On one-loop corrections to matching conditions of Lattice HQET including $1/m_b$ terms

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Motivations

Lattice calculations enter in two estimations of V_{ub}

For processes with $b \to u$ transitions $\Gamma \sim |V_{ub}|^2$:

- exclusive semi-leptonic $B \to \pi l \nu$ (involves $f_+(q^2)$)
- exclusive leptonic $B \to \tau \nu$ (involves f_B)

$\Rightarrow$ high precision lattice calculations needed.

Hierarchy of scales

- $L > 4/m_\pi \approx 6\ \mathrm{fm}$ to suppress finite-size effects for light quarks
- $a < (2m_B)^{-1}$ to control discretization errors for the heavy quark

$\Rightarrow$ solution: HQET on the lattice.

HQET at order $(\Lambda_{\rm QCD}/m_b)$

$\mathscr{L}_{\rm HQET}$

$$\mathscr{L}_{\rm HQET} = \underbrace{\mathscr{L}_{\rm stat}}_{\mathcal{O}(1)} + \underbrace{\omega_{\rm kin}\,\mathscr{L}_{\rm kin} + \omega_{\rm spin}\,\mathscr{L}_{\rm spin}}_{\mathcal{O}(1/m)}$$

with $\mathscr{L}_{\rm stat} = \bar{\psi}_h D_0 \psi_h$.

HQET operators: Example of A_0

$$Z_{A_0}^{\rm QCD}\,(A_I^{\rm QCD})_0 = Z_{A_0}^{\rm QCD}\left[\bar{\psi}_l \gamma_0\gamma_5\psi_b\right]$$

$$\downarrow$$

$$Z_{A_0}^{\rm HQET}\,(A_I^{\rm HQET})_0 = Z_{A_0}^{\rm HQET}\Big[\bar{\psi}_l\gamma_0\gamma_5\psi_h + a c_{A_{0,1}}\bar{\psi}_l \frac{1}{2}\gamma_5\gamma^k(\nabla_k^S - \overleftarrow{\nabla}_k^S)\psi_h +$$

$$+ a c_{A_{0,2}}\bar{\psi}_l \frac{1}{2}\gamma_5\gamma^k(\nabla_k^S + \overleftarrow{\nabla}_k^S)\psi_h + \ldots\Big]$$

$\Rightarrow$ in total 19 HQET parameters are needed.

Matching

Finite volume scheme

In order to match HQET and QCD:

- we use a set of observables $\{\phi_i\}$ defined in a finite volume,
- we evaluate them in QCD and HQET to obtain $\{\phi_i^{\rm QCD}(L, z = mL, a)\}$ and $\{\phi_i^{\rm HQET}(L, a)\}$,
- by equating the corresponding quantities

$$\phi_i^{\rm QCD}(L, z, a = 0) \stackrel{!}{=} \phi_i^{\rm HQET}(L, a) = \eta(L, a)_i + \varphi(L, a)_{ij}\, \omega_j(z, a)$$

we can extract the HQET parameters $\omega_j(z, a)$.

$\Rightarrow$ non-perturbative definition (matching) and evolution of HQET parameters

One-loop lattice perturbation theory for HQET

Why?

Any HQET quantity is a truncated expansion in $1/z$ of the corresponding QCD quantity, hence one would like to make sure that matching doesn't introduce artificially large $1/z^2$ corrections

- already checked at tree-level in $\bar{g}^2$
- in this work: verification at one-loop

How?

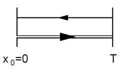
PASTOR [written by D. Hesse]: automatic tool for generation and calculation of lattice Feynman diagrams in the Schrödinger functional framework.

input: discretized action, correlation function, L/a, $z = \bar{m}L$, a

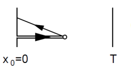
output: Feynman rules, Feynman diagrams, numerical contributions of each diagram

Schrödinger functional correlation functions

- boundary-to-boundary: $F_1(\theta) = \sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}} \langle \bar{\zeta}'_l(\mathbf{u})\gamma_5\zeta'_h(\mathbf{v})\bar{\zeta}_h(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle$



- current insertions: $f_{A_0}(\theta, x_0) = \sum_{\mathbf{u},\mathbf{v}} \langle \bar{\zeta}_h(\mathbf{u})\gamma_5\zeta_l(\mathbf{v})A_0(x_0)\rangle$



Additional parameters: $\psi_h(x+L\hat{k}) = e^{i\theta_h^k}\psi_h(x)$, $\psi_l(x+L\hat{k}) = e^{i\theta_l^k}\psi_l(x)$.

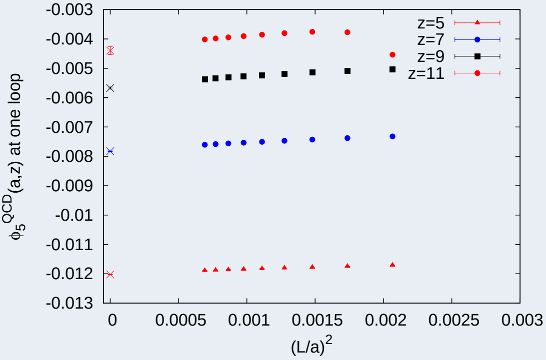
Observable for $c_{A_{0,2}}$

[Della Morte]

$$\phi_5^{\mathrm{QCD}} = \log \frac{f_{A_0}(\theta_l^i = \theta_1, \theta_h^i = \theta_2)}{f_{A_0}(\theta_l^i = \theta_1, \theta_h^i = \theta_3)}$$

$c_{A_{0,2}}$: Continuum extrapolations

Continuum extrapolations of ϕ_5^{QCD}



Naive expectation: tree-level $\sim 1/z \to$ one-loop $\sim \frac{1}{4\pi} 1/z$.

$c_{A_{0,2}}$: Estimating $1/m_b^2$ corrections

The generic structure of a matching condition for an observable which doesn't need to be renormalized is

$$\phi^{\mathrm{QCD}}(z, a=0) = \phi^{\mathrm{stat}} + \sum_t \omega_t(z)\phi_t^{1/m} + \mathcal{O}(1/z^2).$$

The z-dependence of the one-loop correction can be studied using $(\hat{\omega}_t = \bar{m}\omega_t,\ \hat{\phi}_t = L\phi_t)$

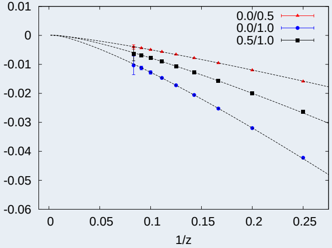
$$R = \frac{\phi^{(1)}_{\mathrm{QCD}}(\theta, z) - \phi^{(1)}_{\mathrm{stat}}(\theta)}{\phi^{(0)}_{\mathrm{QCD}}(\theta, z) - \phi^{(0)}_{\mathrm{stat}}(\theta)} = \frac{\sum_t \hat{\omega}_t^{(0)}\hat{\phi}_t^{(1)}(\theta)}{\sum_t \hat{\omega}_t^{(0)}\hat{\phi}_t^{(0)}(\theta)} + \frac{\sum_t \hat{\omega}_t^{(1)}(z)\hat{\phi}_t^{(0)}(\theta)}{\sum_t \hat{\omega}_t^{(0)}\hat{\phi}_t^{(0)}(\theta)}$$
$$= \alpha(\theta) + \gamma(\theta)\log(z) + \mathcal{O}(1/z)$$

When R is plotted on a linear-log plot, the ratio R measures:

- $1/z^2$ *corrections*: deviations from a linear behaviour,
- *coefficient of the subleading logarithm* : slope of the data.

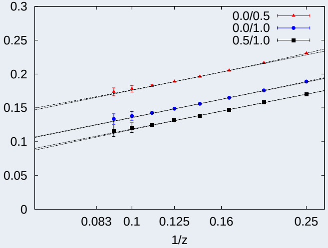
$c_{A_{0,2}}$: Results

$\phi_5^{\mathrm{QCD}}(z)$ at one-loop



(a) Fitting ansatz:
$f(z) = \beta_0/z + \beta_1 \log z/z$.

Ratio R_5



(b) Fitting ansätze: $f(z) = \alpha + \gamma \log z$ and
$f'(z) = \alpha' + \gamma' \log z + \delta'/z$.
Conclusion: $\frac{f(4) - f'(4)}{f(4)} \sim 0.003$.

Observable

The observable used is [Heitger et al.]

$$\phi_6^{\mathrm{QCD}} = \log \frac{f_{A_0}(\theta,\theta)}{\sqrt{F_1(\theta)}}$$

Matching condition

We match the renormalized observables at static order

$$\left(\phi_6^{\mathrm{QCD}}\right)^{\mathrm{ren}}(L,z) = \left(\phi_6^{\mathrm{stat}}\right)^{\mathrm{ren}}(L,z) + \log C_{A_0}^{\mathrm{match}}\left(\bar{g}^2(z)\right) + \mathcal{O}(1/z)$$

and expand both sides in the coupling $\bar{g}^2(z)$

$$\left(\phi_6^{\mathrm{QCD}}\right)^{(0)}(z) = \left(\phi_6^{\mathrm{stat}}\right)^{(0)} + \mathcal{O}(1/z),$$

$$\left(\phi_6^{\mathrm{QCD}}\right)^{(\mathrm{ren},1)}(z) = \left(\phi_6^{\mathrm{stat}}\right)^{(\mathrm{bare},1)} - \gamma_0 \log(a\bar{m}) + B_{A_0} + \mathcal{O}(1/z).$$

$Z_{A_0}^{\mathrm{HQET}}$: Estimating $1/m_b^2$ corrections

Subtracting $\gamma_0 \log z$ from both sides yields

$$\left(\phi_6^{\mathrm{QCD}}\right)^{(\mathrm{ren},1)}(z) - \gamma_0 \log z = \left(\phi_6^{\mathrm{stat}}\right)^{(\mathrm{bare},1)} - \gamma_0 \log\left(\frac{a}{L}\right) + B_{A_0} + \mathcal{O}(1/z)$$
$$= \left(\phi_6^{\mathrm{stat}}\right)^{(\mathrm{ren},1)} + \mathcal{O}(1/z).$$

In order to make visible the $1/m_b^2$ corrections we plot the quantity Q defined as

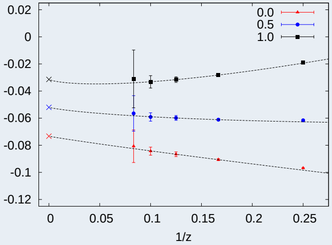
$$Q = z\left[\left(\phi_6^{\mathrm{QCD}}\right)^{(\mathrm{ren},1)}(z) - \gamma_0 \log z\right] - z\left[\left(\phi_6^{\mathrm{stat}}\right)^{(\mathrm{ren},1)}\right]$$
$$= z\left[\mathcal{O}(1/z) + \mathcal{O}(1/z^2)\right] = \alpha_0 + \alpha_1 \log(z) + \mathcal{O}(1/z)$$

When Q is plotted on a linear-log plot:

- the $1/z^2$ *corrections* are seen as deviations from a linear behaviour
- the *coefficient of the subleading logarithm* as the slope of the data

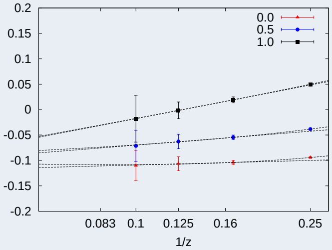
$Z_{A_0}^{\mathrm{HQET}}$: Results

$\phi_6^{\mathrm{QCD}}(z)$ at one-loop



(a) Fitting ansatz:
$f(z) = \beta_0 + \beta_1/z + \beta_2 \log z/z$.

Q_6



(b) Fitting ansätze: $f(z) = \alpha_0 + \alpha_1 \log z$ and $f'(z) = \alpha_0' + \alpha_1' \log z + \alpha_2'/z$.

Conclusion: $\frac{f(4) - f'(4)}{f(4)} \sim 0.01$.

Conclusions

- lattice HQET is a prototype of an effective theory where one can perform a non-perturbative matching

- we have a setup for checking contamination with $1/z^2$ terms of the matching conditions
- we can further optimize the choice of kinematics for the matching conditions
- we presented results for 2 generic matching conditions; results for all 19 matching conditions fixing $\mathscr{L}_{HQET}$, vector and axial currents at order $1/m_b$ are ready