Calculating the $x$ Dependence of Hadron PDFs

Huey-Wen Lin
University of Washington
Probing Hadron Structure

§ Structure function/distribution functions

Annul Deep inelastic scattering (DIS)

\[ \sigma \sim L^{\mu\nu} W_{\mu\nu} , \]

\[ W_{\mu\nu} = i \int d^4 x e^{i q x} \langle N | T \{ J^\mu (x), J^\nu (0) \} | N \rangle \]

\[ x = Q^2 / 2 q \cdot P \]

§ Important for QCD and BSM searches

§ Rely on operator product expansion to extract moments

Annul Moments, e.g. \( \langle x^n \rangle_q \), are commonly calculated; must deal with op. mixings, noise increasing with \( n \), etc.

Annul Hard to get to higher moments \( (n > 3) \)

Annul Tricks: subtraction to remove divergent terms, heavy fields, four-point functions… None is practical enough
§ What can we learn about the $x$-distribution?

Make an ansätz of some smooth form for the distribution and fix the parameters by matching to the lattice moments.

$$xq(x) = ax^b(1-x)^c (1 + \epsilon \sqrt{x} + \gamma x)$$

Cannot separate valence-quark contribution from sea.

New idea needed to access the sea!

The Idea

§ Lightcone quark distribution

\[ q(x, \mu) = \int \frac{d\xi_-}{4\pi} e^{-i\xi_- x P_+} \left[ P \left( \bar{\psi}(\xi_-) \gamma_+ \exp \left( -ig \int_0^{\xi_-} d\eta_- A_+(\eta_-) \right) \psi(0) \right) \right] \]

Renormalization scale \( \mu \)
Lightcone coordinate \( \xi_\pm = (t \pm z) / \sqrt{2} \)

Nucleon momentum \( P_\mu \)

§ Approaching lightcone with large \( P \)

\( \Rightarrow \) Just another limit to take, like taking \( a \to 0 \)
The Idea

§ Finite-momentum quark distribution

\[ q(x, \mu, P_z) = \int \frac{d z}{4 \pi} e^{-i z k_z} \left\langle P \right| \bar{\psi}(z) \gamma_z \exp(-i g \int_0^z dz' A_z(z')) \psi(0) \right| P \right\rangle + O(\Lambda_{QCD}^2 / P_z^2, M_N^2 / P_z^2) \]

\[ x = k_z / P_z \]

Lattice z coordinate

Nucleon momentum \( P_\mu = \{P_0,0,0,P_z\} \)

\( \infty \) In \( P_z \to \infty \) limit, parton distribution is recovered

\( \infty \) For finite \( P_z \), corrections are needed

Xiangdong Ji, 1305.1539 (PRL soon)
Some Lattice Details

§ Exploratory study

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- \( N_f = 2+1+1 \) clover/HISQ lattices (MILC)
  \[ M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \ (L \approx 2.88 \text{ fm}) \]
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separation \( (t_{\text{sep}} \approx 0.96 \text{ and } 1.2 \text{ fm}) \) used

§ Properties known on these lattices

- Lattice \( Z_\Gamma \) for bilinear operator ~ 1
  (with HYP-smearing)
- \( M_\pi L \approx 4.6 \) large enough to avoid finite-volume effects

§ Feasible with today’s computational resources!

- \( O(\text{hour}) \) rewriting three-point insertion code (Chroma)
- 8/16 nodes on UW Hyak cluster

Hyak @ UW
Exploratory study

\[
\left\langle P \left| \overline{\psi}(z) \gamma_z \exp\left(-i g \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle
\]

How many links are needed?

Lattice momenta discretized by finite size of volume

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]
§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left( P \left| \psi(z) \gamma_z \exp\left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right) \]

\[ P_z \in \{1, 2, 3\} \frac{2\pi}{L} \]

Uncorrected bare lattice results

\[ x = \frac{k_z}{P_z} \]
§ Exploratory study

\[
\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle
\]

\[P_z \in \{1, 2, 3\} \frac{2\pi}{L}\]

Distribution gets sharper as \(P_z\) increases
Artifacts due to finite \(P_z\) on the lattice

Improvement?
Work out leading-\(P_z\) corrections
§ Back to the continuum

$$q(x, \mu) = q_{FP}(x, \mu, P_z) + O(\Lambda_{QCD}^{2} / P_z^2) + O(M_N^2 / P_z^2) + O(\alpha_s)$$

What we want

What we calculate on the lattice

$P_z \in \{1, 2, 3\} \cdot \frac{2\pi}{L}$

Smaller $P_z$ correction but complicated twist-4 operator (extrapolate it away)

Xiangdong Ji, 1305.1539 (PRL soon)

Dominant correction (for nucleon); known scaling form

J.-W. Chen

Not included yet; O(20%) systematics

J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)
§ Back to the continuum

\[ q(x, \mu) = q_{FP}(x, \mu, P_z) + O(\Lambda_{QCD}^2 / P_z^2) + O(M_N^2 / P_z^2) + O(\alpha_s) \]

\( P_z \in \{1, 2, 3\} \frac{2\pi}{L} \)

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§ Changes in \( x \), and \( q(x) \)
§ Exploratory study

Take ratios (partially cancel statistical and systematic uncertainty)

\[ q_{\text{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx \, q(x, \mu, P_z)} \]

Removing \( O(M_N^2/4P_z^2) \) errors

No significant finite-momentum effect seen for \( P_z > 1 \)

§ Renormalization needed
Quark Distribution

§ Compare with experiments

§ Compare with experiments

Compared with E866
Too good to be true?

Lost resolution in small-x region
Future improvement to have larger lattice volume

\[ \int dx \frac{\bar{u}(x) - \bar{d}(x)}{g_V} \approx 0.196 (28) \]

\begin{tabular}{|c|c|c|}
\hline
Experiment & \( x \) range & \( \int d_x [\bar{d}(x) - \bar{u}(x)] dx \) \\
\hline
E866 & 0.015 < \( x \) < 0.35 & 0.118 ± 0.012 \\
NMC & 0.004 < \( x \) < 0.80 & 0.148 ± 0.039 \\
HERMES & 0.020 < \( x \) < 0.30 & 0.16 ± 0.03 \\
\hline
\end{tabular}

Rx: Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Transversity Distribution

§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-i z k_z} \left| \begin{array}{c} P \\ \bar{\psi}(z) \sigma_{xy} \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \end{array} \right| P \]

Uncorrected bare lattice results

§ Renormalization needed
**Transversity Distribution**

**§ Exploratory study**

\[
\int \frac{dz}{4\pi} e^{-i z k_z} \left\langle P \left| \overline{\psi}(z) \sigma_{xy} \exp\left(-i g \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle
\]

Removing \( O(M_N^2/4P_z^2) \) errors

**§ Renormalization needed**
Transversity Distribution

§ Exploratory study
☞ We found $\delta \bar{u} < \delta \bar{d}$ with large sea asymmetry
☞ Chiral quark-soliton model

\[ \int d x \frac{\delta \bar{u}(x) - \delta \bar{d}(x)}{g_T} \approx -0.320 \ (18) \]
\[ \int d x (\delta \bar{u}(x) - \delta \bar{d}(x)) \approx -0.082 \]

B. Dressler et al., hep-ph/9809487

P. Schweitzer et al., PRD 64, 034013 (2001)

PRELIMINARY
Pion Distribution Amplitude

§ Exploratory study

\[ \int \frac{dz}{2\pi} e^{-izk_z} \langle 0 | \bar{d}(z) \gamma_z \gamma_5 \exp(-ig \int_0^z d z' A_z(z')) u(0) | \pi^+(P) \rangle \]

Leading mass correction applied

Dominated by \( O(\Lambda_{QCD}^2/P_z^2) \) errors

\( P_z \in \{1, 2, 3\} \frac{2\pi}{L} \)
A NEW HOPE

It is a period of war and economic uncertainty. Turmoil has engulfed the galactic republics. Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful...
Overcoming longstanding obstacle to $x$-distribution

- New idea by Ji for studying full $x$ dependence of PDFs
- Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

Caveats

- Not a precision calculation yet
- Need to complete the other $p_z$ corrections (on-going; possibly done in a couple weeks)
- Systematics due to large momenta (some ideas to improve it)

Need improvement for large-momentum sources

- Better overlapping boosted hadron smearing (asymmetric source)
- Applications: large-$q$ form factors, hadronic and flavor physics, …
Summary and Outlook

Exciting time for hadron structure on the lattice

§ We hope this exploratory study motivates others to give Ji’s method a try

☞ More details in the upcoming paper(s), such as $P_z$ corrections from various sources

§ Hope to see many calculations and more ideas

☞ Like $g_{S,T}$ calculations with PNDME
☞ Many more quantities to study:
  - strange/charm/beauty sea distributions, gluons, TMD…

§ “Working” workshop in Shanghai this fall

☞ More details will be announced via latticenews
Backup Slides
Helicity Distribution

§ Exploratory study

\[ \int \frac{dz}{4\pi} e^{-izk_z} \left( P \left| \bar{\psi}(z) \gamma_z \gamma_5 \exp \left( -ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right) \]

PRELIMINARY

Huey-Wen Lin — Lattice 2013 @ Mainz
§ Exploratory study

\[
\int \frac{dz}{4\pi} e^{-izk_z} \left[ P \left| \overline{\psi}(z) \gamma_z \gamma_5 \exp \left(-ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right]
\]

Corrected to \(O(P_z^{-2})\)

Gray band shows extrapolation of \(P_z^{-4}\) terms

Large \(O(P_z^{-4})\) seen but well fit by extrapolation

\[
\int_{-\infty}^0 dx \frac{\Delta u(x) - \Delta d(x)}{g_A} \approx 0.315 \pm 0.005
\]
Helicity Distribution

§ Model: e.g. chiral quark-soliton model

\[ \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.31 \]

B. Dressler et al. hep-ph/9809487

A. Airapetian et al. (HERMES), Phys.Rev. D71, 012003 (2005)

\[ \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.090 (91) \]

\[ x(\Delta \bar{u} - \Delta \bar{d}) \]

D. De Florian et al., PRL 101 (2008) 072001
§ Model: e.g. chiral quark-soliton model

\[ \int dx (\Delta \bar{u}(x) - \Delta \bar{d}(x)) \approx 0.090 \pm 0.010 \]

§ Experimental comparison

A. Airapetian et al. (HERMES), Phys.Rev. D71, 012003 (2005)

B. Dressler et al. hep-ph/9809487

D. De Florian et al., PRL 101 (2008) 072001