B-physics computations from $N_f = 2$ tmQCD

Petros Dimopoulos

Centro Fermi
& University of Rome "Tor Vergata"

July 29 - August 3, 2013
"31st International Symposium on Lattice Field Theory"
Mainz, Germany

In collaboration with:
N. Carrasco, M. Ciuchini, R. Frezzotti, V. Gimenez, G. Herdoiza,
V. Lubicz, G. Martinelli, C. Michael, E. Picca, G.C. Rossi,
F. Sanfilippo, A. Shindler, L. Silvestrini, S. Simula, C. Tarantino

Outline

- B-physics computations with $N_f = 2$ tmQCD
  - Method based on Ratios of heavy-light $(h, \ell/s)$ observables using relativistic quarks and exact knowledge of static limit for the appropriate ratios
  - Interpolation of $(h, \ell/s)$ observables to the b-region from the charm region and the static limit
  - Ratio method helps for suppressing systematics of the heavy sector
  - Computation of b-quark mass, decay constants, B-Bag parameters for the *complete* 4-f operator basis, ξ

    ⋆ Error budget determined mainly from scale setting and RCs uncertainties

ETMC:
JHEP **1004** (2010) 049 (0909.3187)
JHEP **1201** (2012) 046 (1107.1441)
PoS ICHEP2012 (2012) 428 (1212.0301);
PoS LATTICE2012 (2012) 104 (1211.0568);
PoS LATTICE2012 (2012) 105 (1211.0565)
N. Carrasco *et al.* (in preparation) (2013)

ETMC – $N_f = 2$ twisted-mass formulation

- Mtm lattice regularization for $N_f = 2$ QCD action

$$S^{\rm ph}_{N_f=2} = S^{\rm YM}_L + a^4 \sum_x \bar{\psi}(x) \left[\gamma \cdot \widetilde{\nabla} - i\gamma_5 \tau^3 \left(-\frac{a}{2} r \nabla^* \nabla + M_{\rm cr}(r)\right) + \mu_q\right] \psi(x)$$

[Frezzotti, Grassi, Sint, Weisz, JHEP 2001; Frezzotti, Rossi, JHEP 2004]

- ψ is a flavour doublet, $M_{\rm cr}(r)$ is the critical mass and τ^3 acts on flavour indices
  - Automatic O(a) improvement for the physical quantities
  - Dirac-Wilson matrix determinant is positive

    and (lowest eigenvalue)^2 bounded from below by $\mu_q{}^2$
  - Simplified (operator) renormalization ...
    - Multiplicative quark mass renormalization
    - No RC for pseudoscalar decay constant (PCAC)

- O(a^2) breaking of parity and isospin

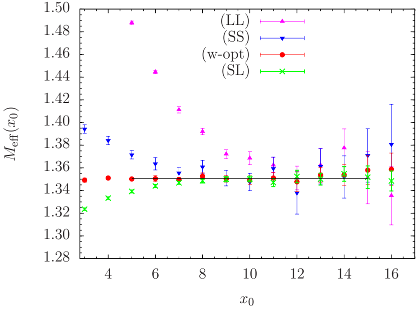
ETMC, Phys.Lett.B 2007
ETMC, Comput.Phys.Commun. 179 (2008)
ETMC, JHEP 1008 (2010) 097

ETMC $N_f = 2$ simulations

β	$(L/a,\ T/a)$	$a\mu_\ell$	$a\mu_s$ (valence)	$a\mu_h$ (valence)
3.80	(24, 48)	0.0080, 0.0110	0.0175, 0.0194 0.0213	0.1982, 0.2331, 0.2742, 0.3225, 0.3793, 0.4461, 0.5246, 0.6170, 0.7257, 0.8536
3.90	(32, 64) (24, 48)	0.0030, 0.0040 0.0040, 0.0064 0.0085, 0.0100	0.0159, 0.0177 0.0195	0.1828, 0.2150, 0.2529, 0.2974, 0.3498, 0.4114, 0.4839, 0.5691, 0.6694, 0.7873
4.05	(32, 64)	0.0030, 0.0080	0.0139, 0.0154 0.0169	0.1572, 0.1849, 0.2175, 0.2558, 0.3008, 0.3538, 0.4162, 0.4895, 0.5757, 0.6771
4.20	(48, 96) (32, 64)	0.0020 0.0065	0.0116, 0.0129 0.0142	0.13315, 0.1566, 0.1842, 0.2166, 0.2548, 0.2997, 0.3525, 0.4145, 0.4876, 0.5734

- $a\ =\ \{0.098,\ 0.085,\ 0.067,\ 0.054\}$ fm
- $m_{ps}^{\ell\ell}\ \in\ \{270,\ 600\}$ MeV
- $L\ \in\ \{1.7,\ \ 2.8\}$ fm , $m_{ps}L \geq 3.5$
- $\mu_\ell \in [\sim m_s/6, \sim m_s/2]$
- $\mu_h \in [\sim m_c, \sim 3m_c]$

$M_{\rm eff}^{(h\ell)}$ plateau quality - optimal interpolating fields



$(\beta = 3.80;\ (L/a, T/a) = (24, 48);\ m_h \sim 2m_c)$

$\Phi^{\rm S} \propto (1 + \kappa_{\rm G} a^2 \nabla^2_{\rm APE})^{N_G} \Phi^{\rm L}$
$\alpha_{\rm APE} = 0.5,\ N_{\rm APE} = 20,\ \kappa_{\rm G} = 4,\ N_{\rm G} = 30$

Improved projection

- Smearing techniques improve signal; reduce the coupling between the ground and excited states; safe good plateaux at earlier times; necessary also for obtaining safe plateau in the calculation of 3-point correlation functions (when large heavy quark mass [$>$ 1 GeV] are employed).
- Employ "optimal" interpolating field: $\Phi_w \propto w\Phi^{\rm S} + (1-w)\Phi^{\rm L}$; w is tunable parameter

Ratio method

- use correlators with *relativistic* quarks
- c-mass region computations are reliable ('small' discr. errors)
- construct HQET-inspired ratios of the observable of interest at consecutive (nearby) values of the heavy quark mass ($\mu_h^{(n)} = \lambda \mu_h^{(n-1)}$)
- ratios show smooth chiral and continuum limit behaviour
- ratios at the ∞-mass (static) point are exactly *known* ($= 1$)
- physical value of the observable at the b-mass point is related to its c-like value by a *chain* of the ratios ending up at the static point: use HQET-inspired interpolation

b-quark mass computation - 1

- observing that $\lim_{\mu_h^{\rm pole}\to\infty}\left(\frac{M_{h\ell}}{\mu_h^{\rm pole}}\right) = \text{constant}$ (HQET)
- construct (taking $\frac{\overline{\mu}_h^{(n)}}{\overline{\mu}_h^{(n-1)}} = \lambda$):

$$
\begin{aligned}
y(\overline{\mu}_h^{(n)},\lambda;\overline{\mu}_\ell,a) &\equiv \frac{M_{h\ell}(\overline{\mu}_h^{(n)};\overline{\mu}_\ell,a)}{M_{h\ell}(\overline{\mu}_h^{(n-1)};\overline{\mu}_\ell,a)}\cdot\frac{\overline{\mu}_h^{(n-1)}}{\overline{\mu}_h^{(n)}}\cdot\frac{\rho(\overline{\mu}_h^{(n-1)},\mu^*)}{\rho(\overline{\mu}_h^{(n)},\mu^*)} = \\
&= \lambda^{-1}\frac{M_{h\ell}(\overline{\mu}_h^{(n)};\overline{\mu}_\ell,a)}{M_{h\ell}(\overline{\mu}_h^{(n)}/\lambda;\overline{\mu}_\ell,a)}\cdot\frac{\rho(\overline{\mu}_h^{(n)}/\lambda,\mu^*)}{\rho(\overline{\mu}_h^{(n)},\mu^*)}\,, \qquad n=2,\dots,N
\end{aligned}
$$

$\mu_h^{\rm pole} = \rho(\overline{\mu}_h,\mu^*)\,\overline{\mu}_h(\mu^*)$ (with $\overline{\mu}_h \leftarrow \overline{\rm MS}$ scheme)

$\rho(\overline{\mu}_h,\mu^*)$ known in PT up to N^3LO – relevant only for the '$1/\overline{\mu}_h$' interpolation

→ In the static limit (and in CL) obviously:

$$\lim_{\overline{\mu}_h\to\infty} y(\overline{\mu}_h,\lambda;\overline{\mu}_\ell,a=0) = 1$$

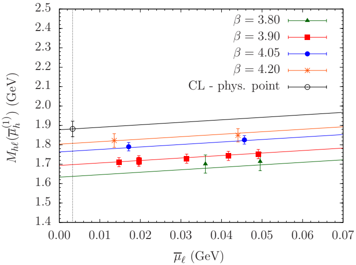
b-quark mass computation - 2

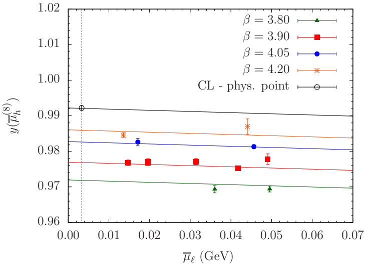
- Aim: construct the chain equation

$$y(\overline{\mu}_h^{(2)})\, y(\overline{\mu}_h^{(3)}) \,\ldots\, y(\overline{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\overline{\mu}_h^{(K+1)})}{M_{hu/d}(\overline{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\overline{\mu}_h^{(1)}, \mu^*)}{\rho(\overline{\mu}_h^{(K+1)}, \mu^*)} \right]$$

- Need: extrapolation in the continuum limit - physical point to obtain
  - *pseudoscalar mass* $M_{hu/d}(\overline{\mu}_h^{(1)})$ ($\leftarrow$ in the *c*-quark region)
  - *ratios* $y(\overline{\mu}_h^{(n)})$ $(n = 2, \ldots, N)$

- smooth fits – discr. effects under control



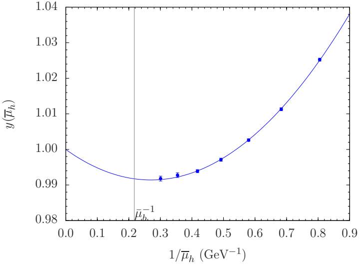


b-quark mass computation - 3

- use the chain equation

$$y(\overline{\mu}_h^{(2)})\, y(\overline{\mu}_h^{(3)}) \,\ldots\, y(\overline{\mu}_h^{(K+1)}) = \lambda^{-K}\, \frac{M_{hu/d}(\overline{\mu}_h^{(K+1)})}{M_{hu/d}(\overline{\mu}_h^{(1)})} \cdot \left[\frac{\rho(\overline{\mu}_h^{(1)}, \mu^*)}{\rho(\overline{\mu}_h^{(K+1)}, \mu^*)} \right]$$

- Evaluate the (*lhs*) with $y(\overline{\mu}_h^{(j)})$ set to the best fit data.
- Adjust $(\lambda, \overline{\mu}_h^{(1)})$ such that K integer and $M_{hu/d}(\overline{\mu}_h^{(K+1)}) \equiv M_B^{expt}$.
  our calculation: $\lambda = 1.1784$ and $\overline{\mu}_h^{(1)} = 1.05$ GeV (in $\overline{\mathrm{MS}}$, 3 GeV)
  $\rightarrow \overline{\mu}_b = \lambda^K \overline{\mu}_h^{(1)}$ $(K = 9)$
- *strong* cancellations of perturbative factors ρ's in the ratios:
  $\rightarrow$ subpercent dependence on ρ's PT-order

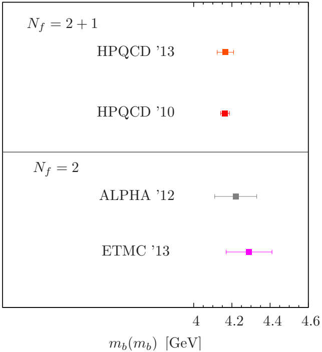


- fit ansatz $y(\overline{\mu}_h) = 1 + \frac{\eta_1}{\overline{\mu}_h} + \frac{\eta_2}{\overline{\mu}_h^2}$ (inspired by HQET)
- curvature denotes a large $1/\overline{\mu}_h^2$ contribution to ratios y

b-quark mass - results

- $m_b(m_b, \overline{\mathrm{MS}}) = 4.29(12)$ GeV

- compatible result for m_b if (hs)-data and M_{Bs}^{expt} as input are used



- Main source of uncertainty of the ETMC result is due to quark mass RC and scale setting uncertainties;

- stats + fit (CL + chiral) $\sim$ 2.1% ;
  latt. scale syst $\sim$ 2.0% ;
  discr. syst. $\sim$ 0.2 % ;
  syst. $1/\overline{\mu}_h$ fit (TL, LO, NLO) $\sim$ 0.4% ;
  syst. RG-running $\sim$ 0.5% .

(for $N_f = 2+1+1$ by ETMC, see talk by E. Picca, Friday)

General case computation - 1

- For any observable $Q_{h\,\ell/s} \equiv f_{Bs},\ f_B,\ f_{Bs}/f_B,\ B_i^{(d/s)},\ \xi$ consider the HQET scaling quantity $\Phi_Q = Q_{h\,\ell/s} \times (\mu_h^{\rm pole})^{\alpha}$ *or* $\tilde{\Phi}_Q = Q_{h\,\ell/s} \times (M_{h\,\ell/s})^{\alpha}$
- observing that $\lim_{\mu_h^{\rm pole}\to\infty} \left(Q_{h\,\ell/s} \times (\mu_h^{\rm pole})^{\alpha}\right) = \text{constant}$

  $\lim_{M_{h\,\ell/s}\to\infty} \left(Q_{h\,\ell/s} \times (M_{h\,\ell/s})^{\alpha}\right) = \text{constant } (\textit{up to log corrections})$

  $\alpha = 1/2$ for $f_{Bs},\ f_B$; $\alpha = 0$ for $f_{Bs}/f_B,\ B_i^{(d/s)},\ \xi$
- construct ratios at near-by $\overline{\mu}_h$ $(\overline{\mu}_h^{(n)} = \lambda \overline{\mu}_h^{(n-1)})$

  $$z_{\Phi}(\overline{\mu}_h^{(n)}, \lambda; \overline{\mu}_{\ell/s}, a) = \frac{\Phi_Q(\overline{\mu}_h^{(n)}; \overline{\mu}_{\ell/s}, a)}{\Phi_Q(\overline{\mu}_h^{(n-1)}; \overline{\mu}_{\ell/s}, a)}\ \frac{C_Q(\overline{\mu}_h^{(n)})}{C_Q(\overline{\mu}_h^{(n-1)})}$$

  and similar definitions when employing $\tilde{\Phi}_Q$

  ⋆ $C_Q(\overline{\mu}_h^{(n)})$ include anomalous dimension in HQET and possible ρ factors
- $\lim_{\overline{\mu}_h\to\infty} z_{\Phi}(\overline{\mu}_h, \lambda; \overline{\mu}_{\ell/s}, a=0) = 1$

  $\lim_{M_{h\,\ell/s}\to\infty} z_{\tilde{\Phi}}(\overline{\mu}_h, \lambda; \overline{\mu}_{\ell/s}, a=0) = 1$

General case computation - 2

- construct the *chain* equation

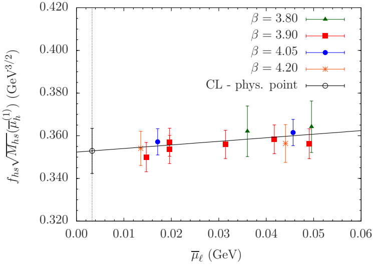
$$z_\Phi(\overline{\mu}_h^{(2)})\, z_\Phi(\overline{\mu}_h^{(3)})\, \ldots\, z_\Phi(\overline{\mu}_h^{(K+1)}) = \lambda^{\alpha K}\, \frac{Q_{h\,d/s}(\overline{\mu}_h^{(K+1)})}{Q_{h\,d/s}(\overline{\mu}_h^{(1)})} \cdot \left[\frac{C_Q(\overline{\mu}_h^{(K+1)})}{C_Q(\overline{\mu}_h^{(1)})}\right] \qquad \textit{or}$$

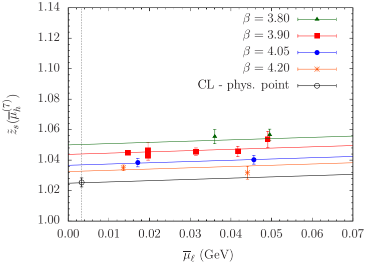
$$z_{\tilde\Phi}(\overline{\mu}_h^{(2)})\, z_{\tilde\Phi}(\overline{\mu}_h^{(3)})\, \ldots\, z_{\tilde\Phi}(\overline{\mu}_h^{(K+1)}) = \frac{Q_{h\,d/s}(\overline{\mu}_h^{(K+1)})}{Q_{h\,d/s}(\overline{\mu}_h^{(1)})} \cdot \left[\frac{\tilde C_Q(\overline{\mu}_h^{(K+1)})}{\tilde C_Q(\overline{\mu}_h^{(1)})}\right]$$

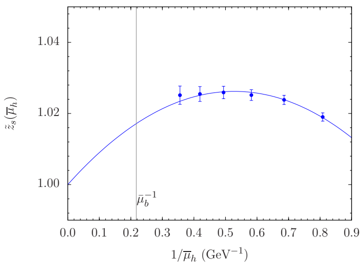
  and, at $\overline{\mu}_b = \lambda^K \overline{\mu}_h^{(1)}$, determine $Q_{b\,d/s} = Q_{h\,d/s}(\overline{\mu}_h^{(K+1)})$.
- (*lhs*) is determined by the best fit data using $z_{..}(\overline{\mu}_h)$-ratios after they have been extrapolated (smoothly) to the CL.
- $Q_{h\,d/s}(\overline{\mu}_h^{(1)})$ determined safely in the c-quark region employing an extrapolation to CL and physical point.

f_{Bs}

$[\ B(B_s \to \ell^+\ell^-) \propto M_{B_s} f_{B_s} |V_{tb}^* V_{tq}|^2)\ ]$



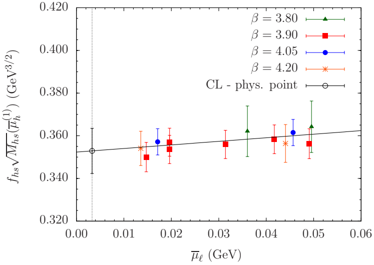


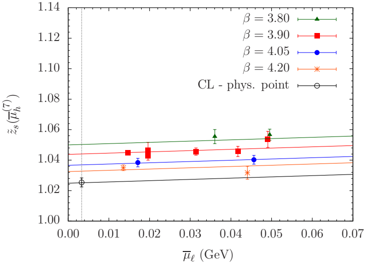


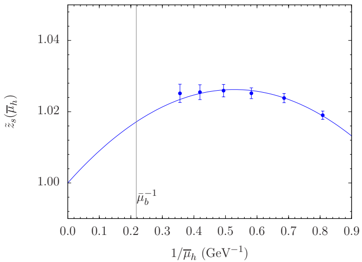
- fit ansatz (inspired by HQET)
  $z_{\tilde{\Phi}}(\overline{\mu}_h) = 1 + \frac{\hat{\eta}_1}{\overline{\mu}_h} + \frac{\hat{\eta}_2}{\overline{\mu}_h^2}$
- *Note:* final results at physical *b*-point show weak dependence ($\lesssim$ 1%) on PT-order (TL, LO, NLO) either employing $\Phi_{f_{Bs}}$ or $\tilde{\Phi}_{f_{Bs}}$.

f_{Bs} (continue)

$[\ B(B_s \to \ell^+\ell^-) \propto M_{B_s} f_{B_s} |V_{tb}^* V_{tq}|^2)\ ]$

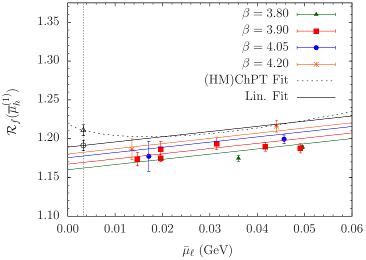


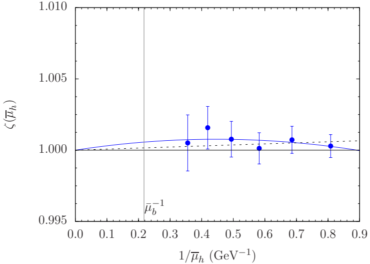




- $f_{Bs} = 228(8)$ MeV
- stats + fit (CL + chiral) $\sim 2.2\%$;
  latt. scale syst $\sim 2.0\%$;
  discr. syst. ~ 1.3 % ;
  syst. '$1/\overline{\mu}_h$' fit $\sim 1.0\%$.

f_{Bs}/f_B

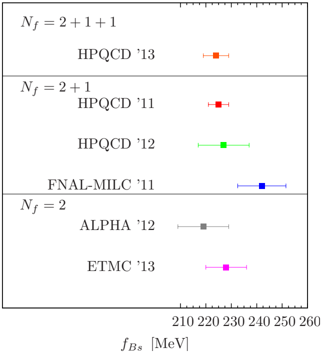


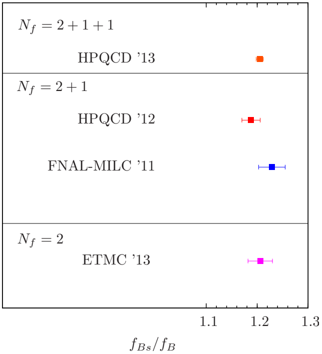


- SU(3) breaking ratio:
  cancellation of systematics;
  sensitive to chiral extrapolation
- Extrapolation in CL + phys. point in the c-quark region. Define:
  $\mathcal{R}_f(\overline{\mu}_h^{(1)}) = [(f_{hs}/f_{h\ell})/(f_{s\ell}/f_{\ell\ell}](f_K/f_\pi)$
  $\longrightarrow$ large cancellation of the chiral logarithmic terms
  [Becirevic *et al.*, Phys.Lett.B 2003]

- $f_{Bs}/f_B = 1.206(24)$
- stats + fit (CL + chiral) $\sim 0.8\%$;
  syst. $\mathcal{R}_f \sim 1.7\%$;
  discr. syst. ~ 0.4 % ;
  syst. '$1/\overline{\mu}_h$' fit $\sim 0.1\%$.

Results & Comparisons - I

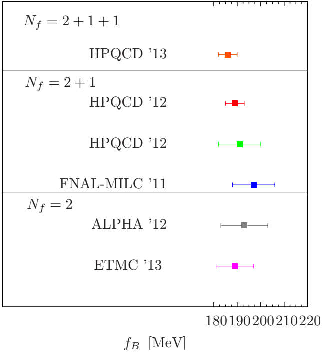




- $f_{Bs}(\mathrm{ETMC} - 2013) = 228(8)$ MeV
- $f_{Bs}/f_B(\mathrm{ETMC} - 2013) = 1.206(24)$

(for $N_f = 2+1+1$ results by ETMC, see talk by E. Picca, Friday)

Results & Comparisons - II



- $f_B(\mathrm{ETMC}-2013) = 189(8)$ MeV
  $(f_B = f_{Bs}/(f_{Bs}/f_B))$

(for $N_f = 2+1+1$ results by ETMC, see talk by E. Picca, Friday)

Ratio method for the $\Delta B = 2$ operators

[Neutral $B_{d/s}$-meson oscillations via loop box diagrams]

- QCD

$$
\begin{aligned}
O_1 &= [\bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
O_2 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1-\gamma_5) q^\beta] \\
O_3 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1-\gamma_5) q^\alpha] \\
O_4 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1+\gamma_5) q^\beta] \\
O_5 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}
$$

$q = d, s$

- HQET

$$
\begin{aligned}
\hat{O}_1 &= [\bar{h}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{h}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
\hat{O}_2 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1-\gamma_5) q^\beta] \\
\hat{O}_3 &= -\tilde{O}_2 - (1/2)\tilde{O}_1 \\
\hat{O}_4 &= [\bar{h}^\alpha (1-\gamma_5) q^\alpha][\bar{h}^\beta (1+\gamma_5) q^\beta] \\
\hat{O}_5 &= [\bar{h}^\alpha (1-\gamma_5) q^\beta][\bar{h}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}
$$

⋆ Matching between QCD and HQET operators:

$$[\mathbf{W}^T_{QCD}(\mu_h, \mu)]^{-1} \langle \vec{O}(\mu) \rangle_{\mu_h} = C(\mu_h)\, [\mathbf{W}^T_{HQET}(\mu_h, \tilde{\mu})]^{-1} \langle \vec{\hat{O}}(\tilde{\mu}) \rangle + \mathcal{O}(1/\mu_h) + \ldots$$

$[\mathbf{W}^T_{\ldots}(\mu_1, \mu_2)]^{-1}$: evolution 5x5 matrices

$C(\mu_h)$: matching matrix

[Becirevic *et al.* JHEP 2002]

Calculation of bag parameters $B_i^{(d/s)}$

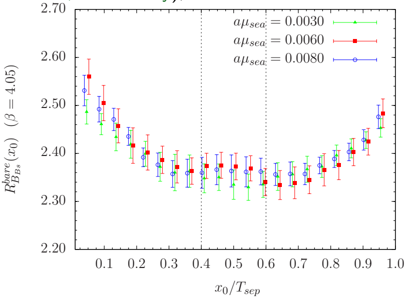
- B-bag parameters encode the non-perturbative QCD contribution to the $B^0_{d/s} - \bar{B}^0_{d/s}$ mixing: (in SM:) $\Delta M_q \propto |V^*_{tq} V_{tb}|^2 M_{B_q} f^2_{B_q} \hat{B}_1^{(q)}$.
  $\Delta M_s / \Delta M_d$ constraints the UT apex.
- ETMC Calculation: use mixed action; Osterwalder-Seiler valence quarks; suitable combinations of maximally twisted valence quarks ensure both

→ continuum-like renormalisation pattern for the 4-fermion operators

→ automatic O(a)-improvement. [Frezzotti and Rossi, JHEP 2004]

→ RCs computed with RI/MOM techniques
(for K-sector: $N_f = 2$ **ETMC**, JHEP 2012, 1207.1287);
$N_f = 2+1+1$ see talk by N. Carrasco, Wednesday);

- $R_{B_1^{(q)}} = \dfrac{C_{PO_1P}(x_0)}{8/3 C_{PA}(x_0) C_{AP}(x_0)} \to B_1^{(q)}$
- similar plateau quality for $R_{B^{(q)}_{2,\ldots,5}}$



Ratio method for the $\Delta B = 2$ operators

- set:

  $(\mathbf{W}^T_{QCD}(\mu_h,\mu) C(\mu_h)[\mathbf{W}^T_{HQET}(\mu_h,\tilde{\mu})]^{-1})^{-1}\langle\vec{O}(\mu)\rangle_{\mu_h} \equiv$

  $[C_B(\mu_h,\mu,\tilde{\mu})]^{-1}\langle\vec{O}(\mu)\rangle_{\mu_h} = \langle\vec{\hat{O}}(\tilde{\mu})\rangle + \mathcal{O}(1/\mu_h) + \ldots$

- $[\mathbf{W}^T_{\ldots}(\mu_1,\mu_2)]^{-1}$ and $C(\mu_h)$ are $(3\times 3 \oplus 2\times 2)$ block-diagonal matrices

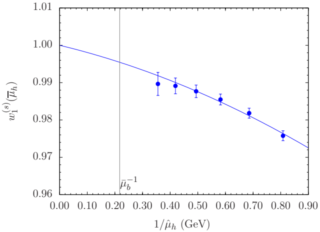
- up to LO O_1 and $\hat{O}_1$ renormalise multipicatively.

- construct ratios at near-by heavy quark masses and form the suitable chain equation. Determine $B_i^{(d/s)}$, $i = 1,\ldots 5$ for the complete 4-f operator basis.

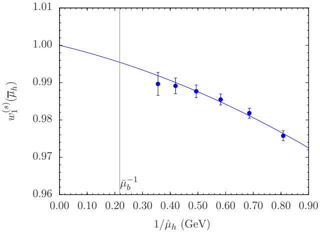
- work in the same way for computing $B_1^{(s)}/B_1^{(d)}$ and $\xi = \frac{f_{Bs}}{f_{Bd}}\sqrt{\frac{B_1^{(s)}}{B_1^{(d)}}}$

fitting ratios against $1/\overline{\mu}_h$

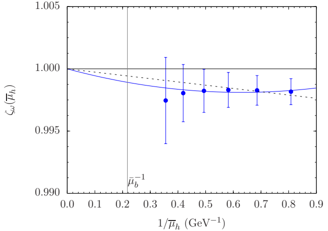
$B_1^{(s)}$



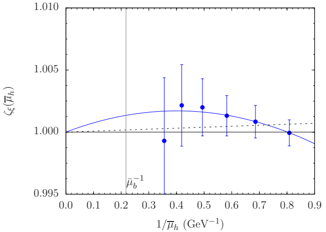
$B_2^{(s)}$



$B_1^{(s)}/B_1^{(d)}$



ξ



Results

$(\overline{\mathrm{MS}}, m_b)$				
$B_1^{(d)}$	$B_2^{(d)}$	$B_3^{(d)}$	$B_4^{(d)}$	$B_5^{(d)}$
0.85(4)	0.72(3)	0.88(13)	0.95(5)	1.47(12)
$B_1^{(s)}$	$B_2^{(s)}$	$B_3^{(s)}$	$B_4^{(s)}$	$B_5^{(s)}$
0.86(3)	0.73(3)	0.89(12)	0.93(4)	1.57(11)

$$B_1^{(s)}/B_1^{(d)} = 1.01(2)$$

$(\overline{\mathrm{MS}}, m_b)$ [MeV]					
i	1	2	3	4	5
$f_{Bd}\sqrt{B_i^{(d)}}$	174(8)	160(8)	177(17)	185(9)	229(14)
$f_{Bs}\sqrt{B_i^{(s)}}$	211(8)	195(7)	215(17)	220(9)	285(14)

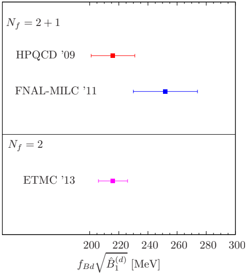
$$\xi = 1.225(31)$$

$$f_{Bd}\sqrt{\hat{B}_1^{(d)}} = 216(10)\ \mathrm{MeV}$$

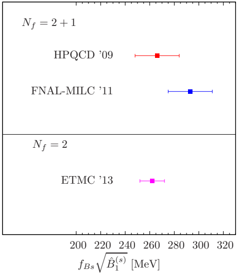
$$f_{Bs}\sqrt{\hat{B}_1^{(s)}} = 262(10)\ \mathrm{MeV}$$

Comparison plots

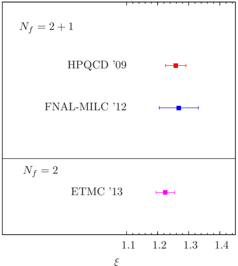
$f_{Bd}\sqrt{\hat{B}_1^{(d)}}$



$f_{Bs}\sqrt{\hat{B}_1^{(s)}}$



ξ



Model independent constraints on $\Delta B = 2$ and NP scale - UT analysis

$$\mathcal{H}_{\rm eff}^{\Delta B=2} = \sum_{i=1}^{5} C_i \mathcal{O}_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{\mathcal{O}}_i$$

$$\begin{aligned}
\mathcal{O}_1 &= [\bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha][\bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta] \\
\mathcal{O}_2 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1-\gamma_5) q^\beta] \\
\mathcal{O}_3 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1-\gamma_5) q^\alpha] \\
\mathcal{O}_4 &= [\bar{b}^\alpha (1-\gamma_5) q^\alpha][\bar{b}^\beta (1+\gamma_5) q^\beta] \\
\mathcal{O}_5 &= [\bar{b}^\alpha (1-\gamma_5) q^\beta][\bar{b}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}$$

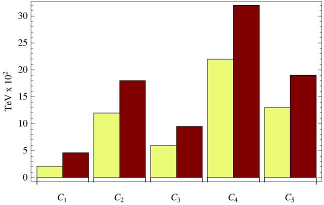
⋆ $C_i(\Lambda)$ extracted from the data (Expt + Theory) – switching on one operator at the time

⋆ $C_i(\Lambda) = \dfrac{LF_i}{\Lambda^2}$, NP generic, $L \sim 1$, $F_i \sim 1$

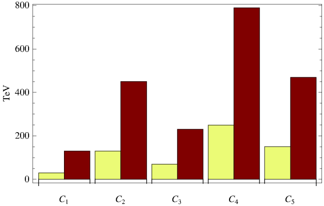
⋆ upper bounds for $C_i(\Lambda)$ translated to lower bounds for NP scale Λ

$$\begin{aligned}
\tilde{\mathcal{O}}_1 &= [\bar{b}^\alpha \gamma_\mu (1+\gamma_5) q^\alpha][\bar{b}^\beta \gamma_\mu (1+\gamma_5) q^\beta] \\
\tilde{\mathcal{O}}_2 &= [\bar{b}^\alpha (1+\gamma_5) q^\alpha][\bar{b}^\beta (1+\gamma_5) q^\beta] \\
\tilde{\mathcal{O}}_3 &= [\bar{b}^\alpha (1+\gamma_5) q^\beta][\bar{b}^\beta (1+\gamma_5) q^\alpha]
\end{aligned}$$

constraints on $B_d^0 - \bar{B}_d^0$



constraints on $B_s^0 - \bar{B}_s^0$



- Brown bars: **this work**; Yellow bars: UTfit , M. Bona *et al.* JHEP 2008 0707.0636
- More stringest constraints on NP scale from $K - \bar{K}^0$ [**ETMC**, JHEP 2013, 1207.1287]

Summary

- Ratio method uses relativistic quarks and an *obvious* value of the static limit. No static calculation needed.
- Ratio method can be used for all observables whose static limit behaviour is known from HQET.
- **ETMC** ($N_f = 2$) results for

  m_b, f_{Bs}, f_B, f_{Bs}/f_B, B_{Bs}, B_{Bd}, B_{Bs}/B_{Bd} and ξ

  are in the same ballpark w.r.t results from other collaborations.
- First ETMC results using $N_f = 2 + 1 + 1$ dynamical quarks and the ratio method (see E. Picca's talk, Friday).

Extra slides

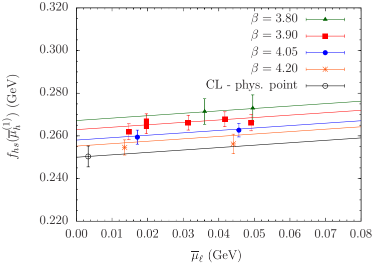
- (HM)ChPT fit ansatz for $(f_{hs}/f_{h\ell})(f_{s\ell}/f_{\ell\ell})$:

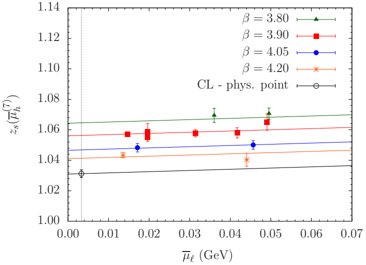
$$\mathcal{R}_f = a_h^{(2)}\left[1 + b_h^{(2)}\overline{\mu}_\ell + \left[\frac{3(1+3\hat{g}^2)}{4} - \frac{5}{4}\right]\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] + D_h^{(2)}a^2$$

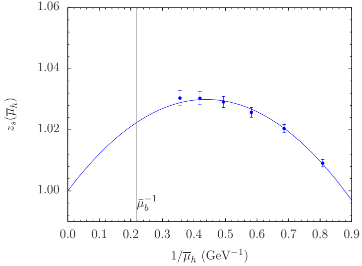
- (HM)ChPT fit ansatz for B_i

$$\begin{aligned}
B_1^{(d)} &= B_1^{\chi}\left[1 + b_1\overline{\mu}_\ell - \frac{(1-3\hat{g}^2)}{2}\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] + D_1 a^2 \\
B_i^{(d)} &= B_i^{\chi}\left[1 + b_i\overline{\mu}_\ell - \frac{(1-3\hat{g}^2\,Y)}{2}\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] + D_i a^2, \quad i=2 \\
B_i^{(d)} &= B_i^{\chi}\left[1 + b_i\overline{\mu}_\ell + \frac{(1+3\hat{g}^2\,Y)}{2}\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\log\left(\frac{2B_0\overline{\mu}_\ell}{(4\pi f_0)^2}\right)\right] + D_i a^2, \quad i=4,5
\end{aligned}$$

f_{Bs} (ratio method analysis using $\Phi_{f_{Bs}}$)

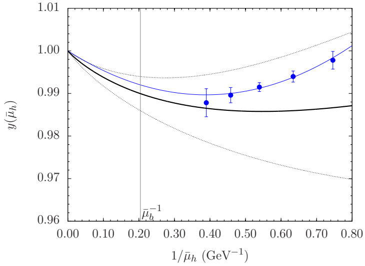






b-quark mass ratios - phenom. indications

[see **ETMC**, JHEP (2012) 046 (1107.1441)]

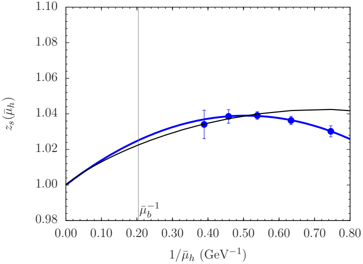


$$y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}$$

- consider (HQET) $M_{h\ell} = \mu_h^{\rm pole} + \bar{\Lambda} - \frac{(\lambda_1+3\lambda_2)}{2}\frac{1}{\mu_h^{\rm pole}} + \mathcal{O}\left(\frac{1}{(\mu_h^{\rm pole})^2}\right)$
- and get $y = 1 - \bar{\Lambda}\frac{\lambda^{\rm pole}-1}{\mu_h^{\rm pole}} + \left(\frac{(\lambda_1+3\lambda_2)}{2}(\lambda^{\rm pole}+1) + \bar{\Lambda}^2\lambda^{\rm pole}\right)\frac{\lambda^{\rm pole}-1}{(\mu_h^{\rm pole})^2}$
  with $\lambda^{\rm pole} = \mu_h^{\rm pole}(\bar{\mu}_h)/\ \mu_h^{\rm pole}(\bar{\mu}_h/\lambda) = \lambda\,\rho(\bar{\mu}_h)/\rho(\bar{\mu}_h/\lambda)$
- use phenomenological estimates for HQET parameters, as e.g.
  $\bar{\Lambda} = 0.39(11)\ \mathrm{GeV}\,, \quad \lambda_1 = -0.19(10)\ \mathrm{GeV}^2\,, \quad \lambda_2 = 0.12(2)\ \mathrm{GeV}^2$

  [M. Gremm, A. Kapustin, Z. Ligeti, M.B. Wise, PhysRevLett 1996]

Ratios for f_{B_s} - Phenomenological Indications



[see **ETMC**, JHEP (2012) 046 (1107.1441)]

$$z_s(\bar{\mu}_h) = 1 + \frac{\zeta_1}{\bar{\mu}_h} + \frac{\zeta_2}{\bar{\mu}_h^2}$$

(for $1/\bar{\mu}_h > 0.60$ estimated uncertainty on the black curve ~ 0.03)

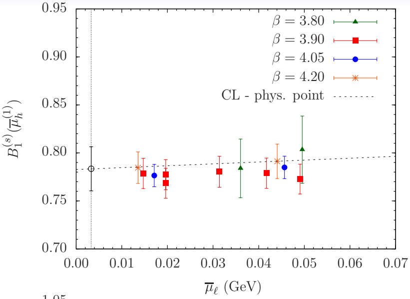
- consider (HQET)

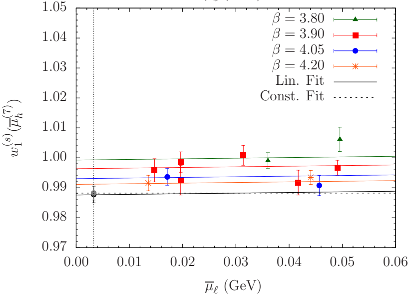
$$\Phi_{hs}(\bar{\mu}_h, \mu_b^*) = \frac{(f_{hs}\sqrt{M_{hs}})^{\mathrm{QCD}}}{C_A^{stat}(\bar{\mu}_h, \mu_b^*)} = \Phi_0(\mu_b^*)\left(1 + \frac{\Phi_1(\mu_b^*)}{\mu_h^{\mathrm{pole}}} + \frac{\Phi_2(\mu_b^*)}{(\mu_h^{\mathrm{pole}})^2}\right) + \mathcal{O}\left(\frac{1}{(\mu_h^{\mathrm{pole}})^3}\right)$$

- and get

$$y_s^{1/2}\, z_s = \frac{\Phi_{hs}(\bar{\mu}_h)}{\Phi_{hs}(\bar{\mu}_h/\lambda)} = 1 - \Phi_1\frac{\lambda^{\mathrm{pole}} - 1}{\mu_h^{\mathrm{pole}}} - \left(\Phi_2(\lambda^{\mathrm{pole}} + 1) - \Phi_1^2\lambda^{\mathrm{pole}}\right)\frac{\lambda^{\mathrm{pole}} - 1}{(\mu_h^{\mathrm{pole}})^2}$$

- use phenomenological values for HQET parameters
$\bar{\Lambda}_s = \bar{\Lambda} + M_{B_s} - M_B$, $\lambda_{1s} = \lambda_1$, $\lambda_{2s} = \lambda_2$, $\Phi_0 = 0.60\ \mathrm{GeV}^{3/2}$ and the estimates
$\Phi_1 = -0.48\ \mathrm{GeV}$, $\Phi_2 = 0.08\ \mathrm{GeV}^2$ ($\rightarrow$ values obtained from inputs at B_s and D_s.)


$\beta = 3.80$
$\beta = 3.90$
$\beta = 4.05$
$\beta = 4.20$
CL - phys. point
$B_1^{(s)}(\overline{\mu}_h^{(1)})$
$\overline{\mu}_\ell$ (GeV)


$\beta = 3.80$
$\beta = 3.90$
$\beta = 4.05$
$\beta = 4.20$
Lin. Fit
Const. Fit
$w_1^{(s)}(\overline{\mu}_h^{(7)})$
$\overline{\mu}_\ell$ (GeV)

$B_i^{(d/s)}$ – Error budget

source of uncertainty (in %)	$B_1^{(d)}$	$B_2^{(d)}$	$B_3^{(d)}$	$B_4^{(d)}$	$B_5^{(d)}$
stat + fit + RCs	3.8	4.0	14	4.6	5.1
syst. of fits	1.6	1.8	7.3	2.7	4.4
syst. from discr. effects	1.7	0.2	0.7	2.0	3.8
syst. due to Λ_{QCD} and N_f	0.2	0.5	0.2	0.1	0.9
Total	4.5	4.4	16	5.7	7.8

source of uncertainty (in %)	$B_1^{(s)}$	$B_2^{(s)}$	$B_3^{(s)}$	$B_4^{(s)}$	$B_5^{(s)}$
stat + fit + RCs	3.1	3.6	11.1	4.3	4.2
syst. of fits	1.3	1.7	6.7	0.2	5.0
syst. from discr. effects	0.5	0.5	2.9	1.2	0.6
syst. due to Λ_{QCD} and N_f	0.2	0.5	0.2	0.1	0.9
Total	3.4	4.0	13.3	4.5	6.6

Optimal interpolating field for f_{ps} and B_i