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Looking at the gluon moment of the nucleon with dynamical twisted mass fermions

Christian Wiese

with Constantia Alexandrou, Vincent Drach, Kyriakos Hadjiyiannakou, Karl Jansen, Bartosz Kostrzewa

DESY Zeuthen

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Outline

- computing $\langle x \rangle_g$ with the Feynman-Hellmann theorem
- direct way of computing the gluon moment $\langle x \rangle_g$ with a disconnected three-point function
- current simulation setup
- status of computations
- future plans

Introduction I

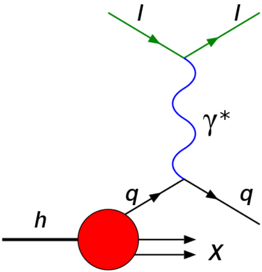
- DIS: existence of partons in the nucleon
- $f_p(x)$ parton distribution function
- probability of finding a parton to have a momentum fraction of x
- first moment of parton distribution function: $\langle x \rangle_p = \int x f_p(x) dx$

$\rightarrow$ average momentum contribution to the nucleon momentum

- $\sum_p \langle x \rangle_p = 1$
- experimentally: $\langle x \rangle_{u+d} \approx 0.4$

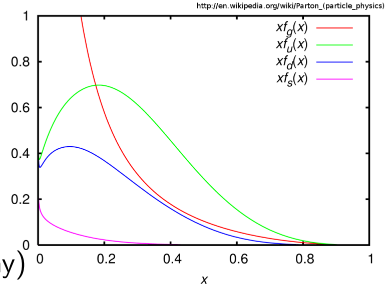
$\rightarrow$ There is something missing.

https://en.wikipedia.org/wiki/Deep_inelastic_scattering



Introduction II

- solution: gluon pdf $f_g(x)$ and gluon moment $\langle x \rangle_g$
- $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1$



- up to now: many computations of fermionic quantities e.g.:

  $\langle x \rangle_{u-d}$ (Talk by C. Alexandrou today)

  g_A (Talk by M. Constantinou today)

  $\sigma_{\pi N}$ (Talk by V. Drach today)
- only few attempts to compute $\langle x \rangle_g$ (with quenched fermions)

Computation of gluon moment I

- We can access the gluon moment via the matrix elements of a suitable operator:
- gluon operator:

$$\mathcal{O}_{\mu\nu} = -\mathrm{tr}_c G_{\mu\rho} G_{\nu\rho}$$

- We can compute the matrix elements with a ratio of a three-point and a two-point function:

$$\frac{\langle H(p,t)\mathcal{O}(\tau)H(p,0)\rangle}{\langle H(p,t)H(p,0)\rangle} \overset{0\ll\tau\ll t}{=} (\mathcal{O})_{H(p)H(p)}$$

Computation of gluon moment II

- two possible forms of the operator
- $\mathcal{A}_i = \mathcal{O}_{i4}$
- $\mathcal{B} = \mathcal{O}_{44} - \frac{1}{3}\mathcal{O}_{jj}$ (euclidean notation for T_{00})
- after form factor decomposition

$$(\mathcal{A}_i)_{N(p)N(p)} = -ip_i \langle x \rangle_g$$

x requires non-zero momentum

$$(\mathcal{B})_{N(p)N(p)} = (m_N + \frac{2}{3E_N}\vec{p}^{\,2})\langle x \rangle_g$$

x subtraction of two terms similar in magnitude

Feynman-Hellmann theorem

- method for computing the matrix elements following a paper of QCDSF and UKQCD Collaborations[a]
- starting point: **Feynman-Hellmann theorem**
- introduce parameter λ into the action: $S \to S(\lambda)$
- then one can derive:

$$\frac{\partial E_N}{\partial \lambda} = (: \frac{\partial \hat{S}(\lambda)}{\partial \lambda} :)_{N(p)N(p)}$$

- :...: means the subtraction of the vacuum term

[a] R. Horsley *et al.* Phys. Lett. B **714** (2012) 312 [arXiv:1205.6410 [hep-lat]].

Feynman-Hellmann theorem II

- we modify the Wilson plaquette action:
- $S(\lambda) = \frac{1}{3}\beta(1+\lambda)\sum_i \mathrm{tr}_c[1-U_{i4}] + \frac{1}{3}\beta(1-\lambda)\sum_{i<j} \mathrm{tr}_c[1-U_{ij}]$

$\rightarrow$ $\lambda = 0$ is the standard plaquette action

- the derivative can be related to the gluon moment:
- $\frac{\partial E_N}{\partial \lambda} = \frac{3}{2}(\mathcal{B})_{N(p)N(p),\lambda}$
- $\frac{\partial E_N}{\partial \lambda}\Big|_{\lambda=0} = -\frac{3}{2}(m_N + \frac{2}{3E_n}\vec{p}^{\,2})\langle x\rangle_g$
- no subtraction here because vacuum term of the operator is zero
- when computing at zero momentum we get

$$\langle x\rangle_g = \frac{2}{3am_N}\frac{\partial am_N}{\partial \lambda}\Big|_{\lambda=0}$$

- we need to compute the nucleon mass for different λ values

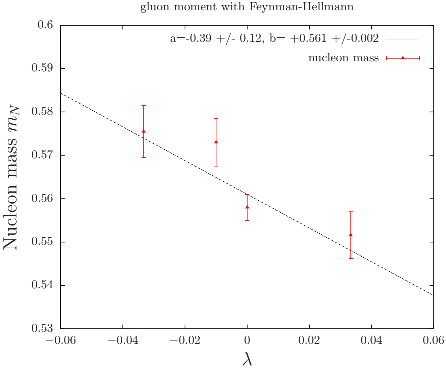
Setup for the Feynman-Hellman theorem

- $N_f = 2 + 1 + 1$ with twisted mass fermions

$\rightarrow$ Iwasaki action!

- $\beta = 1.90$, ($a \approx 0.078$ fm)
- $L^3 \times T = 24^3 \times 48$
- $\mu = 0.0085$ ($m_{PS} \approx 491$ MeV)
- at the moment $O(200)$ configurations
- $\lambda = \{-0.0333, -0.01, +0.0333\}$
- For $\lambda = 0$ we can use existing results for the nucleon.
- problem: We have to tune κ for each λ.

Preliminary data



- visible slope
- Is it linear?

$\rightarrow$ we need more statistics and more λ points

Computation with the direct method

- alternative: directly compute the matrix element with given three- and two-point functions
- For zero zero momentum:

$\rightarrow$ $(\mathcal{B})_{N(0)N(0)} = m_N \langle x \rangle_g$

- write the operator in terms of plaquettes:
  $\mathcal{B}(t) = \frac{4}{9}\frac{\beta}{a}(\sum_i \mathrm{tr}_c[U_{i4}(t)] - \sum_{i<j} \mathrm{tr}_c[U_{ij}(t)])$
- The three-point function is disconnected and can be computed by multiplying the two-point function with the inserted operator:

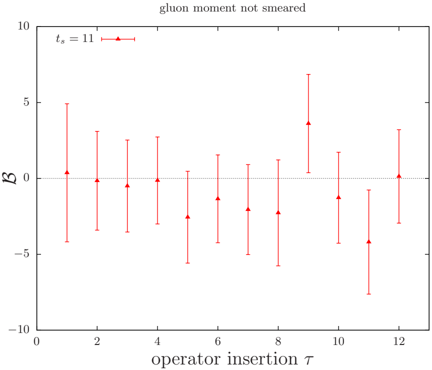
$$\frac{\langle [N(t)N(0)]\mathcal{B}(\tau)\rangle}{\langle N(t)N(0)\rangle} \stackrel{0\ll\tau\ll t}{=} (\mathcal{B})_{N(0)N(0)}$$

Setup for the direct method

- $N_f = 2 + 1 + 1$ with twisted mass fermions
- $\beta = 1.95$ ($a \approx 0.078$ fm)
- $L^3 \times T = 32^3 \times 64$
- $\mu = 0.0055$ ($m_{PS} \approx 393$ MeV)
- $O(2300)$ configurations with 32 two-point functions each (proton and neutron two-point function for 16 different source positions)

$\sim$ 73000 measurements

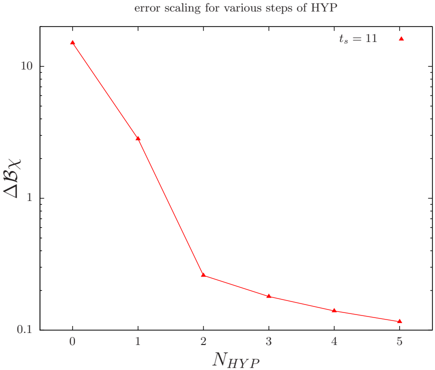
Matrix element of the unsmeared operator



- poor signal, compatible with 0
- try using a HYP smeared operator as suggested in [a](quenched work)

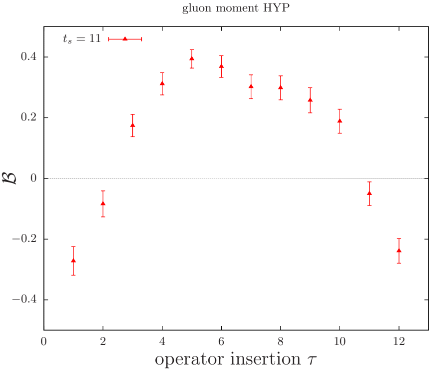
[a]H. B. Meyer and J. W. Negele, Phys. Rev. D **77**, 037501 (2008) [arXiv:0707.3225 [hep-lat]].

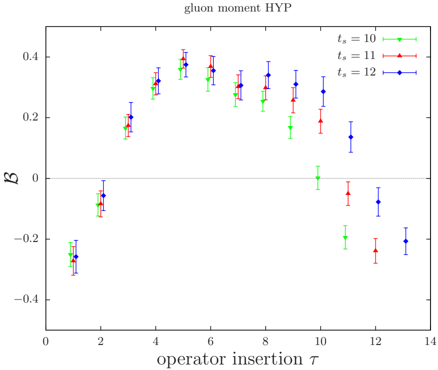
Matrix element of the unsmeared operator



- strong scaling of the error with HYP steps
- using more then one step of HYP smearing seems reasonable
- Our choice: We used 5 steps of HYP smearing with $\alpha_{1,2,3} = \{0.75, 0.6, 0.3\}$.

Matrix element: HYP-smeared operator





- nice signal, possible plateau
- result needs to be normalized and renormalized

Renormalization

- Gluon operators are singlet operators.

$\rightarrow$ mixing with quark singlet

- $\binom{\langle x \rangle_g^{\overline{MS}}}{\sum_q \langle x \rangle_q^{\overline{MS}}} = Z_{2\times 2} \binom{\langle x \rangle_g^{bare}}{\sum_q \langle x \rangle_q^{bare}}$
- $\langle x \rangle_g^{bare} = Z_g \langle x \rangle_g^{lat}$, $\langle x \rangle_q^{bare} = Z_q \langle x \rangle_q^{lat}$

$\rightarrow$ $\langle x \rangle_g^{\overline{MS}} = Z_{bare\, gg}^{\overline{MS}} \langle x \rangle_g^{bare} + [1 - Z_{bare\, qq}^{\overline{MS}}] \sum_q \langle x \rangle_q^{bare}$

- we need Z_g, Z_q, $Z_{bare\, qq}^{\overline{MS}}$, $Z_{bare\, gg}^{\overline{MS}}$, $\langle x \rangle_u^{lat}$ and $\langle x \rangle_d^{lat}$

? different renormalization because of using a HYP smeared operator

Conclusion

- Two methods which can be used to extract $\langle x \rangle_g$:
- Feynman-Hellmann theorem

\+ visible signal with small statistics

\- configurations and two-point functions for λ have to be computed

- direct method:

\+ two-point functions and configurations are available

\+ computation of plaquettes is cheap

\- need large statistics

Future plans and problems

- Which method should be preferred?
- FHt: λ for the improved term of the gauge action?
- How to compute all the renormalization factors?
- physical point, continuum limit
- gluon moment of other hadrons

Thanks

Thank you for your attention and future discussions.