B-physics computations from $N_f = 2$ tmQCD

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July 29 - August 3, 2013

“31st International Symposium on Lattice Field Theory”
Mainz, Germany

In collaboration with:
Outline

- **B-physics computations with** $N_f = 2 \text{ tmQCD}$
  - Method based on **Ratios** of heavy-light $(h, \ell/s)$ observables using relativistic quarks and exact knowledge of static limit for the appropriate ratios
  - **Interpolation** of $(h, \ell/s)$ observables to the b-region from the charm region and the static limit
  - **Ratio method** helps for suppressing systematics of the heavy sector
  - Computation of $b$-quark mass, **decay constants**, $B$-**Bag** parameters for the **complete** 4-f operator basis, $\xi$

  ★ Error budget determined mainly from scale setting and RCs uncertainties

**ETMC:**

JHEP 1004 (2010) 049 (0909.3187)
JHEP 1201 (2012) 046 (1107.1441)
PoS ICHEP2012 (2012) 428 (1212.0301);
PoS LATTICE2012 (2012) 104 (1211.0568);
PoS LATTICE2012 (2012) 105 (1211.0565)
N. Carrasco et al. (in preparation) (2013)
ETMC – $N_f = 2$ twisted-mass formulation

- Mtm lattice regularization for $N_f = 2$ QCD action

\[ S_{N_f=2}^{ph} = S_L^{YM} + a^4 \sum_x \bar{\psi}(x) \left[ \gamma \cdot \tilde{\nabla} - i\gamma_5 \tau^3 \left( -\frac{a}{2} r \nabla^* \nabla + M_{cr}(r) \right) + \mu_q \right] \psi(x) \]

[Frezzotti, Grassi, Sint, Weisz, JHEP 2001; Frezzotti, Rossi, JHEP 2004]

- $\psi$ is a flavour doublet, $M_{cr}(r)$ is the critical mass and $\tau^3$ acts on flavour indices

  - Automatic $O(a)$ improvement for the physical quantities
  - Dirac-Wilson matrix determinant is positive
    and (lowest eigenvalue)$^2$ bounded from below by $\mu_q^2$
  - Simplified (operator) renormalization ...
    - Multiplicative quark mass renormalization
    - No RC for pseudoscalar decay constant (PCAC)

- $O(a^2)$ breaking of parity and isospin
ETMC $N_f = 2$ simulations

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$(L/a, T/a)$</th>
<th>$a\mu_\ell$</th>
<th>$a\mu_s$ (valence)</th>
<th>$a\mu_h$ (valence)</th>
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<tbody>
<tr>
<td>3.80</td>
<td>(24, 48)</td>
<td>0.0080, 0.0110</td>
<td>0.0175, 0.0194</td>
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<td>0.0159, 0.0177</td>
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<td>(24, 48)</td>
<td>0.0085, 0.0100</td>
<td>0.0116, 0.0129</td>
<td>0.0142, 0.13315, 0.1566, 0.1842, 0.2166, 0.2548, 0.2997, 0.3525, 0.4145, 0.4876, 0.5734</td>
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<td>4.05</td>
<td>(32, 64)</td>
<td>0.0020</td>
<td>0.0116, 0.0129</td>
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<td>4.20</td>
<td>(48, 96)</td>
<td>0.0065</td>
<td>0.0116, 0.0129</td>
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</tr>
</tbody>
</table>

- $a = \{0.098, 0.085, 0.067, 0.054\}$ fm
- $m_{ps}^{\ell\ell} \in \{270, 600\}$ MeV
- $L \in \{1.7, 2.8\}$ fm, $m_{ps}L \geq 3.5$
- $\mu_\ell \in [\sim m_s/6, \sim m_s/2]$
- $\mu_h \in [\sim m_c, \sim 3m_c]$
\[ M_{\text{eff}}^{(h \ell)} \] plateau quality - optimal interpolating fields

\[ (\beta = 3.80; (L/a, T/a) = (24, 48); m_h \sim 2m_c) \]

\[ \Phi^S \propto (1 + \kappa_G a^2 \nabla_{\text{APE}}^2)^{N_G} \Phi^L \]

\[ \alpha_{\text{APE}} = 0.5, N_{\text{APE}} = 20, \kappa_G = 4, N_G = 30 \]

Improved projection

- Smearing techniques improve signal; reduce the coupling between the ground and excited states; safe good plateaux at earlier times; necessary also for obtaining safe plateau in the calculation of 3-point correlation functions (when large heavy quark mass \([ > 1 \text{ GeV}]) are employed).

- Employ "optimal" interpolating field: \( \Phi_w \propto w\Phi^S + (1 - w)\Phi^L \);
  \( w \) is tunable parameter
Ratio method

- use correlators with *relativistic* quarks
- $c$-mass region computations are reliable ('small' discr. errors)
- construct HQET-inspired ratios of the observable of interest at consecutive (nearby) values of the heavy quark mass
  \[ (\mu_h^{(n)} = \lambda \mu_h^{(n-1)}) \]
- ratios show smooth chiral and continuum limit behaviour
- ratios at the $\infty$-mass (static) point are exactly known ($= 1$)
- physical value of the observable at the $b$-mass point is related to its $c$-like value by a chain of the ratios ending up at the static point:
  use HQET-inspired interpolation
• observing that \( \lim_{\mu_h^{\text{pole}} \to \infty} \left( \frac{M_{h\ell}}{\mu_h^{\text{pole}}} \right) = \text{constant} \) (HQET)

• construct (taking \( \frac{\mu_h^{(n)}}{\mu_h^{(n-1)}} = \lambda \)):

\[
y(\mu_h^{(n)}, \lambda; \bar{\mu}_\ell, a) \equiv \frac{M_{h\ell}(\mu_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\mu_h^{(n-1)}; \bar{\mu}_\ell, a)} \cdot \frac{\mu_h^{(n-1)}}{\mu_h^{(n)}} \cdot \frac{\rho(\mu_h^{(n-1)}, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)} = \\
= \lambda^{-1} \frac{M_{h\ell}(\mu_h^{(n)}; \bar{\mu}_\ell, a)}{M_{h\ell}(\mu_h^{(n)}; \lambda; \bar{\mu}_\ell, a)} \cdot \frac{\rho(\mu_h^{(n)}/\lambda, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)}, \quad n = 2, \ldots, N
\]

\( \mu_h^{\text{pole}} = \rho(\bar{\mu}_h, \mu^*) \bar{\mu}_h(\mu^*) \) (with \( \bar{\mu}_h \leftarrow \overline{\text{MS}} \) scheme)

\( \rho(\bar{\mu}_h, \mu^*) \) known in PT up to \( \text{N}^3\text{LO} \) – relevant only for the '1/\( \bar{\mu}_h \)' interpolation

→ In the static limit (and in CL) obviously:

\[
\lim_{\bar{\mu}_h \to \infty} y(\mu_h, \lambda; \bar{\mu}_\ell, a = 0) = 1
\]
b-quark mass computation - 2

- Aim: construct the chain equation
  \[ y(\mu_h^{(2)}) y(\mu_h^{(3)}) \ldots y(\mu_h^{(K+1)}) = \lambda - \kappa \frac{M_{hu/d}(\mu_h^{(K+1)})}{M_{hu/d}(\mu_h^{(1)})} \cdot \left[ \frac{\rho(\mu_h^{(1)}, \mu^*)}{\rho(\mu_h^{(K+1)}, \mu^*)} \right] \]

- Need: extrapolation in the continuum limit - physical point to obtain
  - *pseudoscalar mass* \( M_{hu/d}(\mu_h^{(1)}) \) (← in the \( c \)-quark region)
  - *ratios* \( y(\mu_h^{(n)}) \) (\( n = 2, \ldots, N \))

- smooth fits – discr. effects under control
b-quark mass computation - 3

- use the chain equation
  \[ y(\overline{\mu}_h^{(2)}) y(\overline{\mu}_h^{(3)}) \ldots y(\overline{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\overline{\mu}_h^{(K+1)})}{M_{hu/d}(\overline{\mu}_h^{(1)})} \cdot \left[ \frac{\rho(\overline{\mu}_h^{(1)}, \mu^*)}{\rho(\overline{\mu}_h^{(K+1)}, \mu^*)} \right] \]

- Evaluate the (lhs) with \( y(\overline{\mu}_h^{(j)}) \) set to the best fit data.

- Adjust \( (\lambda, \overline{\mu}_h^{(1)}) \) such that \( K \) integer and \( M_{hu/d}(\overline{\mu}_h^{(K+1)}) \equiv M_B^{\text{expt}} \).
  
  **our calculation**: \( \lambda = 1.1784 \) and \( \overline{\mu}_h^{(1)} = 1.05 \) GeV (in \( \overline{\text{MS}} \), 3 GeV)
  
  \( \rightarrow \overline{\mu}_b = \lambda^K \overline{\mu}_h^{(1)} \) (\( K = 9 \))

- **strong** cancellations of perturbative factors \( \rho \)'s in the ratios:
  
  \( \rightarrow \) subpercent dependence on \( \rho \)'s PT-order

- **fit ansatz** \( y(\overline{\mu}_h) = 1 + \frac{\eta_1}{\overline{\mu}_h} + \frac{\eta_2}{\overline{\mu}_h^2} \)
  
  (inspired by HQET)

- curvature denotes a large \( 1/\overline{\mu}_h^2 \)
  contribution to ratios \( y \)
b-quark mass - results

- \( m_b(m_b, \overline{\text{MS}}) = 4.29(12) \text{ GeV} \)

- compatible result for \( m_b \) if (\( hs \))-data and \( M_{Bs}^{\text{expt}} \) as input are used

- Main source of uncertainty of the ETMC result is due to quark mass RC and scale setting uncertainties;

- stats + fit (CL + chiral) \( \sim 2.1\% \);
- latt. scale syst \( \sim 2.0\% \);
- discr. syst. \( \sim 0.2 \% \);
- syst. \( 1/\mu_h \) fit (TL, LO, NLO) \( \sim 0.4\% \);
- syst. RG-running \( \sim 0.5\% \).

(for \( N_f = 2 + 1 + 1 \) by ETMC, see talk by E. Picca, Friday)
General case computation - 1

- For any observable $Q_{h \ell/s} \equiv f_{Bs}, f_B, f_{Bs}/f_B, B_i^{(d/s)}$, $\xi$ consider the HQET scaling quantity $\Phi_Q = Q_{h \ell/s} \times (\mu_{h}^{\text{pole}})^{\alpha}$ or $\tilde{\Phi}_Q = Q_{h \ell/s} \times (M_{h}^{\ell/s})^{\alpha}$

- Observing that
  $$\lim_{\mu_{h}^{\text{pole}} \to \infty} \left( Q_{h \ell/s} \times (\mu_{h}^{\text{pole}})^{\alpha} \right) = \text{constant}$$
  $$\lim_{M_{h}^{\ell/s} \to \infty} \left( Q_{h \ell/s} \times (M_{h}^{\ell/s})^{\alpha} \right) = \text{constant (up to log corrections)}$$

- $\alpha = 1/2$ for $f_{Bs}, f_B$; $\alpha = 0$ for $f_{Bs}/f_B, B_i^{(d/s)}, \xi$

- Construct ratios at near-by $\mu_{h}$ ($\mu_{h}^{(n)} = \lambda \mu_{h}^{(n-1)}$)
  $$z_{\Phi}(\mu_{h}^{(n)}, \lambda; \mu_{\ell/s}, a) = \frac{\Phi_Q(\mu_{h}^{(n)}; \mu_{\ell/s}, a)}{\Phi_Q(\mu_{h}^{(n-1)}; \mu_{\ell/s}, a)} \frac{C_Q(\mu_{h}^{(n)})}{C_Q(\mu_{h}^{(n-1)})}$$

- And similar definitions when employing $\tilde{\Phi}_Q$
  $\star C_Q(\mu_{h}^{(n)})$ include anomalous dimension in HQET and possible $\rho$ factors

- $\lim_{\mu_{h} \to \infty} z_{\Phi}(\mu_{h}, \lambda; \mu_{\ell/s}, a = 0) = 1$
  $\lim_{M_{h}^{\ell/s} \to \infty} z_{\tilde{\Phi}}(\mu_{h}, \lambda; \mu_{\ell/s}, a = 0) = 1$
• construct the chain equation

$$z_\Phi (\overline{\mu}_h^{(2)}) z_\Phi (\overline{\mu}_h^{(3)}) \ldots z_\Phi (\overline{\mu}_h^{(K+1)}) = \lambda^{\alpha K} \frac{Q_{hd/s}(\overline{\mu}_h^{(K+1)})}{Q_{hd/s}(\overline{\mu}_h^{(1)})} \cdot \left[ \frac{C_Q(\overline{\mu}_h^{(K+1)})}{C_Q(\overline{\mu}_h^{(1)})} \right]$$

or

$$z_\tilde{\Phi} (\overline{\mu}_h^{(2)}) z_\tilde{\Phi} (\overline{\mu}_h^{(3)}) \ldots z_\tilde{\Phi} (\overline{\mu}_h^{(K+1)}) = \frac{Q_{hd/s}(\overline{\mu}_h^{(K+1)})}{Q_{hd/s}(\overline{\mu}_h^{(1)})} \cdot \left[ \frac{\tilde{C}_Q(\overline{\mu}_h^{(K+1)})}{\tilde{C}_Q(\overline{\mu}_h^{(1)})} \right]$$

and, at $\overline{\mu}_b = \lambda^K \overline{\mu}_h^{(1)}$, determine $Q_{bd/s} = Q_{hd/s}(\overline{\mu}_h^{(K+1)})$.

• (lhs) is determined by the best fit data using $z_\bullet (\overline{\mu}_h)$-ratios after they have been extrapolated (smoothly) to the CL.

• $Q_{hd/s}(\overline{\mu}_h^{(1)})$ determined safely in the $c$-quark region employing an extrapolation to CL and physical point.
\[ f_{Bs} \]

\[ B(B_s \rightarrow \ell^+ \ell^-) \propto M_{Bs} f_{Bs} |V_{tb}^* V_{tq}|^2 \]

- fit ansatz (inspired by HQET)
  \[ z_\Phi(\bar{\mu}_h) = 1 + \frac{\hat{\eta}_1}{\bar{\mu}_h} + \frac{\hat{\eta}_2}{\bar{\mu}_h^2} \]

- Note: final results at physical \( b \)-point show weak dependence (\( \lesssim 1\%) \) on PT-order (TL, LO, NLO) either employing \( \Phi_{f_{Bs}} \) or \( \tilde{\Phi}_{f_{Bs}} \).
\[ f_{B_s} (\text{continue}) \]

\[ B(B_s \to \ell^+ \ell^-) \propto M_{B_s} f_{B_s} (V_{tb}^* V_{tq})^2 \]

- \( f_{B_s} = 228(8) \text{ MeV} \)
- \( \text{stats + fit (CL + chiral)} \sim 2.2\%; \)
- \( \text{latt. scale syst} \sim 2.0\%; \)
- \( \text{discr. syst.} \sim 1.3\%; \)
- \( \text{syst. '1}/\bar{\mu}_h' \text{ fit} \sim 1.0\%. \)
SU(3) breaking ratio:
cancellation of systematics;
sensitive to chiral extrapolation

Extrapolation in CL + phys. point in the
$c$-quark region. Define:

$$\mathcal{R}_f(\bar{\mu}_h^{(1)}) = \left(\frac{f_{hs}/f_{hel}}{f_{se}/f_{lle}}\right) \left(\frac{f_K/f_\pi}{f_{B_s}/f_B}\right)$$

$\rightarrow$ large cancellation of the chiral
logarithmic terms


$f_{B_s}/f_B = 1.206(24)$

stats + fit (CL + chiral) $\sim 0.8\%$ ;
syst. $\mathcal{R}_f \sim 1.7\%$ ;
discr. syst. $\sim 0.4\%$ ;
syst. '$1/\bar{\mu}_h$' fit $\sim 0.1\%$. 
Results & Comparisons - I

- \( f_{Bs}(\text{ETMC} - 2013) = 228(8) \text{ MeV} \)
- \( f_{Bs}/f_B(\text{ETMC} - 2013) = 1.206(24) \)

(for \( N_f = 2 + 1 + 1 \) results by ETMC, see talk by E. Picca, Friday)
\begin{itemize}
\item $f_B(\text{ETMC} - 2013) = 189(8) \text{ MeV}$
\item $(f_B = f_{Bs}/(f_{Bs}/f_B))$
\end{itemize}

(for $N_f = 2 + 1 + 1$ results by ETMC, see talk by E. Picca, Friday)
Ratio method for the $\Delta B = 2$ operators

[ Neutral $B_{d/s}$-meson oscillations via loop box diagrams ]

- **QCD**
  
  \[
  O_1 = [\bar{b}^\alpha \gamma_\mu (1 - \gamma_5) q^\alpha] [\bar{b}^\beta \gamma_\mu (1 - \gamma_5) q^\beta]
  
  O_2 = [\bar{b}^\alpha (1 - \gamma_5) q^\alpha] [\bar{b}^\beta (1 - \gamma_5) q^\beta] 
  
  O_3 = [\bar{b}^\alpha (1 - \gamma_5) q^\beta] [\bar{b}^\beta (1 - \gamma_5) q^\alpha] 
  
  O_4 = [\bar{b}^\alpha (1 - \gamma_5) q^\alpha] [\bar{b}^\beta (1 + \gamma_5) q^\beta] 
  
  O_5 = [\bar{b}^\alpha (1 - \gamma_5) q^\beta] [\bar{b}^\beta (1 + \gamma_5) q^\alpha] 
  
  q = d, s
  
- **HQET**
  
  \[
  \hat{O}_1 = [\tilde{h}^\alpha \gamma_\mu (1 - \gamma_5) q^\alpha] [\tilde{h}^\beta \gamma_\mu (1 - \gamma_5) q^\beta] 
  
  \hat{O}_2 = [\tilde{h}^\alpha (1 - \gamma_5) q^\alpha] [\tilde{h}^\beta (1 - \gamma_5) q^\beta] 
  
  \hat{O}_3 = -\tilde{O}_2 - (1/2) \tilde{O}_1 
  
  \hat{O}_4 = [\tilde{h}^\alpha (1 - \gamma_5) q^\alpha] [\tilde{h}^\beta (1 + \gamma_5) q^\beta] 
  
  \hat{O}_5 = [\tilde{h}^\alpha (1 - \gamma_5) q^\beta] [\tilde{h}^\beta (1 + \gamma_5) q^\alpha]
  
  \]

  - Matching between QCD and HQET operators:
    
    \[
    [W_{QCD}^T(\mu_h, \mu)]^{-1} \langle \hat{O}(\mu) \rangle_{\mu_h} = C(\mu_h) [W_{HQET}^T(\mu_h, \tilde{\mu})]^{-1} \langle \tilde{O}(\tilde{\mu}) \rangle + O(1/\mu_h) + \ldots
    
    [W_{\ldots}^T(\mu_1, \mu_2)]^{-1}: \text{evolution 5x5 matrices}
    
    C(\mu_h): \text{matching matrix}
    
    [Becirevic et al. JHEP 2002]
Calculation of bag parameters $B_i^{(d/s)}$

- $B$-bag parameters encode the non-perturbative QCD contribution to the $B^0_d/s - \bar{B}^0_d/s$ mixing: (in SM:) $\Delta M_q \propto |V_{tq}^* V_{tb}|^2 M_{Bq} f^2_{Bq} \hat{B}_1^{(q)}$.
  $\Delta M_s/\Delta M_d$ constrains the UT apex.
- ETMC Calculation: use mixed action; Osterwalder-Seiler valence quarks; suitable combinations of maximally twisted valence quarks ensure both
  $\rightarrow$ continuum-like renormalisation pattern for the 4-fermion operators
  $\rightarrow$ automatic $O(a)$-improvement. [Frezzotti and Rossi, JHEP 2004]
  $\rightarrow$ RCs computed with RI/MOM techniques
    (for $K$-sector: $N_f = 2$ ETMC, JHEP 2012, 1207.1287);
    $N_f = 2 + 1 + 1$ see talk by N. Carrasco, Wednesday);

- $R_{B_1^{(q)}} = \frac{C_{PO_1} P(x_0)}{8/3 C_{PA}(x_0) C_{AP}(x_0)} \rightarrow B_1^{(q)}$
- similar plateau quality for $R_{B_2^{(q)},...,5}$

![Graph showing $R_{B_1}$](image.png)
Ratio method for the $\Delta B = 2$ operators

- set:
\[
(W_{QCD}^T(\mu_h, \mu)C(\mu_h)[W_{HQET}^T(\mu_h, \tilde{\mu})]^{-1})^{-1}<\hat{O}(\mu)>_{\mu_h} \\
[C_B(\mu_h, \mu, \tilde{\mu})]^{-1}<\tilde{O}(\mu)>_{\mu_h} = <\tilde{O}(\tilde{\mu})> + O(1/\mu_h) + \ldots
\]

- $[W_{\ldots}(\mu_1, \mu_2)]^{-1}$ and $C(\mu_h)$ are $(3 \times 3 \oplus 2 \times 2)$ block-diagonal matrices

- up to LO $O_1$ and $\hat{O}_1$ renormalise multiplicatively.

- construct ratios at near-by heavy quark masses and form the suitable chain equation. Determine $B_i^{(d/s)}$, $i = 1, \ldots 5$ for the complete 4-f operator basis.

- work in the same way for computing $B_1^{(s)}/B_1^{(d)}$ and $\xi = \frac{f_{Bs}}{f_{Bd}} \sqrt{\frac{B_1^{(s)}}{B_1^{(d)}}}$
fitting ratios against $1/\mu_h$

$B_1^{(s)}$

$B_1^{(s)}/B_1^{(d)}$

$\xi$

$B_2^{(s)}$
## Results

<table>
<thead>
<tr>
<th></th>
<th>$B_1^{(d)}$</th>
<th>$B_2^{(d)}$</th>
<th>$B_3^{(d)}$</th>
<th>$B_4^{(d)}$</th>
<th>$B_5^{(d)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1^{(s)}$</td>
<td>0.85(4)</td>
<td>0.72(3)</td>
<td>0.88(13)</td>
<td>0.95(5)</td>
<td>1.47(12)</td>
</tr>
<tr>
<td>$B_2^{(s)}$</td>
<td>0.86(3)</td>
<td>0.73(3)</td>
<td>0.89(12)</td>
<td>0.93(4)</td>
<td>1.57(11)</td>
</tr>
</tbody>
</table>

$$B_1^{(s)} / B_1^{(d)} = 1.01(2)$$

<table>
<thead>
<tr>
<th></th>
<th>$B_1^{(s)}$</th>
<th>$B_2^{(s)}$</th>
<th>$B_3^{(s)}$</th>
<th>$B_4^{(s)}$</th>
<th>$B_5^{(s)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{Bd} \sqrt{B_i^{(d)}}$</td>
<td>174(8)</td>
<td>160(8)</td>
<td>177(17)</td>
<td>185(9)</td>
<td>229(14)</td>
</tr>
<tr>
<td>$f_{Bs} \sqrt{B_i^{(s)}}$</td>
<td>211(8)</td>
<td>195(7)</td>
<td>215(17)</td>
<td>220(9)</td>
<td>285(14)</td>
</tr>
</tbody>
</table>

$$\xi = 1.225(31)$$

$$f_{Bd} \sqrt{\hat{B}_1^{(d)}} = 216(10) \text{ MeV}$$

$$f_{Bs} \sqrt{\hat{B}_1^{(s)}} = 262(10) \text{ MeV}$$
Comparison plots

$f_{Bd} \sqrt{\hat{B}_1^{(d)}}$

$N_f = 2 + 1$
- HPQCD '09
- FNAL-MILC '11

$N_f = 2$
- ETMC '13

$N_f = 2 + 1$
- HPQCD '09
- FNAL-MILC '11

$N_f = 2$
- ETMC '13

$f_{Bs} \sqrt{\hat{B}_1^{(s)}}$

$N_f = 2 + 1$
- HPQCD '09
- FNAL-MILC '11

$N_f = 2$
- ETMC '13

$\xi$

$N_f = 2 + 1$
- HPQCD '09
- FNAL-MILC '12

$N_f = 2$
- ETMC '13

$\xi$
Model independent constraints on $\Delta B = 2$ and NP scale - UT analysis

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i \mathcal{O}_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{\mathcal{O}}_i$$

$$\mathcal{O}_1 = [\bar{b}^\alpha \gamma_\mu (1 - \gamma_5) q^{\alpha}] [\bar{b}^\beta \gamma_\mu (1 - \gamma_5) q^{\beta}]$$
$$\mathcal{O}_2 = [\bar{b}^\alpha (1 - \gamma_5) q^{\alpha}] [\bar{b}^\beta (1 - \gamma_5) q^{\beta}]$$
$$\mathcal{O}_3 = [\bar{b}^\alpha (1 - \gamma_5) q^{\beta}] [\bar{b}^\beta (1 - \gamma_5) q^{\alpha}]$$
$$\mathcal{O}_4 = [\bar{b}^\alpha (1 - \gamma_5) q^{\alpha}] [\bar{b}^\beta (1 + \gamma_5) q^{\beta}]$$
$$\mathcal{O}_5 = [\bar{b}^\alpha (1 - \gamma_5) q^{\beta}] [\bar{b}^\beta (1 + \gamma_5) q^{\alpha}]$$

- Brown bars: this work; Yellow bars: UTfit, M. Bona et al. JHEP 2008 0707.0636
- More stringent constraints on NP scale from $K - \bar{K}^0$ [ETMC, JHEP 2013, 1207.1287]
Summary

• Ratio method uses relativistic quarks and an *obvious* value of the static limit. No static calculation needed.

• Ratio method can be used for all observables whose static limit behaviour is known from HQET.

• **ETMC** ($N_f = 2$) results for $m_b, f_{B_s}, f_B, f_{B_s}/f_B, B_{B_s}, B_{B_d}, B_{B_s}/B_{B_d}$ and $\xi$ are in the same ballpark w.r.t results from other collaborations.

• First ETMC results using $N_f = 2 + 1 + 1$ dynamical quarks and the ratio method (see E. Picca’s talk, Friday).
Extra slides
\begin{itemize}
  \item (HM)ChPT fit ansatz for \((f_{hs}/f_{h\ell})(f_{s\ell}/f_{\ell\ell})\):

  \[
  \mathcal{R}_f = a_h^{(2)} \left[ 1 + b_h^{(2)} \mu_{\ell} \right] + \left[ \frac{3(1 + 3\hat{g}^2)}{4} - \frac{5}{4} \right] \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \log \left( \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \right) + D_h^{(2)} a^2
  \]

  \item (HM)ChPT fit ansatz for \(B_i\)

  \[
  B_i^{(d)} = B_i^\chi \left[ 1 + b_i\mu_{\ell} - \frac{(1 - 3\hat{g}^2)}{2} \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \log \left( \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \right) \right] + D_i a^2, \quad i = 2, 4, 5
  \]

  \[
  B_i^{(d)} = B_i^\chi \left[ 1 + b_i\mu_{\ell} + \frac{(1 + 3\hat{g}^2 Y)}{2} \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \log \left( \frac{2B_0\mu_{\ell}}{(4\pi f_0)^2} \right) \right] + D_i a^2, \quad i = 2, 4, 5
  \]

\end{itemize}
$f_{Bs}$ (ratio method analysis using $\Phi_{f_{Bs}}$)

\begin{align*}
\beta &= 4.20 \\
\beta &= 4.05 \\
\beta &= 3.90 \\
\beta &= 3.80
\end{align*}

\begin{align*}
\mu_\ell \text{ (GeV)} \\
f_{h_3}(\mu(1)_{h}) \text{ (GeV)}
\end{align*}

\begin{align*}
\beta &= 3.80 \\
\beta &= 3.90 \\
\beta &= 4.05 \\
\beta &= 4.20
\end{align*}

\begin{align*}
\bar{\mu} - 1 \\
\mu_h^{-1} \text{ (GeV)}
\end{align*}
b-quark mass ratios - phenom. indications

[see ETMC, JHEP (2012) 046 (1107.1441)]

\[ y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2} \]

- consider (HQET) \( M_{h\ell} = \mu_h^{\text{pole}} + \bar{\Lambda} - \frac{(\lambda_1+3\lambda_2)}{2} \frac{1}{\mu_h^{\text{pole}}} + \mathcal{O}\left(\frac{1}{(\mu_h^{\text{pole}})^2}\right) \)

- and get \( y = 1 - \bar{\Lambda} \lambda^{\text{pole}} \frac{1}{\mu_h^{\text{pole}}} + \left(\frac{(\lambda_1+3\lambda_2)}{2}(\lambda^{\text{pole}} + 1) + \bar{\Lambda}^2 \lambda^{\text{pole}} \right) \frac{\lambda^{\text{pole}} - 1}{(\mu_h^{\text{pole}})^2} \)
  with \( \lambda^{\text{pole}} = \frac{\mu_h^{\text{pole}} (\bar{\mu}_h)}{\mu_h^{\text{pole}} (\bar{\mu}_h/\lambda)} = \lambda \rho(\bar{\mu}_h)/\rho(\bar{\mu}_h/\lambda) \)

- use phenomenological estimates for HQET parameters, as e.g.
  \( \bar{\Lambda} = 0.39(11) \text{ GeV}, \quad \lambda_1 = -0.19(10) \text{ GeV}^2, \quad \lambda_2 = 0.12(2) \text{ GeV}^2 \)

Ratios for $f_{Bs}$ - Phenomenological Indications

[see ETMC, JHEP (2012) 046 (1107.1441)]

\[ z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2} \]

(for $1/\bar{\mu}_h > 0.60$ estimated uncertainty on the black curve $\sim 0.03$)

- consider (HQET)
  \[ \Phi_{hs}(\bar{\mu}_h, \mu_b^*) = \left( \frac{f_{hs}\sqrt{M_{hs}}}{C_A^{stat}(\bar{\mu}_h, \mu_b^*)} \right)_{QCD} = \frac{\Phi_0(\mu_b^*)}{\mu_{pole}} \left( 1 + \frac{\Phi_1(\mu_b^*)}{\mu_{pole}} + \frac{\Phi_2(\mu_b^*)}{(\mu_{pole})^2} \right) + \mathcal{O}\left( \frac{1}{(\mu_{pole})^3} \right) \]

- and get
  \[ y_s^{1/2} z_s = \frac{\Phi_{hs}(\bar{\mu}_h)}{\Phi_{hs}(\bar{\mu}_h/\lambda)} = 1 - \Phi_1 \frac{\lambda_{pole}}{\mu_{pole}} - \left( \Phi_2(\lambda^2_{pole} + 1) - \Phi_1^2 \lambda_{pole} \right) \frac{\lambda_{pole}}{(\mu_{pole})^2} \]

- use phenomenological values for HQET parameters
  \( \bar{\Lambda}_s = \bar{\Lambda} + M_{Bs} - M_B, \quad \lambda_{1s} = \lambda_1, \quad \lambda_{2s} = \lambda_2, \quad \Phi_0 = 0.60 \text{ GeV}^{3/2} \) and the estimates
  \( \Phi_1 = -0.48 \text{ GeV}, \quad \Phi_2 = 0.08 \text{ GeV}^2 \) (\( \rightarrow \) values obtained from inputs at $B_s$ and $D_s$. )
$B_i^{(d/s)}$ – Error budget

<table>
<thead>
<tr>
<th>source of uncertainty (in %)</th>
<th>$B_1^{(d)}$</th>
<th>$B_2^{(d)}$</th>
<th>$B_3^{(d)}$</th>
<th>$B_4^{(d)}$</th>
<th>$B_5^{(d)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stat + fit + RCs</td>
<td>3.8</td>
<td>4.0</td>
<td>14</td>
<td>4.6</td>
<td>5.1</td>
</tr>
<tr>
<td>syst. of fits</td>
<td>1.6</td>
<td>1.8</td>
<td>7.3</td>
<td>2.7</td>
<td>4.4</td>
</tr>
<tr>
<td>syst. from discr. effects</td>
<td>1.7</td>
<td>0.2</td>
<td>0.7</td>
<td>2.0</td>
<td>3.8</td>
</tr>
<tr>
<td>syst. due to $\Lambda_{QCD}$ and $N_f$</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Total</td>
<td>4.5</td>
<td>4.4</td>
<td>16</td>
<td>5.7</td>
<td>7.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>source of uncertainty (in %)</th>
<th>$B_1^{(s)}$</th>
<th>$B_2^{(s)}$</th>
<th>$B_3^{(s)}$</th>
<th>$B_4^{(s)}$</th>
<th>$B_5^{(s)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>stat + fit + RCs</td>
<td>3.1</td>
<td>3.6</td>
<td>11.1</td>
<td>4.3</td>
<td>4.2</td>
</tr>
<tr>
<td>syst. of fits</td>
<td>1.3</td>
<td>1.7</td>
<td>6.7</td>
<td>0.2</td>
<td>5.0</td>
</tr>
<tr>
<td>syst. from discr. effects</td>
<td>0.5</td>
<td>0.5</td>
<td>2.9</td>
<td>1.2</td>
<td>0.6</td>
</tr>
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<td>0.2</td>
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<td>0.2</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>Total</td>
<td>3.4</td>
<td>4.0</td>
<td>13.3</td>
<td>4.5</td>
<td>6.6</td>
</tr>
</tbody>
</table>
Optimal interpolating field for $f_{ps}$ and $B_i$