JULY 29 – AUGUST 03 2013
MAINZ, GERMANY

Gutenberg University, Mainz, Germany
Monday, July 29 —Saturday, Aug 3

The form factor for $B \to \pi$ semileptonic decay from 2+1 flavors of domain-wall fermions

Taichi Kawanai (RBC-UKQCD collaboration)

Why we compute the $B \to \pi$ form factor on the Lattice

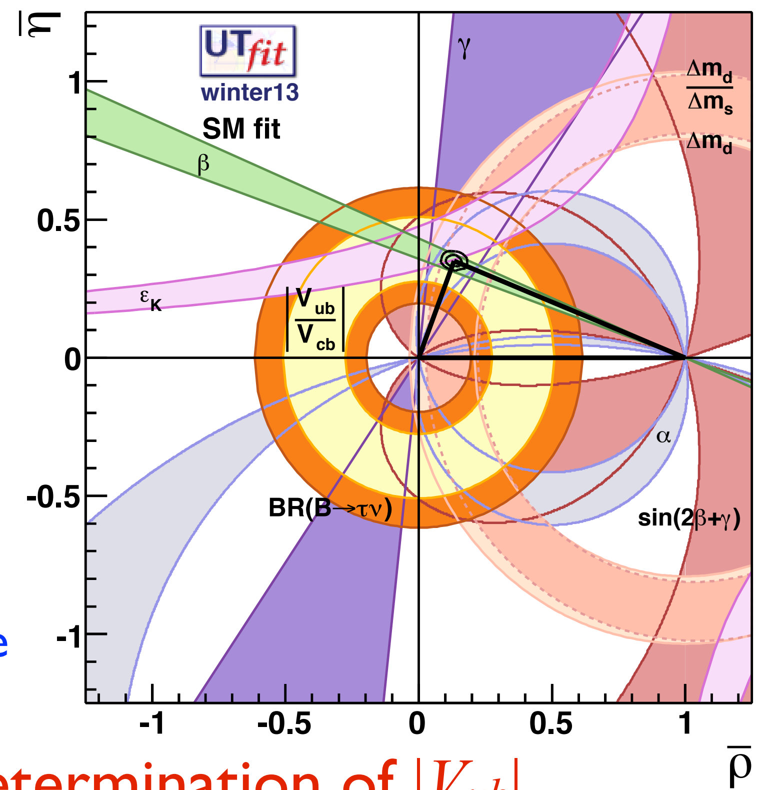
A precise determination of V_{ub} allows a strong test of the standard model

The constraint on the apex $(\bar{\rho},\ \bar{\eta})$ of the CKM triangle from $|V_{ub}|$ will strengthen tests of the Standard-Model CKM framework.

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{1-\lambda^2/2}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

- $\lambda = |V_{ud}|$ known to ~ 1 %
- $|V_{cb}|$ known to ~2 %

Dominant error (orange ring) comes from the uncertainty of $|V_{ub}|$. ~7% inclusive / ~10% exclusive



There has been a long standing puzzle in the determination of $|V_{ub}|$

- $\sim 3\sigma$ discrepancy between exclusive ($B\to\pi l\nu$) and inclusive ($B\to X_u l\nu$) determination.

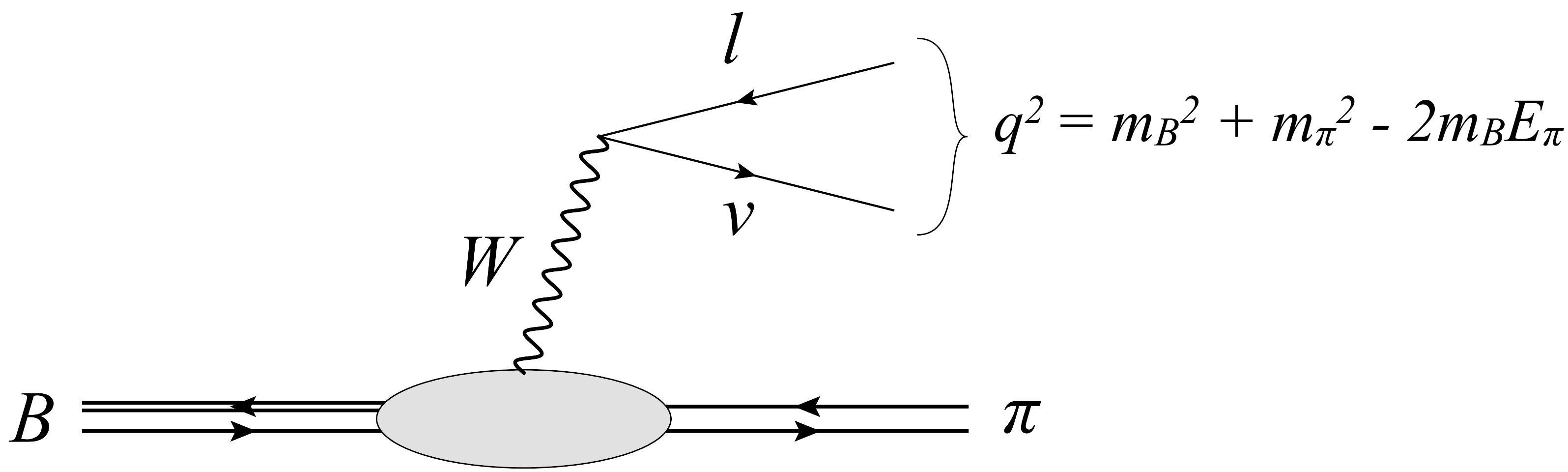
J. Laiho, E. Lunghi, and R. S. Van de Water, Phys. Rev. D81, 034503 (2010)

- BR($B\to\tau\nu$) leads to larger $|V_{ub}|$ which disagrees with an average of $|V_{ub}|_{\mathrm{excl}}$ and $|V_{ub}|_{\mathrm{incl}}$ by more than 2σ. (although the most recent Belle hadronic tagging measurement is 2σ lower than the previous experimental average and in better agreement with other determinations of $|V_{ub}|$)

E. Lunghi and A. Soni, Phys.Lett. B697, 323 (2011)

Why we compute the $B \to \pi$ form factor on the Lattice

$f_+(q^2)$ is crucial for the determination of the CKM matrix element $|V_{ub}|$



•The exclusive $B \to \pi l \nu$ semileptonic decay allows the determination of |Vub| via:

Goal

$$\boxed{\frac{d\Gamma}{dq^2}} = \frac{G_F^2}{192\pi^3 m_B^3}\left[(m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2\right]^{3/2} \times |\mathbf{f_+(q^2)}|^2 \times |\mathbf{V_{ub}}|^2$$

Experiment Known factor Hadronic part CKM matrix

•Experiment can only measure the CKM matrix element times hadronic form factor.

•The hadronic form factor must be computed nonperturbatively via lattice QCD.

How to calculate $f_{+/0}(q^2)$ from Lattice QCD

- Non-perturbative form factors $\boldsymbol{f_+(q^2)}$ and $\boldsymbol{f_0(q^2)}$ parametrize the hadronic matrix element of the $b \to u$ quark flavor-changing vector current V_μ .

$$\boxed{\langle\pi|V_\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu}$$

- On the lattice, we calculate the form factors $f_{||}$ and $f_\perp$.
  - ▸ Proportional to vector current matrix elements in the B-meson rest frame:

$$\begin{aligned} f_{||}(E_\pi) &= \langle\pi|V_0|B\rangle/\sqrt{2m_B} \\ f_\perp(E_\pi)p_i &= \langle\pi|V_i|B\rangle/\sqrt{2m_B} \end{aligned}$$

  - ▸ Easy to relate to the desired form factor $\boldsymbol{f_+(q^2)}$ and $\boldsymbol{f_0(q^2)}$.

$$\begin{aligned} f_0(q^2) &= \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2}\left[(m_B - E_\pi)f_{||}(E_\pi) + (E_\pi^2 - m_\pi^2)f_\perp(E_\pi)\right] \\ f_+(q^2) &= \frac{1}{\sqrt{2m_B}}\left[f_{||}(E_\pi) + (m_B - E_\pi)f_\perp(E_\pi)\right] \end{aligned}$$

Lattice action and parameters

- We use the 2+1 flavor dynamical **domain-wall** fermion gauge field configurations generated by the RBC/UKQCD Collaborations.

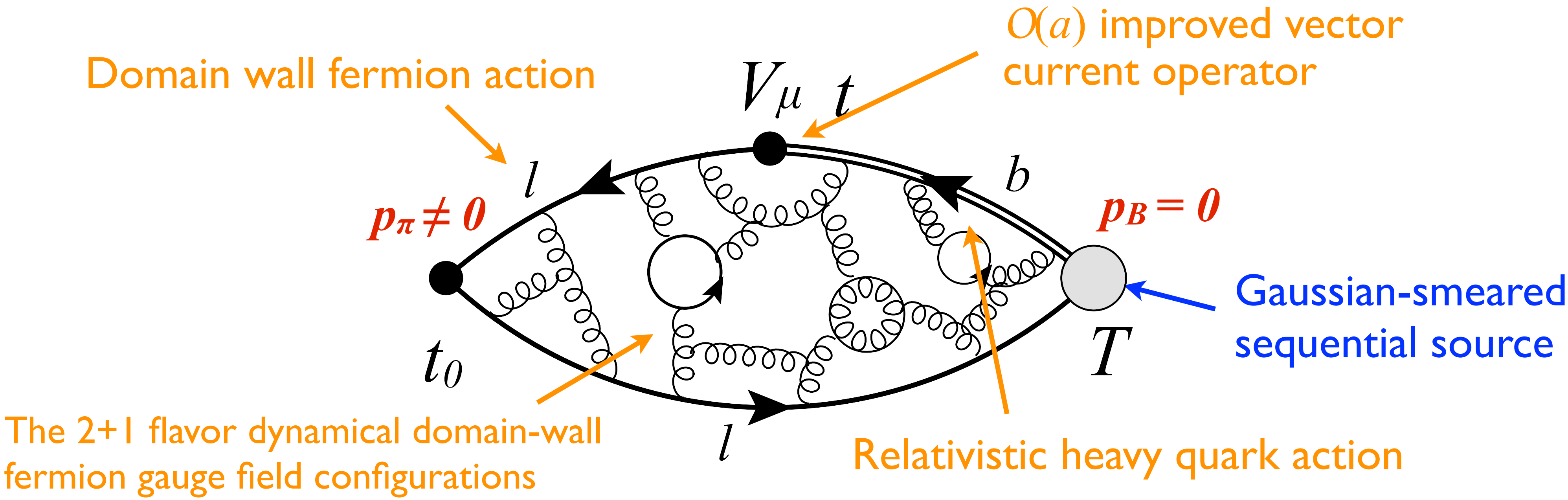
C. Allton et al. (RBC-UKQCD), Phys. Rev. D78, 114509 (2008)
Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)

	$L \times T$	a [fm]	m^{ud}_{sea}	m^{s}_{sea}	m^{π}_{sea} [MeV]	# of configs.	# of sources
Fine Lattice	32 × 64	≈ 0.08	0.004	0.030	289	628	2
	32 × 64	≈ 0.08	0.006	0.030	345	445	2
	32 × 64	≈ 0.08	0.008	0.030	394	544	2
Coarse Lattice	24 × 64	≈ 0.11	0.005	0.040	329	1636	1
	24 × 64	≈ 0.11	0.010	0.040	422	1419	1

- For the b-quark we use the relativistic heavy quark (RHQ) action developed by Li, Lin, and Christ in Refs. N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

- We use the nonperturbative determinations of the parameters of the RHQ action obtained in Y.Aoki et. al Phys. Rev. D 86, 116003 (2012).

How to calculate $f_{+/0}(q^2)$ from Lattice QCD



- Extract the lattice form factor from the ratio of the 3pt function to 2pt functions:

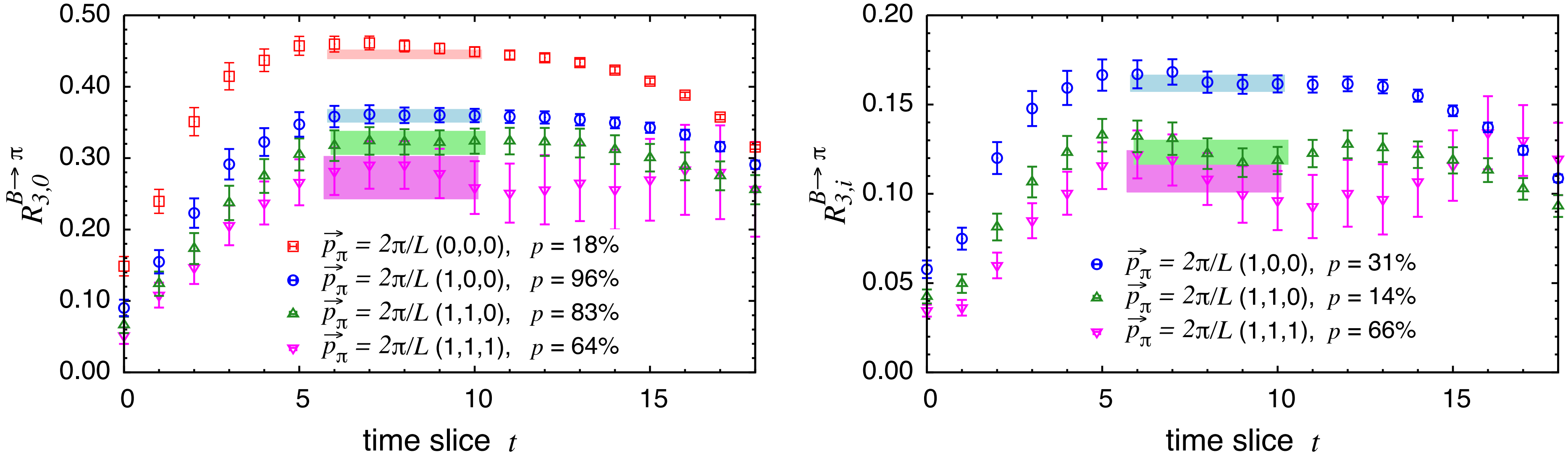
J. A. Bailey et al. (MILC Collaborations), Phys. Rev. D79, 054507 (2009).

$$R_{3,\mu}^{B\to\pi}(t,T) = \frac{C_{3,\mu}^{B\to\pi}(t,T)}{\sqrt{C_2^{\pi}(t)C_2^{B}(T-t)}}\sqrt{\frac{2E_0^\pi}{e^{-E_0^\pi t}e^{-m_0^B t}}}$$

$$f_{\parallel}^{\rm lat} = \lim_{t,T\to\infty} R_0^{B\to\pi}(t,T)$$

$$f_{\perp}^{\rm lat} = \lim_{t,T\to\infty} \frac{1}{p_\pi^i} R_i^{B\to\pi}(t,T)$$

The ratio of 3pt over 2pt functions



- After a careful study of source-sink separations, we use $T = t_B - t_\pi = 20$.
- We fit the ratio to a plateau in the region $0 \ll t \ll T$.
- The fit range is chosen such that the individual pion and B-meson 2-points have decayed to the ground state and the correlated constant fit has a good χ^2/d.o.f.

Form factors $f_{||}$ and $f_\perp$

- The continuum form factors are given by

$$f_\perp(E_\pi) = Z^{bl}_{V_i} \lim_{t,T\to\infty} \frac{1}{p^i_\pi} R^{B\to\pi}_{3,i}(E_\pi, t, T)$$

$$f_{||}(E_\pi) = Z^{bl}_{V_0} \lim_{t,T\to\infty} R^{B\to\pi}_{3,0}(E_\pi, t, T)$$

- We calculate the heavy-light current renormalization factor Z_V^{bl} using the mostly nonperturbative method. A. X. El-Khadra et al. Phys.Rev. D64, 014502 (2001)

compute with 1-loop lattice perturbation theory

[See talk by C.Lehner 14:40~]

$$Z^{bl}_{V_\mu} = \underbrace{\boxed{\rho^{bl}_{V_\mu}}}_{\approx 1}\sqrt{\boxed{Z^{bb}_V Z^{ll}_V}}$$

compute nonperturbatively

- Z_V^{ll} obtained by the RBC/UKQCD collaborations by exploiting the fact $Z_A=Z_V$ for domain-wall fermions. Y. Aoki et al. (RBC/UKQCD Collaboration), Phys.Rev. D83, 074508 (2011)
- Z_V^{bb} obtained from the matrix element of the $b\to b$ vector current between two Bs mesons.

Chiral-continuum extrapolated $f_{||}$ and $f_{\perp}$

- Correlated simultaneous chiral-continuum fit ($m_\pi \rightarrow m_\pi^{\text{phys}}$, $a \rightarrow 0$)
  - to $f_\perp$ data using NLO SU(2) HMχPT
  - to $f_{||}$ data using NLO SU(2) HMχPT plus NNLO analytic terms to interpolate in E_π^2

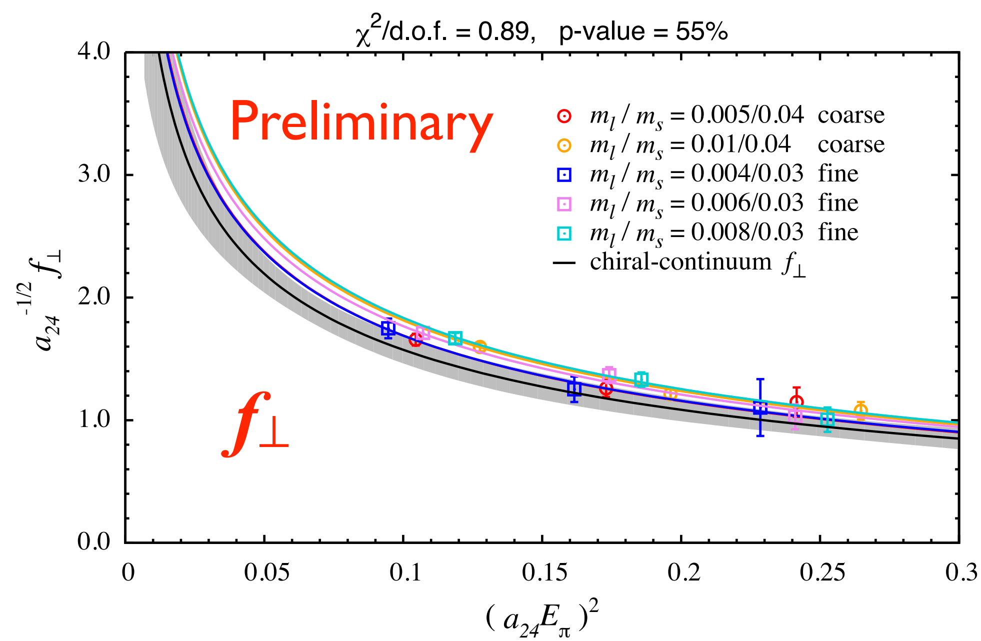
D. Bećirević et al, Phys. Rev. D 68, 074003 (2003)
ETM Collaboration, arXiv:1104.0869

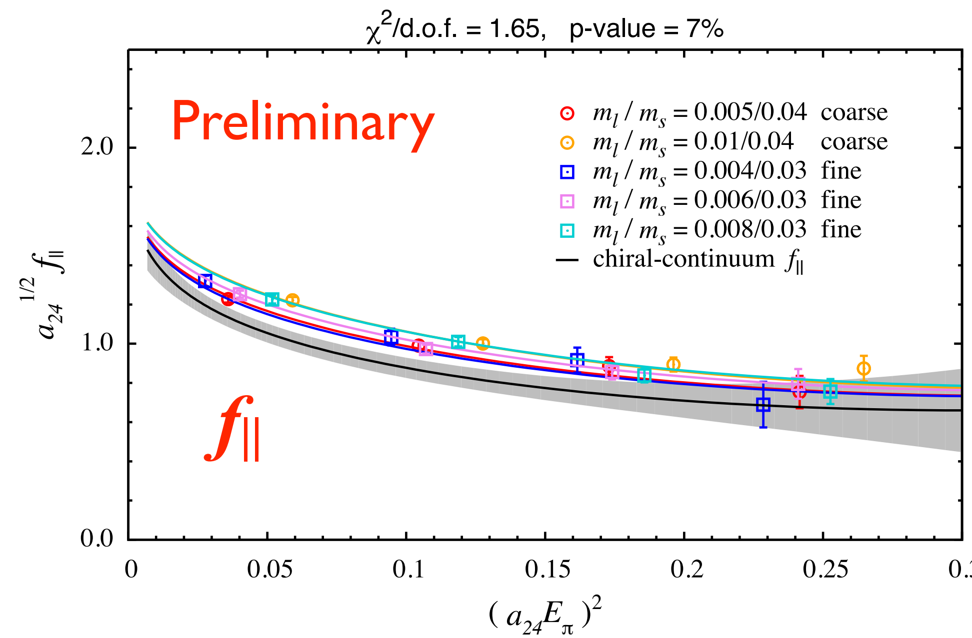
$$f_\perp(m_l, E_\pi, a^2) = \frac{1}{E_\pi + \Delta} c_\perp^{(0)} \left(1 + \delta f_\perp + c_\perp^{(1)} m_l^2 + c_\perp^{(2)} E_\pi + c_\perp^{(3)} E_\pi^2 + c_\perp^{(4)} a^2\right)$$

$$f_{||}(m_l, E_\pi, a^2) = c_{||}^{(0)} \Big(1 + \delta f_{||} + c_{||}^{(1)} m_l + c_{||}^{(2)} E_\pi + c_{||}^{(3)} E_\pi^2 + c_{||}^{(4)} a^2$$

$$\text{NNLO} \quad \underline{+ c_{||}^{(5)} m_l E_\pi + c_{||}^{(6)} m_l E_\pi^2 + c_{||}^{(7)} E_\pi^3}\Big)$$

The function δf indicate non-analytic "log" functions of the pseudoscalar meson masses.

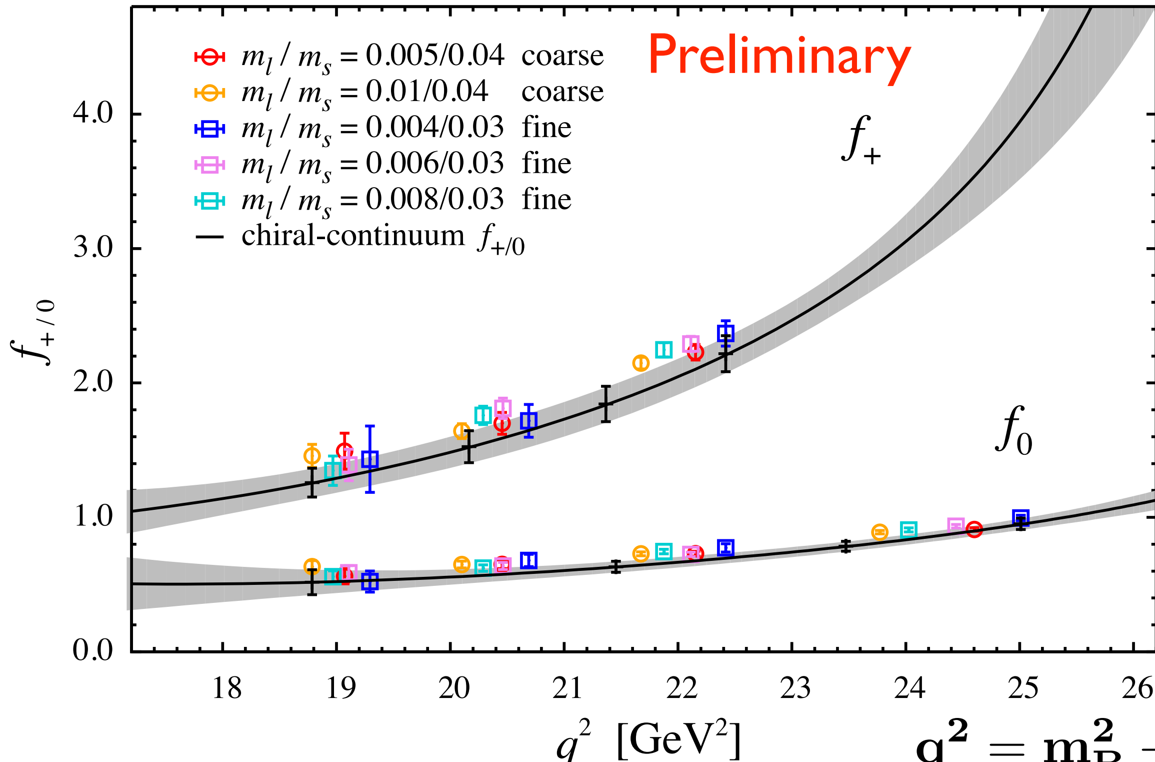




Black curves show chiral-continuum extrapolated $f_{||}$ and $f_\perp$ with statistical errors.

Form factors f_+ and f_0

- The form factors f_+ and f_0 are easily obtained from $f_{||}$ and $f_\perp$.
  - By definition, the form factors satisfy the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$.



$$\mathbf{q^2 = m_B^2 + m_\pi^2 - 2m_B E_\pi}$$

- We must extrapolate the lattice data to lower q^2 (larger E_π^2) to reach the kinematic region where experimental measurements are most precise.
- Using chiral-continuum extrapolated lattice data, in the range of simulated pion energies, we generate four synthetic data points of f_+ and f_0 (**black**) used in q^2 extrapolation to full kinematic range.

z-expansion of f_+ and f_0

Boyd, Grinstein, Lebed, Phys.Rev.Lett. 74 (1995) 4603

We employ the model-independent z-expansion fit to extrapolate to low momentum transfer.

- Consider mapping the variable q^2 onto a new variable z.

  semileptonic region
  $0 < q^2 < t_-$ $\rightarrow$ $-0.34 < z < 0.22$ (when we choose $t_0 = 0.65 t_+$)

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

- The form factor $f(q^2)$ is analytic in the semileptonic region except at B^* pole.
  $\rightarrow$ $f(q^2)$ can be expressed as convergent power series.

$$f(q^2) = \frac{1}{P(q^2)\phi(q^2, t_0)} \sum_{k=0}^{\infty} a^{(k)}(t_0) z(q^2, t_0)^k$$

contains subthreshold poles

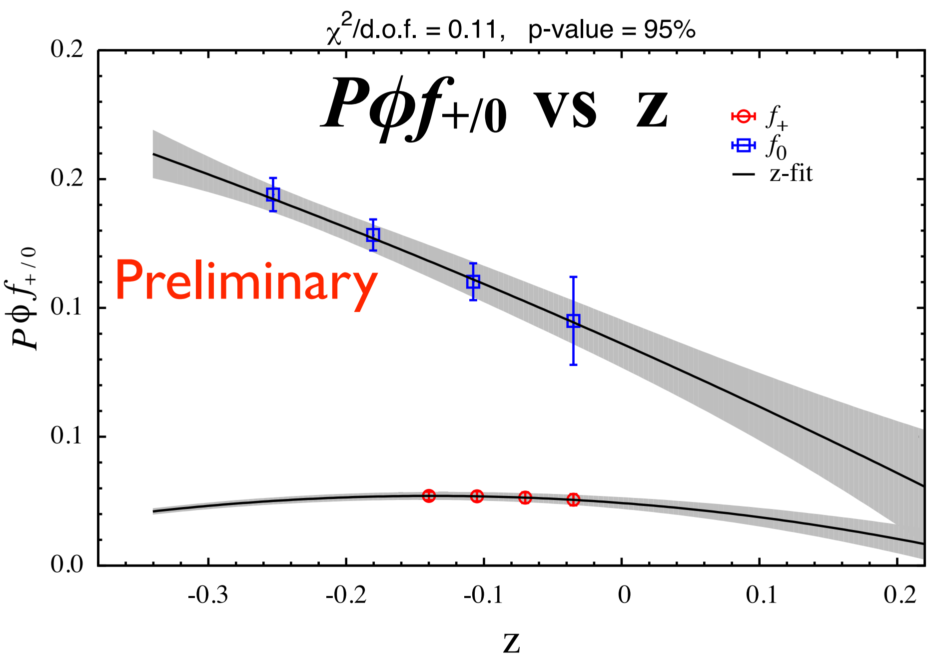
Arbitrary analytic function which affects the numerical values of the series coefficients

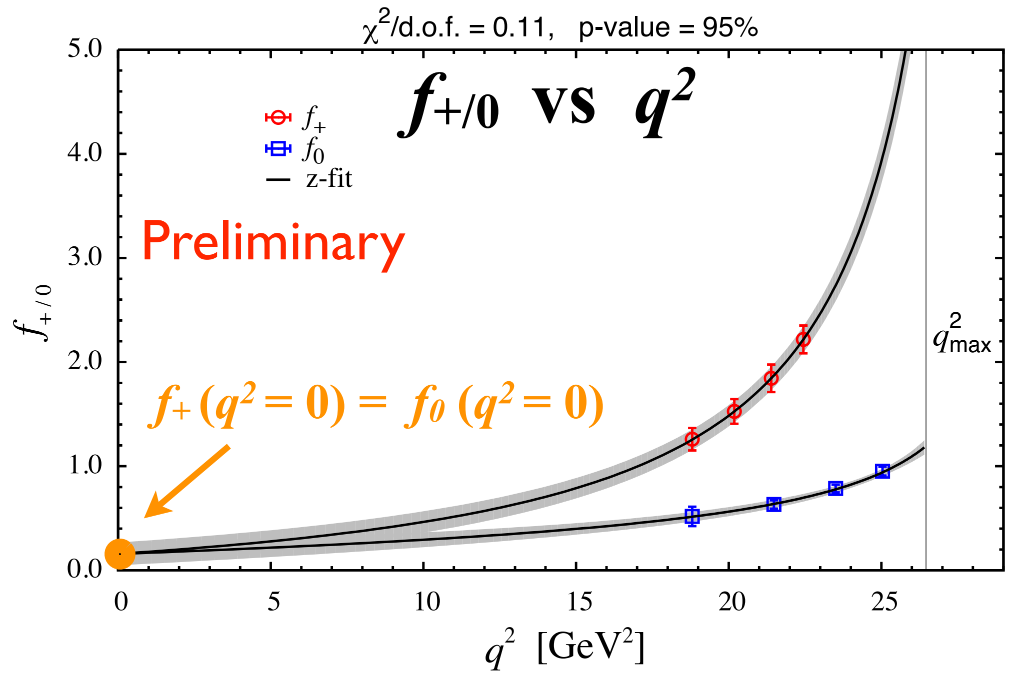
- The sum of the series coefficients is bounded by unitarity.

$$\sum_{k=0}^{N} {a^{(k)}}^2 \leq 1$$

- Therefore this bound combined with the small |z| ensures that only a small number of terms is needed to accurately describe the shape of the form factor.

z-expansion of f_+ and f_0





- We show z-expansion fits imposing the kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$.
- Our z-fit includes terms up to z^2. $P\phi f = a^{(0)} + a^{(1)} z + a^{(2)} z^2$
- The resulting slope and curvature of the lattice data of the $B \to \pi l \nu$ form factor are

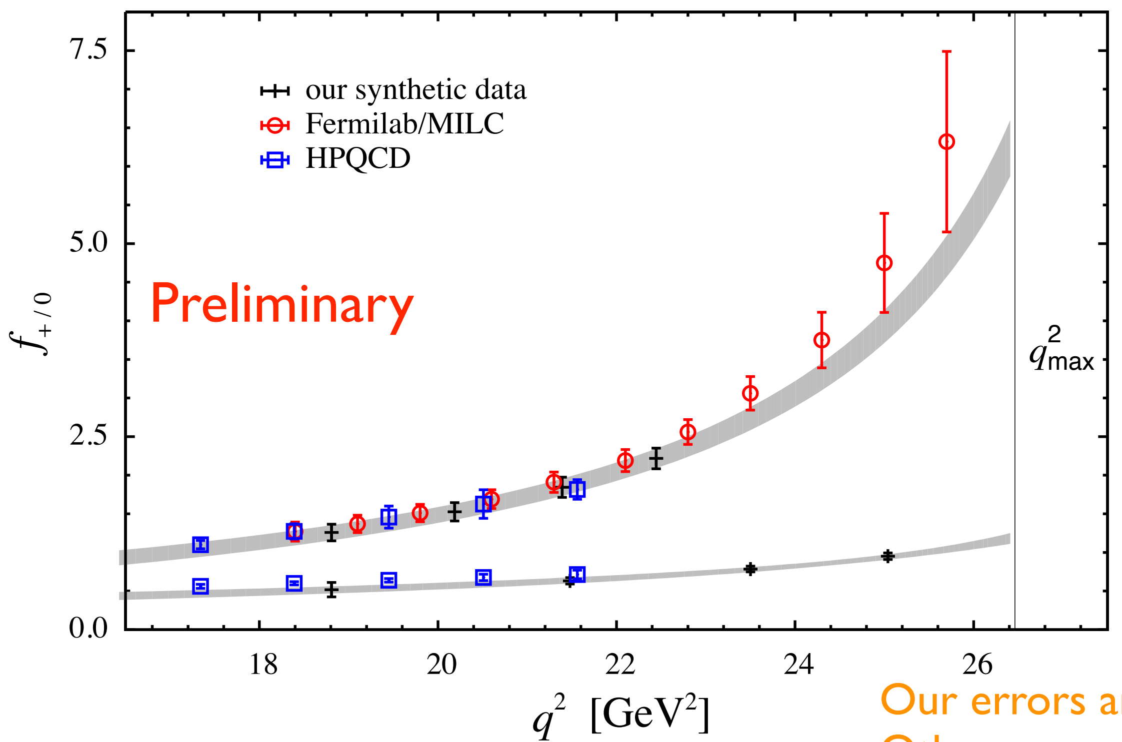
$$\boxed{\begin{array}{l} a_+^{(1)} / a_+^{(0)} = -1.66 \pm 0.70 \\ a_+^{(2)} / a_+^{(0)} = -6.0 \pm 1.5 \end{array}}$$

cf. The slope of the BABAR experiment

$a_+^{(1)} / a_+^{(0)} = -1.60 \pm 0.26$

Phys. Rev. D79, 054507 (2009)

Comparison with other calculations



Our errors are only statistical error.
Others are combined statistical and chiral extrapolation errors.

- Two calculations of the $B\to\pi l\nu$ form factor using **MILC gauge configurations** that include 2+1 flavors of dynamical improved staggered quarks.
  - Fermilab/MILC collaboration uses relativistic (Fermilab) b-quarks.
    Phys. Rev. D79, 054507 (2009)
  - HPQCD collaboration uses nonrelativistic (NRQCD) b-quarks.
    Phys. Rev. D73, 074502 (2006), Erratum-ibid.D75:119906 (2007)

Conclusions and future prospects

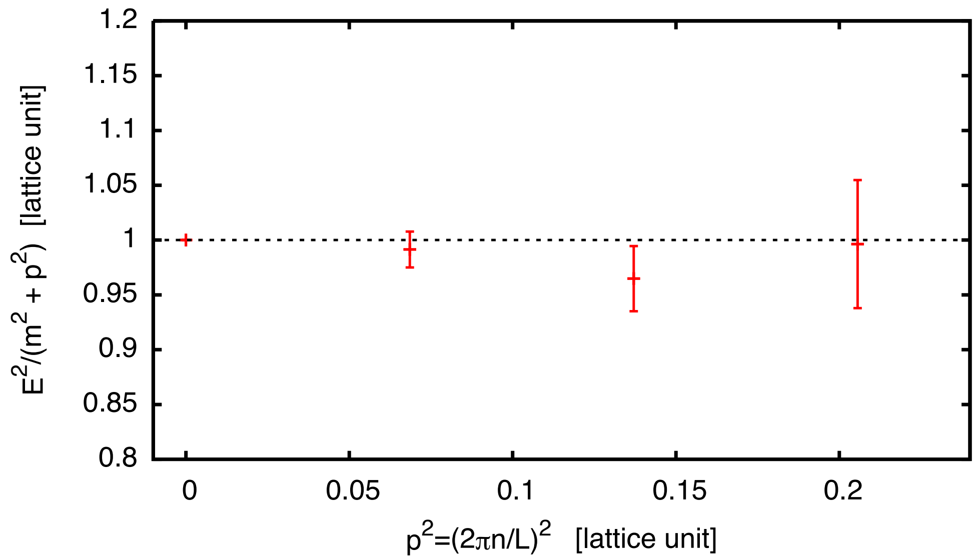
- We have calculated the $B \to \pi$ form factors $f_{||}$ and $f_{\perp}$ using 2+1 flavor dynamical domain-wall fermion gauge field configurations with relativistic heavy quark action on fine ($24^3\times64$, $a\sim0.11$fm) lattice and coarse ($32^3\times64$, a~0.08fm) lattice.
- Perform the simultaneous chiral-continuum extrapolation to all data of $f_{||}$ and $f_{\perp}$ using SU(2) HMχPT formula.
- Perform the z-expansion fits with kinematic constraint $\boldsymbol{f_+ = f_0}$ at $\boldsymbol{q^2 = 0}$ in order to extrapolate to low momentum transfer (high E_π^2).
- Will provide important independent check on existing calculations using staggered light quarks.

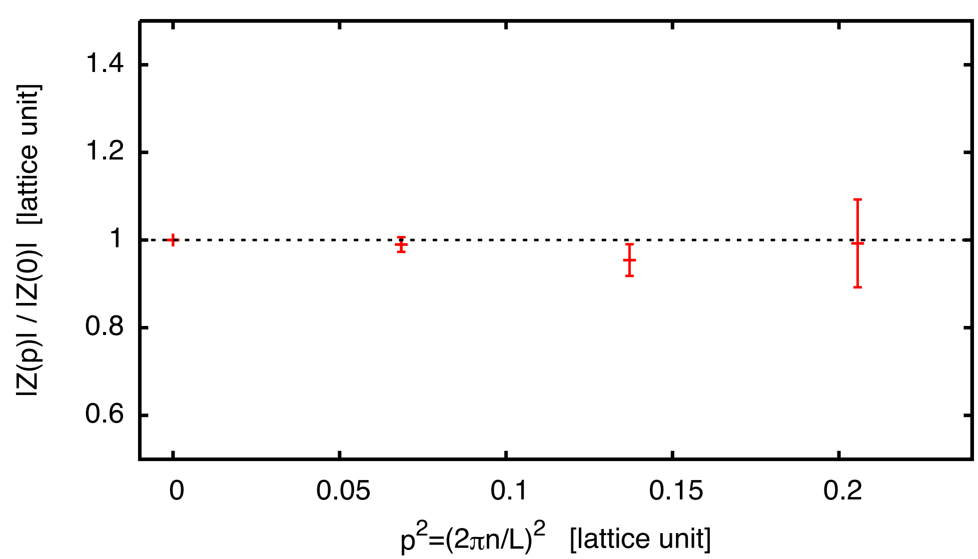
Work still in progress:

- Include partially quenched data points in our analysis.
- Estimate the systematic uncertainties in $\boldsymbol{f_+}$ and $\boldsymbol{f_0}$.
- Compare with experimental data from Babar and Belle in order to obtain |Vub|.
- Perform $\boldsymbol{q^2}$ extrapolation using alternative z-parameterization from Bourrely, Caprini, and Lellouch [Phys.Rev. D79 (2009) 013008, Erratum-ibid. D82 (2010) 099902]

Backup slides

Dispersion relation and amplitude Z_π





- The pion energies satisfy the continuum dispersion relation: $E_\pi^2 = |\vec{p}_\pi|^2 + m_\pi^2$
- The pion amplitude $Z_\pi = |\langle 0|\mathcal{O}_\pi|\pi\rangle|$ is independent of momentum

$$Z_\pi(E) = \lim_{t\to\infty} \left\{C_2^\pi(t) \times 2Ee^{Et}\right\}^{\frac{1}{2}}$$

$O(a)$ improved vector current operator

[See talk by C.Lehner 14:40~]

The heavy-light current operator at tree level is

$$V_{\mu,0}(x) = \bar{q}(x)\mathcal{O}_{\mu,0}Q(x), \quad \mathcal{O}_{\mu,0} = \gamma_\mu$$

Four single derivative operators are needed for O(a) improvement.

$$\boxed{\begin{aligned} \mathcal{O}_{1,\mu} &= 2\overrightarrow{D}_\mu \\ \mathcal{O}_{2,\mu} &= 2\overleftarrow{D}_\mu \\ \mathcal{O}_{3,\mu} &= 2\gamma_\mu\gamma_i\overrightarrow{D}_i \\ \mathcal{O}_{4,\mu} &= 2\gamma_\mu\gamma_i\overleftarrow{D}_i \end{aligned}}$$

The O(a) improved vector current operator is given by

temporal ($\mu = 0$): $\mathcal{O}_0^{\mathrm{imp}} = \mathcal{O}_{0,0} + c_3^{V_0}\mathcal{O}_{0,3} + c_4^{V_0}\mathcal{O}_{0,4}$

spatial ($\mu = i$): $\mathcal{O}_i^{\mathrm{imp}} = \mathcal{O}_{i,0} + c_1^{V_i}\mathcal{O}_{i,1} + c_2^{V_i}\mathcal{O}_{i,2} + c_3^{V_i}\mathcal{O}_{i,3} + c_4^{V_i}\mathcal{O}_{i,4}$

Coefficients are determined by 1-loop lattice perturbation theory.

Relativistic heavy quark action for b-quarks

Heavy quark mass introduces discretization errors of $O((ma)^n)$.

- At bottom quark mass, it becomes severe: $m_b \sim 4$ GeV and $1/a \sim 2$ GeV, then $m_b a > O(1)$.

Relativistic heavy quark action (RHQ action)

$$S^{\mathrm{RHQ}} = \sum_{n,n'} \bar{\psi}_n \left\{ \mathbf{m_0} + \gamma_0 D_0 - \frac{aD_0^2}{2} + \zeta \left[\vec{\gamma} \cdot \vec{D} - \frac{a\vec{D}^2}{2} \right] - a \sum_{\mu\nu} \frac{i\mathbf{c_P}}{4} \sigma_{\mu\nu} F_{\mu\nu} \right\}_{n,n'} \psi'_n$$

- The Fermilab group showed that you can remove all errors of $O((ma)^n)$ by appropriately tuning the parameters of the anisotropic clover action

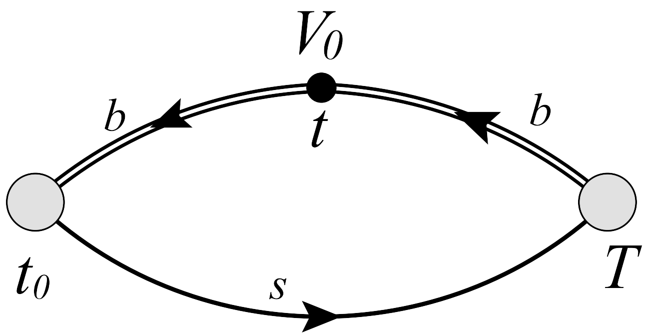
  A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D55, 3933 (1997)

- Errors are of $O(a^2p^2)$.
- Li, Lin, and Christ showed that the parameters $\{m_0, \zeta, c_P\}$ can be tuned nonperturbatively.

  N. H. Christ, M. Li, and H.-W. Lin, Phys.Rev. D76, 074505 (2007)
  H.-W. Lin and N. Christ, Phys.Rev. D76, 074506 (2007)

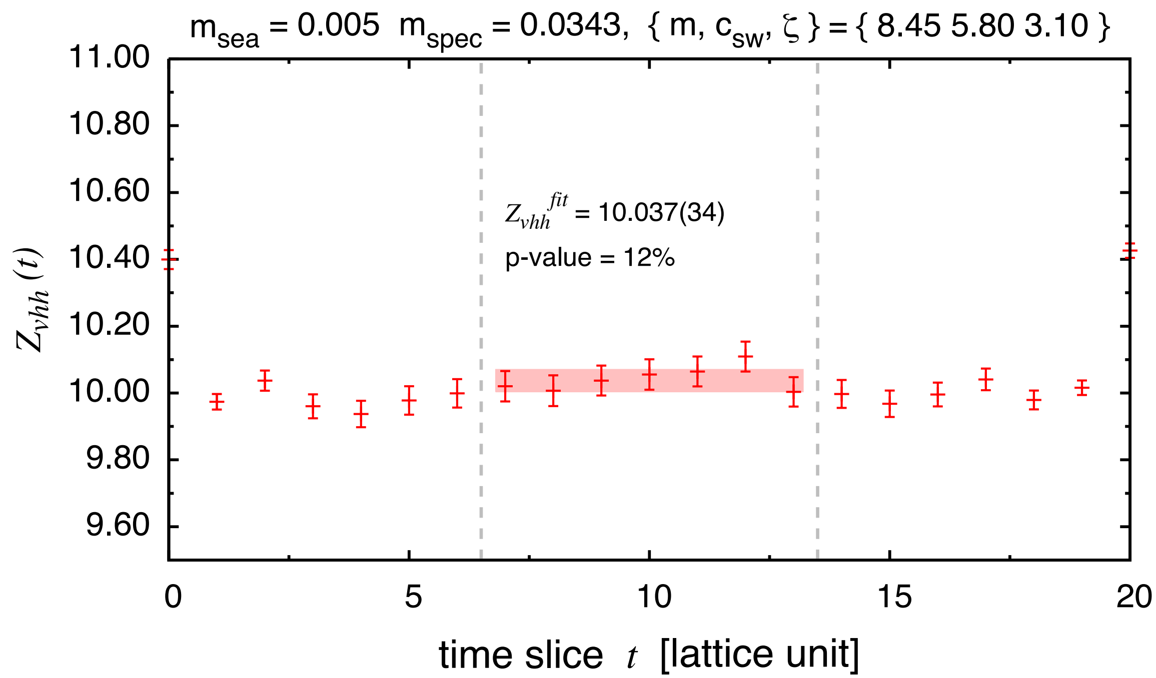
- We use the results for the parameters of the RHQ action obtained for b-quarks in Y. Aoki et. al Phys. Rev. D 86, 116003 (2012)

Renormalization factor $Z_V{}^{bb}$



$$Z_V^{bb} \times \langle m_0^B | V_0^{bb} | m_0^B \rangle = 2m_B$$

$$\frac{C_2^B(T)}{C_{3,\mu}^{B\to B}(t,T)} \xrightarrow{t,T\to\infty} Z_V^{bb}$$



At tree level, the expression of $Z_V{}^{bb}$ is given by

$$Z_V^{bb} = u_0 \exp(M_1), \quad M_1 = \log[1 + \tilde{m}_0], \quad \tilde{m}_0 = \frac{m_0}{u_0} - (1+3\zeta)(1 - \frac{1}{u_0})$$

Here $m_0 = 7.80$, $\zeta = 3.20$, $u_0 = 0.8757$.

$$\boxed{\begin{array}{rcl} \textbf{NP} & : & Z_V^{bb} = 10.037(34) \\ \textbf{tree level} & : & Z_V^{bb} = 9.993 \end{array}}$$