B-physics with Nf=2 Wilson fermions

Bernardoni Fabio

In collaboration with: B. Blossier, J. Bulava, M. Della Morte, P. Fritzsch, N. Garron, A. Gerardin, J.Heitger, H. Simma, R. Sommer

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Outline

- **Motivation** well known:
  - matrix elements for B decays (cfr $|V_{ub}|$)
  - $m_b(m_b)$ for perturbative computations
  - reproduce B mesons mass spectrum
- **Method**
  - CLS configurations
  - HQET renormalization, matching, improvement
  - large volume computations
- **Results**

predictions $f_B,\ f_{B_s},\ \dfrac{f_{B_s}}{f_B}$

postdictions $m_b^{\overline{MS}}(m_b),\ m_{B_s} - m_B,\ m_{B^*} - m_B,\ m_{B_s^*} - m_{B_s}$

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Treatment of light quarks

$N_f = 2$ sea Wilson quarks

- Volume effects exponentially suppressed:

  $$Lm_\pi \geq 4.0$$

- Light quark mass chiral extrapolation:

  $$(190 \lesssim m_\pi \lesssim 450)\ \mathrm{MeV}$$

- Discretization effects:
  - 3 lattice spacings a

    $$0.048, 0.065, 0.075\ \mathrm{fm}$$

  - NP renormalization
  - NP $O(a)$ improvement

CLS based

id	L/a	a [fm]	m_π[MeV]	$m_\pi L$
A4	32	0.0755	380	4.7
A5			330	4.0
B6	48		270	5.2
E5	32	0.0658	440	4.7
F6	48		310	5.0
F7			270	4.3
G8	64		190	4.1
N5	48	0.0486	440	5.2
N6			340	4.0
O7	64		270	4.2

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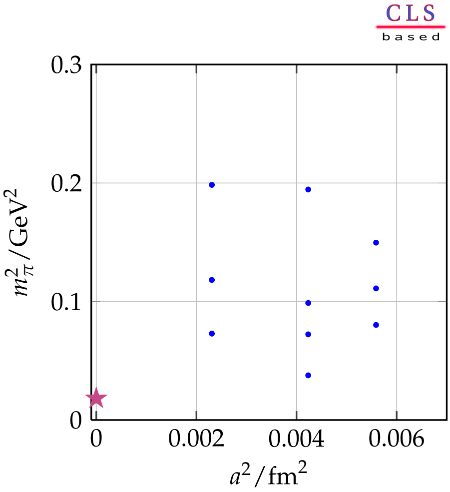


Treatment of b quark

$m_b \gg \Lambda_{QCD}$: b treated in HQET

- expansion in $1\,/\,m_b$

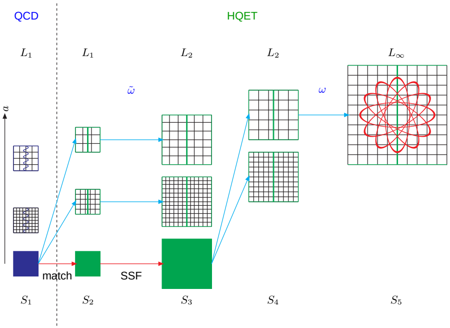
$$\mathcal{L}_{\mathrm{HQET}}(x) = \mathcal{L}_h^{stat} - \omega_{\mathrm{kin}}\mathcal{O}_{kin}(x) - \omega_{\mathrm{spin}}\mathcal{O}_{spin}(x)$$
$$= \overline{\psi}_{\mathrm{h}}(x)\,D_0\,\psi_{\mathrm{h}}(x) - \omega_{\mathrm{kin}}\overline{\psi}_{\mathrm{h}}(x)\mathbf{D}^2\psi_{\mathrm{h}}(x) - \omega_{\mathrm{spin}}\overline{\psi}_{\mathrm{h}}(x)\sigma\cdot\mathbf{B}\psi_{\mathrm{h}}(x)$$

- restrict to processes such that $p \ll m_b$
- power divergences in a^{-1} $\Rightarrow$ need NP renormalization
  [Maiani, Martinelli, Sachrajda 92]
- renormalizable at every order in $1\,/\,m_b$
  $\Rightarrow$ safe estimate of discretization effects

$$\langle\mathcal{O}\rangle = \langle\mathcal{O}\rangle_{stat} + \omega_{\mathrm{kin}}\sum_x\langle\mathcal{O}\mathcal{O}_{kin}(x)\rangle_{stat} + \omega_{\mathrm{spin}}\sum_x\langle\mathcal{O}\mathcal{O}_{spin}(x)\rangle_{stat}$$

$$\langle\mathcal{O}\rangle_{stat} = \frac{1}{\mathcal{Z}}\int_{fields}\mathcal{O}\exp\left(-a^4\sum_x[\mathcal{L}_{light} + \mathcal{L}_h^{stat}]\right)$$

ALPHA strategy for NP renormalization & matching



[Blossier et al. 12]

- match QCD and HQET at $a^{-1} \gg M_b$
  small volume, however $z = LM_b \gg 1$
  determine NP: $\vec{\omega}(z) = m_{\rm bare}(z), Z_{\rm A}^{\rm HQET}(z), c_{\rm A}^{(1)}(z), \omega_{\rm kin}(z), \omega_{\rm spin}(z)$
- step scaling to a used in large volumes

$\Rightarrow$ determine M_b dependence of large volume observables

The b-quark's mass

1. (left) extrapolate to physical point ($m_{PS} \to m_\pi$)

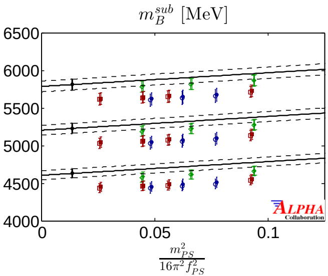
$$m_B\left(z, m_{\rm PS}, a, {\rm HYPn}\right) + \frac{3\hat{g}^2}{32\pi}\left(\frac{m_{\rm PS}^3}{f_{PS}^2} - \frac{m_\pi^3}{f_\pi^2}\right) = B(z) + Cm_{\rm PS}^2 + D_{\rm HYPn}a^2$$

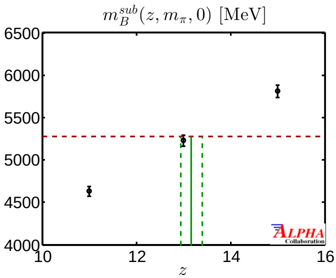
$$\hat{g} = 0.51(2) \text{ [Bulava et al. 10]}$$

2. (right) Interpolate $m_B(z)$ to get M_b:

$$m_B(z, m_\pi, a)\big|_{z=z_{\rm b}} \equiv m_{\rm B}^{\rm exp} = 5279.5{\rm MeV}$$

3. we get $z_{\rm b} = 13.17(23)(13)_z$ or equivalently $m_{\rm b}^{\overline{MS}}(m_{\rm b}) = 4.23(11)(3)_z$ GeV.





Observables in HQET

1. interpolate $\vec{\omega}(z)$ to get $\vec{\omega}(z_b)$
2. compute matrix elements in HQET

$$p^{stat} = \lim_{x_0\to\infty}\{2e^{E_{stat}x_0}\sum_{\vec{x}}\langle A_0(x)A_0(0)\rangle\}^{1/2} \quad E_{stat} = -\lim_{x_0\to\infty}\partial_0 \ln\sum_{\vec{x}}\langle A_0(x)A_0(0)\rangle$$

$$\ldots \qquad \ldots$$

3. combine matrix elements and $\vec{\omega}(z_b)$

$$\ln(f_{\mathrm{B}}\sqrt{m_{\mathrm{B}}/2}) = \ln(Z_{\mathrm{A}}^{\mathrm{HQET}}) + \ln(p^{\mathrm{stat}}) + b_{\mathrm{A}}^{\mathrm{stat}}am^{l}_{\mathrm{PCAC}} + \omega_{\mathrm{kin}}p^{\mathrm{kin}} + \omega_{\mathrm{spin}}p^{\mathrm{spin}} + c_{\mathrm{A}}^{(1)}p^{\mathrm{A}^{(1)}}$$

$$A_{0,\mathrm{R}}^{\mathrm{HQET}} = Z_{\mathrm{A}}^{\mathrm{HQET}}\,[A_0^{\mathrm{stat}} + c_{\mathrm{A}}^{(1)}A_0^{(1)}]\,, \qquad A_0^{\mathrm{stat}} = \overline{\psi}_{\mathrm{l}}\,\gamma_0\gamma_5\,\psi_{\mathrm{h}}\,,$$

$$A_0^{(1)} = \overline{\psi}_{\mathrm{l}}\,\gamma_5\gamma_i\frac{1}{2}(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\,\psi_{\mathrm{h}}$$

Treatment of the excited states

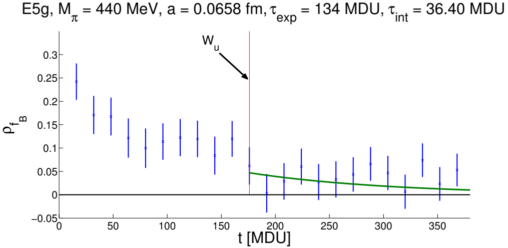
- GEVP with 3 light quark wavefunctions (levels of smearing)
- plateau average only where excited states contribution negligible

$$\sigma_{sys}(t_{min}) \ll \sigma_{stat}(t_{min})$$

Statistical analysis

Use methods described in: [Schaefer et al. 10]

- all correlations taken into account (eg with a, $\vec{\omega}(z)$, m_π, f_π)
- for an observable $\mathcal{O}$: $$\delta\mathcal{O} \propto \tau_{int} = \frac{1}{2} + \sum_{1}^{\infty} \rho_{\mathcal{O}}(t)$$
- in practice restrict sum up to W, but $$\rho_{\mathcal{O}}(t) \xrightarrow{t\to\infty} A_{\mathcal{O}} e^{-t/\tau_{exp}}$$

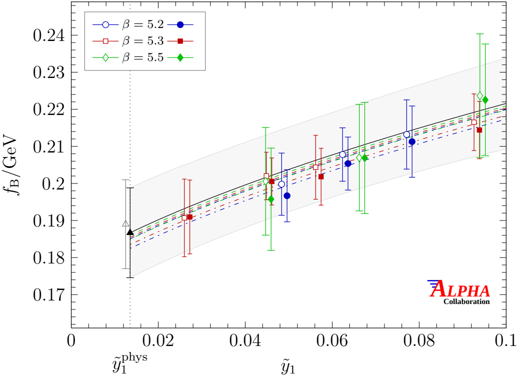


- typically $W < \tau_{exp}$
- attach a tail to $\rho_{\mathcal{O}}$

$$\tau_{int}^{u}(\mathcal{O}) = \tau_{int}(\mathcal{O}, W_u) + \tau_{exp}\,\rho_{\mathcal{O}}(W_u)$$

Results: f_B

$$f_B\sqrt{m_B} = A\left(1+\frac{3+9\widehat{g}^2}{8}\left(\tilde{y}_1 \ln \tilde{y}_1(PS) - \tilde{y}_1 \ln \tilde{y}_1(\pi)\right)\right) + B\left(\tilde{y}_1(PS) - \tilde{y}_1(\pi)\right) + C_{HYPn}\, a^2$$



- continuum and chiral extrapolation (NLO HMChPT):

$\Rightarrow$ error from $a \to 0$ in stat

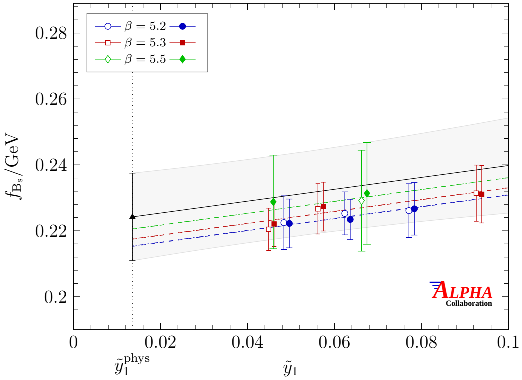
- $\tilde{y}_1(PS) = \dfrac{m_{PS}^2}{16\pi^2 f_{PS}^2}$
- ChPT: NLO vs. linear extrapolation
- tiny cutoff effects

$$f_B = 187(12)_{stat}(2)_{ChPT}\ \text{MeV}$$

HYP1 open symbols / dashed lines HYP2 filled symbols / dash-dotted lines

Results: f_{B_s}

$$f_{B_s}\sqrt{m_{B_s}} = A + B\,(\tilde{y}_1(PS) - \tilde{y}_1(\pi)) + C_{HYPn}\,a^2$$

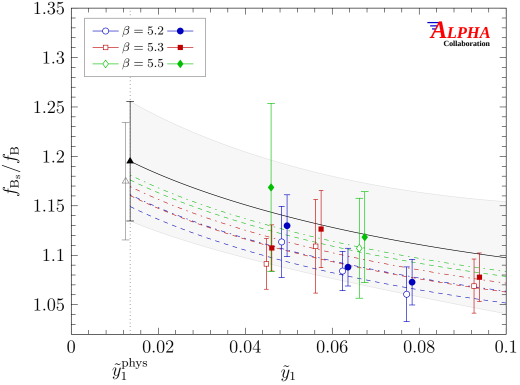


- NLO HMChPT has no log for f_{B_s} in PQ
- κ_s from scale setting by f_K [Fritzsch et al. 12]
- less statistics wrt f_B
- small cutoff effects

$$f_{B_s} = 224(13)_{stat}\ \mathrm{MeV}$$

Results: f_{B_s} / f_B

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_B\sqrt{m_B}} = A\left(1 - \frac{3+9\hat{g}^2}{8}\left(\tilde{y}_1 \ln \tilde{y}_1(PS) - \tilde{y}_1 \ln \tilde{y}_1(\pi)\right)\right) + B\left(\tilde{y}_1(PS) - \tilde{y}_1(\pi)\right) + C_{HYPn}a^2$$

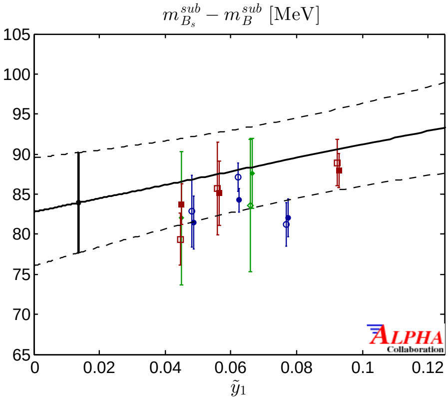


- ChPT: NLO vs. linear extrapolation
- less statistics wrt f_B
- small cutoff effects

$$f_{B_s} / f_B = 1.195(61)_{stat}(20)_{ChPT}$$

Results: $m_{B_s} - m_B$

$$m_{B_s} - m_B - \frac{9\hat{g}^2}{128\pi}\left(\frac{m_{PS}^3}{f_{PS}^2} - \frac{m_\pi^3}{f_\pi^2}\right) = A + B\left(\tilde{y}_1(PS) - \tilde{y}_1(\pi)\right) + C_{HYPn}a^2$$

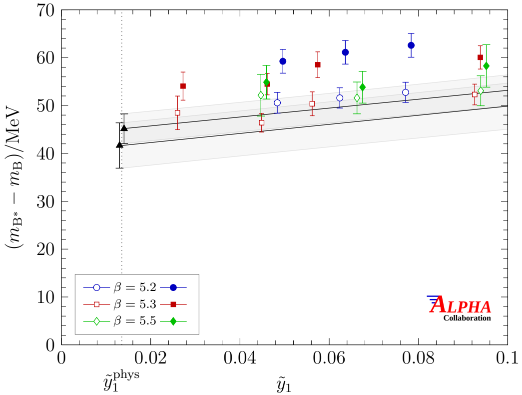


- ChPT: NLO vs. linear extrapolation
- a: fit with or without $\beta = 5.5$

$$m_{B_s} - m_B = 83.9(6.3)_{stat}(6.9)_{ChPT}(0.8)_a \text{ MeV}$$

Results: $m_{B^*} - m_B$

$$m_{B^*} - m_B = A + B\,(\tilde{y}_1(PS) - \tilde{y}_1(\pi)) + C_{HYPn}\,a$$

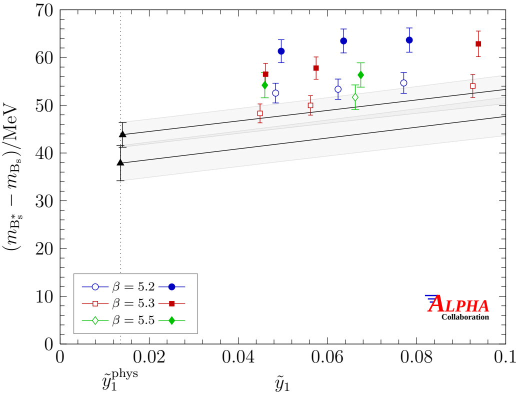


- $m_{B^*} - m_B = O(1/m_b)$
- despite $O(a)$ improvement, expect $O(a/m_b)$ effects
- a: fit in a^2 vs. fit in a

$$m_{B^*} - m_B = 41.7(4.7)_{stat}(3.4)_a \text{ MeV}$$

Results: $m_{B_s^*} - m_{B_s}$

$$m_{B_s^*} - m_{B_s} = A + B\left(\tilde{y}_1(PS) - \tilde{y}_1(\pi)\right) + C_{HYPn}a$$



- $m_{B_s^*} - m_{B_s} = O(1/m_b)$
- despite $O(a)$ improvement, expect $O(a/m_b)$ effects
- a: fit in a^2 vs. fit in a

$$m_{B_s^*} - m_{B_s} = 37.9(3.7)_{stat}(5.9)_a \text{ MeV}$$

Comparison with experimental results: postdictions

Observable	ALPHA	Exp.	Method
m_B[MeV]	input	5279.5	e^+e^- scat.
$m_b^{\overline{MS}}(m_b)$ [GeV]	$4.23(11)(3)_z$	4.18(3)	smeared $\sigma(e^+e^- \to b\bar{b})$+ PT
$m_{B_s} - m_B$ [MeV]	$83.9(6.3)(6.9)_a$	87.35(0.23)	pp, $p\bar{p}$ scat.
$m_{B^*} - m_B$ [MeV]	$41.7(4.7)(3.4)_a$	45.3(0.8)	e^+e^- scat.
$m_{B_s^*} - m_{B_s}$ [MeV]	$37.9(3.7)(5.9)_a$	48.7(2.3)	e^+e^- scat.

- reproducing well known experimental results is a reason to be confident in our method
- systematics still relevant for some quantities

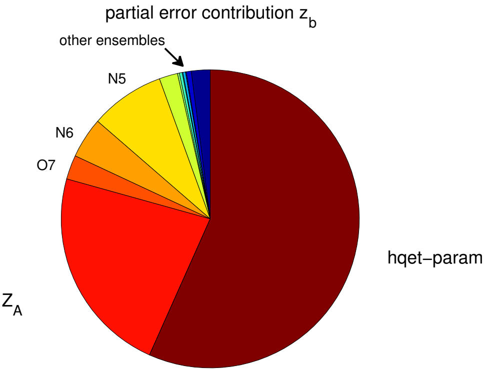
Comparison with Lattice averages: predictions

Obs.	ALPHA	Lat. Av.[1]	Experiment
f_B [MeV]	$187(12)(2)_{ChPT}$	190.6(4.7)	$BR(B \to \tau\nu)_{ALPHA} = 1.065(21) \times 10^{-4}$ $BR(B \to \tau\nu)_{exp} = 1.05(25) \times 10^{-4}$
f_{B_s} [MeV]	224(13)	227.6(5.0)	$BR(B_s \to \mu^+\mu^-)_{ALPHA} = 3.15(27) \times 10^{-9}$ $BR(B_s \to \mu^+\mu^-)_{exp} = 2.9(0.7) \times 10^{-9}$
f_{B_s}/f_B	$1.195(61)(20)_{ChPT}$	1.201(17)	

- $BR(B \to \tau\nu)_{ALPHA}$ uses $|V_{ub}|$ from PDG 12 (inclusive decays, $BR(B \to \pi l \nu)$)
- $BR(B_s \to \mu^+\mu^-)_{ALPHA}$ uses $|V_{tb}^* V_{ts}|$ from CKM fit (mainly B_s^0 splitting)
- agreement with Lattice Averages
- agreement with Experiment

[1] www.latticeaverages.org

Partial contributions to the total error for z_b



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