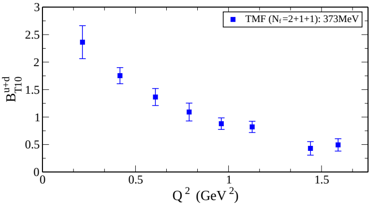


exp. point:

$$A^{exp}_{T10}(0.8\mathrm{GeV}^2) = 0.34^{+0.18}_{-0.36} \;\dagger$$

(at $\mu^2 = 110\mathrm{GeV}^2$)





[T. Bhattacharya et al., Phys. Rev. **D85** 5 (2012)]

† (M. Anselmino et al., Nucl. Phys. Proc. Suppl. 191 (2009) 98)

Future Work

- ★ Increase the statistics for $m_\pi = 126,\, 213$ MeV
- ★ Compute non-perturbative renormalization function for one-D tensor and subtract $\mathcal{O}(a^2)$ terms
- ★ Include a 3^{rd} projector in the 3-pt function (Stochastic Method)

Talk by K. Hadjigiannakou

Thursday 17:30

THANK YOU